Simulation Approach to Prediction of Highway Structure Conditions

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Various techniques, either statistical or stochastic, have been applied to predict highway structure conditions. Researchers found the stochastic approach more appropriate than the statistical approach in highway project selections using dynamic optimization techniques. However, it was also found that condition predictions using the Markov chain could be biased, depending on the values of transition probabilities. In an attempt to minimize the bias in Markov chain predictions, the Monte Carlo simulation technique was applied in the present study in combination with transition probabilities obtained from Markov chain approaches. This study showed that the simulation method could produce more realistic predictions than the analytical Markov chain approach. The Monte Carlo simulation method is described and compared with the analytical Markov chain method. An application example is presented to show the mechanism of the Monte Carlo simulation method and to compare the results of the simulation and Markov chain predictions.

Stochastic processes, such as the Markov chain, have been successfully applied to predict pavement and bridge conditions (*1,2*). Advantages of the stochastic approach over the statistical approach were exhibited in highway project selections using dynamic optimization techniques (*3*). However, as with any other prediction techniques, uncertainty, randomness, and unrealistic assumptions are also involved in the stochastic techniques. It was found in this study that condition predictions using the Markov chain could be biased, depending on the values of transition probabilities. In an attempt to minimize bias in the Markov chain predictions, the Monte Carlo simulation technique was applied in combination with the transition probabilities obtained from Markov chain approaches. The Monte Carlo simulation method generates random numbers and compares these random numbers with transition probabilities of the Markov chain to determine the future condition of highway structures. The present study showed that the simulation method could produce more realistic predictions than the Markov chain approach. The simulation method is described here and is compared with the analytical Markov chain method. Although this prediction technique can be used for estimating conditions of any highway structures, bridge condition predictions are made in this paper for demonstration purpose. The Markov chain prediction model developed earlier for the Indiana Bridge Management System (*2*) is therefore briefly described to introduce the Markov chain transition probabilities and to compare the results of the two approaches.

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MARKOV TRANSITION PROBABILITIES

The Markov chain as applied to bridge performance prediction is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition changing from one state to another (*2*). These probabilities are represented in a matrix form that is called the transition probability matrix, or transition matrix, of the Markov chain. Knowing the present state of bridges, or the initial state, the future conditions can be predicted through multiplications of initial state vector and the transition probability matrix.

According to the FHWA bridge rating system, bridge inspectors rate each inspected bridge with a number between 0 and 9, with 9 being the maximum rating number for the condition of a new bridge (*4*). The condition ratings below 3 need not be included in the Markov chain transition matrixes because the lowest rating number before a bridge is repaired or replaced is generally taken to be 3. Seven bridge condition ratings can be defined as seven states with each condition rating corresponding to one of the states. For example, condition Rating 9 is defined as State 1, Rating 8 as State 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability of condition changing from one state, say i, to another state, j, during a given period of time, which is denoted by $p_{i,j}$

Let the transition probability matrix of the Markov chain be P, given by

$$p = \begin{vmatrix} P_{1,1} & P_{1,2} & \cdot & \cdot & \cdot & P_{1,7} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ P_{7,1} & P_{7,2} & \cdot & \cdot & \cdot & P_{7,7} \end{vmatrix} \quad (1)$$

The state vector for any time T, $Q_{(T)}$, can be obtained by the multiplication of initial state vector $Q_{(0)}$ and the Tth power of the transition probability matrix P:

$$Q_{(T)} = Q_{(0)} * P * P * \ldots * P = Q_{(0)} * P^T \quad (2)$$

where $Q_{(0)}$ and $Q_{(T)}$ are the vector expressions of condition ratings at time 0 and T, respectively, and can be converted to condition rating values (*2*). Because the present condition $[Q_{(0)}]$ is known, the future condition at any given time T can be predicted as long as the transition matrix P is given.

The inspection of bridges includes ratings of individual components, such as deck, superstructure, and substructure, as well as of the overall bridge condition. Unless rehabilitation or repair is applied, bridge structures gradually deteriorate, so that the bridge condition ratings are either unchanged or changed to a lower number during a given time period. That is, a bridge condition rating should decrease or remain the same as the bridge ages. Therefore, the probability $p_{i,j}$ is null for $i > j$, where i and j represent the states in the Markov chain.

Because the rate of deterioration of bridge condition is different at different bridge ages, the transition process of bridge conditions is not homogeneous with respect to bridge age. However, a Markov process requires a presumption of homogeneity (*5*). Therefore, if only one transition matrix were used throughout a bridge's life span, the inaccuracy of condition estimation would occur as a result of nonhomogeneity of the condition transition process. To avoid overestimating or underestimating the bridge condition, an approach called zoning technique (*1*) was used to obtain the transition matrix.

A 1-year transition period was used in developing Markov chain transition matrixes. In other words, $p_{i,j}$ was the transition probability from State i to State j during 1 year. Bridge age was divided into groups, and within each age group the Markov chain was assumed to be homogeneous. A 6-year group was found appropriate for the data base as well as for solving equations of unknown probabilities. A separate transition matrix was developed for each group.

To make the computations simple, an assumption was made that the bridge condition rating would not drop by more than one state in a single year. Thus, the bridge condition would either stay in its current state or fall to the next lower state in 1 year. Therefore, the transition matrix of condition ratings has the following form:

$$P = \begin{vmatrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad (3)$$

where $q(i) = 1 - p(i)$. $p(i)$ is corresponded to $p_{i,i}$ and $q(i)$ to $p_{i,i+1}$ in Equation 1. Therefore, $p(1)$ is the transition probability from Rating 9 (State 1) to Rating 9, and $q(1)$, from Rating 9 to Rating 8, and so on.

Because the lowest rating number before a bridge is repaired or replaced is 3, the corresponding transition probability $p(7)$ equals 1. For each age group the transition probabilities were obtained by minimizing the absolute distance between the average bridge condition rating at a certain age and the predicted bridge condition for the corresponding age generated by the Markov chain (*2*). With the obtained transition matrixes, the future condition can be predicted by using Equation 2.

To show the process of Markov chain prediction, a simple example is presented as follows. Suppose a concrete bridge on an Interstate highway is 35 years old now and has a deck condition rating of 6. It is desired to predict the deck condition in the next year. The transition matrix for the deck of this type of bridges of 31 to 36 years old was obtained (*2*):

$$P = \begin{vmatrix} 0.44 & 0.56 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.50 & 0.50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.40 & 0.60 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.40 & 0.60 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.20 & 0.80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{vmatrix} \quad (4)$$

The deck condition rating is 6, and the initial state vector $Q_{(0)}$ of the bridge deck is [0 0 0 1 0 0 0], where the numbers are the probabilities of the condition ratings being 9, 8, 7, . . . , and 3, respectively. Because it is known that the current condition rating is 6, the number corresponding to Rating 6 in $Q_{(0)}$ is 1, and others are 0. Thus, $Q_{(1)}$ can be predicted using Equation 2:

$$Q_{(1)} = Q_{(0)} \times P$$

$$= [0001000] \begin{vmatrix} 0.44 & 0.56 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.50 & 0.50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.40 & 0.60 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.40 & 0.60 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.20 & 0.80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{vmatrix} \quad (5)$$

or

$$Q_{(1)} = [0\ 0\ 0\ 0.40\ 0.60\ 0\ 0]$$

$Q_{(1)}$ can then be converted to a rating number, r_1, by multiplying a vector of condition ratings R:

$$r_1 = Q_{(1)} \times R$$

$$= [0\ 0\ 0\ 0.40\ 0.60\ 0\ 0] \begin{vmatrix} 9 \\ 8 \\ 7 \\ 6 \\ 5 \\ 4 \\ 3 \end{vmatrix} = 5.4 \approx 5 \quad (6)$$

Therefore, the deck condition rating in the next year is predicted as 5. It should be noted that the predicted value was rounded to its nearest integer number because the rating system uses only integers as rating numbers.

An examination of matrixes $Q_{(0)}$ and P reveals that $Q_{(0)}$ has only one nonzero element, and each row, except the last row, in P has only two nonzero elements. This indicates that the Markov chain prediction of the next year's condition rating r_1 is affected by only the transition probabilities corresponding to the current condition rating r_0, or $p(i)$ and $q(i) = 1 - p(i)$, where i is the condition state of r_0 as defined in Equation 3. Therefore, the previous computations can be simplified as follows:

$$r_1 = r_0 \times p(i) + (r_0 - 1) \times q(i) \quad (7)$$

where i is the condition state corresponding to the given condition rating. Thus, the prediction of the deck condition rating can be made in one step:

$$r_1 = 6 \times 0.4 + 5 \times 0.6 = 5.4 \approx 5. \quad (8)$$

SIMULATION APPROACH

Simulation techniques are widely used by engineers and researchers to analyze the behavior of real systems using computers. The Monte Carlo method (*6*) is one of the most commonly used simulation techniques for engineering modeling. Through the Monte Carlo method, a decision is made by comparing a random number generated by computer to a known probability value of the given problem.

Bridge condition deterioration is a probabilistic process and not a deterministic one. Therefore, the Monte Carlo method is suitable for predicting bridge conditions as long as the probabilities of condition changes are determined. If the deck condition rating in the previous example is predicted by the Monte Carlo method, the following steps would be necessary.

1. Generate a random number from a uniform distribution in the interval [0.0, 1.0] using a computer or any other method (such as a random number table).
2. If the random number ≤ 0.40, the predicted condition rating is 6. If the random number >0.40, the predicted condition rating is 5.

In this simulation, each of the uniform random numbers in the interval [0.0, 1.0] has an equal chance of occurring. Therefore, the probabilities of a random number falling into the interval [0.0, 0.40] and the interval [0.40 1.0] are 0.40 and 0.60, respectively, exactly the same as the given probabilities.

Transition probabilities for different types of bridges at different bridge ages were developed in an earlier study (*2*). The Monte Carlo technique was applied in bridge condition prediction using these probabilities. In this study, a simulation program in FORTRAN 77 was developed on a UNIX computer to predict bridge conditions. Random numbers can be generated by the program using a FORTRAN random number subroutine. Figure 1 shows a flow chart of the simulation prediction model. RNUN is an IMSL (*7*) subroutine which generates a uniformly distributed random number once it is called. This program can be used to predict the future conditions of a number of bridges. It can be modified to predict the conditions of highway structures other than bridges, such as pavements. To do so, a user needs to obtain the appropriate probabilities of condition deterioration of the structure, incorporate these probabilities into the program, and change the appropriate IF-THEN conditions.

COMPARISON OF THE TWO APPROACHES

Both the Monte Carlo simulation and Markov chain analytical methods use transition probabilities to estimate bridge conditions. However, the results of the predictions are generally not the same because of the different mechanisms involved.

A random number generated in the Monte Carlo simulation has an equal chance of falling into any point in the interval [0.0, 1.0]. It is therefore expected that Monte Carlo estimation will closely reflect the given probability value when the number of bridges involved is reasonably large. For example, if there are 100 bridges with deck condition ratings of 6, the Monte Carlo simulation [$p(4) = 0.4$ from Equation 4] would yield a prediction that about 40 bridge decks will remain in

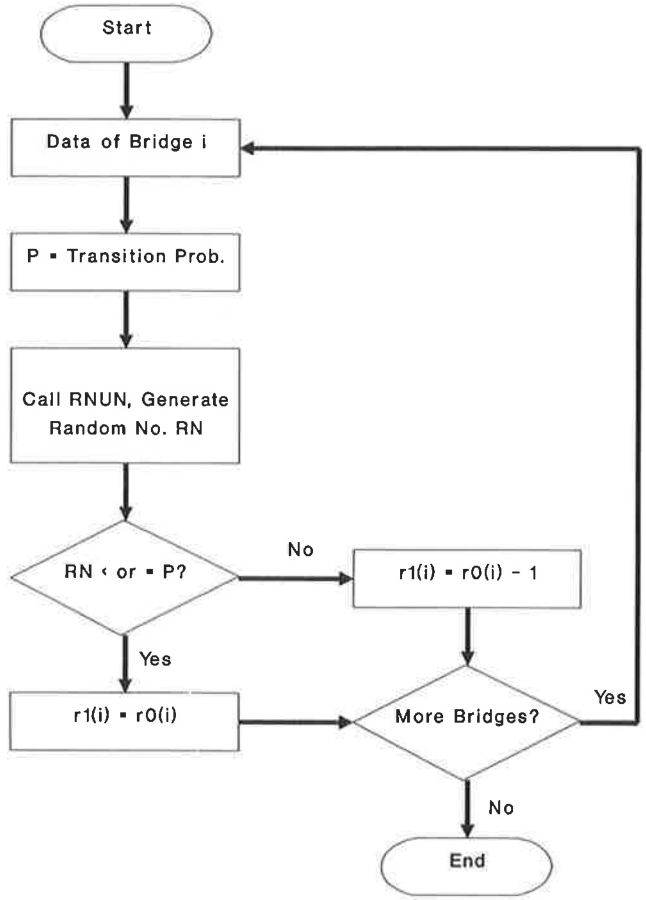

FIGURE 1 Flowchart of simulation prediction program.

Rating 6, and nearly 60 decks will deteriorate to Rating 5 in the next year.

On the other hand, as indicated by Equations 7 and 8, the Markov chain method predicts the deck condition ratings of all the 100 bridges as 5. The transition probability of $p(4) = 0.4$ means that for about 40 out of 100 bridges, or 40 percent, the deck condition rating would remain at 6 after 1 year. However, the future condition ratings of all the 100 bridge (100 percent) decks are predicted at 5 by the Markov chain method (Equation 8). The Markov chain method would, in this case, lead to an overestimation of bridge needs. As a result, the estimated budget and other resources needed for the coming year would be higher than what might be needed. Depending on the value of a transition probability, the Markov chain method can also underestimate the number of bridges that would deteriorate to a lower condition rating. This can be shown by writing Equation 7 as follows because $q(i) = 1 - p(i)$:

$$r_1 = r_0 + p(i) - 1 \qquad (9)$$

Therefore, if $p(i) \geq 0.5$, r_1 is rounded to r_0, and if $p(i) < 0.5$, r_1 is rounded to $r_0 - 1$. In the former case [$p(i) \geq 0.5$], the number of bridges that would deteriorate to deck rating $r_0 - 1$ will be underestimated by the Markov chain method. In the latter case, the number will be overestimated.

To demonstrate the differences between the two prediction methods, 30 bridges were selected from the Indiana bridge inventory file for estimation of deck conditions and repair costs if rehabilitation was found to be necessary. Table 1 presents the condition and cost information of these bridges and the prediction results of the Monte Carlo simulation and Markov chain methods. All the bridges had deck condition ratings of 6 and were 31 to 36 years old. The corresponding transition probabilities for these bridge decks were $p(4) = 0.4$ and $q(4) = 1 - p(4) = 0.6$. If a deck condition rating was equal to or less than 5, the bridge deck was considered a candidate for rehabilitation. To schedule the bridge rehabilitation activities for the next year, it was therefore necessary to estimate the number of bridge decks that would have a rating value of 5 the following year. The predictions of the condition ratings and the associated rehabilitation costs for the next year, made by both simulation and Markov chain methods, are also included in Table 1.

Using the Monte Carlo method, the deck condition of each bridge was predicted by generating a random number and comparing it with the transition probability 0.4. If the random number was less than or equal to 0.4, the predicted deck rating was 6; otherwise, the rating was 5. For the Markov chain method, Equation 7 was used to predict the future deck condition ratings. Because the decks with ratings of 6 would not be rehabilitated, their corresponding repair costs were estimated as $0 for the next year. However, if a deck rating was predicted to fall to 5, its estimated rehabilitation cost was included in the next year's total rehabilitation cost.

As shown in Table 1, the simulation method predicted that the rating of 13 bridge decks would remain at 6, and the ratings of 17 bridge decks would deteriorate 5 after one year. The predicted percentages of bridge decks remaining at Rating 6 and dropping to Rating 5 were 43 percent and 57 percent,

TABLE 1 Results of Simulation and Markov Chain Predictions

Bridge No.	Current Deck Rating (r_0)	Rehab. Cost ($\$10^3$)	Simulation Prediction: Random No.	Simulation Prediction: r_1	Simulation Prediction: Cost ($\$10^3$)	Markov Prediction: r_1	Markov Prediction: Cost ($\$10^3$)
1	6	235	0.2682	6	0	5	235
2	6	276	0.4435	5	276	5	276
3	6	387	0.9589	5	387	5	387
4	6	281	0.0986	6	0	5	281
5	6	107	0.5558	5	107	5	107
6	6	121	0.2997	6	0	5	121
7	6	210	0.1469	6	0	5	210
8	6	400	0.9883	5	400	5	400
9	6	257	0.6276	5	257	5	257
10	6	330	0.4300	5	330	5	330
11	6	270	0.2014	6	0	5	270
12	6	201	0.9986	5	201	5	201
13	6	205	0.0605	6	0	5	205
14	6	476	0.0528	6	0	5	476
15	6	102	0.1994	6	0	5	102
16	6	154	0.8356	5	154	5	154
17	6	621	0.1956	6	0	5	621
18	6	176	0.6856	5	176	5	176
19	6	124	0.1284	6	0	5	124
20	6	159	0.2720	6	0	5	159
21	6	247	0.1352	6	0	5	247
22	6	169	0.8433	5	169	5	169
23	6	93	0.4900	5	93	5	93
24	6	800	0.7173	5	800	5	800
25	6	500	0.6396	5	500	5	500
26	6	635	0.2340	6	0	5	635
27	6	545	0.8986	5	545	5	545
28	6	385	0.8000	5	385	5	385
29	6	193	0.9178	5	193	5	193
30	6	288	0.4251	5	288	5	288
Total		8,947			5,261		8,947

respectively. They were close to the given transition probabilities of 0.4 (or 40 percent) and 0.6 (or 60 percent). However, results from the Markov chain method predicted that the ratings of all 30 bridge decks (or 100 percent) would decrease from 6 to 5 in the next year (Equation 8).

The total cost of repairing all 30 bridges was $8,947,000. The total expected cost for the next year can be computed using the transition probabilities (0.4 and 0.6) and estimated costs ($0 for $r_1 = 6$; c_i for $r_1 = 5$) of individual bridges:

$$\text{Total expected cost} = \sum_{i+1}^{30} (0.4 \times \$0 + 0.6 \times C_i)$$
$$= \$5{,}368{,}200 \quad (10)$$

where c_i is the estimated rehabilitation cost of Bridge *i*. Compared with the total expected cost ($5,368,200), the total cost predicted by the simulation method ($5,261,000) is apparently a more reasonable estimation than the total cost predicted by the Markov chain method ($8,947,000).

The results of the predictions indicate that the simulation approach has advantages over the Markov chain approach in estimating the rehabilitation costs as well as the number of bridges to be repaired. Because a Monte Carlo simulation prediction is based on generated random numbers, the result varies with each different operation of the computer program. However, it is also generally true that for each of the runs of the computer program the percentage of bridges selected for rehabilitation will be close to the given transition probability, and the total cost predicted will also be close to the total expected cost. This is because the chances that the uniformly distributed random numbers fall into any subinterval of [0.0, 1.0] are proportional to the length of the subinterval.

To compare the results of different operations of the simulation program, the program was run 20 times. The results of the 20 predictions are shown in Figures 2 and 3. Figure 2

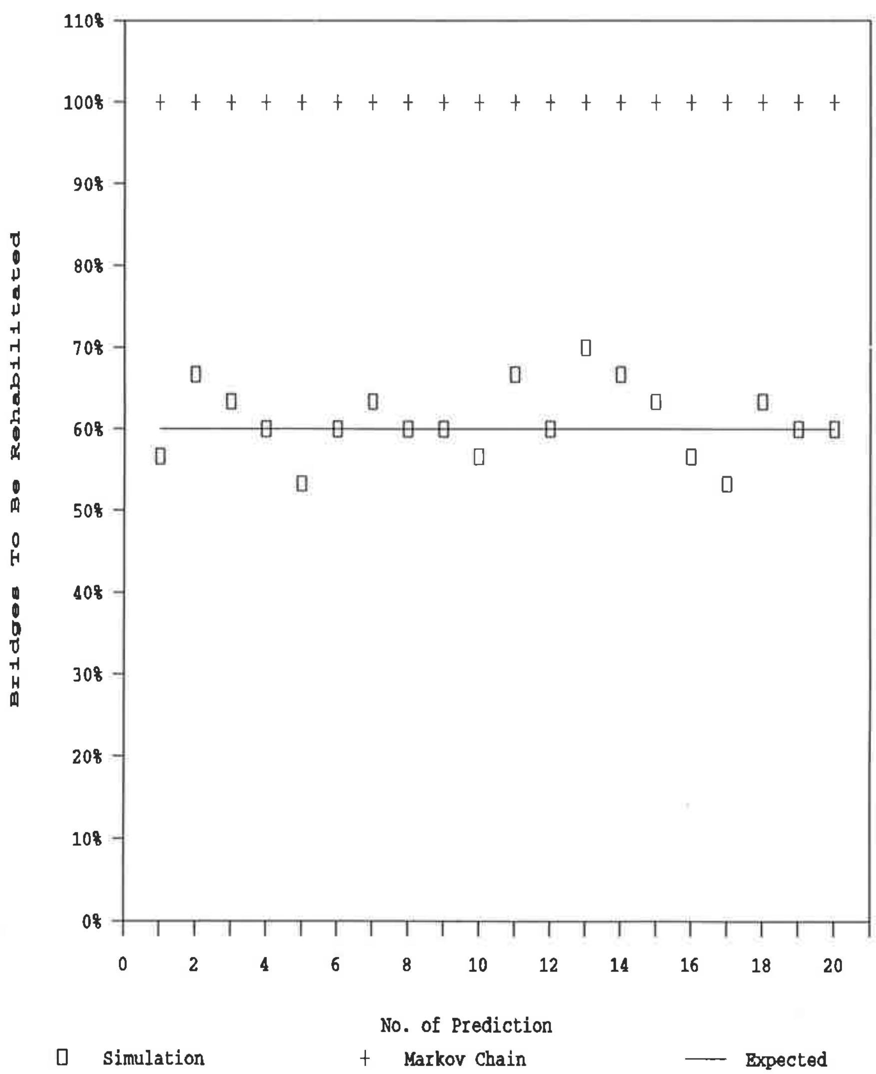

FIGURE 2 Predictions of bridges to be rehabilitated by simulation and Markov chain methods.

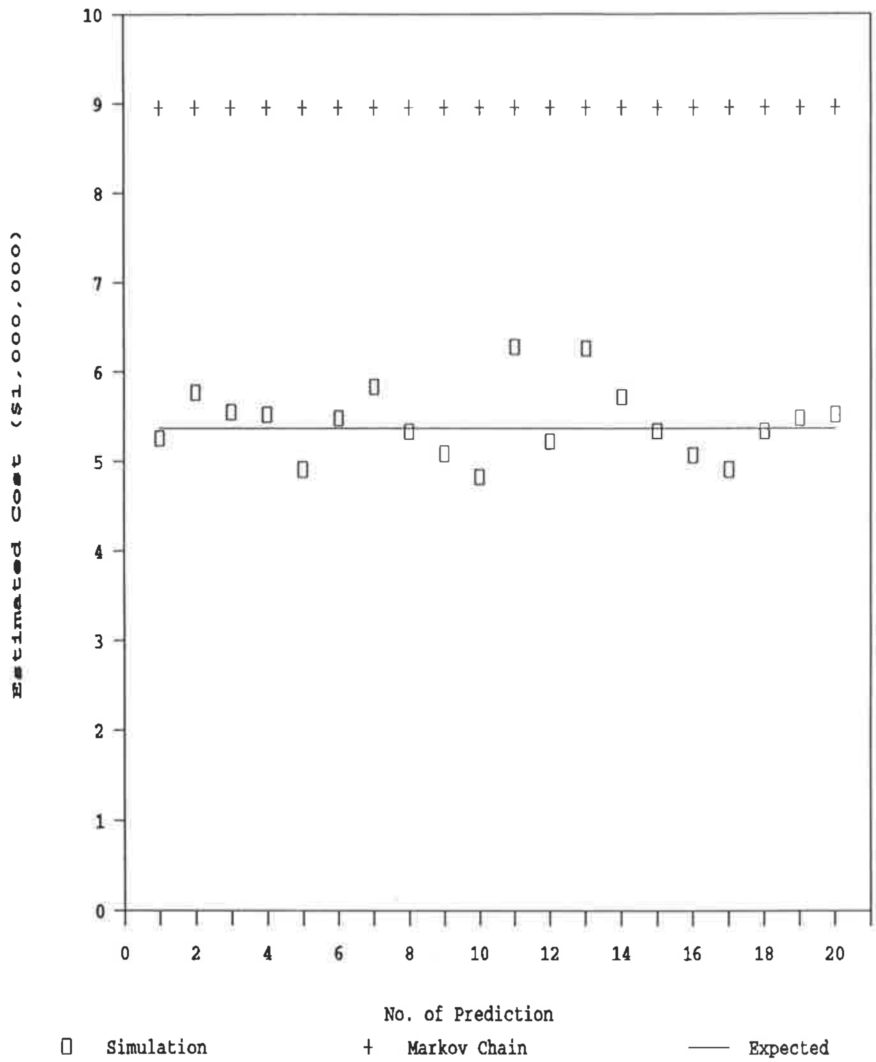

FIGURE 3 Predictions of rehabilitation costs by simulation and Markov chain methods.

expresses the predicted number of bridges to be rehabilitated (the ones with deck Rating 5 in the next year) as the percentage of the total bridges. The total estimated rehabilitation cost for these predictions is plotted in Figure 3. The results of the Markov chain predictions and the expected percentage (60 percent) and average cost ($5,368,200) are also included in the figures for comparison. The two figures illustrate that the simulation predictions of both the percentages and the total rehabilitation costs were in the close neighborhood of the expected values, whereas the Markov chain predictions were consistently higher than the expected values.

The previous example showed that the simulation predictions were reasonably close to the given transition probabilities. The simulation predictions will reflect transition probabilities more closely if a large number of highway structures is involved. For pavement or bridge management, the number of projects is usually sufficiently large. Therefore, the simulation method will be an appropriate approach for predicting facility conditions of these management systems. This method would be especially useful in updating conditions of highway structures if dynamic optimization techniques are applied for project selections (*3*).

CONCLUSION

A highway structure condition prediction method using the Monte Carlo simulation technique was presented here. This method is suitable for pavement and bridge management. Results of the study showed that the Monte Carlo simulation method could provide more accurate predictions than the Markov chain method. The simulation prediction model can be incorporated into a dynamic optimization program to update structural conditions at each stage of the optimization computation. It can also be used separately to program rehabilitation activities of highway structures. The simulation predictions of number of projects, budget, and other resources needed for a given program period would be close to reality as long as the transition probabilities are reasonable.

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