

Probabilistic Approach to Local Bridge Pier Scour

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A method is presented for evaluating the risk of failure of bridge structures due to pier scour during flood events. The method takes into account the hydrodynamics of the flow field in the vicinity of the pier. A newly developed entropy-based velocity distribution is used to define the effective depth of the scour-producing activity in the vortex near the pier. This velocity distribution is integrated to obtain an upper bound on the energy in the vortex, which is then used to estimate the local pier scour. The scour-estimation procedure is used with a flood probability distribution to assign an exceedance probability to estimated scour values. This scour probability distribution can be used along with information about the pier parameters to assign a risk of failure corresponding to any discharge value. The procedure allows for the development of a system of risk assessment priorities to aid in scheduling bridge monitoring and repair.

The undermining of bridge piers by scour resulting from relatively infrequent flood flows has come to be recognized as a major threat to highway safety. In fact, FHWA has urged state highway departments to evaluate the scour potential of their primary bridges. As a part of this procedure, priorities are being set for inspection of sites that demonstrate the most serious potential scour problems. In this paper, a probabilistic method of evaluating bridge sites for possible scour problems is derived and demonstrated using recorded scour data. Using this approach, priority setting for bridge inventories can be based on risk assessments that will identify bridges with a high risk of scour potential.

Research on scour at bridge piers has been proceeding in the United States and elsewhere for more than 50 years. The mechanics by which scour occurs at bridge piers is fairly well understood. However, past research has concentrated on the development of relationships between observed scour (usually in flume experiments) and flow, sediment, and pier variables by regression analyses. Studies of this type abound (1-4). This research in general has been summarized by Melville (5).

Another technique of scour analysis would be to model the complex hydrodynamic flow system near a bridge pier. Attempts to do so are under way, but successful results have not yet been reported. The technique developed in this study is a combination of the classic regression method and the model approach. It is known that pier scour results from a complex flow pattern in the vicinity of the pier known as the horseshoe vortex. However, in past regression studies, little attention has been given to the dynamics of this system as approach flow conditions are usually employed. In this study,

hydraulic variables are selected after consideration is given to the importance of the horseshoe vortex in scour development.

In this approach, the energy in the part of the flow that is physically causing the scour is determined. A probabilistic approach is used to estimate the velocity profile in the scour-producing flow field. This profile is integrated to determine the energy in the flow region, and this energy is used to estimate the resulting scour depth.

The purpose of this project is to produce a methodology for a stochastic risk analysis of scour. The goal is to assign a probability of pier scour corresponding to any river stage observed at a bridge structure. This probability value is then used in conjunction with a flood frequency distribution (probability of the occurrence of different stages) to assign an ultimate risk of bridge pier scour with corresponding confidence limits.

HYDRAULIC CONSIDERATIONS

The hydrodynamic picture of the flow system around bridge piers is very complex. The flow around a single pier is shown schematically in Figure 1. The obstruction of the pier in the flow field causes water to back up on the upstream side of the pier. This small surge is called the bow wave. The pressure differential between the bow wave and the shallower downstream flow causes a strong vertical component in the velocity vector to develop upstream.

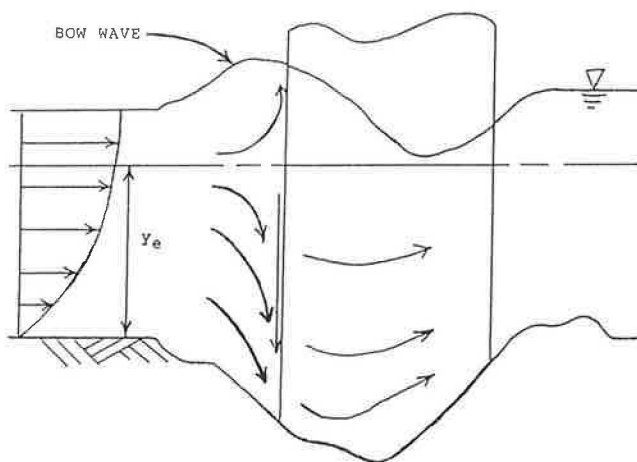


FIGURE 1 Hydrodynamics of flow in vortex.

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Concurrently, an acceleration of the average velocity is caused by the pier constriction and a momentum transfer between the water and the pier surface and channel boundary. The momentum transfer with the pier surface also contributes to the development of the vertical velocity profile. Of course, there is also an energy loss due to friction and boundary drag.

Studies (6) have shown that the vertical velocity profile does not extend all the way to the surface but begins at some point below. The region of strong turbulence and vertical velocities around the pier is called the horseshoe vortex. The activity within the horseshoe vortex is the immediate cause of the local scour around the pier.

In this study, a different approach is taken to relate the activity in the vortex to the pier scour. First, a very careful and accurate horizontal velocity profile is derived for the region in the vicinity of the pier, particularly with respect to near bed velocities. Next, the development of vertical velocities is used as an indicator of the energy in the vortex. This zone will define the effective depth of the vortex. The energy in this region is then found by integrating the velocity distribution, and the scour is determined from the energy of the vortex.

The ultimate goal of this analysis is to determine the risk of bridge failure due to pier scour during flood events. To do this, the following steps are necessary: (a) develop a flood probability model; (b) develop an accurate velocity profile in the vicinity of the pier; (c) determine the effective depth of the vortex; (d) determine the energy contained within this depth; (e) determine the relationship between this energy and the local maximum scour; and (f) verify and calibrate field data.

FLOOD PROBABILITY DISTRIBUTION

Work on the development of flood frequency procedures has recently been focused in the area of regional frequency estimation. One of the most popular of these procedures is the index flood method developed by Dalrymple (7). In this procedure, an assumed probability distribution is fitted to the observed flood records at each location within a homogeneous region. One standardizes the statistics of the at-site distributions by dividing by the local mean in each case. Regional estimates of the statistics are obtained by averaging the standardized local statistics. The regional distributional parameters are obtained by relating the regional standardized statistics to the distributional parameters. These regional parameters are then used to generate flood magnitudes for the site of interest and are subsequently readjusted to account for differences in scale between watersheds.

This procedure has been developed using the Extreme Value Type 1 (EV1) as the base distribution by Greis and Wood (8); it has also been successfully used with the Generalized Extreme Value (GEV) base distribution by Naghavi et al. (9). In a study of currently used regional frequency techniques, Potter and Lettenmair (10) found the GEV-based procedure to be superior in terms of predictive robustness to any other method tested.

The index method was extended to ungauged watersheds by Naghavi et al. (9), who developed relationships between the mean flood in each homogeneous region and the geo-

physical watershed characteristics within the region. When this technique is used, discharges corresponding to any desired exceedance probability can be estimated for any site at which scour estimates are needed.

DEVELOPMENT OF HORIZONTAL VELOCITY PROFILE

A horizontal velocity profile has been developed by Barbé et al. (11), who extended a method previously proposed by Chiu (12-14). This method is based on the principle of maximum entropy (15,16). Entropy can be used as a measure of the information content imbued in a system.

The Shannon entropy functional (SEF) was the first mathematical representation of entropy for use in information theory (17,18). Consider a probability density function $f(x)$ for a continuous random variable x . Then

$$\int_0^{\infty} f(x) dx = 1 \quad (1)$$

and $f(x)$ is positive for all values of x .

SEF is defined as

$$I(f) = - \int_0^{\infty} f(x) \ln[f(x)] dx \quad (2)$$

$I(f)$ may be thought of as the expected value or mean of $-\ln[f(x)]$.

The entropy can be maximized by minimizing the a priori assumptions about the system subject to broad physical constraints. For bridge pier scour, the constraints are the laws of conservation of mass, momentum, and energy that must be satisfied by any physical system.

From boundary shear considerations, the classical method of describing the velocity profile is by relating it to the depth. In open channel flow with depth D , the velocity monotonically increases from zero at the bed, because of maximum boundary shear at the bed, to a maximum value at the surface, because of minimum boundary shear when the water-air interface is neglected. Let u be the velocity at a distance y above the channel bed. Then, the probability of the velocity being less than or equal to u is y/D ; the cumulative distribution function is

$$F(u) = \frac{y}{D} \quad (3)$$

and the probability density function is

$$f(u) = \frac{1}{D} \frac{dy}{du} \quad (4)$$

By writing the physical constraints in their most general integral form, Barbé (19) was able to derive and estimate the solution for velocity profiles corresponding to all of the constraints listed above as well as any combination of them. Let u_D be the maximum velocity that occurs at the surface, and

the constraints used by Barbé et al. (11) become

Constraint 1 (from conservation of mass):

$$\int_0^{u_D} uf(u) du = u_m \quad (5)$$

where u_m is the mean velocity (depth averaged).

Constraint 2 (from conservation of momentum):

$$\int_0^{u_D} u^2 f(u) du = \frac{M}{\rho D} \quad (6)$$

where

- M = momentum flux per unit width of channel,
- ρ = mass density of water, and
- D = depth of flow in the channel.

Constraint 3 (from conservation of energy):

$$\int_0^{u_D} u^3 f(u) du = (E - D) 2gu_m \quad (7)$$

where E is the energy flux per unit width of channel and g is the acceleration due to gravity.

Applying the principle of maximum entropy, the probability density function derived is given as

$$f(u) = \exp\{L_1 - 1 + L_2 u + L_3 u^2 + L_4 u^3\} \quad (8)$$

where $L_1, L_2, L_3,$ and L_4 are parameters of the distribution.

Equation 8 represents the entropy-derived velocity distribution based on the conservation of mass, momentum, and energy. In the vicinity of boundaries, such as channel beds or pier surfaces, it may be difficult to account for energy losses

accurately. For this reason, the energy conservation constraint is removed by setting L_4 equal to zero. Then the velocity profiles are obtained by substituting Equation 8 (with $L_4 = 0$) into Equation 4 and obtaining approximate solutions to the resulting integral equations (11). The derived profiles were compared to other standard velocity profiles as well as to observed data published by Davoren (6) (Figures 2 through 4). These data represent the only currently available observations of velocity and scour measurements taken concurrently in the field during high discharge events. The new three-parameter entropy method using constraints of continuity and momentum results in an excellent description of the entire velocity profile. In particular, note the accurate fit near the channel bed, which validates the selection of constraints. This method was used in this study to obtain the energy in the horseshoe vortex.

The velocity profile used is given by

$$\begin{aligned} \exp(A) \left\{ \exp(L_2 u) + L_3 \left[\exp(L_2 u) \left(u^2 - \frac{2u}{L_2} + \frac{2}{L_2^2} \right) \right] \right\} \\ = \frac{y}{D} + \exp(A) \left\{ \frac{1}{L_2} + \frac{2L_3}{L_2^3} \right\} \quad (9) \end{aligned}$$

Equation 9 represents a three-parameter velocity profile whose parameters ($A, L_2,$ and L_3) are determined through an approximation technique using only observed mean and maximum (surface) velocities (11). In other words, using the principle of maximum entropy, the information content from the conservation of momentum can be added without increasing the number of parameters. The needed parameters remain the obtainable mean and maximum (surface) velocities.

To summarize: the concept of entropy was used to derive a velocity profile subject to the physical laws of conservation of mass and momentum. Thus, no assumptions are necessary

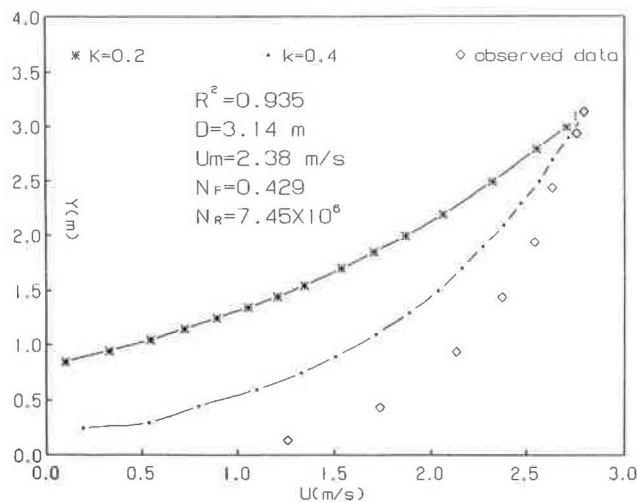


FIGURE 2 Prantl-Von Karman velocity distribution.

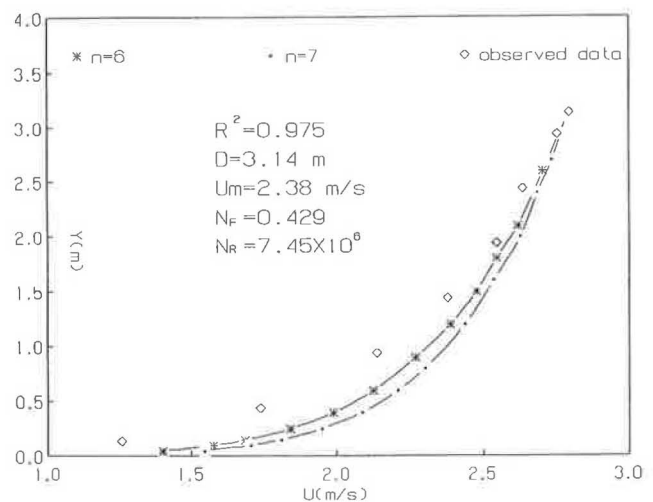


FIGURE 3 Power law velocity distribution.

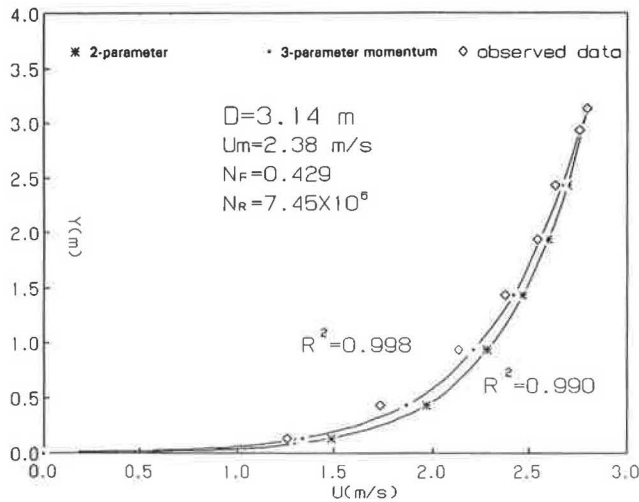


FIGURE 4 Entropy-based velocity distribution.

in deriving this profile. In the event that constraints can be determined representing the conservation of momentum or energy at the pier, then velocity profiles at the pier can be directly determined by this method. In addition, Figures 2 through 4 show that the entropy method resulted in superior fits to observed data, particularly near the bed.

DETERMINATION OF EFFECTIVE DEPTH OF HORSESHOE VORTEX

The effective depth is defined as that part of the approach flow that is deflected downward—that is, the part of the flow that is effective in causing scour (see Figure 1). Barbé (19) defined the upper limit of the effective depth as the point at which the vertical velocity component amounted to 3 percent of the horizontal component. However, this definition was based on observations of vertical velocities contained in Davoren's data (6).

Alternatively, the point at which vertical velocities become significant can be determined as the point at which the horizontal velocity components diverge significantly from those projected from the entropy distribution if no pier were present. This distribution has been derived by Chiu (12) and is currently available. A comprehensive study of the vertical dimensions of the horseshoe vortex was reported by Kothiyari et al. (20). On the basis of data reported by many previous investigators, a relationship was developed between the effective depth of the vortex and approach flow depth and pier diameter. Their results are published (21) in both mathematical and graphical forms and provide a basis for determining the effective depth when observed data are not available.

In this case, based on Davoren's (6) observations, Barbé (19) developed relationships between the effective depth and the total approach flow depth. The resulting equations produced for the effective depth were

$$y_e = D - 0.6945D^2 + 0.2633D^3 \quad (0 \leq D \leq 0.7 \text{ m}) \quad (10)$$

and

$$y_e = 0.45 + 0.4147(D - 0.7) - 0.1416(D - 0.7)^2 + 0.0193(D - 0.7)^3 \quad (0.7 \text{ m} \leq D \leq 3.14 \text{ m}) \quad (11)$$

where y_e = the effective depth of horizontal flow (in meters) and D = the total approach flow depth (in meters). When compared with the results of Kothiyari et al. (20), these values appeared to fall in the range of effective depths predicted by their equations.

DETERMINATION OF SCOUR RELATION

Many conventional scour formulas relate conditions in the upstream approach flow to the scour at the bridge pier. The energy in the approach flow field has been shown to be well related to field scour observations (22). However, most of these studies used the total depth and velocity head in the upstream section. This formulation neglects energy losses and momentum transfers within the effective depth region in the vicinity of the pier. These losses, as well as the energy lost in the formation of the bow wave, do not translate into the production of scour. A better relation could be expected to exist if only the energy within the effective depth of the vortex were used. This approach was taken in this study: the effective depth was defined as described previously, and the kinetic energy of the vortex turbulence field was derived by integrating the velocity distribution derived from maximum entropy concepts. This energy was then added to the potential energy in the vortex (the effective depth) to obtain an expression of the form

$$d_s = K_1 Y_e + K_2 \frac{\left(\int_0^{y_e} u^3 dy \right)}{\left(2g \int_0^{y_e} u dy \right)} \quad (12)$$

where

- d_s = maximum scour depth,
- K_1, K_2 = empirical coefficients,
- Y_e = effective depth of downward vertical velocity formation, and
- u = velocity in vortex obtained from entropy expression.

Equation 12 is cast in a form commonly encountered among scour formulas. The energy in the effective depth represents an upper bound on energy available to produce scour. The major differences between Equation 12 and conventional energy-based equations are that only the energy within the effective depth is used and the velocity head is based on a velocity profile method that requires no a priori assumptions. The profile can be obtained from observations of only mean and surface velocities and has been shown to be particularly accurate near the channel bed. Also, because the velocity head is based on conservation of mass and momentum, no inconsistencies are introduced into the analysis by neglecting

energy losses in the derivation of the velocity profile. The momentum transfers are accounted for in the effective depth term and the coefficients. The coefficients K_1 and K_2 represent energy loss coefficients due to boundary drag and momentum transfers as horizontal velocities are converted to vertical velocities. The coefficients and the effective depth will be functions of hydraulic and sediment characteristics and bridge geometry. In this way, parameters such as pier width and sediment size will enter the analysis. It is anticipated that flume studies can be used to determine values for these coefficients corresponding to various scenarios. Once the equation has been calibrated to these scenarios, it should function as an effective general scour formula. Equation 12 represents a consistent attempt to relate actual scour-producing energy in the vortex to the resultant scour, based on a minimum of assumptions.

EVALUATION OF NEW SCOUR-ESTIMATION METHOD

A field study of bridge pier scour was performed by Davoren (6), who measured vertical and horizontal velocities at the pier as well as real-time scour. The study site consisted of a single 1.5-m-diameter hollow steel cylindrical pier established in the Ohau River downstream of a hydroelectric power station. The study reach had bed sediment with $D_{50} = 20$ mm and a standard deviation of $\sigma_g = 5.3$. Observations were conducted during sustained periods of steady releases from the plant. These conditions represent clear water scour observations.

For demonstration purposes, Equation 12 was evaluated using Davoren's data base (6). The constants K_1 and K_2 in the scour equation were determined by a two-parameter least-squares method to fit the data. The coefficients computed were $K_1 = 1.793$ and $K_2 = -2.514$; therefore, the new scour equation for Davoren's site is

$$d_s = 1.793 Y_e - 2.514 \frac{\left(\int_0^{y_e} u^3 dy \right)}{\left(2g \int_0^{y_e} u dy \right)} \quad r^2 = .924 \quad (13)$$

These predicted values were compared with Davoren's published observed scour data (6). The results are given in Table 1. Runs 1, 3, and 4 were used to calibrate the coefficients K_1 and K_2 , and the scour corresponding to Runs 2, 5, and 6 was computed. The table shows that the predicted values compare very well with the observations.

SCOUR RISK ANALYSIS

In applying the procedure, a flood probability distribution is obtained either from observed data or using a regional approach as described previously. A water surface profile method can be used to estimate mean and maximum velocities for any flood magnitude of interest. The entropy method is then used to obtain the velocity profiles. The velocity distribution and the estimated effective depth values are used to obtain

TABLE 1 COMPUTED VERSUS OBSERVED SCOUR

Run	Computed Scour (m)	Observed Scour (m)
1	1.16	1.28
2	1.28	1.20
3	1.01	0.88
4	0.57	0.72
5	0.61	0.59
6	0.25	0.26

the maximum scour-producing energy within the effective depth region. This energy is used to estimate scour depth corresponding to each flood magnitude.

EXAMPLE APPLICATION

For example, Davoren (6) placed probability estimates on the discharges corresponding to his Runs 1 through 6 using a Gumbel (EV1) distribution based on 55 years of record. These values and the estimated scour values corresponding to each flood magnitude are given in Table 2.

From this table, the scour estimate corresponding to any exceedance probability up to .01 (the 100-year event) can be interpolated. Using this information in conjunction with data about the pier parameters, this bridge could be ranked relative to other bridges for its scheduled monitoring and maintenance.

TABLE 2 PROBABILISTIC ANALYSIS OF DISCHARGES AND SCOUR

Run	Estimated Scour (m)	Discharge (m^3/s)	Exceedance Probability
1	1.16	480	.0303
2	1.28	600	.01
3	1.01	360	.145
4	.57	280	.364
5	.61	180	.833
6	.26	300	.294

CONCLUSIONS

A procedure is presented to aid in the evaluation of scour potential for existing and proposed bridge structures. For existing bridges, hydraulic variables and pier geometry can be used to determine the effective depth of scour-producing activity and the velocity profile in this region. From this information, the energy available to produce scour can be determined. Energy loss coefficients need to be evaluated from either observed data or the results of flume experiments.

For proposed bridge sections, the anticipated hydraulic variables can be determined from backwater models such as WSPRO, and the effective depth can be obtained from the results of Kothyari et al. (20,21). It might also help to place experimental piers at the proposed bridge site to collect data to calibrate the equations and determine loss coefficients. In any case, flume studies should be performed to calibrate the proposed procedure to a variety of pier geometries and sediment characteristics. In this way, the procedures can be made general enough to be applicable to any real-world situation in which either existing or proposed bridges must be evaluated for scour vulnerability.

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