Fixed-Point Approach To Estimating Freeway Origin-Destination Matrices and the Effect of Erroneous Data on Estimate Precision

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A fixed-point approach was applied to the problem of estimating freeway origin-destination (OD) matrices given historical sequences of input and output counts. This estimator was compared with other previously tested estimators in simulation experiments that demonstrated the properties of the chosen estimator and the effect of erroneous data on the precision of the OD estimates. The simulation results indicated that the fixed-point estimator produced the most accurate OD estimates of those tested and that data with measurement error (e.g., from malfunctioning loop detectors) severely affect the precision of OD estimates.

Research on improved methods of control and management of traffic on urban freeways has been gaining increased attention because of growing freeway congestion coupled with limited foreseeable investment in new infrastructure. Attention is now concentrated on efforts to improve the efficiency of existing facilities through better means of freeway surveillance, ramp metering, incident detection, and so forth. To achieve maximum effectiveness in these areas, we need the capability of anticipating traffic problems, such as bottlenecks, before they occur. Therefore, models that produce accurate short-term forecasts of freeway flow are a top priority. Such models usually require an accurate estimate of the freeway origin-destination (OD) matrix. An origin-destination matrix gives the magnitude of travel during a given interval of time from each of the trip origins (on-ramps) to each of the trip destinations (off-ramps). In practice, the true OD matrix is seldom available because the collection of OD data is costly, time consuming, and less accurate than the more easily collected traffic volume data. Consequently, there has been considerable research interest in the development of models or techniques that are capable of estimating freeway OD matrices from input and output counts. Such data are usually collected automatically through loop detectors installed at different sections of the freeway. Since these counts are collected continuously, models that could use these counts to estimate OD patterns could also provide important information on changes in trip patterns over time to traffic and transit planners.

Research in this area of model development can be grouped into two main categories. In the first category (static models), only a single set of input and output counts is used for estimation. The estimation problem here is underdetermined, and a prior OD matrix is required to produce the "updated" estimate. The estimation process involves updating the prior OD matrix in such a way that the updated estimate reproduces the selected set of input and output counts. However, the quality of such updated matrices depends on the quality of the prior estimate, which, in most cases, is poor and difficult to obtain (*1*). Studies in this category of model development include Van Zuylen and Willumsen (*2*), Willumsen (*3*), Van Zuylen (*4*), Nguyen (*5*), Cascetta (*6*), Maher (*7*), Stokes and Morris (*8*), and Hendrickson and McNeil (*9*). In the second category of model development research (dynamic models), historical sequences of input and output counts are considered. The use of time series data here causes these estimation problems to be overdetermined. Studies in this category include Cremer and Keller (*10,11*) and Nihan and Davis (*1,12*). These authors present a number of algorithms that are based on prediction-error minimization methods to estimate movement volumes for a single intersection given time series of entering and exiting counts at each intersection leg. There has been limited success, to date, in extending the application of the second category of models to more complicated networks. This paper addresses the application of such models to a simple freeway network.

Most of the models developed for estimating OD matrices assume the availability of error-free data. However, recent studies by Jacobson et al. (*13*) and Chen and May (*14*) indicate a number of ways in which loop detectors can malfunction and provide erroneous data [e.g., stuck sensors, chatting, pulse breakup, hanging (on or off), and intermittent malfunctioning]. Since our interest lies in estimating the OD matrix parameters from time series of input and output counts and since the estimation assumes conservation of flow in each time period, it is important that the data observed be as error-free as possible. This was the motivation for the second part of our study, which addressed the potential impact of measurement error in resulting OD estimates.

This paper attempts to accomplish two tasks. The first is the development of an estimation technique based on the "fixed-point problem" (FPP) approach that is capable of estimating the freeway OD matrix given time series of input and output counts. The second involves exploration of the effect of measurement error in input and output counts (e.g., due to faulty loop detectors) on the precision of estimates

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and the asymptotic properties of estimators used. The malfunction of loop detectors is simulated by adding a measurement error to traffic counts at selected entry or exit points. Loop detectors in good working condition (reliable traffic counts) are represented as having zero measurement error, whereas malfunctioning loop detectors are simulated by adding a measurement error term to the data.

PROBLEM DESCRIPTION AND MODEL FORMULATION

Problem Description

The objective of this research was the development of an algorithm that could accurately estimate the proportion of flow from each on-ramp to each off-ramp given a section of freeway and time-series of entering (on-ramp) and exiting (off-ramp) counts. Specifically, the objective was the estimation of the OD matrix proportions $b_{ij}(t)$ given time series counts of $\mathbf{q}(t)$ and $\mathbf{y}(t)$ (input and output counts respectively) so that Constraints 1 and 2 are satisfied.

$$\sum_{j=1}^{N} b_{ij}(t) = 1.0 \qquad i = 1, 2, \ldots, M \tag{1}$$

$$b_{ij}(t) \geq 0 \qquad i = 1, 2, \ldots, M \qquad j = 1, 2, \ldots, N \tag{2}$$

where

$b_{ij}(t)$ = the proportion of vehicles originating at i and destined for j at time t,
M = total number of origin points (on-ramps and upstream mainline), and
N = total number of destination points (off-ramps and downstream mainline).

The first two constraints ensure conservation of flow during each time interval and elimination of any negative OD volumes, respectively. A third constraint prohibits flow from an on-ramp to an upstream off-ramp:

$$b_{ij}(t) = 0 \qquad (i, j) \in Z \tag{3}$$

where Z is the set of OD pairs that are known a priori to have zero flow.

Cremer and Keller (*10,11*) showed that the output counts can be expressed as weighted sums of input counts.

$$\mathbf{y}'(t) = \mathbf{q}'(t)\mathbf{B}(t) + \mathbf{e}'(t) \tag{4}$$

$$E[y_j(t)] = \textstyle\sum_i b_{ij} q_i(t) \tag{5}$$

where

$\mathbf{q}(t)$ = $m \times 1$ vector of input counts for time t,
$\mathbf{y}(t)$ = $n \times 1$ vector of output counts for time t,
$\mathbf{B}(t)$ = $m \times n$ matrix with elements $b_{ij}(t)$ = proportion of trips from i to j during time t,
$\mathbf{x}(t)$ = $m \times n$ matrix of OD volumes for time t, and
$\mathbf{e}'(t)$ = transpose of the $n \times 1$ vector of prediction errors (assumed independent and normally distributed).

Although this formulation was developed for intersection models, where the travel time from origin to destination is very short, it was assumed that this could also be applied to the freeway OD problem, provided that the time interval, t, were long enough to accommodate increased travel times. It was further assumed that an interval three or four times longer than the longest OD travel time for the study section would be acceptable. This would allow most trips that originated at some point in the freeway section during that interval to be completed within the same interval. This simple, first-stage assumption avoided the necessity for a more complex formulation including lagged input variables.

Previous Estimation Approaches

Typically, there are two possible approaches to estimating the OD matrix parameters from time series counts: recursive (on-line) and nonrecursive (off-line). In the nonrecursive approach it is assumed that the $\mathbf{B}(t)$ matrix is time invariant and that the OD parameter estimates apply for the entire period. In the recursive approach, the $\mathbf{B}(t)$ matrix is allowed to vary with time, and a new set of OD estimates is produced for each interval in the time period.

Nonrecursive Estimators

If the OD matrix can be assumed to be time invariant, Equation 4 becomes a standard linear regression equation, and, since the time series counts of inputs and outputs are known, the ordinary least squares (OLS) estimator can be used to estimate the OD matrix. The objective is to choose the $\mathbf{B}(t) = \mathbf{B}$ matrix that minimizes the sum of the squared prediction errors:

$$SS_j = \textstyle\sum_t [y_j(t) - \mathbf{q}'(t)\mathbf{b}_j(t)]^2 \tag{6}$$

A constrained least squares (CLS) approach that ensures that Equations 1 and 2 are satisfied can also be applied in the same manner.

An alternative estimation method, the expectation maximization (EM) algorithm (*15*), has also been applied to the time invariant OD problem. Given that we only observe $q_i(t)$ and $y_j(t)$, the EM algorithm lends itself nicely to this underdetermined problem. For nonrecursive estimators, assuming that the input counts are generated by random variables that are independent across time, the likelihood of the OD movements $[x_{ij}(t) = b_{ij} q_i(t)]$ can be given by

$$L_t \{x_{ij}(t), b_{ij}, i = 1, \ldots, M, j = 1, 2, \ldots, N\} = \textstyle\prod_i \{[q_i(t)!/\prod_j x_{ij}(t)!][\prod_j b_{ij} x_{ij}(t)]\} \tag{7}$$

It can be shown that the maximum likelihood estimator of log (L) is

$$\hat{b}_{ij} = \textstyle\sum_t x_{ij}(t) / \sum_t q_i(t) \qquad i = 1, 2, \ldots, M \quad j = 1, 2, \ldots, N \tag{8}$$

In an intersection application of the EM approach, Nihan and Davis (*1*) treated the turning movements $x_{ij}(t)$'s as outcomes of multinomial random variables, one for each entering leg. This was under the assumptions that the $\mathbf{B}(t)$ matrix was time invariant, the input counts $q_i(t)$ were known, that each driver arriving at Leg i of the intersection during t made the turning movement decision independently of all other drivers arriving during t, and that all vehicles entering during Interval t also exited during t (conservation of flow). With these assumptions the expected value for each movement from Origin i to Destination j was given by

$$E[x_{ij}(t)] = b_{ij}(t)q_i(t) \tag{9}$$

where $x_{ij}(t)$ is the number of vehicles entering at i and exiting at j during Interval t.

The EM algorithm begins by estimating the conditional expectation of the turning movements $x_{ij}(t)$'s given an initial estimate of the $\mathbf{B}(t)$ matrix and all input and output counts.

$$\sum_t \hat{x}_{ij}(t) = \sum_t \{E[x_{ij}(t)] | \mathbf{B}(t), \mathbf{q}(t), \mathbf{y}(t), \quad t = 1, 2, \ldots, T\} \tag{10}$$

The $\mathbf{B}(t)$ matrix is then reestimated by replacing $\Sigma x_{ij}(t)$ in Equation 8 with $\Sigma \hat{x}_{ij}(t)$. The EM algorithm iterates between Equations 8 and 10 until convergence is achieved.

Applied to a four-leg isolated intersection with 100 simulated data sets, the EM estimates of the $\mathbf{B}(t)$ matrix showed much lower variances than a least squares–based estimator. However, these estimates did have significant biases. Moreover, the EM algorithm required high computational demands because both the inverse of the random vector $\mathbf{y}(t)$'s covariance matrix and the covariance matrix between $x_{ij}(t)$ and $\mathbf{y}(t)$ (shown below) had to be calculated at each iteration (*1*).

$$C[y_j(t), y_k(t)] = \begin{cases} -\sum_i b_{ij} b_{ik} q_i(t) & j \neq k \\ \sum_i b_{ij}(1 - b_{ij}) q_i(t) & j = k \end{cases}$$

$$C[x_{ij}(t), y_k(t)] = \begin{cases} -\ b_{ij} b_{ik} q_i(t) & j \neq k \\ b_{ij}(1 - b_{ij}) q_i(t) & j = k \end{cases}$$

Recursive Estimators

Cremer and Keller (*10*) developed an algorithm for dynamic estimation of intersection turning movements. The algorithm could be used to estimate the $\mathbf{B}(t)$ matrix using the recursion equations, which have the form of a stochastic gradient algorithm:

$$b_{ij}(t) = b_{ij}(t - 1) + q_i(t)[y_i(t) - \mathbf{q}'(t)\mathbf{b}_j(t - 1)] \tag{11}$$

$$\mathbf{b}_j(t) = \mathbf{b}_j(t - 1) + (1/t)(\mathbf{R}^{-1})[y_j(t) - \mathbf{q}'(t)\mathbf{b}_j(t - 1) \tag{12}$$

$$\mathbf{R}(t) = \mathbf{R}(t - 1) + (1/t)[\mathbf{q}(t)\mathbf{q}'(t) - \mathbf{R}(t - 1)] \tag{13}$$

Other dynamic approaches considered by Cremer and Keller (*10*) and Nihan and Davis (*1*) include recursive least squares (RLS), which is basically the application of OLS to sequential least squares equations, and normalized recursive least squares (RLSN), which includes the satisfaction of Constraints 1 and 2.

Preliminary Test Using Nonrecursive Estimators with Freeway Data

To date, the preceding approaches have seen limited application and have been primarily used in estimation of turning movements for isolated intersections. When used to estimate the $\mathbf{B}(t) = \mathbf{B}$ matrix of an isolated intersection, the OLS and CLS methods gave consistent, unbiased estimates and low computational demands (*1*). This inspired us to adopt the standard linear regression model as a starting point for estimating the $\mathbf{B}$ matrix of a freeway section.

Figure 1 shows a schematic representation of a section of Interstate 5 in north Seattle. The section consists of six origins (O1–O6) and three destinations (D1–D3). A data set for this section (one time series count for each point of input and output) was obtained from Traffic Systems Management Center of the Washington State Department of Transportation. These counts were automatically collected through loop detectors installed on the freeway. To account for traffic congestion and travel times from origins to destinations, the data were aggregated to 15-min counts for 24 hr, thus giving a time series length of 96.

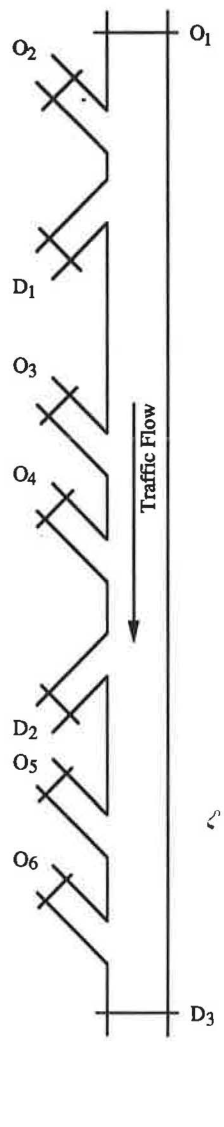

FIGURE 1 Section of Interstate 5 in north Seattle.

Table 1 gives the estimates of the $\mathbf{B}(t) = \mathbf{B}$ matrix using both OLS and CLS. Although the actual $\mathbf{B}$ matrix is not known, it is clear that OLS failed to produce reasonable estimates (both Constraints 1 and 2 were violated). The estimates produced by CLS satisfy Constraints 1 and 2 but are not realistic. For example, they suggest that about 45 percent of traffic that originates from On-Ramp 2 is destined to Off-Ramp 1 and the rest (55 percent) is destined to Off-Ramp 2. Examination of actual data indicated that conservation of flow was never achieved at most time periods, an obvious indication of loop detector error. This warranted conducting simulation scenarios to investigate the effect of erroneous data on the precision of estimates for the various estimators. Before conducting these simulation scenarios, an additional estimator based on the FPP was developed and included in subsequent evaluations.

Fixed-Point Estimation Approach

As discussed earlier, the EM algorithm was successfully used (*1*) on an isolated four-leg intersection to estimate turning movements. However, the algorithm required high computational demands. Furthermore, to operationalize this algorithm for the intersection problem, all U turns were prohibited to avoid having a singular covariance matrix of the random vector $\mathbf{y}(t)$ [since $\Sigma_i q_i(t) = \Sigma_j y_j(t)$]. To operationalize the EM algorithm to estimate the OD matrix of a given freeway section, one needs to prohibit at least one OD movement (e.g., O_4 to D_2). This is realistic since on-off movements of such short distances are expected to be rare.

To simplify calculations, the estimation problem was structured in such a way that each cell of the OD matrix could be estimated separately. This means replacing both the covariance matrix of the random vector $\mathbf{y}(t)$ by its variance $\{\Sigma_i b_{ij}(1 - b_{ij})q_i(t)\}$ and the covariance matrix of $x_{ij}(t)$ and $\mathbf{y}(t)$ by its variance $\{b_{ij}(1 - b_{ij})q_i(t)\}$. The conditional expectation of the turning movements $\{x_{ij}(t)\text{'s}\}$ are then given by

TABLE 1 Performance of OLS and CLS Estimators on Actual Freeway Data

Movement	OLS bij	CLS bij
11	0.028	0.014
12	0.090	0.005
13	0.695	0.977
21	0.387	0.446
22	0.567	0.551
23	-0.162	0.000
32	-0.045	0.000
33	0.957	1.001
42	-0.723	0.000
43	2.366	1.002
53	3.022	1.001
63	1.056	1.004

$$E\{x_{ij}(t)|\mathbf{B}, \mathbf{q}(t), y_j(t)\} = b_{ij}q_i(t) + C\{x_{ij}(t), y_j(t)\} \times \mathrm{Var}^{-1}\{y_j(t)\}\{y_j(t) - \textstyle\sum_i b_{ij}q_i(t)\} \tag{14}$$

where

$$C\{x_{ij}(t), y_j(t)\} = \{b_{ij}(1 - b_{ij})q_i(t)\} \text{ and}$$
$$\mathrm{Var}\{y_j(t)\} = \textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)$$

Equation 14 becomes

$$E\{x_{ij}(t)|\mathbf{B}, \mathbf{q}(t), y_j(t)\} = b_{ij}q_i(t) + \{b_{ij}(1 - b_{ij})q_i(t)\} \times \{\textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)\}^{-1}\{y_j(t) - \textstyle\sum_i b_{ij}q_i(t)\} \tag{15}$$

Summing over t,

$$E[\textstyle\sum_t x_{ij}(t)|\mathbf{B}, \mathbf{q}(t), y_i(t)] = b_{ij}\textstyle\sum_t q_i(t) + \textstyle\sum_t\{b_{ij}(1 - b_{ij})q_i(t)\{y_j(t) - \textstyle\sum_i b_{ij}q_i(t)\} \div \textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)\} \tag{16}$$

Having structured the estimation problem in such a way that each cell in the $\mathbf{B}(t)$ matrix was estimated separately, it was decided to treat each function as an FPP (*16,17*). This essentially involves solving for the convergence point of a recursive estimation algorithm of the form $b_{ij}^{k+1} = g(b_{ij}^k)$ (i.e., the point where $b_{ij}^{k+1} = b_{ij}^k$. Given $f(b_{ij})$ where $0 \le b_{ij} \le 1$, the objective is to find values s such that $f(s) = 0$. Let $g(b_{ij})$ be an auxiliary function such that $s = g(s)$ wherever $f(s) = 0$. The problem of finding s such that $s = g(s)$ is known as the FPP, and s is said to be a fixed point of $g(b_{ij})$. Thus, finding a fixed point for $g(b_{ij})$, $0 \le b_{ij} \le 1$, means finding a zero of $f(b_{ij})$, $0 \le b_{ij} \le 1$.

From equations 8 and 16, we can represent the recursive function as

$$b_{ij}(k + 1) = \textstyle\sum_t x_{ij}(t)/\textstyle\sum_t q_i(t) = b_{ij}(k) + d/\textstyle\sum_t q_i(t) \tag{17}$$

where

$$d = \textstyle\sum_t\{b_{ij}(1 - b_{ij})q_i(t)[y_j(t) - \textstyle\sum_i b_{ij}q_i(t)]/\textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)\}$$

Defining

$$f(b_{ij}) = \textstyle\sum_t\{b_{ij}(1 - b_{ij})q_i(t)[y_j(t) - \textstyle\sum_i b_{ij}q_i(t)]/\textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)\}$$

form $s = g(s)$ reduces to

$$\textstyle\sum_t\{b_{ij}(1 - b_{ij})q_i(t)[y_j(t) - b_{ij}q_i(t)] \div \textstyle\sum_i b_{ij}(1 - b_{ij})q_i(t)\} = 0 \tag{18}$$

Thus, the problem is reduced to solution of a set of n nonlinear equations in n variables. A NAG routine C05NCF (*18*) that uses a modification of the Powell hybrid iterative method is used to obtain a numerical solution, thereby giving the $\mathbf{B}(t)$ estimates.

ALGORITHM TESTING

In this section we investigate the accuracy of the estimators for freeway OD problems and the effect of detector malfunction on the precision of the OD estimates and the properties of the estimators chosen. In addition to the newly developed estimator (FPP), other estimators already developed are considered. Cremer and Keller (*10*) and Nihan and Davis (*1*) present a family of estimators based on the principle of prediction-error minimization that are also included. Thus, in this paper the following estimators are evaluated:

1. OLS,
2. CLS,
3. FPP,
4. RLS, and
5. RLSN.

Since these models require time series data of entering and exiting counts, it is important that the loop detectors provide us with accurate measurement of traffic counts. To investigate the effect of measurement error on the estimates of the $\mathbf{B}(t)$ matrix, we consider two scenarios. The first deals with the case of no measurement error (i.e., no faulty or malfunctioning loop detectors at entry or exit points). In the second scenario measurement error is introduced in both the input and the output counts.

Scenario 1

To test the properties of the algorithms with no measurement error in input or output counts, 50 simulated data sets were generated. Each data set consisted of five simulated days, for 480 time periods. For the selected section shown in Figure 1, input counts (O1–O6) with a counting interval of 15 min were generated for 24 hr, giving six time series of length 96, one for each origin. An OD matrix was assumed as part of the process to simulate the exiting counts. This simulated matrix was taken as the "true" OD matrix. The OD volumes (x_{ij}'s) were generated by the IMSL subroutine GGMTN (IMSL) and summed to produce simulated exiting counts $y_j(t)$. Hence, each simulated day consisted of the same entering counts but different exiting counts. The difference between the total inputs and total outputs for each time period was zero (i.e., conservation of flow was attained).

The FPP estimator was then applied using two simulated entering $[q_i(t)]$ and exiting $y_j(t)$ counts to produce estimates of the true OD values. Table 2 gives the averages of the OD estimates and the standard deviations determined across 50 simulated data sets. These results indicate that the FPP estimator, in general, produced lower variances than did the least squares–based estimators. Furthermore, the estimates were generally unbiased and similar to those produced by OLS, CLS, and RLS. Examining the averages of the $\mathbf{B}(t)$ matrix across 50 simulated data sets, we see that all estimators satisfied Constraints 1 and 2. Table 3 shows the absolute

TABLE 3 Percent Absolute Difference Between $\hat{b}_{ij}$ and True b_{ij} for Scenario 1 (No Measurement Error)

Movement	OLS	CLS	FPP	RLS	RLSN
11	0.10	0.10	0.10	0.10	8.10
12	0.80	1.00	0.70	0.80	9.50
13	0.25	0.11	0.15	0.25	2.20
21	1.20	1.20	0.40	1.20	19.4
22	7.60	8.80	7.20	7.60	29.00
23	0.60	0.60	0.60	0.60	0.60
32	10.90	11.90	10.00	10.90	133.00
33	1.56	1.32	1.44	1.56	14.79
42	1.25	2.40	1.40	1.25	48.35
43	1.00	0.60	0.75	1.00	12.09
53	0.20	0.00	0.00	0.20	0.00
63	0.30	0.00	0.00	0.30	0.00

TABLE 2 Performance of Five Estimators on Simulated Data (No Measurement Error)

Movement	True b_{ij}	OLS		CLS		FPP		RLS		RLSN	
		$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}
11	0.100	0.0999	0.0028	0.0999	0.0028	0.0999	0.0025	0.0999	0.0028	0.0919	0.0083*
12	0.100	0.1008	0.00414	0.1010	0.0042*	0.1007	0.0037	0.1008	0.00414	0.0905	0.0042*
13	0.800	0.798	0.0061*	0.7991	0.0049	0.7988	0.0050*	0.798	0.0061*	0.8176	0.0077*
21	0.050	0.0506	0.0101	0.0506	0.0102	0.0502	0.009	0.0506	0.0101	0.0597	0.0053*
22	0.050	0.0538	0.0112*	0.0544	0.0121*	0.0536	0.0103*	0.0538	0.0112*	0.0355	0.0098*
23	0.900	0.895	0.0164*	0.895	0.0145*	0.8950	0.0140*	0.8953	0.0164*	0.905	0.0146*
32	0.100	0.0891	0.0408*	0.0881	0.0422*	0.0900	0.039*	0.0891	0.0408*	0.233	0.0374*
33	0.900	0.914	0.0543*	0.9119	0.0421*	0.913	0.0496*	0.9141	0.0543*	0.7669	0.0374*
42	0.200	0.1975	0.0268	0.1952	0.0286	0.1972	0.024	0.1975	0.0268	0.2967	0.0301*
43	0.800	0.808	0.0469	0.8048	0.0287	0.8060	0.0345	0.8080	0.0469	0.7033	0.0301*
53	1.000	1.002	0.059	1.000	0.000	1.000	0.000	1.002	0.059	1.000	0.0000
63	1.000	0.997	0.020	1.000	0.000	1.000	0.000	0.9973	0.0200	1.000	0.0000

Results of Scenario 1. Averages (b_{ij}) and standard deviations (s_{ij}) for offline estimators.

*Significant difference (0.05 level) between b_{ij} and true b_{ij}.

percentage difference between b_{ij} and the true b_{ij} for each movement. In general, the FPP estimator showed lower differences compared with the least squares–based estimators. The normalized recursive least squares estimators produced the highest differences. Figures 2 and 3 show that both the recursive least squares and the normalized recursive least squares estimators were asymptotically unbiased. However, RLSN had a slower convergence than the RLS. This was to be expected, since the constraints had to be satisfied each time period. Figures 4 and 5 show that both the RLS and RLSN were asymptotically consistent since the variances approached zero. Again the RLSN had slower convergence.

Scenario 2

In this scenario, measurement error was added to selected entry and exit counts. It was assumed that the loop detectors at Origins 1 and 3 and Destinations 1 and 3 (see Figure 1) were malfunctioning. The measurement error at each entry and exit point was generated separately by an IMSL subroutine GGNML, such that the variance of the measurement error at Origin 1 and Exit 3 was set to be 1.5 times the mean of the input counts, whereas the variance at Origin 3 and Exit 1 was designed to be equal to the mean of the input counts. With the introduction of measurement error, the conservation of flow was no longer satisfied at each time period (i.e., the difference between the total inputs and total outputs at each time period was not zero). Table 4 gives the averages of the OD estimates and the standard deviations determined across all data sets. The results indicate that all estimators produced biased estimates. The FPP estimator, however, gave the lowest variances compared with other estimators. Furthermore, the unconstrained estimators (OLS, RLS, and FPP) produced estimates that did not satisfy Constraints 1 and 2. Figures 6 and 7 show the effect of measurement error on the asymptomatic properties of RLS and RLSN. Although RLSN had slower convergence to the true value, it did not have a persistent bias as did the RLS. In terms of consistency, Figures 8 and 9 show that both estimators RLS and RLSN had slow convergence to zero compared with the case of no measurement error (Figures 4 and 5). However, the RLS estimator showed faster convergence to zero than did the RLS (at least for movement b_{13}). Although CLS provided the smallest sum of the absolute difference ($\Sigma|b_{ij} - b_{ij}|$), it produced very large percentage differences, particularly for Movements 22, 32, and 42. Table 5 gives the absolute percentage difference ($|\hat{b}_{ij} - b_{ij}|/b_{ij}$) between the estimated and true OD parameters. The FPP estimator produced the second-lowest sum of absolute difference and generally the smallest percentage difference.

CONCLUDING REMARKS

In addressing the problem of estimating freeway OD matrices from sets of input/output counts, several estimators were tested. The fixed-point estimator developed in this paper showed generally lower variances and more accurate estimates compared with four least squares–based estimators. The paper

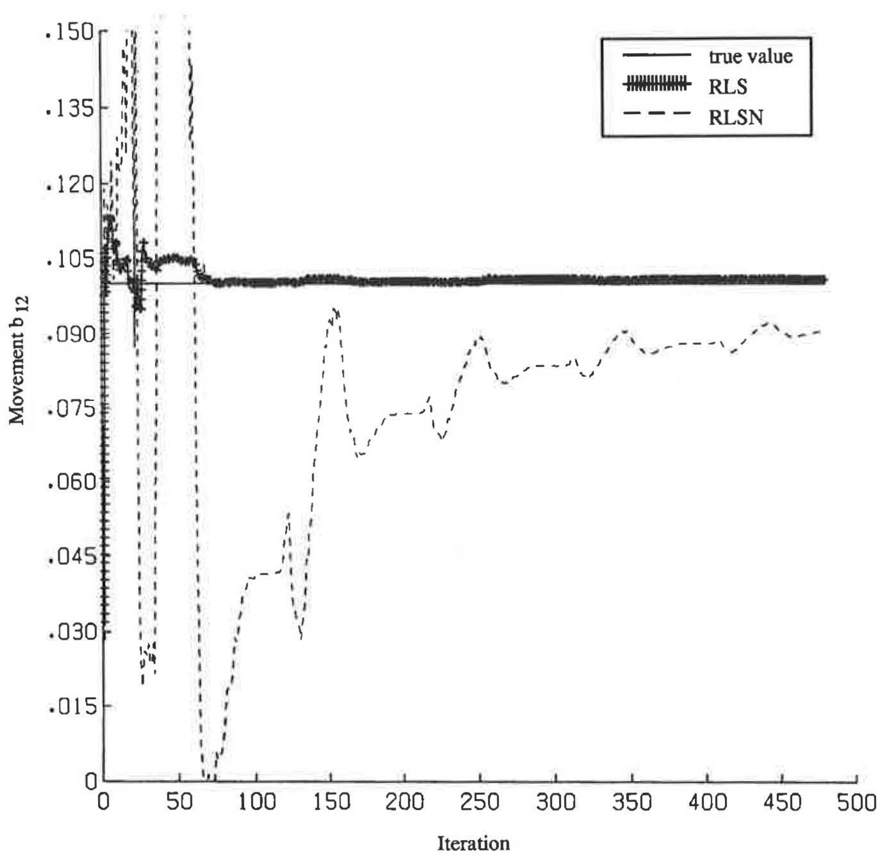

FIGURE 2 Average across simulations of Movement b_{12} computed by two recursive estimators for Scenario 1.

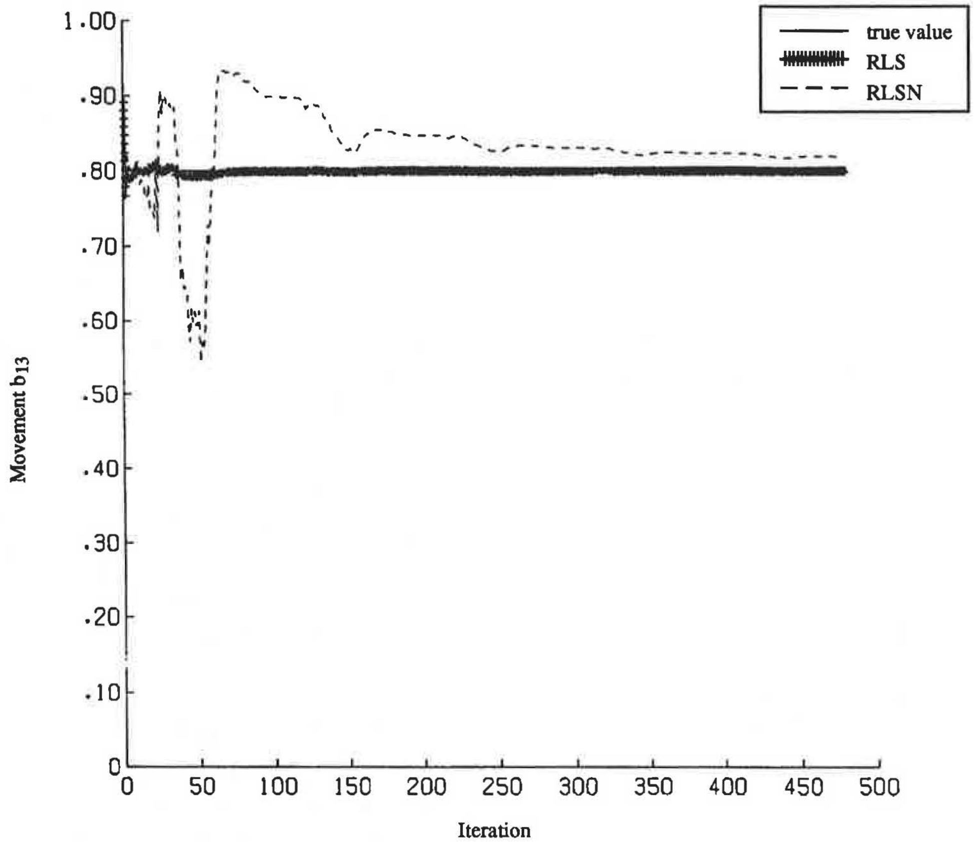

FIGURE 3 Average across simulations of Movement b_{13} computed by two recursive estimators for Scenario 1.

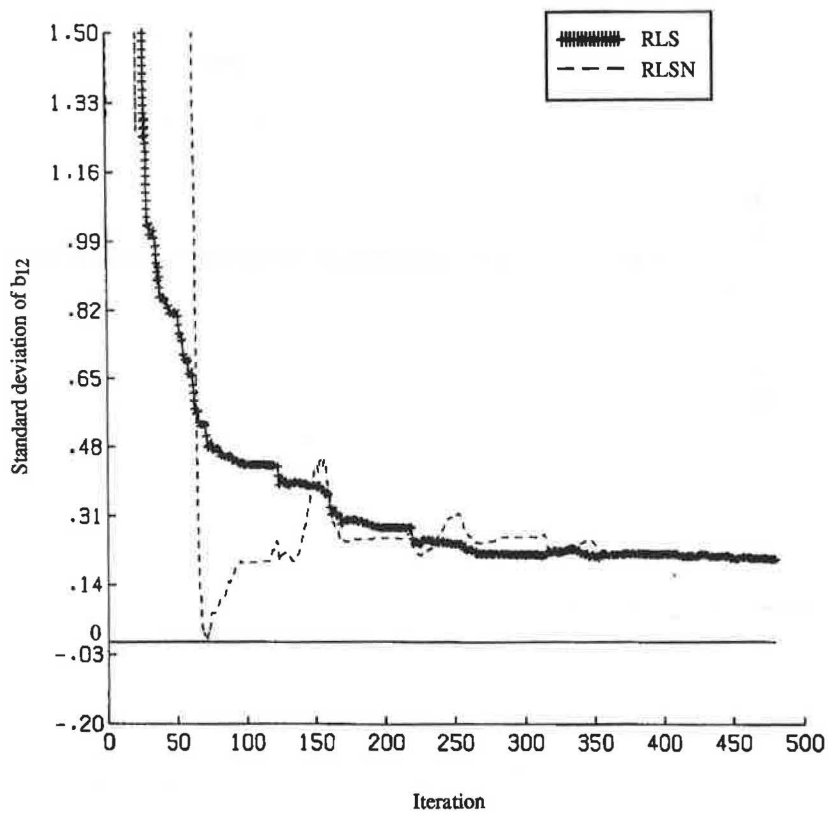

FIGURE 4 Standard deviation across simulations for estimates of Movement b_{12} for Scenario 1.

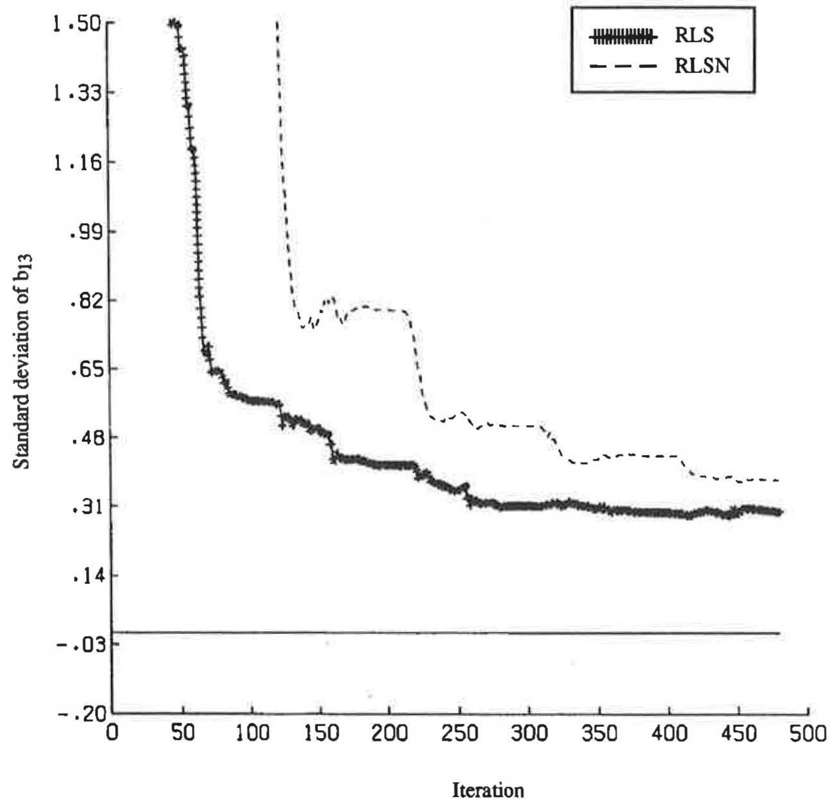

FIGURE 5 Standard deviation across simulations for estimates of Movement b_{13} for Scenario 1.

TABLE 4 Performance of Five Estimators on Simulated Data with Measurement Error at Entry Points 1 and 3 and Exit Points 1 and 3

		OLS		CLS		FPP		RLS		RLSN	
Movement	True b_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	$\hat{b}_{ij}$	s_{ij}	b_{ij}	s_{ij}
11	0.100	0.099	0.0027*	0.105	0.004*	0.098	0.0026*	0.099	0.0027*	0.101	0.015*
12	0.100	0.097	0.004*	0.105	0.008*	0.096	0.003*	0.097	0.004*	0.089	0.005*
13	0.800	0.769	0.022*	0.787	0.012*	0.773	0.018*	0.769	0.022*	0.810	0.014*
21	0.050	0.055	0.009*	0.032	0.013*	0.058	0.009*	0.055	0.009*	0.051	0.017
22	0.050	0.056	0.013*	0.031	0.023*	0.056	0.0102*	0.056	0.013*	0.035	0.019*
23	0.900	0.981	0.055*	0.940	0.031*	0.967	0.052*	0.981	0.055*	0.914	0.022*
32	0.100	0.088	0.038*	0.170	0.086*	0.091	0.030*	0.088	0.038*	0.278	0.108*
33	0.900	0.744	0.191*	0.831	0.086*	0.760	0.162*	0.900	0.191*	0.722	0.108*
42	0.200	0.228	0.032*	0.122	0.063*	0.231	0.027*	0.228	0.032*	0.297	0.063*
43	0.800	0.884	0.165*	0.878	0.064*	1.005	0.129*	0.884	0.165*	0.703	0.063*
53	1.000	1.250	0.261*	1.000	0.001	1.000	0.000	1.250	0.261*	1.000	0.0000
63	1.000	0.993	0.059*	1.000	0.003	1.000	0.000	0.993	0.059	1.000	0.0000

Results of Scenario 2. Averages (b_{ij}) and standard deviations (s_{ij}) for offline estimators.

*Significant difference (0.05 level) between $\hat{b}_{ij}$ and true b_{ij}.

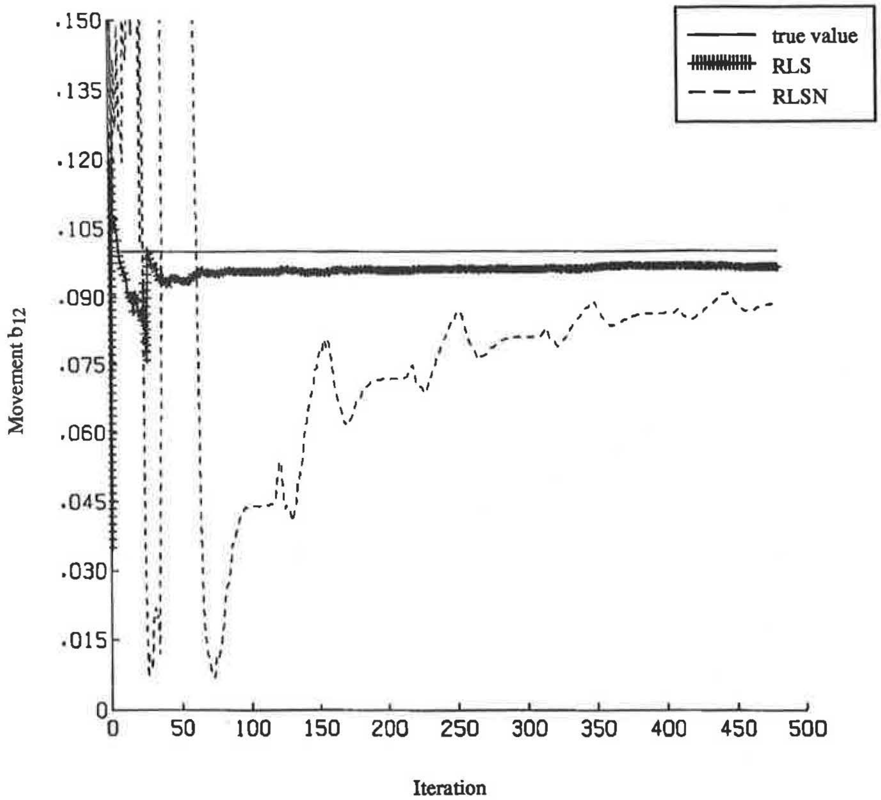

FIGURE 6 Average across simulations of Movement b_{12} computed by two recursive estimators for Scenario 2.

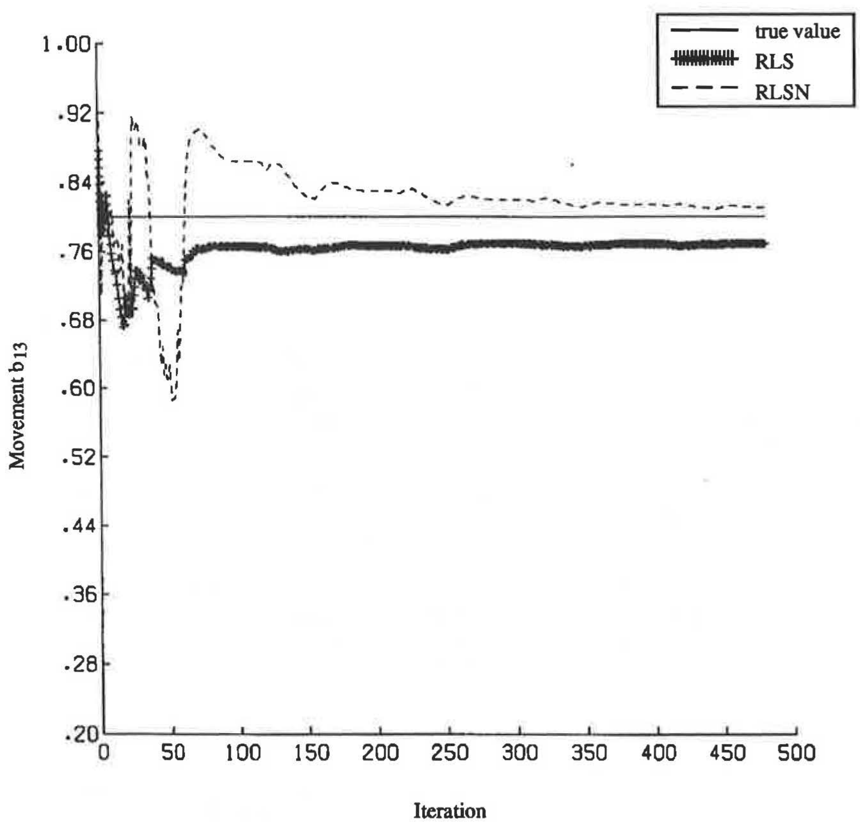

FIGURE 7 Average across simulations of Movement b_{13} computed by two recursive estimators for Scenario 2.

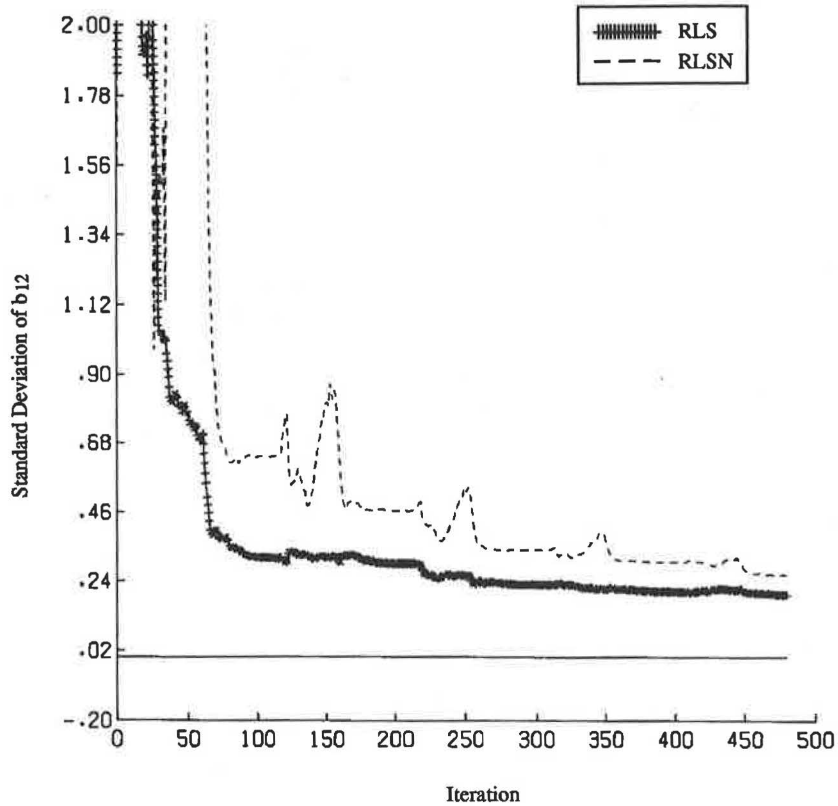

FIGURE 8 Standard deviation across simulations for estimates of Movement b_{12} for Scenario 2.

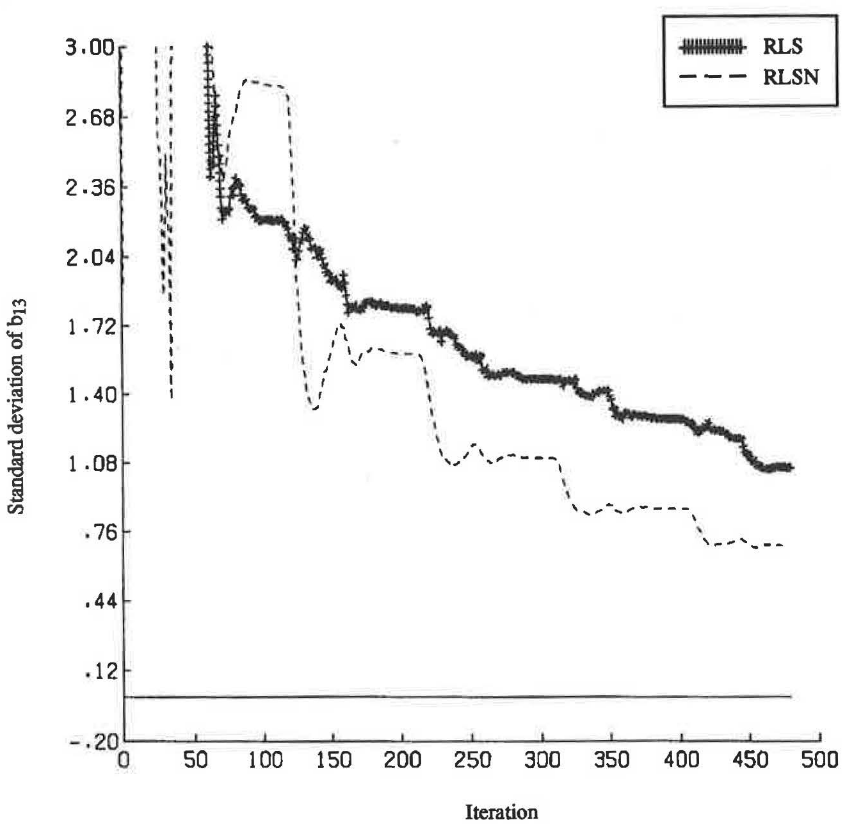

FIGURE 9 Standard deviation across simulations for estimates of Movement b_{13} for Scenario 2.

TABLE 5 Percent Absolute Difference Between $\hat{b}_{ij}$ and True b_{ij} for Scenario 2

Movement	OLS	CLS	FPP	RLS	RLSN
11	1.20	4.50	1.90	1.20	1.20
12	3.50	5.00	4.40	3.50	11.40
13	3.90	1.60	3.40	3.90	1.30
21	9.80	35.60	16.00	9.80	1.60
22	12.40	37.80	12.40	12.40	29.80
23	9.00	4.40	7.40	9.00	1.60
32	12.00	69.60	8.80	12.00	17.80
33	17.30	7.70	15.60	17.30	19.80
42	14.00	39.0	15.50	14.00	48.50
43	10.50	9.80	26.60	10.50	12.20
53	25.00	0.00	0.00	25.00	0.00
63	0.70	0.20	0.00	0.70	0.00

also investigated the effect of erroneous data on the precision of the estimates and the properties of the estimators used by considering two scenarios. The first scenario represented loop detectors that produce accurate traffic counts; in the second scenario, selected entry and exit points were chosen as having faulty or malfunctioning loop detectors. Results indicated that measurement errors severely influenced the precision of OD matrix parameter estimates (percentage errors were significantly increased; Constraints 1 and 2 were no longer satisfied) and the asymptotic properties of these estimators. For example, the RLS became persistently biased when measurement error was introduced. Although these preliminary results are based on simulated scenarios, they highlight the need for theoretical models that account for erroneous data.

With present loop detector technology, erroneous traffic volume counts can be expected from time to time. As illustrated here, the presence of erroneous data severely affects the precision of OD matrix estimates. Therefore, to obtain reasonable estimates, estimators must be capable of handling data with measurement error. Another alternative would be the use of technology that does not inherit the same problems as loop detectors do. Research in this area is active in the United States and Europe. One other alternative currently pursued at different institutions is the detection and diagnosing of erroneous data from loop detectors (*13,14*) before use in forecasting models.

REFERENCES

1. N. Nihan and G. Davis. Application of Prediction-Error Minimization and Maximum Likelihood To Estimate Intersection OD Matrices from Traffic Counts. *Transportation Science*, Vol. 23, No. 2, 1989.
2. H. Van Zuylen and L. Willumsen. The Most Likely Trip Matrix Estimated from Traffic Counts. *Transportation Research*, Vol. 14B, 1980, pp. 281–293.
3. L. Willumsen. Simplified Transport Models Based on Traffic Counts. *Transportation*, Vol. 10, 1981, pp. 257–278.
4. H. Van Zuylen. Some Improvements in the Estimation of an OD Matrix from Traffic Counts. In *Eighth International Symposium on Transportation and Traffic Theory*, University of Toronto Press, Toronto, Ontario, Canada, 1981, pp. 656–671.
5. S. Nguyen. Estimating Origin-Destination Matrices from Observed Flows. In *Transportation Planning Models* (M. Florain, ed.), North-Holland, Amsterdam, the Netherlands, 1984, pp. 363–380.
6. E. Cascetta. Estimation of Trip Matrices from Traffic Counts and Survey Data: A Generalized Least-Squares Estimator. *Transportation Research*, Vol. 18B, 1984, pp. 289–299.
7. M. Maher. Inferences on Trip Matrices from Observations on Link Volumes: A Bayesian Statistical Approach. *Transportation Research*, Vol. 17B, 1983, pp. 435–447.
8. R. Stokes and D. Morris. Applications of an Algorithm for Estimating Freeway Trip Tables. In *Transportation Research Record 976*, TRB, National Research Council, Washington, D.C., 1984, pp. 21–25.
9. C. Hendrickson and S. McNeil. Estimation of Origin-Destination Matrices with Constrained Regression. In *Transportation Research Record 976*, TRB, National Research Council, Washington, D.C., 1984, pp. 25–32.
10. M. Cremer and H. Keller. Dynamic Identification of Flows from Traffic Counts at Complex Intersections. In *Eighth International Symposium on Transportation and Traffic Theory*, University of Toronto Press, Toronto, Ontario, Canada, 1983, pp. 121–142.
11. M. Cremer and H. Keller. A New Class of Dynamic Methods for the Identification of Origin-Destination Flows. *Transportation Research*, Vol. 21B, 1987, pp. 117–132.
12. N. Nihan and G. Davis. Recursive Estimation of Origin-Destination Matrices from Input-Output Counts. *Transportation Research*, Vol. 21B, 1987, pp. 149–163.
13. L. Jacobson, N. Nihan, and J. Bender. Detecting Erroneous Loop Detector Data in a Freeway Traffic Management System. Presented at the 69th Annual Meeting of the Transportation Research Board, Washington, D.C., 1990.
14. L. Chen and A. May. Traffic Detector Errors and Diagnostics. In *Transportation Research Record 1132*, TRB, National Research Council, Washington, D.C., 1987.
15. A. P. Dempster, N. M. Laird, and D. B. Dubin. Maximum Likelihood from Incomplete Data via the EM Algorithm. *J.R. Stat. Soc.*, Ser. B39, 1977, pp. 1–38.
16. J. Traub. *Iterative Methods for the Solution of Equations*. Prentice-Hall, Englewood Cliffs, N.J., 1964.
17. K. Border. *Fixed Point Theorems with Applications to Economic and Game Theory*. Cambridge University Press, Cambridge, 1985.
18. *NAG Fortran Library Manual*. Mark 9, NAG.

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