Bearing Capacity Determination Method for Strip Surface Footings Underlain by Voids

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A method of bearing capacity determination for strip surface footings subjected to vertical central loading and underlain by a continuous circular void with its axis parallel with the footing axis is presented. A nomograph is also presented for ease in application of the developed equations. For equation development, the performance of strip surface footings with and without an underground void was investigated using a plane-strain finite element computer program. In the analysis, the foundation soil was characterized as a nonlinear elastic, perfectly plastic material that obeys the Drucker-Prager yield criterion. A wide range of soil properties, footing widths, void sizes, and void locations including depth to void and void eccentricity was considered. The ultimate bearing capacity values were obtained from the results of analysis and were related graphically with the influencing factors investigated. The bearing capacity equations were then developed through curve fitting to these graphical relationships. The effectiveness of the developed equations was evaluated by comparing the computed bearing capacity values with the model footing test results. Good agreement between the two sets of data was shown. It was therefore concluded that the developed equations together with the nomograph may become an effective tool for analysis and design of strip surface footing underlain by a continuous circular void, at least within the conditions investigated.

Despite its origin, either naturally formed or man-made, an underground void may occur under a foundation. The frequency of its presence under the foundation and the seriousness of its effect on foundation stability have been pointed out in previous papers (*1,2*). To approach such a problem, options such as filling the void, excavating the overlying soil and placing the foundation below the void, using piles or piers, reinforcing the overlying soil layer, relocating the foundation site, and possibly others are considered. These options are often difficult and very costly to implement. To ensure its long-term stability, the foundation must be originally designed with a thorough understanding of the void effect. In response to this need, many research studies have been conducted (*1–9*).

There is no methodology currently available for analysis and design of such a foundation system. The core of the methodology requires equations for determination of the ultimate bearing capacity of the foundation. The development of such bearing capacity equations for analysis of strip surface footing underlain by a continuous circular void is presented.

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FOOTING PERFORMANCE ANALYSIS

The analysis of strip surface footing performance under vertical central loading was performed using the finite element method. A two-dimensional plane-strain finite element computer program was developed. In the computer program, the footing is characterized as a linear elastic material and the foundation soil as a nonlinear elastic, perfectly plastic material that obeys Hooke's law and the Drucker-Prager yield criterion (*10*). A hypothetical layer to model possible slips at the interface is placed between the vertical sides of the footing and the soil. The nonlinear solution is accomplished through the use of Reyes's incremental stress-strain relation (*11*) together with the tangential stiffness method.

The computer program differs from those previously developed by Baus (*12*), Badie (*13*), and Azam (*14*) in that it contains a preprocessor, the main program, and a postprocessor. The preprocessor is for input data preparation, and the postprocessor is for providing the results of finite element analysis in the desired graphical form. The preprocessor adopts a step-by-step procedure using the question-and-answer interactive format to guide the user to provide the necessary input. It contains various subroutines for generating the finite element mesh and for specifying boundary conditions, material properties, external loading, and type of analysis. The program also incorporates the numerical scheme of Siriwardane and Desai (*15*) for keeping the state of stress during yielding on the yield surface, and the numerical computation uses the Gauss-Jordan elimination method to solve the symmetrical BANDED global stiffness matrix. A detailed description of program development is given by Hsieh (*16*).

The computer program was validated by using the model footing test data obtained by Baus (*12*), Badie (*13*), and Azam (*14*) and the data published by Siriwardane and Desai (*15*) and Whitman and Hoeg (*17*). The finite element analysis was performed using an IBM 4090. The generation of element mesh and other input data preparation was done through a VAX 8550 or a PC, and a VAX 8550 was used to obtain the results of analysis. For plotting software, PLOT 10 was used.

FOUNDATION SYSTEM INVESTIGATED

The strip surface footing analyzed was a reinforced concrete footing with a width varying between 2 in. and 6 ft. Three different soils support the footing—commercial kaolin, silty clay, and clayey sand. The underground void was circular in cross section and continuous with its axis parallel to the footing axis. Figure 1 shows a schematic view of the footing/soil/void system together with the symbols used for defining footing size and the size and location of the void. As shown, B, D, E, and W represent footing width, depth to void, void eccentricity, and void diameter, respectively.

In the analysis, the size (W) and location (D and E) of the void were expressed as ratios to footing width (B). Three levels each of void size (W/B = 0.67, 1, and 2), void eccentricity (E/B = 0, 1, and 2), and numerous levels of depth to void (D/B = 1 to 14) were analyzed. The material properties used in the analysis are summarized in Table 1. The concrete footing properties in Table 1 are obtained from Bowles (*18*), and the properties of silty clay, kaolin, and clayey sand are from Baus (*12*), Badie (*13*), and Azam (*14*), respectively.

ULTIMATE BEARING CAPACITY

Results of the finite element analysis were used to evaluate the mechanistic behavior of the foundation system. Among the behaviors investigated are principal stress distribution, displacement fields, propagation of plastic yielding, deformed configurations of the void, and the footing pressure versus settlement relation. From these data, the ultimate bearing capacity of each condition analyzed was obtained.

The ultimate bearing capacity data were analyzed further with respect to various factors considered including soil type, footing width, void size, and void location. It was found that the effect of soil type and footing width on bearing capacity for footing underlain by a void can be properly considered by nondimensionalizing both the independent and dependent variables: the ratios of q_o/q_{nv}, D/B, W/B, and E/B, in which q_o and q_{nv} are the ultimate bearing capacities of with and without void conditions, respectively (*16*). Furthermore, the ultimate bearing capacity values can be related with void locations for different void sizes in the form shown in Figures 2 through 6 regardless of soil type. Note that the data in

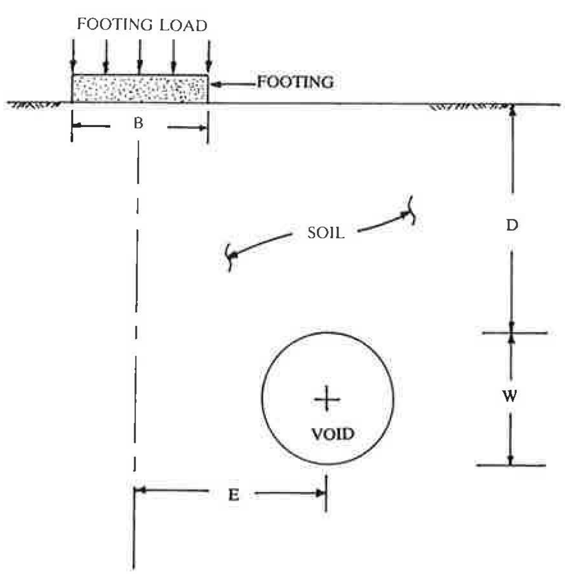

FIGURE 1 Schematic view of footing/soil/void system.

TABLE 1 PROPERTIES OF FOUNDATION SOILS AND CONCRETE FOOTING USED IN FINITE ELEMENT ANALYSIS

Material Parameters	Kaolin	Silty Clay	Clayey Sand	Concrete Footing
Internal Friction Angle, deg	8.0	13.5	31.0	N/A
Unit Cohesion (psi)	23.0	9.5	1.3	N/A
Initial Modulus in Compression (psi)	2880	677	6100	$3.3x10^6$
Initial Modulus in Tension (psi)	7000	1505	11300	$3.3x10^6$
Poisson's Ratio	0.39	0.28	0.32	0.20
Dry Unit Weight, pci	0.058	0.052	0.061	0.087

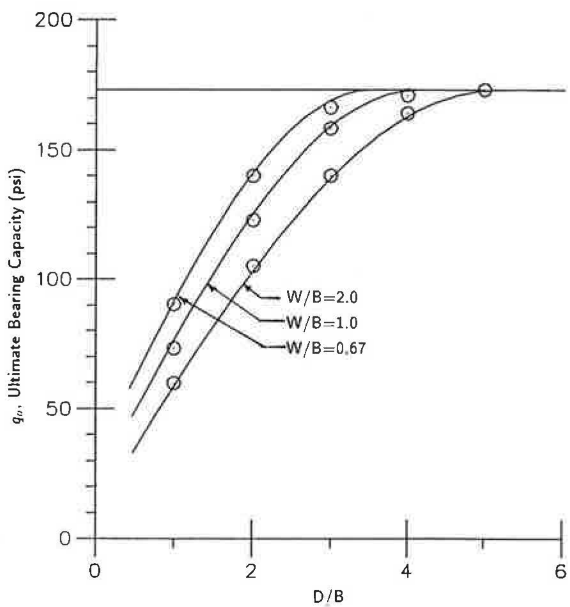

FIGURE 2 Variation of ultimate bearing capacity with *D/B* for kaolin with *E/B* = 0 and *W/B* = 0.67, 1.0, and 2.0.

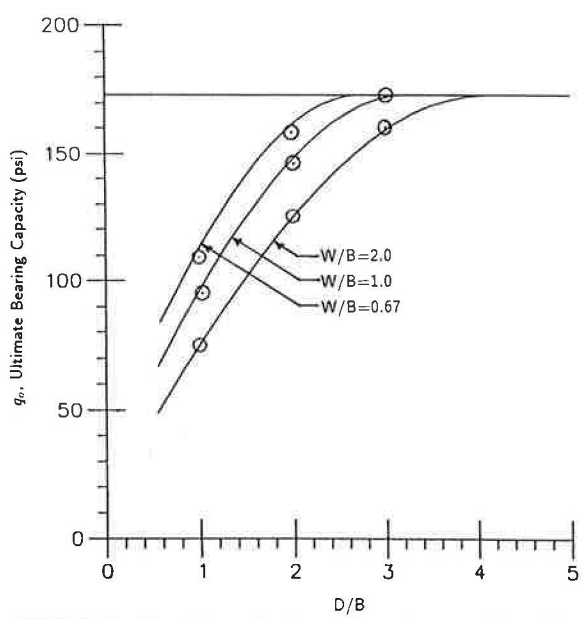

FIGURE 3 Variation of ultimate bearing capacity with *D/B* for kaolin with *E/B* = 1.0 and *W/B* = 0.67, 1.0, and 2.0.

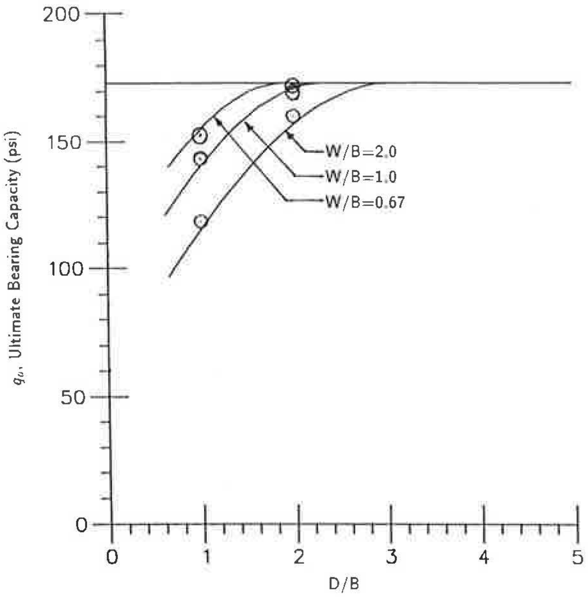

FIGURE 4 Variation of ultimate bearing capacity with *D/B* for kaolin with *E/B* = 2.0 and *W/B* = 0.67, 1.0, and 2.0.

Figures 2, 3, and 4 are for kaolin with $E/B = 0$, 1, and 2, respectively; Figures 5 and 6 are respectively for silty clay and clayey sand with $E/B = 0$. Other figures are available elsewhere (*16*). Thus, there is a total of nine figures. These graphs form the data base for the development of the ultimate bearing capacity equation.

According to Figures 2 through 6 and the other figures, the ultimate bearing capacity of strip footing varies with void size and location so that the bearing capacity increases as the depth to void increases, void size decreases, and void eccentricity increases, while other factors are constant. The data indicate that there is a depth to void beyond which the presence of a void has no effect on the ultimate bearing capacity. This depth to void, termed as the critical depth (D_c) in previous papers (*1,2*), varies with soil type, void size, and void eccentricity. The critical depth to void is an important factor required in the ultimate bearing capacity equation.

EQUATION FORMULATION

The ultimate bearing capacity equation was developed by fitting the graphical relations in Figures 2 through 6 and the other figures. In the fitting process, the main features of the curves were first identified. Functions having such features were then selected to fit the curves. Through trial and error, the function best fitting the curve was adopted. Further, the coefficient, amplitude, and argument of the function were determined for each curve. The various sets of coefficients for all curves were analyzed to determine the variation between each coefficient and the influencing factors.

For the curves in Figures 2 through 6, different functions were tried, such as polynomial, hyperbolic secant, arc tangent, and sine. Of the various functions attempted, the one-quarter-cycle sine function best fit the curves. The function contains two coefficients—one for the intercept on the vertical axis and the other for the amplitude. By using this function, the SAS nonlinear regression analysis was performed to determine the coefficients that best fit each curve. It was found that the argument in the sine function equals $\frac{\pi D}{2D_c}$, and the coefficients vary with the void size, void location, and shear strength property of soil.

The equation relating the coefficients with void size, void location, and soil strength was developed first by plotting the coefficients against *W/B* for each *E/B* and soil type. All the relation curves were then fitted by a factored hyperbolic secant function that has a maximum value equal to q_{nv}, the ultimate bearing capacity value of no-void condition, and is asymptotic to a constant value as *W/B* approaches infinity. Furthermore, to generalize the equation that fits the curves in Figures 2 through 6, the ultimate bearing capacity is expressed as q_o/q_{nv}. In the analysis, the value of q_{nv} is computed

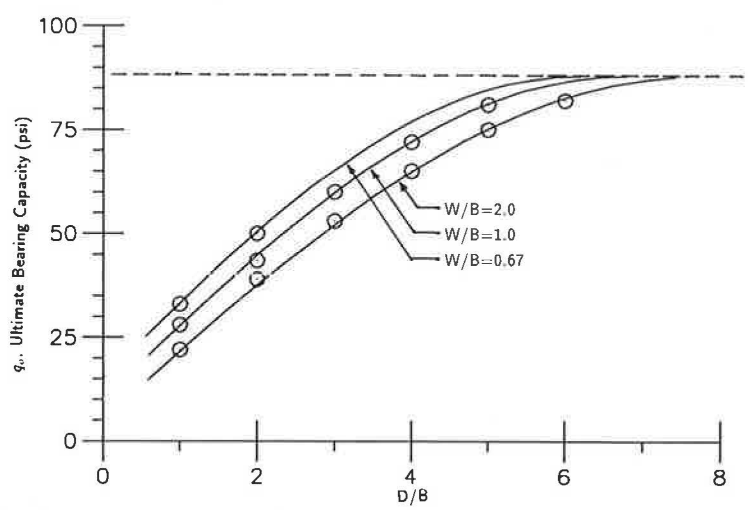

FIGURE 5 Variation of ultimate bearing capacity with *D/B* for silty clay with *E/B* = 0 and *W/B* = 0.67, 1.0, and 2.0.

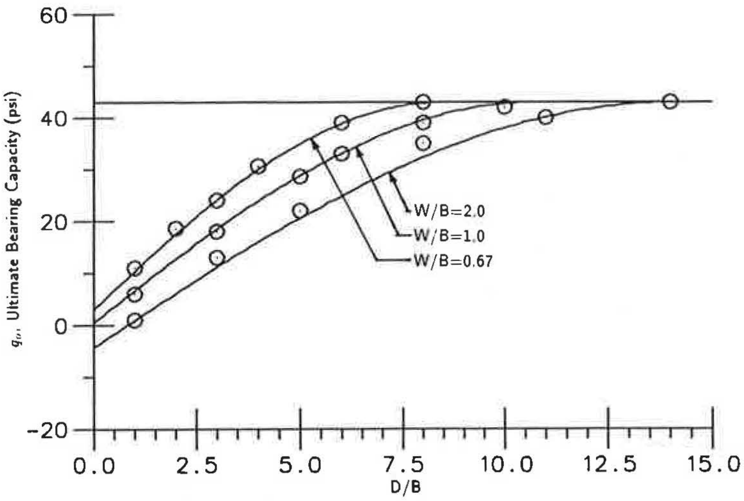

FIGURE 6 Variation of ultimate bearing capacity with *D*/*B* for clayey sand with *E*/*B* = 0 and *W*/*B* = 0.67, 1.0, and 2.0.

by using the conventional bearing capacity equation together with Meyerhof's coefficients.

Because the argument of the one-quarter-cycle sine function contains the critical depth to void (D_c), an equation for predicting D_c is required. From Figures 2 through 6 and the others, the critical depth to void for each condition is obtained from the point where the curve reaches the no-void bearing capacity value. The values of D_c thus obtained are plotted against W/B for each level of E/B. The plots reveal that the D_c/B versus log (W/B) relation can be approximated by a linear function. The coefficients contained in the linear function of D_c/B versus log (W/B) are then further related with void location and soil strength property.

Details on equation formulation are documented elsewhere (*16*). The final ultimate bearing capacity equation is as follows, where ζ is the bearing capacity ratio:

$$q_o = \zeta q_{nv} \tag{1}$$

In Equation 1,

$$\zeta = \sin\delta + K(1 - \sin\delta) \tag{2}$$

and

$$\delta = \frac{\pi}{2}\frac{D}{D_c} \tag{3}$$

$$D_c = B[D_1 + D_2 \log(W/B)] \tag{4}$$

$$D_1 = 16.3\sin 2\phi \cos^2\phi - 2.93(E/B)\sin 2\phi \tag{5}$$

$$D_2 = 22.5\sin\phi - 3.5(E/B)\tan\phi \tag{6}$$

$$K = K_p + \operatorname{sech}\{[2.9 - \tan^2\phi - 0.4(E/B)^2\cos 2\phi] + [2.5 - 1.5\tan^2\phi - 0.58(E/B)\cos^2\phi]\log(W/B)\} \tag{7}$$

$$K_p = \begin{cases} -0.42\tan^2\phi & \text{if } 2cB \geq \gamma B^2 \\ 0 & \text{if } 2cB < \gamma B^2 \end{cases}$$

In the preceding equations, ϕ = internal friction angle of soil, c = cohesion of soil, and γ = unit weight of soil.

NOMOGRAPH AND EXAMPLE

For ease in application of the developed equations one nomograph for determination of K is presented in Figure 7. There are three sets of curves—Figures 7a, b, and c for internal friction angle (ϕ) equal to 10, 20, and 30 degrees, respectively. A minimum of three levels for each variable is also presented in the nomograph for ease in data interpolation when necessary.

As an example of the use of the nomograph and equations, a 5-ft-wide strip surface footing is supported by cohesive soil. The foundation soil has a unit weight, cohesion, and internal friction angle of 130 pcf, 15 psi, and 20 degrees, respectively. A 5-ft diameter continuous circular void having its axis parallel with the footing axis is 3 ft from the footing axis and 10 ft below the footing base. The ultimate bearing capacity of the footing can be determined as follows.

For the conditions given, B = 5 ft, E = 3 ft, W = 5 ft, and D = 10 ft, γ = 130 pcf, c = 15 psi, and ϕ = 20 degrees.

1. Determine D_c and compute δ. From Equations 4, 5, and 6, for $E/B = 0.6$, $W/B = 1$, and ϕ = 20 degrees,

$$D_C \cong 44 \text{ ft}$$

$$\delta = \frac{\pi}{2}\frac{D}{D_c} \cong \frac{180°}{2}\frac{10}{44} \cong 20.45°$$

2. Determine coefficient K. From Figure 7(b), for W/B = 1, E/B = 0.6, ϕ = 20 degrees, and $2cB = 2\,(15 \times 144)(5) = 21600$, which is greater than $\gamma B^2 = (130)(5)^2 = 3250$,

$$K \cong 0.08$$

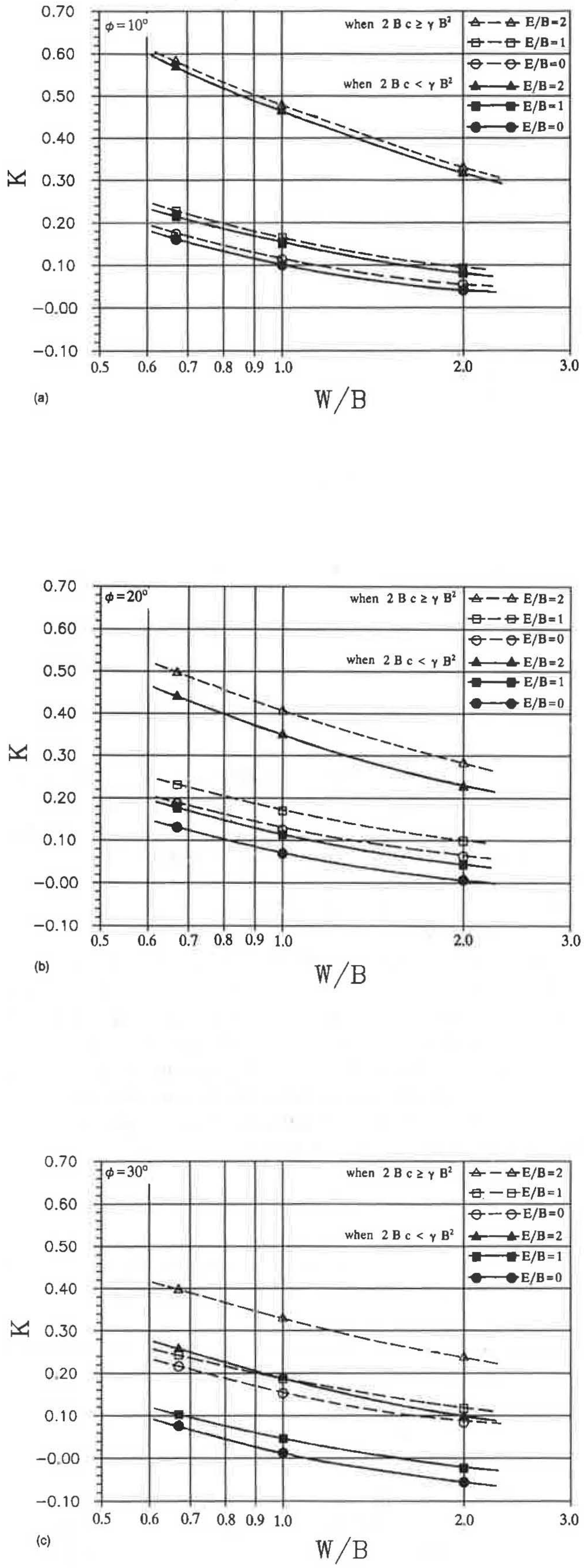

FIGURE 7 Variation of coefficient K with W/B for internal friction angle of (a) 10, (b) 20, and (c) 30 degrees.

3. Compute bearing capacity ratio ζ.

$$\zeta = \sin \delta + K(1 - \sin \delta)$$
$$= \sin 20.45° + 0.08 (1 - \sin 20.45°)$$
$$\cong 0.40$$

4. Compute the ultimate bearing capacity of no-void condition, q_{nv}. For $\phi = 20$ degrees, $N_c \cong 14.83$, $N_q \cong 6.40$, and $N_\gamma \cong 2.90$,

$$q_{nv} = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$$
$$\cong (15)(14.83) + 0 + \frac{1}{2}\left(\frac{130}{1728}\right)(5 \times 12)(2.90)$$
$$\cong 229.0 \text{ psi}$$

5. Compute the required ultimate bearing capacity.

$$q_o = \zeta q_{nv}$$
$$= (0.40)(229.0) = 91.6 \text{ psi}$$

COMPARISONS

To demonstrate the effectiveness of the developed equations, the ultimate bearing capacity of model test footings was computed and compared with the test results obtained by Badie (*13*) for kaolin and Azam (*14*) for clayey sand. The model test footing was a steel plate 2.0 in. wide by 5.25 in. long by 0.5 in. thick, tested under the plane-strain loading in a test tank approximately 32 in. high, 60 in. long, and 5.5 in. wide. A circular void was at various locations. The conditions, including void location and soil type used for comparison, test results, and computed bearing capacity values, are given in Table 2. A fairly good agreement between the two sets of data is seen, indicating that the developed equations can provide accurate bearing capacity data for strip surface footing underlain by a continuous void at least within the range of conditions investigated.

SUMMARY AND CONCLUSIONS

The need to develop a methodology for determining the ultimate bearing capacity of shallow foundation underlain by

TABLE 2 COMPARISON BETWEEN COMPUTED ULTIMATE BEARING CAPACITY AND MODEL FOOTING TEST RESULT

Test Soil	D/B	W/B	E/B	Ultimate Bearing Capacity (psi)	
				Computed	Test Result
Kaolin	2.0	2.41	0	95.6	117.4
	3.0	2.41	0	130.9	137.9
	4.5	2.41	0	165.2	164.3
	2.0	2.41	1.5	133.3	125.1
Clayey Sand	2.0	2.41	0	11.5	17.0

an underground void was identified. A method of bearing capacity determination for strip surface footing overlying a continuous circular void with its axis parallel to the footing axis was presented.

For bearing capacity equation development, the performance of strip surface footing subjected to a vertical central loading with and without an underground void was investigated using a plane-strain finite element computer program. In the analysis, the foundation soil was characterized as a nonlinear elastic, perfectly plastic material that obeys the Drucker-Prager yield criterion. To cover a wide range of soil property, three different soils were analyzed—kaolin, silty clay, and clayey sand. A range of footing width, varying void sizes, and void locations including the depth to void and void eccentricity were considered. The ultimate bearing capacity of each condition analyzed was obtained from the footing performance data. These ultimate bearing capacity values were then related graphically with the various influencing factors investigated including void size, void location, and shear strength properties of the soil. The bearing capacity equations were developed through curve fitting of the graphical relationships. For these equations, one nomograph was presented for ease in equation application.

The developed equations were used to determine the bearing capacity of some model test footings, and the results were compared with the test data. A good agreement between the prediction and test data was obtained. On the basis of this comparison, it may be concluded that the developed bearing capacity equation may become an effective tool for analysis and design of strip surface footing underlain by a circular void, at least within the range of conditions considered.

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REFERENCES

1. R. L. Baus and M. C. Wang. The Bearing Capacity of Strip Footing Located Above a Void in Cohesive Soils. *Journal of Geotechnical Engineering Division*, ASCE, Vol. 109, No. 1, 1983, pp. 1–14.
2. A. Badie and M. C. Wang. Stability of Spread Footing Above Void in Clay. *Journal of Geotechnical Engineering Division*, ASCE, Vol. 110, No. 11, 1984, pp. 1591–1605.
3. M. C. Wang and R. L. Baus. Settlement of Footings Above a Void. *2nd Conference on Ground Movements and Structures*, The Department of Civil Engineering and Building Technology, UWIST, 1980.
4. M. C. Wang and A. Badie. Effect of Underground Void on Foundation Stability. *Journal of Geotechnical Engineering*, ASCE, Vol. 111, No. 8, 1985, pp. 1008–1019.
5. M. C. Wang, C. S. Yoo, and C. W. Hsieh. Effect of Void on Footing Behavior under Eccentric and Inclined Loads. *Foundation Engineering: Current Principles and Practices, Vol. 2, Proc. of the Congress*, ASCE, 1989, pp. 1226–1239.
6. L. A. Wood and W. J. Larnach. The Behavior of Footings Located Above Voids. *Proc., Eleventh International Conference on Soil Mechanics and Foundation Engineering*, Vol. 4, San Francisco, Calif., 1985, pp. 2273–2276.
7. G. A. H. Abdellah and M. H. Abdalla. The Interaction Between a Tunnel/Cavity and Nearby Structures. *Proc., VI Australian Tunnelling Conference*, 1987, Vol. 1, pp. 183–189.
8. M. C. Wang and C. W. Hsieh. Collapse Load of Strip Footing Above Circular Void. *Journal of Geotechnical Engineering*, ASCE, Vol. 113, No. 5, 1987, pp. 511–515.
9. A. Badie and M. C. Wang. Stability of Underground Cavity Subjected to Surface Loads. *International Symposium on Unique Underground Structures*, Denver, Colo., 1990.
10. D. Drucker and W. Prager. Soil Mechanics and Plastic Analysis in Limit Design. Quarterly of Applied Mathematics, Vol. 10, No. 2, 1952, pp. 157–165.
11. S. F. Reyes. *Elastic-Plastic Analysis of Underground Openings by the Finite Element Method*. Ph.D. thesis. University of Illinois, Urbana, 1966.
12. R. L. Baus. *The Stability of Shallow Continuous Footings Located Above Voids*. Ph.D. thesis. The Pennsylvania State University, University Park, 1980.
13. A. Badie. *Stability of Spread Footing Supported by Clay Soil with Underground Void*. Ph.D. thesis. The Pennsylvania State University, University Park, 1983.
14. G. Azam. *Stability of Shallow Continuous Footings Supported by Two-Layer Soil Deposits with an Underground Void*. Ph.D. thesis. The Pennsylvania State University, University Park, 1990.
15. H. J. Siriwardane and C. S. Desai. Computational Procedures for Nonlinear Three-Dimensional Analysis with Some Advanced Constitutive Laws. *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 7, No. 2, 1983, pp. 143–171.
16. C. W. Hsieh. *Development of Methodology for Stability Analysis of Surface Strip Footing Above Continuous Circular Void*. Ph.D. dissertation. Department of Civil Engineering, The Pennsylvania State University, University Park, 1991.
17. R. V. Whitman and K. Hoeg. Development of Plastic Zone Beneath a Footing. *Report to U.S. Army Engineering Waterways Experiment Station*, Department of Civil Engineering, The Massachusetts Institute of Technology, Cambridge, 1966.
18. J. E. Bowles. *Foundation Analysis and Design* (4th ed.). McGraw-Hill Book Co., New York, 1988.

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