Structure-Fracture Toughness Relationships of Asphalt Concrete Mixtures

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A methodology to characterize the resistance of asphalt concrete mixtures to fatigue crack propagation has been developed to construct fundamental structure-fracture toughness relationships. Experimental techniques to evaluate parameters controlling the crack propagation process are presented. These parameters include crack driving force and the change in work done on the crack tip damage (active zone). A linear relationship between the width of the active zone and the crack length is assumed in order to quantify the amount of damage at each crack length. A constitutive equation is used to extract the specific energy of damage (γ'), a material parameter characteristic of the asphalt concrete mixture's resistance to crack propagation, and a dissipative coefficient (β'). The capability of this approach to discriminate the subtle effect introduced by different chemical structures of asphalts on γ' is demonstrated on Elvax-modified AC-5 and AC-20 concrete mixtures. The validity of the current approach to describe the fatigue behavior of asphalt concrete mixtures over the entire range of energy release rate is verified. The practical utilization of this methodology is evident. It forms a basis on which structure- or processing condition–fracture toughness relationships of asphalt concrete mixtures can be constructed. Such relationships can guide the development of asphalt concrete mixtures with superior crack resistance and aid in the design of primary and secondary pavements.

The limitation of fatigue cracking in paving mixtures and flexible pavement systems is a major design criterion in various transportation facilities and military airfields. The repetitive nature of traffic loading on pavements in service has led to laboratory investigations of their performance under cyclic loading. These investigations have been carried out through two main approaches: a phenomenological approach and a fracture mechanics–based approach.

The phenomenological approach that is based on the endurance concept using Wohler techniques (*1*, p. 199) has been used by various researchers (*2*, p. 188; *3*; *4*, p. 310; *5*). It correlates the number of cycles to failure (N_f) to the applied stress (σ) or strain (ε) through empirical constants. A familiar relationship of this approach that represents the fatigue response is expressed as

$$N_f = C_1\ (1/\varepsilon)\ C_2 \qquad (1)$$

where C_1 and C_2 are regression constants. Because of its simplicity, this approach has been widely adopted. However, it carries severe limitations. It does not address the number of cycles to failure for both crack initiation and propagation of which the relative magnitude of each is extremely important from a structural design viewpoint. Moreover, the constants C_1 and C_2 are indeed regression constants and are influenced by many variables such as the type and rate of loading. Therefore, they are not material constants.

The fracture mechanics approach adopted by many workers essentially deals with crack propagation (*6–8*). This approach identifies the crack driving force (X) and correlates it with the average crack speed (da/dN) in a power law based on the equation derived by Paris (*9*, p. 528; *10*). This equation is given as

$$da/dN = A(X)^n \qquad (2)$$

X can be the stress intensity factor K (*11*) or the stress intensity factor range ΔK. The parameters A and n are empirical constants.

Alternatively, the elastic energy release rate may be invoked as the crack driving force for linear elastic conditions with limited crack tip plasticity. This is expressed as

$$G = K^2(l - \nu^2)/E \qquad (3)$$

For elastic-plastic conditions the energy release rate J can be obtained (*12*) from the load displacement curve associated with crack extension, based on the J-integral proposed by Rice (*13*, p. 379). To include unloading effects, J may be evaluated from the change in potential energy (the area above the unloading curve) with respect to the crack length, divided by the thickness of the specimen (*14*). Again A and n in Equation 2 are empirical constants that may under some conditions be constant but that are not material properties. This view is shared by many researchers. It is stated by Majidzadeh et al., who are advocates of the power law type analysis, that at low temperature for sand asphalt and asphaltic concrete beams, A in Equation 2 becomes a material constant (*6*). However, at high temperature, A and n can no longer be considered as material constants.

Schapery developed a theoretical analysis to link material properties such as the creep compliance, tensile strength, and fracture energy to determine the A and n in the Paris equation for viscoelastic materials (*15–17*). This theoretical relationship has been examined using asphalt concrete mixtures by researchers including Little et al. (*14*) and Germann and Lytton (*8*). Little et al. found that the crack speed of various

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asphalt concrete specimens at two different crack lengths (1 and 2 in.) were identical when they were calculated from either the viscoelastic analysis or the linear elastic approach. They concluded that both analyses yield the same results when using A and n from either approach to determine the crack speed. Germann and Lytton found that the calculated values of A and n agree fairly well with those determined experimentally for samples with high asphalt content. At lower asphalt content, the theoretical and experimental values differ significantly.

Even if A and n can be related to some material properties as Schapery suggested, they are still used in a power law relationship that at best can describe only a linear region of fatigue crack propagation; that is, it will not describe fatigue crack propagation over the entire range of the crack driving force.

Thus, it appears that the phenomenon of fatigue cracking in paving mixtures has been treated in a rather empirical manner. Both the phenomenological approach and the fracture mechanics–based approach use regression constants that usually cannot be considered as material parameters. Evidently material parameters are needed to discriminate the effects introduced by the different chemical structures of asphalts, asphalt content, the type of additive used, additive dosage, aging, gradation size, and frequency. Such parameters ought to account for micromechanical effects, that is, the magnitude of matrix cracking and cracking at the matrix-aggregate interface.

The present work outlines an innovative approach to characterize the resistance of asphalt concrete mixtures to fatigue crack propagation (FCP). The philosophy behind this approach is that the resistance of asphalt concrete mixtures to crack propagation depends on the energy expended on irreversible processes (damage) in the vicinity of the crack tip. On this basis, the objective of crack propagation studies is to identify and determine parameters responsible for a mixture's resistance to crack propagation.

THEORETICAL

A fundamental equation has recently been developed (*18*, p. 97) and successfully applied to characterize the resistance of highly strained nonlinear materials to FCP (*19*, p. 167; *20*, p. 98; *21*, p. 285). This equation expresses the rate of FCP as

$$da/dN = \dot{D}/(\gamma^* R_1 - J) \qquad (4)$$

where

da/dN = cyclic rate of FCP,
$\dot{D}$ = cyclic rate of energy dissipation on material transformation associated with active zone evolution,
J = crack driving force (energy release rate),
R_1 = resistance moment that accounts for the amount of damage associated with crack advance, and
γ^* = specific energy of damage.

The last one, γ^*, is a material parameter characteristic of its resistance to crack propagation that is to be extracted from the results of FCP experiments.

Crack Driving Force

The crack driving force can be evaluated at increments of crack length from either the area above the unloading curve (potential energy) or the area under the loading curve (strain energy). Current analysis of the FCP data obtained from experiments conducted on various asphalt mixtures shows that invoking the potential energy principle to evaluate the crack driving force J is more appropriate in stress controlled fatigue. Moreover, because undetected crack propagation may occur during loading, the area above the unloading curves is a more reliable presentation of the potential energy. Thus,

$$J = (\partial P/\partial a)/B \qquad (5)$$

where

P = potential energy (area above the unloading curve),
B = specimen thickness, and
a = crack length.

Resistance Moment

The resistance moment, R_1 in the proposed model, accounts for the amount of damage associated with the crack advance. Video monitoring of crack propagation in various asphalt concrete mixtures reveals damage in the form of "minicracks" emanating from the main crack as shown in Figure 1. Postfailure microscopic analysis of the fractured specimens shows that the minicracks exist on both surfaces of the tested specimens as well as inside the specimens. An important feature of the minicracks is that they mainly exist in the matrix or at the aggregate-matrix interface. This phenomenon of minicracking has been observed in pavement in service under fatigue loading (*22*).

It has also been observed that the amount of transformed (damaged) material (due to minicracks) in the vicinity of the crack tip increases with the crack advance. Following the Irwin plastic zone size (*23,24*) it can be assumed that the width of the transformed material ahead of the crack tip (active zone) is a linear function of the crack length, a, up to its critical length, a_c. This has been shown to be true in various other

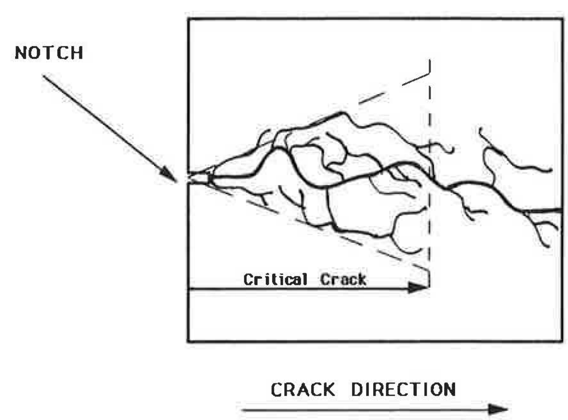

FIGURE 1 Postfailure assembly of damage in form of minicracks in AC-20 asphalt concrete specimen after fatigue.

materials, for example, metals (*25*), polymers (*26*, p. 263; *27*, p. 1377), and polymer composites (*28*, p. 381). Thus, the resistance moment can be evaluated as follows:

$$R_1 = (\partial V/\partial a)/B \tag{6}$$

where V is the volume of the damaged material (active zone) at crack length a.

$$V = (ahB)/2 \tag{7}$$

where h is the width of the active zone and is expressed as

$$h = Ca \tag{8}$$

where C is a constant. Thus,

$$V = (CBa^2)/2 \tag{9}$$

On this basis,

$$R_1 = (CBa)/B = Ca \tag{10}$$

$$C = 2\tan(\phi/2) \tag{11}$$

where ϕ is the active zone angle. Therefore, γ^*R_1 in Equation 4 can be evaluated as

$$\gamma^*R_1 = \gamma^*Ca = \gamma'a \tag{12}$$

where γ' replaces $\gamma^* \cdot \gamma'$ is a material constant characteristic of the material's resistance to crack propagation.

Energy Dissipation

The quantity $\dot{D}$ in the aforementioned model is the cyclic rate of energy dissipation on submicroscopic processes leading to damage formation. It has previously been shown (*19*) that for a strain control fatigue experiment, the value of $\dot{D}$ in Equation 4 can be expressed as

$$\dot{D} = dD/dN = \beta J^2 \tag{13}$$

where β is the coefficient of energy dissipation that expresses the portion of the change in work per cycle expended on damage formation for strain control. For stress-controlled experiments, it is found that the value of $\dot{D}$ can be extracted from the area of the hysteresis loop. This is approached as follows:

$$\dot{D} = dD/dN = \beta'\, dW_i/dN = \beta'\dot{W}_i \tag{14}$$

where β' is the coefficient of energy dissipation on submicroscopic processes associated with the stress control loading configuration. The relationship between β and β' for the same material has yet to be developed. It is expected that both β and β' may be dependent on temperature, strain rate, and a characteristic time of the process (*29*). The quantity $\dot{W}_i$, which is the "change in work," is measured directly as the area of the hysteresis loop at any crack length, a, minus the area of the loop just before crack initiation. In viscoelastic materials, $\dot{W}_i$ includes work expended on damage processes associated with crack growth and history-dependent viscous dissipation processes, both of which are irreversible. In the current analysis, the quantity $\dot{D}$ in Equation 14, which is the rate of energy dissipation on submicroscopic processes leading to damage formation, is expressed as a portion of this total change in work, $\dot{W}_i$. This value has successfully been evaluated for various asphalt concrete mixtures and will be employed in the current study.

EXPERIMENTAL

Asphalt concrete beams 15 in. long, 2 in. wide, and 3.5 in. high were prepared in substantial agreement with ASTM D3202-83. Two types of asphalt were considered in this study: Elvax-modified AC-5 and AC-20. The beams were prepared at an asphalt content of 8 percent by dry weight and a target asphalt concrete unit weight of 149 pcf. The following gradation was used:

Sieve	*Total Passing (%)*
½ in.	100
⅜ in.	95
No. 4	59
No. 16	28
No. 50	9
No. 200	0

The AC-5 specimens were modified with 6 percent Elvax (percentage of the asphalt content). The aggregate, the asphalt cement, the mold, and the compaction hammer were all preheated to 300°F before blending and compaction. Beam compaction was achieved in accordance with the Marshall hammer method (ASTM D1559-89). Beams were allowed to cool off in the compaction mold and were usually tested 7 days after preparation and conditioning. Conditioning consisted of subjecting each beam to a constant temperature of 140°F for 1 day. The air void content for the specimens was approximately 1.3 percent, and the voids in mineral aggregate (VMA) was about 20 percent.

Four-point bend fatigue testing on notched beams of each mixture was conducted on a pneumatic testing machine. The support span was 10.2 in. and the midspan (between the inner load points) was 3.4 in. An initial straight notch with a total depth of 0.25 in. was inserted at the middle of the specimens with a 0.156 in. saw with a round tip of radius 0.094 in. The tests were conducted at a constant frequency of about 0.5 Hz under load control with a constant maximum load of 65 lb. Each cycle consisted of a 0.2-sec load duration. The specimens were cycled from zero to the maximum load. All tests were conducted at 70°F. Multiple identical specimens (three from each mixture) were tested under the same set of experimental conditions. A hysteresis loop was recorded at ¼-in. intervals of crack growth. Software has been developed to digitize graphical data and to calculate pertinent areas within the load-deflection curves obtained during fatigue testing. These areas are considered important in the current analysis of asphalt concrete under fatigue loading. A traveling video camera equipped with a zoom lens was used to monitor the crack propagation and capture any damage events associated with the fracture process.

RESULTS

Substituting Equations 12 and 14 into Equation 4 yields the following relationship:

$$da/dN = [\beta'\dot{W}_i/(\gamma' a - J)] \tag{15}$$

Equation 15 calls on the accurate measurement of the crack speed (da/dN), the crack driving force (J), and the change in work ($\dot{W}_i$). On this basis, the specific energy of damage (γ') and the dissipation coefficient (β') can be extracted from Equation 15. Analysis of results of FCP experiments generated for typical specimens of the modified AC-5 and AC-20 paving mixtures are presented in the following sections. The average values of γ' and β' were then obtained based on the three tested specimens from each mixture.

Crack Speed

In order to determine the crack speed, the crack length is monitored with respect to the number of cycles. This monitoring is a well-established procedure, and many techniques are available to measure the crack length in FCP experiments. In the current experiment, an accurate transparent flexible scale was attached to the specimens to measure the crack length. Typical curves for crack length (a) versus the number of cycles (N) for the modified AC-5 and AC-20 asphalt mixture specimens are shown in Figures 2 and 3, respectively. In the modified AC-5 specimen (Figure 2), a crack initiated at about 2,000 cycles and advanced in a stable manner, reaching ultimate failure at about 3,600 cycles. In the AC-20 specimen, crack initiation started at about 1,400 cycles. The crack then advanced faster than in the modified AC-5, reaching ultimate failure at about 2,700 cycles. The slope of the curves in Figures 2 and 3 is the average crack speed at the corresponding crack length. The number of cycles for both crack initiation and propagation is larger for the modified AC-5 mixture than for the AC-20 mixture. The initiation and propagation life times

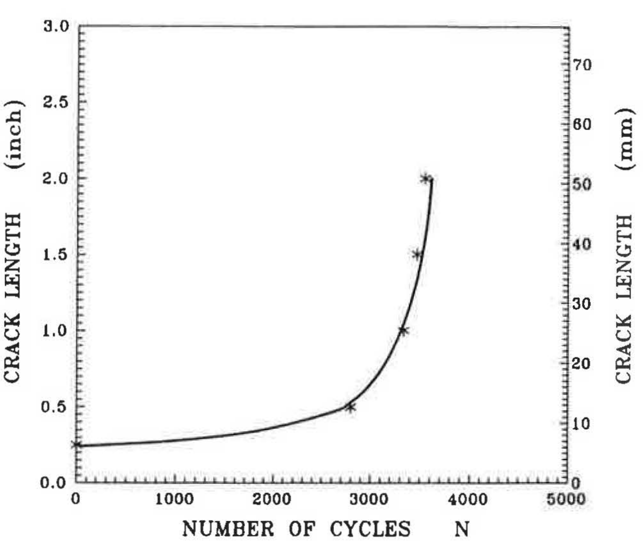

FIGURE 2 Crack length versus number of cycles in modified AC-5 asphalt concrete mixture.

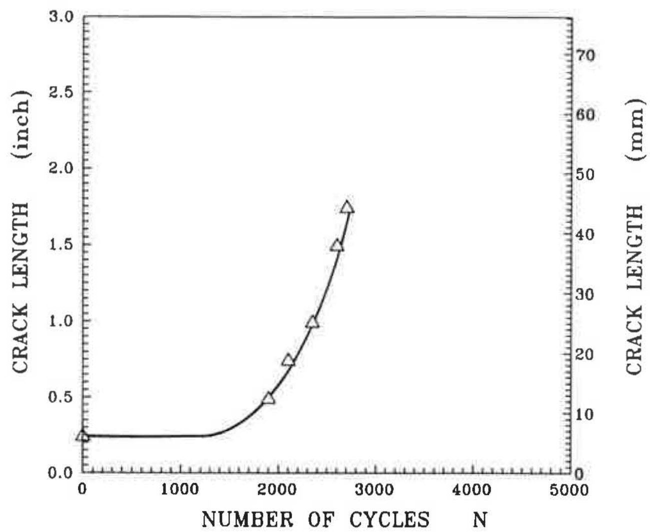

FIGURE 3 Crack length versus number of cycles in AC-20 asphalt concrete mixture.

of the modified AC-5 are about 1.4 and 1.2 times higher than the AC-20, respectively.

Energy Release Rate

Invoking the potential energy principle, Equation 5 is used to evaluate the energy release rate (J) at increments of crack length for the modified AC-5 and AC-20 specimens. This is shown in Figure 4 for typical specimens of each asphalt mixture. It is evident from Figure 4 that the value of J for the modified AC-5 mixture is always higher than that of the AC-20 for the same crack length.

As it is shown in Figure 4, the energy release rate for the modified AC-5 mixture evolves at a faster rate with respect to the crack length than that of the AC-20 asphalt concrete

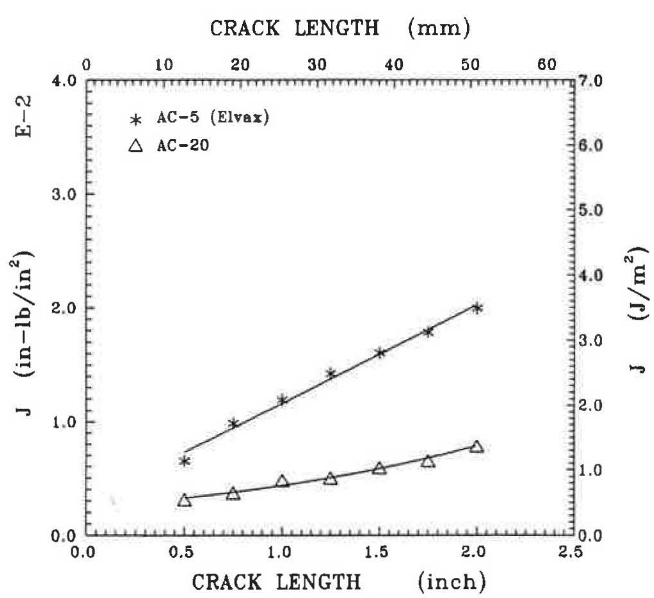

FIGURE 4 Energy release rate (J) versus crack length for modified AC-5 and AC-20 asphalt concrete mixtures.

mixture. The ductile fracture behavior of the AC-5 mixture causes a larger increase in the resistance of this mixture to crack propagation with the crack length. As a result, the energy release rate (J) must increase in order to maintain crack growth. The slow evolution of J with respect to the crack length in the case of the AC-20 mixture is indicative of the "brittle" fracture behavior of this mixture in comparison with the AC-5. Since the energy release rate is always higher for the modified AC-5 mixture, it is expected to be tougher than the AC-20 mixture.

Change in Work

The change in work ($\dot{W}_i$) for the modified AC-5 and AC-20 specimens is evaluated from the area of the hysteresis loops at increments of crack length. The evolution of the quantity $\dot{W}_i$ with the crack length for typical specimens of both materials is shown in Figure 5. The value of $\dot{W}_i$ for the modified AC-5 specimen is higher than that of the AC-20 specimen at each crack length. Thus, it appears that more total work has been expended on both damage formation and history-dependent viscous dissipation processes within the active zone of the modified AC-5 mixture.

DISCUSSION OF RESULTS

The new methodology developed through the work outlined will be used to determine the parameters that characterize the resistance of the two asphalt concrete mixtures to crack propagation. These parameters (γ' and β'), which control the fracture process, can be determined by rearranging Equation 15. Thus,

$$J/a = \gamma' - \beta'\{\dot{W}_i/[(da/dN)a]\} \quad (16)$$

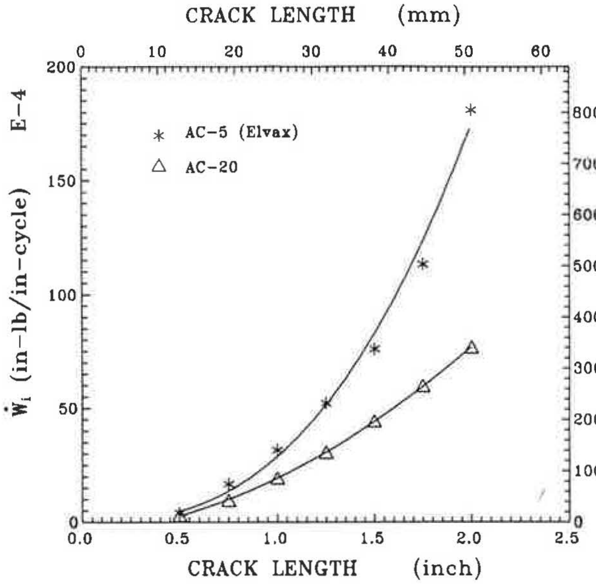

FIGURE 5 Change in work ($\dot{W}_i$) versus crack length for modified AC-5 and AC-20 asphalt concrete mixtures.

If the experimental results of each asphalt mixture tested are in accord with the proposed model, a plot of J/a versus $\{\dot{W}_i/[(da/dN)a]\}$ should give a straight line with γ' the intercept and β' the slope. Indeed, when the results of the experiments previously presented were plotted on the basis of Equation 16, the experimental points for both the modified AC-5 and AC-20 mixtures make nearly a straight line. This is shown in Figures 6 and 7 for a typical specimen of the modified AC-5 and AC-20 mixtures respectively. Thus, it is safe to conclude that the model is in accord with the results of the experiments.

Curves plotted on the basis of Equation 15 using the obtained values of γ' and β' with the experimental results are shown in Figures 8 and 9 for the typical specimens of the modified AC-5 and AC-20 mixtures, respectively. The theoretically obtained curves describe the experimental results very well. This has also been found to be true for the other two specimens of each asphalt mixture. It has been observed that the fatigue crack propagation kinetics of all of the tested specimens of the two asphalt concrete mixtures under consideration display the familiar S-shaped character. Three stages of crack propagation kinetics are obvious. The threshold stage

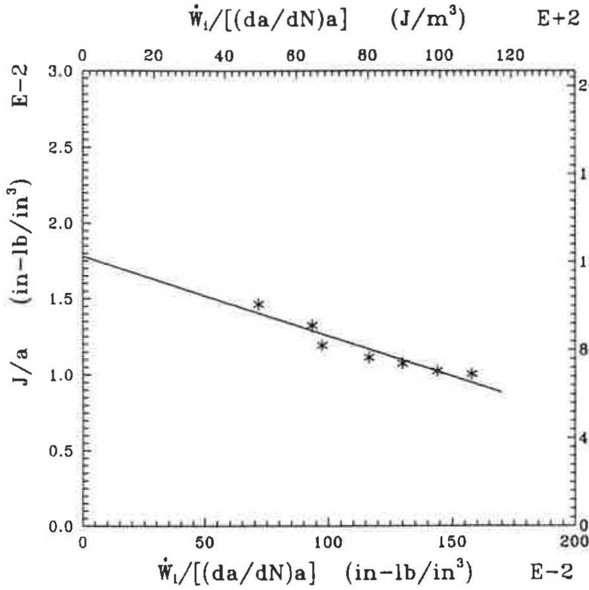

FIGURE 6 FCP behavior of modified AC-5 mixture plotted in form of proposed model to obtain γ' and β'.

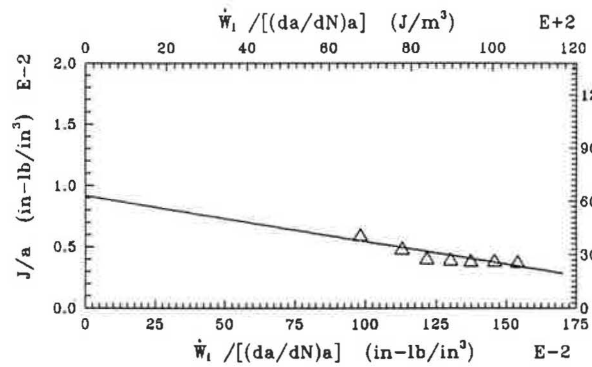

FIGURE 7 FCP behavior of AC-20 mixture plotted in form of proposed model to obtain γ' and β'.

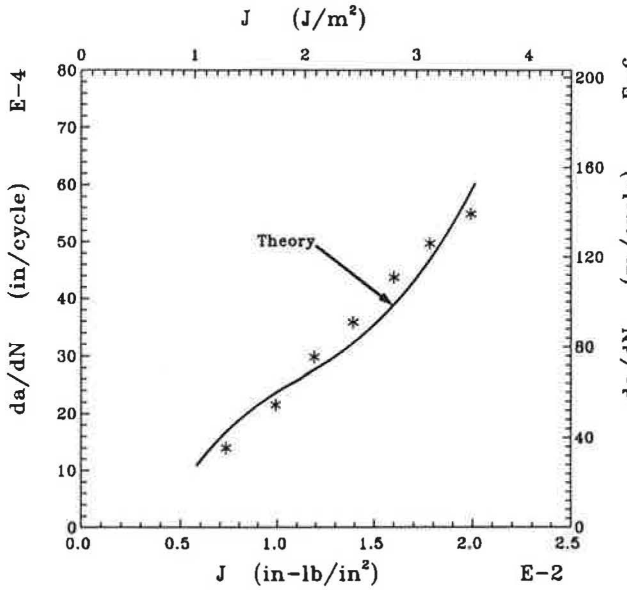

FIGURE 8 Theoretically predicted FCP speed based on proposed model (with experimental data) for modified AC-5 mixture.

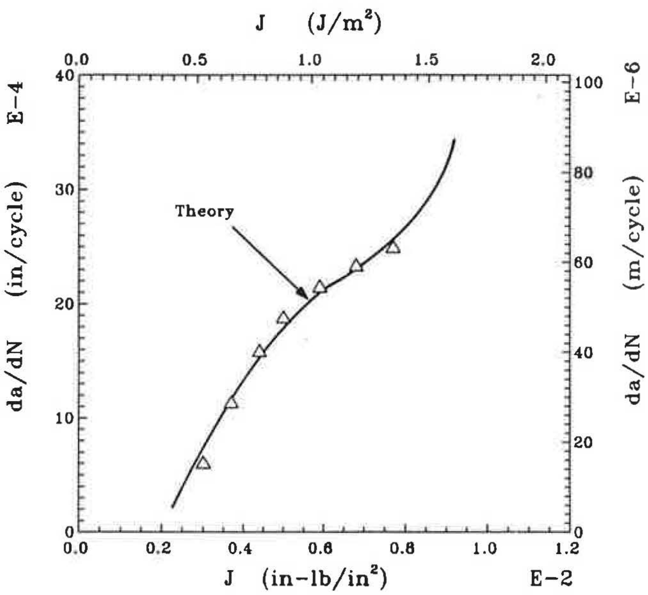

FIGURE 9 Theoretically predicted FCP speed based on proposed model (with experimental data) for modified AC-20 mixture.

is followed by a stage of reduced acceleration and then the stage of unstable crack propagation. This behavior indicates the evolution of crack tip damage.

The average values of γ' and β' for the three Elvax-modified AC-5 specimens tested are $1.79 \pm 0.09 \times 10^{-2}$ (in.-lb/in.^{3}) and $4.8 \pm 0.37 \times 10^{-3}$, respectively. The average values of γ' and β' for the AC-20 specimens are $1.17 \pm 0.41 \times 10^{-2}$ (in.-lb/in.^{3}) and $2.71 \pm 0.70 \times 10^{-3}$, respectively. As can be seen, the average value of γ' is higher for the modified AC-5 mixture than for the AC-20. Thus, more energy is required to cause a unit volume of the AC-5 to change from undamaged to damaged material. On this basis AC-5 is more resistant to crack propagation than AC-20. The larger value of β' for the modified AC-5 mixture reflects the larger percentage of energy expended on dissipative processes and damage growth within the active zone. The standard deviation in the values of γ' and β' for the AC-5 are surprisingly low for this type of heterogeneous material. The large standard deviation for the AC-20 mixture values probably can be attributed to the "brittleness" of the AC-20 compared with the AC-5. To compare the data further, the two curves of Figures 8 and 9 are plotted together in Figure 10. It is evident that the modified AC-5 mixture is superior to the AC-20 mixture, as is reflected by the larger value of γ'.

It should also be mentioned that the fracture processes are not as sensitive to β' as they are to γ'. The denominator of Equation 15 is the energy barrier that controls the fracture process. As can be seen from analysis of the AC-5 mixture, $\dot{W}_i$ and β', which make up the numerator of Equation 15, are higher than for the AC-20. Nevertheless, da/dN is lower. This attests to the sensitivity of the fracture processes to the value of $(\gamma' a - J)$, and in turn γ' rather than β'. Therefore γ' is a candidate material parameter characteristic of the material's resistance to fracture and β' is a dissipative coefficient. The dependency of β' on the strain rate, temperature, and time characteristic of the process is still unresolved and will be the subject of further research.

CONCLUDING REMARKS

A methodology has been developed and successfully applied to characterize the resistance of asphalt concrete mixtures to fatigue crack propagation. This methodology reasonably describes fatigue crack propagation behavior of the pavements over the entire range of the energy release rate. Parameters controlling the fracture process were extracted from fatigue crack propagation experiments on two asphalt concrete mixtures (Elvax-modified AC-5 and AC-20) with defined chem-

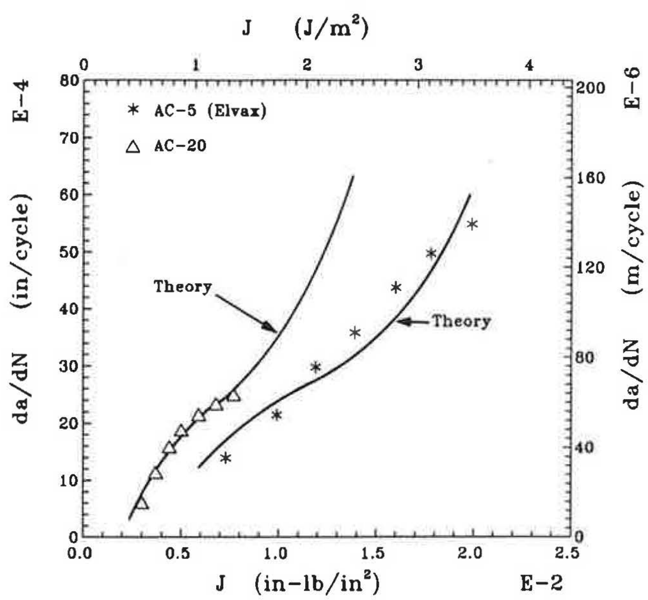

FIGURE 10 Theoretically predicted FCP speed for modified AC-5 and AC-20 mixtures (with experimental data).

ical structures. These are the specific energy of damage (γ'), a candidate material parameter characteristic of the mixtures resistance to FCP, and β', a dissipative coefficient.

The current study reveals that γ' reflects the toughness of the material. A higher value of γ' gives a lower crack speed over the entire range of the energy release rate. Although β' represents the portion of the change in work expended on damage processes, it does not play as important a role in controlling the fracture process as γ' does. Knowing γ' and β' for an asphalt concrete mixture can serve an important practical purpose. Useful relationships can be established between γ' and β' and structure and processing conditions. Such relationships can guide the development of asphalt mixtures with superior resistance to cracking. Research activities to achieve this goal are currently in progress.

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