Mixed-Integer Programming Model for AASHTO Flexible Pavement Design

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A mixed-integer programming model is described that is based on the AASHTO design procedure for flexible pavements and formulated for selecting pavement materials and determining surface, base, and subbase course thicknesses. The objective of the model is to minimize the total cost of pavement structures while meeting the constraints of AASHTO flexible pavement design equations and user-defined criteria. The Flexible Pavement Optimal Design computer program that interfaces with the optimization package LINDO has been developed to obtain quick solutions. Two solutions are given by the program: nonintegers and integers. The program can be used for flexible pavement thickness design in cases in which one or more materials are available for each of three layers if the layer characteristics and material properties are known.

In flexible pavement design, there are usually several material types available for surface, base, and subbase courses. There are many combinations of layer thicknesses for each of the three layers when AASHTO design equations (*1*) are used. In pavement construction, a small reduction in the unit cost of pavement structures can result in considerable savings for the entire project. Therefore, obtaining the best materials at minimum cost is important.

Because the AASHTO DNPS86 program (*2*) has no optimization function, the solutions given by the program may not be the least expensive. Nicholls (*3*) developed a nonlinear optimization program (DNPS86O) using DNPS86 as a subroutine. A minimum-cost solution for the whole design period is obtained by changing design reliability, performance period of initial pavement, and two of the three thicknesses of flexible pavements. Rouphail (*4*) formulated a mixed-integer–linear programming model for minimum-cost design of flexible pavements by changing the number, type, and thickness of paving materials. But the problem of material selection is not addressed in either model.

The flexible pavement design problem is here formulated as a mixed-integer programming model (*5*). The model can select the best combination of different pavement materials for the three layers of pavement structure and give the minimum-cost solution for the selected materials accordingly, while meeting constraints of the AASHTO design equations and user-defined criteria (given a certain level of reliability, performance period of initial pavement, and other input data). A computer program interfacing with the optimization package LINDO (*6*) is developed to get quick solutions. Besides the minimum-cost noninteger solution (layer thicknesses are not rounded to nearest ½ in.), the minimum-cost integer solution (layer thicknesses are integers in inches) can also be obtained from the program. Sensitivity analysis has demonstrated that great benefits can be obtained by using this program.

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AASHTO FLEXIBLE PAVEMENT DESIGN EQUATIONS

In the design guide for flexible pavements, the following equations are used to compute the structural number and layer thicknesses:

$$\log(W_{18}) = Z_r S_0 + 9.36 \log(\text{SN} + 1) - 0.20 + \frac{\log\left[\frac{p_0 - p_t}{4.2 - 1.5}\right]}{0.40 + \frac{1{,}094}{(\text{SN} + 1)^{5.19}}} + 2.32 \log M_r - 8.07 \quad (1)$$

$$\text{SN} = a_1 D_1 + a_2 m_2 D_2 + a_3 m_3 D_3 \quad (2)$$

$$D_1^* \geq \frac{\text{SN}_1}{a_1} \quad (3)$$

$$\text{SN}_1^* = a_1 D_1^* \geq \text{SN}_1 \quad (4)$$

$$D_2^* \geq \frac{\text{SN}_2 - \text{SN}_1^*}{a_2 m_2} \quad (5)$$

$$\text{SN}_2^* = a_2 m_2 D_2^* \quad (6)$$

$$\text{SN}_1^* + \text{SN}_2^* \geq \text{SN}_2 \quad (7)$$

$$D_3^* \geq \frac{\text{SN}_3 - (\text{SN}_1^* + \text{SN}_2^*)}{a_3 m_3} \quad (8)$$

where

W_{18} = predicted number of 18-kip equivalent single axle load applications;
Z_r = standard normal deviate;
S_0 = combined standard error of the traffic prediction and performance prediction;
p_0 = initial design serviceability index;
p_t = design terminal serviceability index;
M_r = resilient modulus (psi);
SN = structural number indicative of total pavement thickness required;

SN_i = structural number corresponding to modulus of base ($i = 1$), subbase ($i = 2$) and roadbed soil ($i = 3$, $SN_3 = SN$);
a_i = ith layer coefficient;
m_i = ith layer drainage coefficient;
D_i = ith layer thickness in inches; and
D_i^*, SN_i^* = values actually used ($i = 1, 2, 3$).

Equation 1 shows that the structural number required for the total pavement structure can be uniquely determined under the same traffic condition and at a certain level of reliability, but that different materials for base and subbase courses with different resilient moduli will have different structural numbers. Equation 2 shows that the thicknesses of surface, base, and subbase courses depend on layer coefficients, drainage coefficients, and a structural number associated with different layers. Equations 3 through 8 are actually used for the computation of layer thicknesses.

It can be seen from these equations that there are many solutions to layer thicknesses for a particular problem with given traffic, environment, reliability, and materials. An optimal solution with minimum total cost for a pavement structure can be found with trial-and-error methods, but it may take much design time. There is no simple method such as using the ratio of SN to unit cost for quick thickness design.

In cases in which several materials are available for each of the three layers, a simple method using the ratio of the layer coefficient multiplying by the drainage coefficient to unit cost can be used to select the types of materials for the design of noninteger layer thickness, but it may not be true for the design of integer layer thickness. This will be illustrated in a later example.

MIXED INTEGER PROGRAMMING MODEL

Let m, n, r be the number of types of surface, base, and subbase courses available for a project in which the resilient moduli, layer coefficients, drainage coefficients, and unit costs corresponding to each material are known. Then the material selection and thickness design problems can be formulated as follows:

Objective function:

Minimize

$$\sum_{i=1}^{m} C_{1i}D_{1i} + \sum_{j=1}^{n} C_{2j}D_{2j} + \sum_{k=1}^{r} C_{3k}D_{3k} \quad (9)$$

In optimization, the objective value of Equation 9 is divided by 36 to get the unit cost of dollars per square yard.

Subject to

1. Constraints of AASHTO equations:

$$SN_1^* \geq SN_1 \quad (10)$$

$$SN_1^* + SN_2^* \geq SN_2 \quad (11)$$

$$SN_1^* + SN_2^* + SN_3^* \geq SN_3 \quad (12)$$

$$SN_1^* = \sum_{i=1}^{m} a_{1i}D_{1i} \quad (13)$$

$$SN_2^* = \sum_{j=1}^{n} a_{2j}m_{2j}D_{2j} \quad (14)$$

$$SN_3^* = \sum_{k=1}^{r} a_{3k}m_{3k}D_{3k} \quad (15)$$

2. Constraints of structural number

$$SN_1 \geq \sum_{i=1}^{m} SN_{1i}X_{1i} \quad (16)$$

$$SN_2 \geq \sum_{j=1}^{n} SN_{2j}X_{2j} \quad (17)$$

$$SN_3 \geq \sum_{k=1}^{r} SNX_{3k} \quad (18)$$

3. Constraints of maximum thicknesses

$$D_{1i} \leq D_{1max}X_{1i} \quad (i = 1, 2, \ldots, m) \quad (19)$$

$$D_{2j} \leq D_{2max}X_{2j} \quad (j = 1, 2, \ldots, n) \quad (20)$$

$$D_{3k} \leq D_{3max}X_{3k} \quad (k = 1, 2, \ldots, r) \quad (21)$$

4. Constraints of minimum thicknesses

$$\sum_{i=1}^{m} D_{1i} \geq D_{1min} \quad (22)$$

$$\sum_{j=1}^{n} D_{2j} \geq D_{2min} \quad (23)$$

$$\sum_{k=1}^{r} D_{3k} \geq D_{3min} \quad (24)$$

5. Constraints of surface, base and subbase course

$$\sum_{i=1}^{m} X_{1i} = 1 \quad (25)$$

$$\sum_{j=1}^{n} X_{2j} = 1 \quad (26)$$

$$\sum_{k=1}^{r} X_{3k} = 1 \quad (27)$$

where

C_{1i} = unit cost of ith type of surface course in dollars per cubic yard;
C_{2j} = unit cost of jth type of base course in dollars per cubic yard;
C_{3k} = unit cost of kth type of subbase course in dollars per cubic yard;
D_{1i} = layer thickness of ith type of surface course in inches;

D_{2j} = layer thickness of jth type of base course in inches;
D_{3k} = layer thickness of kth type of subbase course in inches;
a_{1i} = layer coefficient of ith type of surface course;
a_{2j} = layer coefficient of jth type of base course;
a_{3k} = layer coefficient of kth type of subbase course;
m_{2j} = drainage coefficient of jth type of base course;
m_{3k} = drainage coefficient of kth type of subbase course;
$D_{s\,\max}$ = maximum thicknesses of surface ($s = 1$), base ($s = 2$), and subbase ($s = 3$) course in inches;
$D_{s\,\min}$ = minimum thicknesses of surface ($s = 1$), base ($s = 2$), and subbase ($s = 3$) course in inches;
SN_s = minimum structural number corresponding to modulus of selected types of base course ($s = 1$), subbase ($s = 2$) course, and effective resilient modulus of roadbed soil ($s = 3$);
SN_s^* = structural number actually used in the models ($s = 1, 2, 3$);
SN_{1i} = structural number corresponding to ith base course calculated using AASHTO Equation 1;
SN_{2j} = structural number corresponding to jth subbase course calculated using AASHTO Equation 1;
SN = structural number corresponding to effective resilient of roadbed soil calculated using AASHTO Equation 1;
X_{1i} = 1 if ith type of surface course is selected, otherwise $X_{1i} = 0$;
X_{2j} = 1 if jth type of base course is selected, otherwise $X_{2j} = 0$;
X_{3k} = 1 if kth type of subbase course is selected, otherwise $X_{3k} = 0$; and
m, n, r = number of surface, base, and subbase courses available, respectively ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, r$).

In the model above, Equations 10 through 15 correspond to Equations 3 through 8, which are used to compute layer thicknesses required once material types of the three layers are selected. Equations 16 through 18 are used to select the structural numbers computed by Equation 1 for different layers. Equations 19 through 21 ensure that the layer thicknesses of selected materials are no more than the maximum thicknesses specified for the materials, while those of materials not selected are equal to zero. Equations 22 through 24 ensure that the layer thicknesses of selected materials are no less than the minimum thicknesses specified for the materials. Equations 25 through 27 ensure that only one material is selected for surface, base, and subbase courses, respectively.

This model is able to select the best combination of materials for the three layers and determine the optimal layer thicknesses for the selected materials. If there is only one available type of material for each layer, that is, $m = n = k = 1$, the problem is simplified only to the optimization of layer thicknesses; the formulation of the simplified model is then:

Objective function:

Minimize

$$C_1D_1 + C_2D_2 + C_3D_3 \qquad (28)$$

Subject to

$$SN_1^* \geq SN_1 \qquad (29)$$

$$SN_1^* + SN_2^* \geq SN_2 \qquad (30)$$

$$SN_1^* + SN_2^* + SN_3^* \geq SN_3 \qquad (31)$$

$$SN_1^* = a_1D_1 \qquad (32)$$

$$SN_2^* = a_2m_2D_2 \qquad (33)$$

$$SN_3^* = a_3m_3D_3 \qquad (34)$$

$$D_1 \geq D_{1\min} \qquad (35)$$

$$D_2 \geq D_{2\min} \qquad (36)$$

$$D_3 \geq D_{3\min} \qquad (37)$$

$$D_1 \leq D_{1\max} \qquad (38)$$

$$D_2 \leq D_{2\max} \qquad (39)$$

$$D_3 \leq D_{3\max} \qquad (40)$$

where C_1, C_2, C_3 are the unit costs of surface, base, and subbase course (dollars per cubic yard), and all other variables are as defined before.

A computer program—Flexible Pavement Optimal Design (FPOD)—was developed on the basis of AASHTO equations and the model. The program interfaces with the optimization package LINDO, which can obtain quick solutions. In terms of the *AASHTO Guide for Design of Pavement Structures* and from a practical point of view, all the layer thicknesses should be rounded to the nearest ½ in. (integer solution). For this reason, the program gives two types of solution (nonintegers and integers). It will be demonstrated next that integer solutions always cost more than noninteger solutions.

NUMERICAL EXAMPLE

Consider a flexible pavement design for which three types of materials are available for each of the three layers, respectively. The default data of DNPS86 program are used except for those of pavement layer characteristics, material properties, and costs. The minimum base and subbase thicknesses are set to 6 in. FPOD printouts are shown in Figures 1 through 3. Figures 1 and 2 list all the input data, and Figure 3 presents the optimal solutions.

As presented in Figure 3, for noninteger solution, FPOD selects Asphalt Concrete Type C, Aggregate Type A G4, and Aggregate Type F for surface, base, and subbase course material. The thicknesses of the layers are 9.11, 6.00, and 17.31 in., respectively. With regard to integer solution, the model selects Aggregate Type G instead of Aggregate Type F for subbase course material as in the case of the noninteger solution. The total costs of the pavement structure for noninteger solution and integer solution are $21.72/yd^2 and $21.80/yd^2, respectively. In this case, the integer solution costs 0.37 percent more than the noninteger solution.

AASHTO Flexible Pavement Optimal Design Program CTR UT Austin
FPOD [v1.0 June 1991]

INPUT DATA REPORT (1)

Input File: TEST11.DAT
Report Date: 07/10/1991 Time: 22:33:36 Page 3-1

Project: Example
Road: XXXX From: XXXX To: XXXX
Start Station: 100.000 End Station: 101.000

Performance Period	15	Yrs
Traffic Growth Rate	2	%
Initial Yearly Two Way 18 kips ESAL	2000000	
Directional Distribution Factor	50	%
Lane Distribution Factor	85	%
DESIGN TRAFFIC (ESAL)	14699404	
Design Reliability	95	%
Standard Deviation	0.49	
PSI after initial construction	4.50	
PSI at end of performance	2.50	

ROADBED SOIL SWELLING

Potential Vertical Rise	1.20	inch
Swelling Probability	84	%
Swell Rate Constant	0.075	

FROST HEAVE

Maximum Potential Serviceability Loss	1.00	
Frost Heave Probability	10	%
Frost Heave Rate	30.00	mm/d

Total PSI Loss Due To Swelling & Frost Heave 0.33

ROADBED SOIL RESILIENT MODULI

No	Moduli (psi)	No	Moduli (psi)	No	Moduli (psi)	No	Moduli (psi)
1	6500	7	5000	13	-	19	-
2	30000	8	5000	14	-	20	-
3	2500	9	5000	15	-	21	-
4	4000	10	5000	16	-	22	-
5	4000	11	6500	17	-	23	-
6	5000	12	6500	18	-	24	-

Effective Resilient Modulus of Roadbed Soil 4542 psi

FIGURE 1 FPOD input data report (1).

AASHTO Flexible Pavement Optimal Design Program CTR UT Austin
FPOD [v1.0 June 1991]

INPUT DATA REPORT (2)

Input File: TEST11.DAT
Report Date: 07/10/1991 Time: 22:33:36 Page 3-2

Project: Example
Road: XXXX From: XXXX To: XXXX
Start Station: 100.000 End Station: 101.000

LAYERS	MATERIAL DESCRIPTION	MODULI (psi)	LAYER COEFF	DRAINAGE COEFF	UNIT COST ($/CY)
Surface	1 ASPH CONC TY A	420000	0.40	1.00	51.61
	2 ASPH CONC TY B	430000	0.42	1.00	52.00
	3 ASPH CONC TY C	450000	0.44	1.00	54.00
Base	1 AGGR(TY A GR4)	30000	0.10	1.20	16.50
	2 AGGR(TY B GR4)	32000	0.11	1.20	19.40
	3 AGGR(TY PB G4)	34000	0.12	1.20	20.00
Subbase	1 AGGR TYPE E	11000	0.08	1.20	9.80
	2 AGGR TYPE F	13000	0.09	1.20	11.00
	3 AGGR TYPE G	14000	0.10	1.20	12.50

FIGURE 2 FPOD input data report (2).

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AASHTO Flexible Pavement Optimal Design Program          CTR UT Austin
FPOD [v1.0 June 1991]

                         OPTIMAL SOLUTION REPORT

 Input File: TEST11.DAT
Report Date: 07/10/1991       Time: 22:33:36                  Page 3-3
=====================================================================
Project: Example
Road: XXXX      From: XXXX      To: XXXX
Start Station:  100.000     End Station:   101.000
---------------------------------------------------------------------
                         1. NONINTEGER SOLUTION

NO          LAYERS      MATERIAL              THICKNESS       UNIT COST
                        DESCRIPTION           (inches)         ($/CY)

1         3 Surface     ASPH CONC TY C           9.11           54.00
2         1 Base        AGGR(TY A GR4)           6.00           16.50
3         2 Subbase     AGGR TYPE F             17.31           11.00

            Total Cost:    21.72 ($/SY)
---------------------------------------------------------------------
                         2. INTEGER SOLUTION

NO          LAYERS      MATERIAL              THICKNESS       UNIT COST
                        DESCRIPTION           (inches)         ($/CY)

1         3 Surface     ASPH CONC TY C           9.00           54.00
2         1 Base        AGGR(TY A GR4)           6.00           16.50
3         3 Subbase     AGGR TYPE G             16.00           12.50

            Total Cost:    21.80 ($/SY)
=====================================================================
```

FIGURE 3 FPOD optimal solution report.

From this example, it can be seen that the layer materials selected from the model may not be the same in the two solutions. Therefore, the ratio of layer coefficient to unit cost mentioned before cannot be used to select materials for integer solution in some cases.

SENSITIVITY ANALYSIS

The differences between FPOD and other nonoptimization programs such as DNPS86 are (a) FPOD can select the best combination of the materials for a problem if more than one type of material is available for each of the three layers, and (b) FPOD takes unit costs of the three layers into account in the process of thickness design. In other words, the optimal solutions to the thicknesses should be sensitive to unit costs of the three layers. The major consideration is that optimal thicknesses change with the changes of unit costs in the model, so we focus on the sensitivity analysis of change of unit costs versus change of optimal thicknesses of the three layers and compare the FPOD solutions with DNPS86 solutions. In the sensitivity analysis, the default data (given in Figure 1) of AASHTO DNPS86 program is used by changing the moduli of elasticity, layer coefficients, and drainage coefficients of paving materials to average ones. Unit costs of paving materials for surface, base, and subbase courses range from $40/yd^3$ to $60/yd^3$, $15/yd^3$ to $25/yd^3$, and $8/yd^3$ to $15/yd^3$, respectively (as shown in Table 1).

Optimal Solutions Versus Unit Costs

Table 2 presents the sensitivity analysis results for the 8-in. minimum base and subbase thickness. In this case, the layer thicknesses of surface, base, and subbase courses obtained from the DNPS86 program are 9.97, 11.41, and 21.75 in., respectively. In Table 2, the first column lists the unit costs of the three layer materials, Column 2 lists the total costs of DNPS86 solutions, and Column 7 and Column 12 list the total cost changes of FPOD noninteger solutions and integer solutions as compared with DNPS86 solutions. Finally, Column 13 lists the total cost increase of integer solutions as compared with noninteger solutions. In Part 1 of Table 2, the unit costs of base and subbase courses are fixed to $20/yd^3$ and $10/yd^3$, respectively; the unit cost of surface course changes from $40/yd^3$ to $60/yd^3$. In Part 2 of Table 2, the unit costs of surface and subbase courses are fixed to $50/yd^3$ and $10/yd^3$, respectively; the unit cost of base course changes from $15/yd^3$ to $25/yd^3$. Finally, in Part 3 of Table 2, the unit costs of surface and base courses are fixed to $50/yd^3$ and $20/yd^3$, respectively; the unit cost of the subbase course changes from $8/yd^3$ to $15/yd^3$.

Table 2 shows that optimal noninteger and integer solutions change two or more times when the unit costs of the materials for any two layers are fixed and the unit cost of the material for another layer changes within the unit cost range specified. The thickness of the surface course decreases and the thicknesses of the base and subbase courses increase with the increase of unit cost of surface course. As a general rule, the degree of change depends on layer coefficients, drainage coefficients, and resilient moduli. The smaller the layer and drain-

TABLE 1 INPUT DATA FOR SENSITIVITY ANALYSIS

Layers	Elastic Modulus (psi)	Layer Coefficients	Drainage Coefficients	Range of Unit Costs ($/CY)
Surface	450000	0.35	1.00	40 - 60
Base	30000	0.12	1.00	15 - 25
Subbase	15000	0.08	1.00	8 - 15

TABLE 2 SENSITIVITY ANALYSIS (BASE AND SUBBASE ≥ 8 in.)

Unit Cost ($/CY)	DNPS86 Solutions	FPOD Noninteger Solutions					FPOD Integer Solutions					
	Total Costs ($/SY)	Layer Thickness (in)			Total Cost ($/SY)	(6)-(2) / (2) %	Layer Thickness (in)			Total Cost ($/SY)	(11)-(2) / (2) %	(11)-(6) / (6) %
		Surface	Base	Subbase			Surface	Base	Subbase			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1. SURFACE COURSE COST CHANGES, BASE COURSE COST 20$/CY, SUBBASE COURSE COST 10$/CY												
40	23.46	14.28	8.00	8.00	22.53	-3.96	14	8	10	22.78	-2.90	+1.11
41	23.74	14.28	8.00	8.00	22.93	-3.41	12	8	18	23.11	-2.70	+0.78
42	24.01	14.28	8.00	8.00	23.33	-2.83	12	8	18	23.44	-2.37	+0.47
43	24.29	14.28	8.00	8.00	23.73	-2.31	12	8	18	23.77	-2.14	+0.16
44	24.57	11.14	8.00	21.75	24.10	-1.91	12	8	18	24.11	-1.87	+0.04
45	24.84	11.14	8.00	21.75	24.41	-1.73	12	8	18	24.44	-1.61	+0.12
46	25.12	11.14	8.00	21.75	24.72	-1.59	12	8	18	24.78	-1.35	+0.24
47	25.40	11.14	8.00	21.75	25.03	-1.46	12	8	18	25.11	-1.14	+0.32
48	25.67	11.14	8.00	21.75	25.34	-1.29	12	8	18	25.44	-0.90	+0.39
49	25.95	11.14	8.00	21.75	25.65	-1.16	12	8	18	25.78	-0.66	+0.47
50	26.23	11.14	8.00	21.75	25.96	-1.03	12	8	18	26.11	-0.46	+0.58
51	26.50	11.14	8.00	21.75	26.27	-0.87	11	9	21	26.42	-0.30	+0.57
52	26.78	11.14	8.00	21.75	26.58	-0.75	11	9	21	26.72	-0.22	+0.53
53	27.06	11.14	8.00	21.75	26.89	-0.63	11	9	21	27.02	-0.15	+0.48
54	27.34	11.14	8.00	21.75	27.19	-0.55	11	9	21	27.33	-0.04	+0.51
55	27.61	11.14	8.00	21.75	27.50	-0.40	11	9	21	27.64	+0.11	+050
56	27.89	11.14	8.00	21.75	27.81	-0.29	11	9	21	27.94	+0.18	+0.47
57	28.17	11.14	8.00	21.75	28.12	-0.18	11	9	21	28.25	+0.28	+0.46
58	28.44	11.14	8.00	21.75	28.43	-0.04	11	9	21	28.56	+0.42	+0.45
59	28.72	9.97	11.41	21.75	28.74	0	11	9	21	28.86	+0.49	+0.42
60	29.00	9.97	11.41	21.75	29.02	0	11	9	21	29.17	+0.59	+0.52
2. BASE COURSE COST CHANGES, SURFACE COURSE COST 50$/CY, SUBBASE COURSE COST 10$/CY												
15	24.64	9.97	20.58	8.00	24.64	0	10	20	9	24.73	+0.37	+0.37
16	24.96	9.97	11.41	21.75	24.96	0	10	12	21	25.06	+0.40	+0.40
17	25.28	9.97	11.41	21.75	25.28	0	11	9	21	25.36	+0.32	+0.32
18	25.59	11.14	8.00	21.75	25.51	-0.27	11	9	21	25.61	+0.12	+0.39
19	25.91	11.14	8.00	21.75	25.74	-0.66	11	9	21	25.86	-0.19	+0.47
20	26.23	11.14	8.00	21.75	25.96	-1.03	12	8	18	26.11	-0.46	+0.58
21	26.54	11.14	8.00	21.75	26.18	-1.36	12	8	18	26.33	-0.79	+0.57
22	26.86	11.14	8.00	21.75	26.40	-1.71	12	8	18	26.56	-1.12	+0.61
23	27.18	11.14	8.00	21.75	26.63	-2.02	12	8	18	26.78	-1.47	+0.56
24	27.50	11.14	8.00	21.75	26.84	-2.40	12	8	18	27.00	-1.82	+0.60
25	27.81	11.14	8.00	21.75	27.07	-2.67	12	8	18	27.22	-2.12	+0.55
3. SUBBASE COURSE COST CHANGES, SURFACE COURSE COST 50$/CY, BASE COURSE COST 20$/CY												
8	25.02	11.14	8.00	21.75	24.75	-1.08	11	9	21	24.95	-0.28	+0.81
9	25.62	11.14	8.00	21.75	25.35	-1.05	11	9	21	25.53	-0.35	+0.70
10	26.23	11.14	8.00	21.75	25.96	-1.03	12	8	18	26.11	-0.46	+0.58
11	26.83	11.14	8.00	21.75	26.58	-0.93	12	8	18	26.62	-0.78	+0.15
12	27.44	14.28	8.00	8.00	26.94	-1.82	12	8	18	27.11	-1.20	+0.63
13	28.04	14.28	8.00	8.00	27.17	-3.10	14	9	8	27.33	-2.53	+0.59
14	28.64	14.28	8.00	8.00	27.39	-4.36	14	9	8	27.56	-3.77	+0.62
15	29.25	14.28	8.00	8.00	27.61	-5.61	14	9	8	27.78	-5.03	+0.61

DNPS86 SOLUTION: Surface Course: 9.97 Inches
Base Course: 11.41 Inches
Subbase Course: 21.75 Inches

age coefficients and the larger the resilient moduli, the more the magnitude of change.

Similarly, the thickness of the base course decreases and the thicknesses of the surface and subbase courses increase with the increase of unit cost of base course.

With the increase of the unit cost of the subbase course, for noninteger solutions, the thickness of the base course remains the same; thicknesses of surface and subbase courses increase and decrease, respectively: for integer solutions, base course thickness decreases from $9/yd^3, and then increases from $13/yd^3; surface and subbase course thicknesses change in the same way they do for noninteger solutions.

Optimal Integer Versus Optimal Noninteger Solutions

Table 2 and Figures 4 through 6 show that integer solutions always cost more than noninteger solutions. In this example, the total costs of an integer solution increase up to 1.3 percent compared with a noninteger solution (Figure 6, minimum base and subbase thickness of 10 in.). As a rule, the thicker the minimum thicknesses of base and subbase courses specified, the larger the difference between the two types of solution.

Optimal Versus Nonoptimal Solutions

Figures 7 through 9 show the percentage reduction of total costs of optimal noninteger solutions given by FPOD compared with nonoptimal solutions given by DNPS86. Within a certain range of unit costs, DNPS86 and FPOD get the same solutions; in other words, DNPS86 can also give optimal solutions sometimes. For example, if the unit cost of the surface course is equal to or above $58/yd^3 (Table 2 and Figure 7), or the unit cost of the base course is between $15/yd^3 and $17/yd^3 (Table 2 and Figure 8), both DNPS86 and FPOD give the same solutions, regardless of the minimum thicknesses of base and subbase courses specified. That means DNPS86 so-

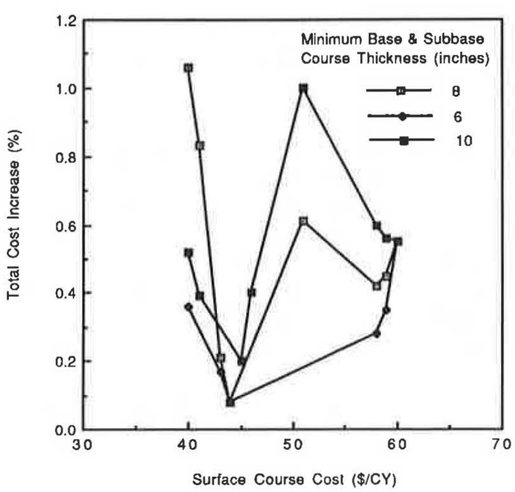

FIGURE 4 Surface course cost versus total cost increase, comparison of integer solutions with noninteger solutions (base course cost, $20/yd^3; subbase course cost, $10/yd^3).

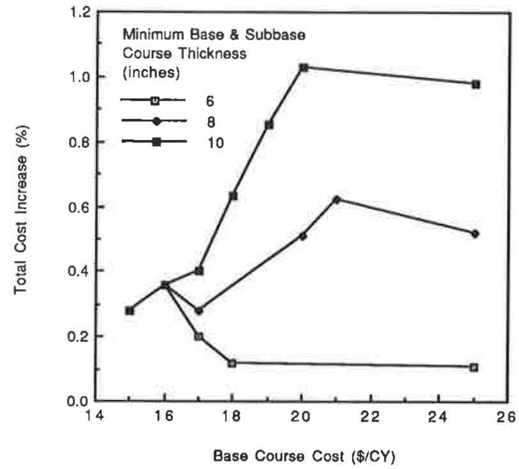

FIGURE 5 Base course cost versus total cost increase, comparison of integer solutions with noninteger solutions (surface course cost, $50/yd^3; subbase course cost, $10/yd^3).

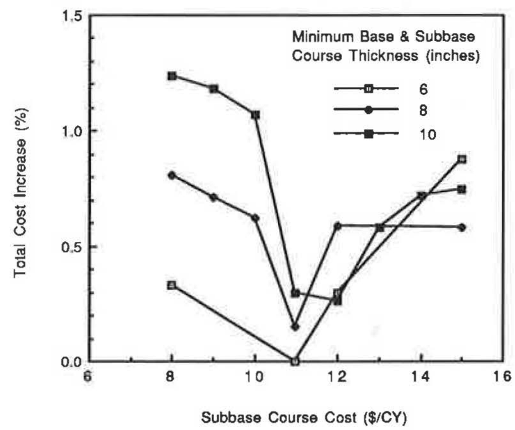

FIGURE 6 Subbase course cost versus total cost increase, comparison of integer solutions with noninteger solutions (surface course cost, $50/yd^3; base course cost, $20/yd^3).

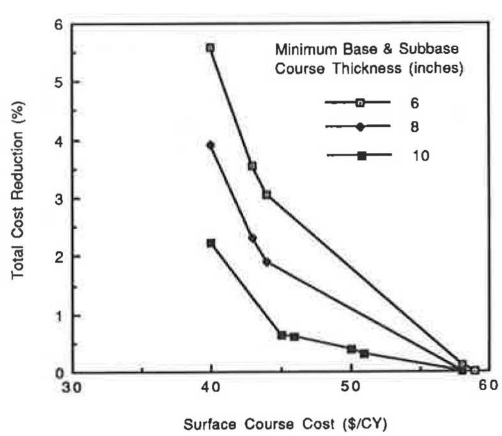

FIGURE 7 Surface course cost versus total cost reduction, comparison of FPOD noninteger solutions with DNPS86 solutions (base course cost, $20/yd^3; subbase course cost, $10/yd^3).

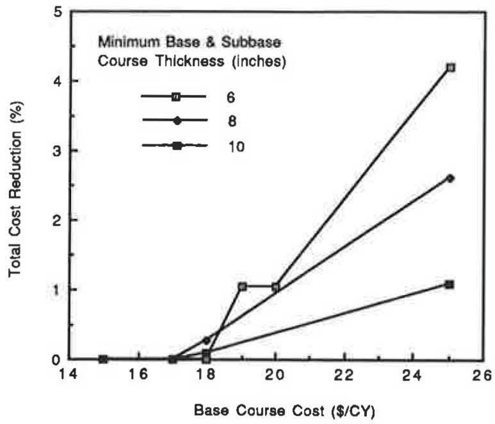

FIGURE 8 Base course cost versus total cost reduction, comparison of FPOD noninteger solutions with DNPS86 solutions (surface course cost, $50/yd³; subbase course cost, $10/yd³).

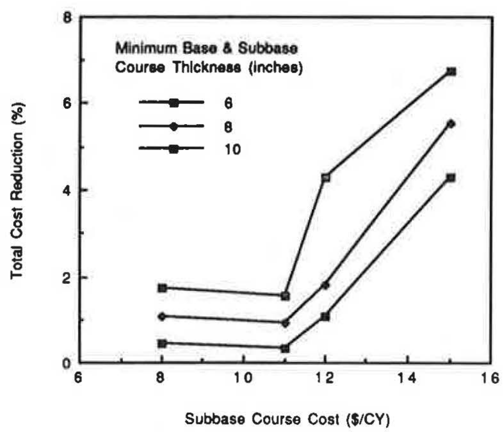

FIGURE 9 Subbase course cost versus total cost reduction, comparison of FPOD noninteger solutions with DNPS86 solutions (surface course cost, $50/yd³; base course cost, $20/yd³).

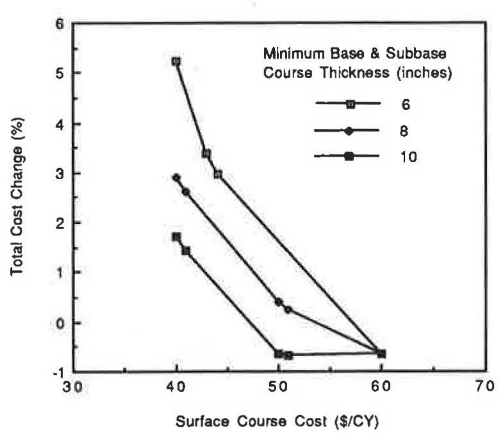

FIGURE 10 Surface course cost versus total cost change, comparison of FPOD integer solutions with DNPS86 solutions (base course cost, $20/yd³; subbase course cost, $10/yd³).

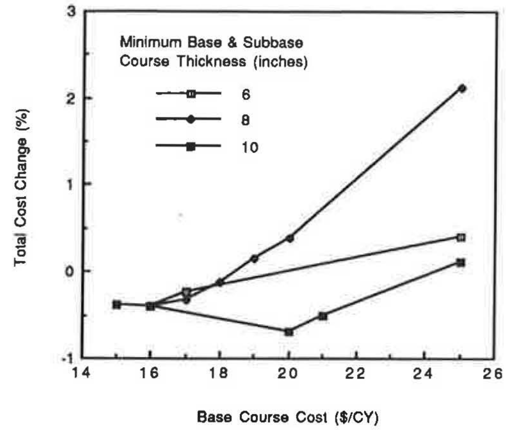

FIGURE 11 Base course cost versus total cost change, comparison of FPOD integer solutions with DNPS86 solutions (surface course cost, $50/yd³; subbase course cost, $10/yd³).

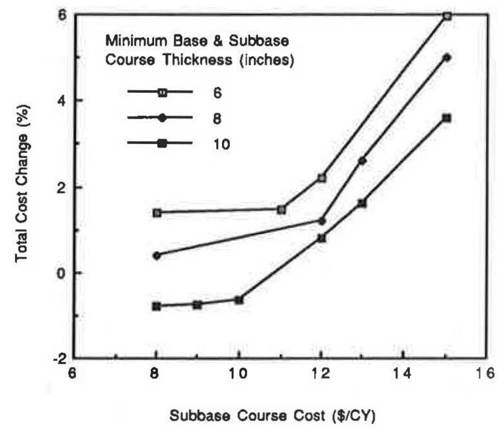

FIGURE 12 Subbase course cost versus total cost change, comparison of FPOD integer solutions with DNPS86 solutions (surface course cost, $50/yd³; base course cost, $20/yd³).

lutions are also optimal for those cases. The savings realized by using FPOD is determined by the minimum thicknesses of the base and subbase courses. Generally speaking, the thinner the base course, the larger the savings will be. Figure 7 shows the reduction of total costs can be as large as 5.8 percent for cases in which the minimum thickness is 6 in. If no base course is allowed, the largest saving can be obtained in some cases.

For noninteger solutions, at least one of the optimal thicknesses of base and subbase is the minimum value specified in most cases, but this may not be true for integer solutions.

Figures 10 through 12 show the percentage change of total costs of optimal integer solutions given by FPOD as compared with nonoptimal solutions given by DNPS86. In some cases, the total costs are more than those of the nonoptimal noninteger solutions.

SUMMARY AND CONCLUSIONS

A mixed-integer programming model was formulated for flexible pavement design problems, and an FPOD computer program was developed accordingly. The FPOD program can give the best combination of various paving materials for all three layers and at the minimum-cost thicknesses. It searches for the minimum-cost solution when the costs of the paving materials change. This capability is desirable when the minimum-cost solution is required (the DNPS86 program gives only one solution, regardless of the costs of the paving materials). FPOD gives two types of solution: noninteger and integer. In terms of the *AASHTO Guide for Design of Pavement Structures* and from a practical point of view, an integer solution should be used, but it costs more than a noninteger solution. The additional cost of using integer solutions depends on the minimum thickness of the base and subbase courses and differs from problem to problem. It is recommended that the decision to select one of the solutions be made in terms of the cost ratio of the two solutions and construction experience.

The minimum-cost solutions for flexible pavements very much depend on the minimum thicknesses of the base and subbase courses set by users. As mentioned in the AASHTO guide, the minimum thickness of all the three layers depends somewhat on local practice and conditions.

The present version of FPOD is used only in the design of new flexible pavement; life-cycle analysis has not been taken into account.

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