Adaptive Filter Forecasting System for Pavement Roughness

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Forecasting pavement roughness conditions can facilitate decision making within a pavement management system at project and network levels. Because pavement roughness change over time is caused by some important conditions and certain stochastic factors, a parameter and dynamic forecasting model is more appropriate for forecasting roughness with respect to linear, static, and nonparameter forecasting models. Thus, an adaptive filter forecasting system is presented that forecasts pavement roughness conditions by means of an adaptive filter using roughness history. The concept of an adaptive filter forecasting system is introduced, along with its mathematical derivation and least-mean-square algorithm. In testing the system's validity, a given mathematical function is used to simulate changing pavement roughness conditions. In addition, a practical application of the adaptive filter forecasting system is presented. The roughness index used is the root-mean-square vertical acceleration of a response-type roadroughness measuring system. Finally, choice of the adaptive filter structure and its stability, based on roughness data collected from Austin Test Sections, are discussed. The structure of system should be decided before each application by experimental results with certain criteria. This is a major limitation of the system.

Measurement of pavement roughness is an important exercise within a pavement management system (PMS) at project and network levels, because it relates to pavement evaluation, maintenance, and rehabilitation (1,2). In addition, pavement roughness measurements have been used in predicting vehicle operating cost, predicting road performance, evaluating road safety, and evaluating passenger degree of comfort (3-8; Darlington, unpublished data). Since the AASHO Road Test, much roughness research has been conducted, including studies on measuring techniques, index development, evaluation, specification, and prediction.

However, pavement roughness measurements can reflect only existing states. Unless adequate forecasting models are used to predict future roughness conditions, existing roughness cannot provide reliable information on which to base future planning, maintenance, rehabilitation, and other PMS activities.

Two concepts concerning roughness prediction must be distinguished. The first, which has been the subject of much research (5,9-11), can be described by the following equation:

$$R_{k} = F(k, D_{k}, M_{k}, T_{k}, E_{k}, \text{ etc.})$$
(1)

where

- R_k = roughness at time k,
- D_k = pavement structure at time k,
- M_k = pavement materials at time k,
- T_k = traffic condition at time k, and
- E_k = environment at time k.

Roughness at time k is estimated using these existing conditions but disregarding past information.

The second concept can be described by another equation:

$$R_{k} = F(R_{k-1}, R_{k-2}, \dots, R_{k-N})$$
(2)

Roughness at time k, R_k , is forecast using historical roughness records at time k - 1, k - 2, ..., k - N. This is a timeseries modeling problem. It appears that Equation 2 does not consider conditions affecting roughness except past roughness. Conditions of pavement structure, materials, traffic, and environment are time-variable; certain changing trends over time are reflected in the past roughness data sequence. Conditions thus are forecast by understanding the changing processes of past conditions of pavement structure, materials, traffic, environment, and such. In this study, this concept is called time-series forecasting of roughness. It provides better information for decision making in planning, maintenance, and rehabilitation because the future roughness state has been forecast.

Traditionally, linear regression and extrapolation models have been used for forecasting (12-15). These are nonparameter estimation models and are usually considered static estimators. It is understood that the changing process of pavement roughness consists of certain trend caused by some conditions and unpredictable stochastic factors. These stochastic factors make a linear static estimator inadequate for forecasting. Linear regression and extrapolation models also have limitations in forecasting pavement roughness.

In the past two decades, several important parameter forecasting models and probability-based models have been applied to transportation areas (15-17). Mathematically, the parameter forecasting models most often used are Kalman filtering (18), time-series prediction (19), spectral analysis (20), and adaptive forecasting (21).

Recent studies have used an adaptive filter model to forecast roughness. This can be considered a dynamic parameter estimation model—that is, the pavement roughness condition forecast at time step k is a function of past conditions at time step $k - 1, k - 2, \ldots, k - M$ where M < k, and M and k are positive integers and a set of parameters estimated by the adaptive filter forecasting system.

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Mathematically, the objective function of this system, minimizing the resulting mean-square error, might be similar to that of the Kalman filtering and time-series prediction models. However, it uses a simplified least-mean-square (LMS) algorithm to search for optimal filter weights or states. This difference means that dynamic response of the system could be better, needing less data storage space than the earlier prediction models. Intuitively speaking, an adaptive filter forecasting system is viewed as one whose structure is adjustable in such a way that its performance improves through contact with its environment.

This paper focuses on a time-series forecasting method for pavement roughness using an adaptive filter forecasting system. The basic concept of the system is introduced, and then its mathematical derivation is described. Results of experiments based on simulation and real roughness data, which is root-mean-square vertical acceleration (RMSVA) (22) collected by the Automatic Road Analyzer (ARAN) (23), are presented and discussed.

BASIC PRINCIPLES OF ADAPTIVE FORECASTING SYSTEM

Figures 1 and 2 show an adaptive forecasting system and its processors, respectively. In these figures, Z^{-1} is an one-step delay factor, and Z^{-s} is an *s*-step delay factor (where *s* is a positive integer). Mathematically, $q(k)Z^{-1} = q(k - 1)$, and $q(k)Z^{-s} = q(k - s)$. As can be seen from Figure 1, the system's core is the adaptive processors, in which all of the parameters (weights) at step *k* are adjustable. The error of forecast e(k) controls adjustment of the system. From Figures 1 and 2, the following equation can be derived:

$$\hat{q}(k) = \sum_{j=0}^{N} W_{jk} q(k-s-j)$$
 $(k = s + 1, s + 2, ...)$ (3)

Equation 3 indicates that $\hat{q}(k)$ is the linear weighted combination of q(k - s), q(k - s - 1), ..., q(k - s - N). The weights are W_{0k} , W_{1k} , ..., W_{Nk} , and the index k denotes the time step. If $\hat{q}(k)$ is used to forecast q(k), then the error



FIGURE 1 Adaptive forecasting system.

of forecast at step k is

$$e(k) = q(k) - \hat{q}(k) \qquad (k = s + 1, s + 2, \ldots)$$
(4)

The purpose of using an adaptive processors is to adjust the weights at each step k so that the mean square error $E[e^2(k)]$ is minimized. The vectors \mathbf{W}_k and \mathbf{Q}_{k-s} are defined as follows:

$$\mathbf{W}_{k} = [W_{0k}, W_{1k}, \dots, W_{Nk}]^{T}$$
$$\mathbf{Q}_{k-s} = [q(k-s), q(k-s-1), \dots, q(k-s-N)]^{T}$$

With these definitions, Equation 3 can be expressed using vector notation:

$$\hat{q}(k) = \mathbf{Q}_{k-s}^T \mathbf{W}_k = \mathbf{W}_k^T \mathbf{Q}_{k-s}$$
(5)

Now that operation of the adaptive processor has been described, one can consider how the adaptive processor adapts—that is, how the vector W_k is adjusted as the timestep index k changes.

From Equations 4 and 5, Equation 6 can be derived:

$$e(k) = q(k) - \mathbf{W}_k^T \mathbf{Q}_{k-s} = q(k) - \mathbf{Q}_{k-s}^T \mathbf{W}_k$$
(6)

By squaring Equation 6, the instantaneous squared error can be obtained.

$$e^{2}(k) = [q(k) - \mathbf{W}_{k}^{T}\mathbf{Q}_{k-s}][q(k) - \mathbf{Q}_{k-s}^{T}\mathbf{W}_{k}]$$
$$= q^{2}(k) + \mathbf{W}_{k}^{T}\mathbf{Q}_{k-s}\mathbf{Q}_{k-s}^{T}\mathbf{W}_{k} - 2q(k)\mathbf{Q}_{k-s}^{T}\mathbf{W}_{k}$$
(7)



FIGURE 2 Adaptive processors $[W_{ik} \text{ is adjusted by } e(k), i = 0 \text{ to } N]$.

To find the expected value of Equation 7 over k, it is assumed that e(k) and q(k) are statistically stationary. This assumption can usually be approximately satisfied for the particular pavement roughness conditions. Then the expectation of $e^2(k)$ is

$$E[e^{2}(k)] = E[q^{2}(k)] + \mathbf{W}_{k}^{T} E[\mathbf{Q}_{k-s} \mathbf{Q}_{k-s}^{T}] \mathbf{W}_{k}$$
$$- 2E[q(k)\mathbf{Q}_{k-s}^{T}] \mathbf{W}_{k}$$
(8)

Let **R** be defined as the square matrix

$$\mathbf{R} = \mathbf{E}[\mathbf{Q}_{k-s}\mathbf{Q}_{k-s}^T] \tag{9}$$

Thus **R** is the correlation matrix of q(k - s) with dimension $N \times N$. Let **P** be defined as the column vector

$$\mathbf{P} = \mathbf{E}[q(k)\mathbf{Q}_{k-s}^{T}]$$

= $E[q(k)q(k-s), q(k)q(k-s-1), \dots, q(k)q(k-s-N)]^{T}$ (10)

This vector is the set of autocorrelation of q(k). **R** and **P** thus are the second-order statistics of the random variable q(k - s) at step k. By the definitions of **R** and **P**, Equation 6 can be expressed as

$$E[e^{2}(k)] = E[q^{2}(k)] + \mathbf{W}_{k}^{T}\mathbf{R}\mathbf{W}_{k} - 2\mathbf{P}^{T}\mathbf{W}_{k}$$
(11)

According to the assumption that q(k) is statistically stationary, **R** and **P** are a constant matrix and vector, respectively. In this case, $E[e^2(k)]$ is a quadratic function of the weight vector \mathbf{W}_k . If the adaptive processor has the ability of "self-study" to seek the minimum $E[e^2(k)]$ by adjusting \mathbf{W}_k , and if $E[e^2(k)]$ tends to be minimal when \mathbf{W}_k tends to be optimal solution \mathbf{W}_k^* , then the forecast of the processors will be optimal. The question is how to find the optimal solution of \mathbf{W}_k so that $E[e^2(k)]$ is minimized at each step k. This can be solved by the gradient method. The gradient of the mean square error $E[e^2(k)]$ is designated ∇_k and can be expressed by

$$\nabla_k = 2\mathbf{R}\mathbf{W}_k - 2\mathbf{P}$$

To obtain the optimal solution \mathbf{W}_{k}^{*} so that $E[e^{2}(k)]$ is minimized, it is necessary to let

$$\nabla_k = 0 = 2\mathbf{R}\mathbf{W}_k^* - 2\mathbf{P}$$

or

$$\mathbf{W}_{k}^{*} = \mathbf{R}^{-1}\mathbf{P} \tag{12}$$

Equation 12 is the optimal solution of W_k . By substituting Equation 12 into Equation 11, and noting that the correlation matrix is symmetric, then

$$E[e^{2}(k)]_{\min} = E[q^{2}(k)] + [\mathbf{R}^{-1}\mathbf{P}]^{T}\mathbf{R}\mathbf{R}^{-1}\mathbf{P} - 2\mathbf{P}^{T}\mathbf{R}^{-1}\mathbf{P}$$
$$= E[q^{2}(k)] + \mathbf{P}^{T}\mathbf{R}^{-1}\mathbf{P} - 2\mathbf{P}^{T}\mathbf{R}^{-1}\mathbf{P}$$
$$= E[q^{2}(k)] - \mathbf{P}^{T}\mathbf{R}^{-1}\mathbf{P} \qquad (13)$$

Although Equation 12 is the optimal solution of W_k , in a practical sense W_k^* is not estimated by Equation 12. In the next section, the algorithm to estimate W_k^* is discussed.

LEAST-MEAN-SQUARE ALGORITHM

Recall in Equation 12 that

$$\nabla_k = 2\mathbf{R}\mathbf{W}_k - 2\mathbf{P}$$

or

$$\mathbf{W}_k^* = \mathbf{R}^{-1}\mathbf{P}$$

By combining these equations, Equation 14 is obtained:

$$\mathbf{W}_k^* = \mathbf{W}_k - 0.5\mathbf{R}^{-1}\nabla_k \tag{14}$$

It can be changed into an adaptive algorithm as follows:

$$\mathbf{W}_{k+1} = \mathbf{W}_k - 0.5 \mathbf{R}^{-1} \nabla_k \tag{15}$$

If the vector of weight W_k is adjusted in the direction of the gradient at each step k and a constant μ ($0 < \mu < 1$) is defined, then Equation 15 can be simplified as follows:

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu(-\nabla_k) \tag{16}$$

where μ regulates step size (from k to k + 1) and has dimensions of reciprocal signal power.

To develop the LMS algorithm, $e^2(k)$ itself can be taken as an estimate of $E[e^2(k)]$; then the estimate of the gradient ∇_k can be expressed by

$$\hat{\nabla} = \begin{bmatrix} \frac{\partial e^2(k)}{\partial W_{0k}} \\ \vdots \\ \frac{\partial e^2(k)}{\partial W_{Nk}} \end{bmatrix} = 2e(k) \begin{bmatrix} \frac{\partial e(k)}{\partial W_{0k}} \\ \vdots \\ \frac{\partial e(k)}{\partial W_{Nk}} \end{bmatrix} = -2e(k)\mathbf{Q}_{k-s}$$
(17)

With this simple estimate of the gradient, the LMS algorithm can be specified by Equations 16 and 17:

$$\mathbf{W}_{k+1} = \mathbf{W}_{k-\mu} \nabla_k = \mathbf{W}_k + 2\mu e(k) \mathbf{Q}_{k-s}$$
(18)

In this research effort, another parameter—AL1—was defined:

$$AL1 = \frac{1}{2\mu}$$

where AL1 is called an attenuate factor. Thus Equations 5 and 18 constitute the adaptive forecast model. Equation 18 indicates that the LMS algorithm can be implemented in a practical system without squaring, averaging, or differentiation and is elegant in its simplicity and efficiency. Lu et al.



FIGURE 3 Comparison between simulated and forecast RMSVA.

SIMULATION OF ADAPTIVE FILTER FORECASTING SYSTEM

A simulation experiment was conducted to evaluate the performance of the adaptive filter forecasting system. The experiment was conducted by inputing a given mathematical function as a simulation of RMSVA to the system. This was done to prove the applicability of the system for practical purposes. In the experiment, a mathematical function was used to simulate RMSVA as a function of time, or RMSVA(t):

RMSVA(t) = 500 - 300 cos(
$$\pi t/100$$
)
(t: month, t = 1, 2, ..., 100) (19)

The experimental results are shown in Figure 3, with $AL1 = 6.52 \times 10^5$, N = 3, and S = 1. It is clear from the graph that the forecast RMSVA follows the true (simulated) RMSVA. For this kind of deterministic RMSVA, the system can precisely predict future characteristics of RMSVA by understanding the past process of RMSVA. This ability could be due to the continuously differentiable nature of the sine function input.

APPLICATIONS OF SYSTEM

As stated in the introduction, roughness conditions are forecast by using past roughness data, RMSVA. The amount of past data that must be stored in the forecasting system depends on the order of the adaptive filter. To forecast future roughness, a certain quantity of initial roughness data should be available. Then, after the forecasting system is in use, initial data will be continuously updated by measured data.

Field Data Collection and Preparation

During the study, the adaptive filter forecasting system was applied to forecasts of RMSVA of Austin Test Sections (ATS). Roughness conditions have been monitored by a K. J. Law profilometer at 20 mph since July 1982. The original index is serviceability index (SI) collected every 3 months. However, because the forecasting system is designed for forecasting RMSVA with past RMSVA data measured by the ARAN unit, original data had to be changed to corresponding RMSVA data by a correlation model between the Law profilometer and the ARAN unit. The correlation model has the following form (23):

SI (profilometer) = $5.297 - 4.742 \ 10^{-3}$ RMSVA (ARAN)

οг

$$RMSVA (ARAN) = 1117 - 210.9 SI (profilometer)$$
(20)

General experience indicates that the measured data include certain systematic and operational errors. A good data processing technique to reduce the errors is data smoothing. In this study, a three-order smoothing filter was used to smooth the measured data sequence.

Results of Forecasting Roughness Data, RMSVA

Although past roughness had been measured, it was impossible to forecast pavement conditions precisely. This result is different from the simulation experiment. The adaptive filter forecasting system can figure statistical characteristics of pavement roughness conditions using the adaptive processor and past roughness data, RMSVA, for optimal forecasting of roughness conditions; that is, statistically the adaptive filter forecasting system's performance is optimal.

Figures 4 and 5 show results of forecasting RMSVA at Austin Test Sections ATS36 and ATS40, with given adaptive



FIGURE 4 Comparison between measured and forecast RMSVA of ATS36.



FIGURE 5 Comparison between measured and forecast RMSVA of ATS40.

filter structures (N = 3, $AL1 = 5.42 \times 10^5$, S = 1) and (N = 3, $AL1 = 1.82 \times 10^5$, S = 1), respectively. Averaged absolute forecast errors are 9.777 for ATS36 and 5.359 for ATS40.

Figures 6 and 7 show results of forecasting RMSVA of ATS07 and ATS38 by the adaptive filter forecasting system with the structures (N = 3, $AL1 = 1.02 \times 10^5$, S = 1) and (N = 3, $AL1 = 1.118 \times 10^6$, S = 1), respectively. These graphs show that some maintenance or rehabilitation activities, such as overlay, took place during the monitoring period, so that the roughness level RMSVA dropped after that work. However, it should be mentioned that the historical roughness



FIGURE 6 Comparison between measured and forecast RMSVA of ATS07.



FIGURE 7 Comparison between measured and forecast RMSVA of ATS38.

data before major maintenance or rehabilitation should not be used to forecast subsequent roughness. Averaged absolute forecast errors for ATS07 and ATS38 are 3.949 and 11.082, respectively.

ADAPTIVE FILTER STRUCTURE CHOICE AND STABILITY

For given pavement roughness conditions, the forecast is affected mainly by the adaptive filter structure (N, AL1). In this study, performance was associated with forecast errors, stability, and such. Adequate choice of the order N and attenuate factor AL1 of the adaptive filter can result in a relatively accurate forecast and good stability. For a given pavement section, tests thus should be conducted to choose the optimal pair of N and AL1 by minimizing the forecast errors, and N and AL1 must be updated further when data on the new roughness condition RMSVA are collected. Table 1 gives optimal pairs of N and AL1 for ATS40. The resulting optimal N and AL1 are based on roughness data RMSVA collected since July 1982. The index E is the averaged absolute forecast error. Any other choice of N and AL1 will result in larger E.

It can be understood that for a new pavement without any existing roughness data, the optimal pairs of N and AL1 cannot be decided and certain initial readings are needed for forecasting. However, roughness readings from other pavement with closely similar conditions can be used to predict roughness of this new pavement. After several readings have been obtained, the forecasting system will gradually get into optimal state by continuously updating its structure.

Like other dynamic systems, the adaptive filter forecasting system also has the problem of stability. A simple definition of stability adopted in this study is that if the averaged absolute forecast error E is always smaller than a given number or critical value, A, the adaptive filter forecasting system is said to be stable; otherwise it is unstable.

Stability of the system depends mainly on AL1 and N. In the plane of (AL1, N) a zone should exist where the system should be stable, or it would be unstable. Figure 8 shows the

TABLE 1OPTIMAL PAIRS OF N AND AL1 TOMINIMIZE FORECAST ERROR FOR ATS40

N	2	3	4	5	6
AL1	350000	542000	750000	958000	1154000
Е	5.3624	5.3594	5.3664	5.3942	5.4185



FIGURE 8 Stable and unstable areas in (AL1, N) plane.

stable and unstable zones based on RMSVA data collected from ATS40 and A = 20. If (AL1, N) belongs to the area above the straight line, the system is stable; otherwise it is unstable. In fact, as long as the system is continuously updated, stability will not be a problem because the optimal Nand AL1 guarantee that the system is in stable zone.

CONCLUSIONS

The adaptive filter forecasting system can be used as a dynamic time-series predictor of pavement roughness conditions. System performance depends both on roughness conditions and structure of the adaptive filter (AL1 and N). In choosing AL1 and N, consideration should be given to stability of the system. To forecast roughness conditions on a specific pavement section, an adequate number of tests should be run to obtain optimal AL1 and N.

The system, like other forecasting models mentioned in this paper, has some limitations for practical application. One of the most critical problems seems to be the convergence that has been discussed in some works (15, 19). Although in certain situations, the adaptive filter forecasting system could converge to the optimal states with given model structures (i.e., AL1 and N), in others the adaptive prediction system might not converge with the same model structures.

In this study the direct application of the adaptive filter forecasting system is to forecast RMSVA. However, in principle this system can be applied to forecasts of other roughness indices, such as SI, international roughness index, and mean absolute slope.

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