Estimating Surface Area for Aggregate in the Size Range 1 mm or Larger

James R. Carr, Manoranjan Misra, and James Litchfield

Measures of surface area are available for small particles up to 0.5 mm in diameter, using a calorimeter to measure heat of immersion, and for very small particles using nitrogen adsorption. These methods have limited or no applicability for determining surface area of particles 1 mm in diameter or larger, in part due to the sensitivity of the analytical equipment. An empirical method based on the fractal dimension is described for estimating surface area of particles with diameters greater than or equal to 1 mm. This method is compared to an existing technique to judge the quality of the surface area estimates. For aggregate pieces having isotropic shape, a single perspective is photographed in silhouette to determine the fractal dimension of the particle’s surface, from which surface area is estimated. For an aggregate piece of anisotropic shape, two or three mutually orthogonal perspectives are photographed in silhouette to determine the fractal dimensions of surfaces comprising the particle. Areas are then determined for these surfaces and summed to derive the total surface area for the sample. A practical procedure for estimating surface area is discussed.

Determining surface area for particles no larger than 0.5 mm is possible using several laboratory procedures, such as nitrogen adsorption or measurement of heat of immersion. With nitrogen adsorption, a sample is typically pulverized prior to testing. Using this procedure, for instance, the RJ concrete aggregate (one of the SHRP reference aggregates) has a surface area equal to 1.32 m²/g of sample (1).

Nitrogen adsorption and heat of immersion are methods not applicable to determining the surface area of particles having a diametrical dimension of 1 mm or larger. With respect to heat of immersion, determined using a calorimeter, particles having diameters larger than 0.5 mm are associated with almost no measurable heat of immersion, whereas smaller particles are associated with a significant and measurable heat of immersion. From the amount of heat emitted through immersion, a surface area can be determined for the particle. For particles larger than 0.5 mm, so little heat is emitted that it is not possible to accurately determine surface area.

Instead, an empirical procedure is sought to estimate surface area for particles 1 mm or larger in diameter. Aggregate roughness is usefully described using the fractal dimension (2). A rough aggregate has a greater surface area in comparison to a smooth one for pieces of approximately the same size and shape. Hence, the use of the fractal dimension to estimate surface area is a logical extension of the previous work.

This method based on the fractal dimension, though, is new. Therefore, another surface area estimation method is used as a comparator. The method chosen uses Ferets and Martin diameters (3).

BACKGROUND ON THE FRACTAL DIMENSION

Mandelbrot (4) illustrates the use of fractals in determining the length of the west coast of Great Britain. Beginning at any arbitrarily located point on this coastline, the length to another arbitrarily located point is measured using a ruler of length \( y \), computed as

\[ L = Ny + f \]  \hspace{1cm} (1)

where

\[ L = \text{length between the two points}, \]
\[ N = \text{number of rulers placed end to end between the two points}, \] and
\[ f = \frac{\text{fraction of the ruler length remaining at the end}}{2}. \]

Equation 1 is expressed in a slightly different form:

\[ L = (N + f)y \]  \hspace{1cm} (2)

The reason for expressing Equation 1 in this form will become clear momentarily.

If a shorter ruler is used to repeat the measurement of the length between the two arbitrarily located points, the length, \( L \), computed using Equation 2 increases. The shorter ruler resolves the bays and inlets bridged by the longer ruler. Thus, the scale of the ruler used to measure length is important.

The relationship shown in Equation 2 is expressed using a superscript, \( D \):

\[ L = (N + fy)^D \]  \hspace{1cm} (3)

where \( D \) is the fractal dimension (4). If we rearrange Equation 3 such that

\[ Ly^{-D} = N + fy \]  \hspace{1cm} (4)

then, by taking the log of both sides of Equation 4, \( D \) can be solved as the slope of the plot of \( \log(N + fy) \) versus \( \log(y) \). That is,

\[ \log(L) - D \log(y) = \log(N + fy) \]  \hspace{1cm} (5)
is equivalent to Equation 4. The fractal dimension, $D$, is the absolute value of the slope of the plot of $\log(N + f/y)$ versus $\log(y)$. The value, $\log(L)$, simply shifts the plot along the horizontal axis [associated with $\log(y)$] and hence is not a necessary piece of information when determining the fractal dimension, $D$.

Three experiments to measure the length between two arbitrarily located points on the west coast of Great Britain are presented in Table 1. These results are shown in Figure 1, from which the fractal dimension, $D$, is determined to be 1.3. Experiments similar to that presented in Table 1 are used subsequently to resolve fractal dimensions of silhouettes of concrete aggregate.

**BACKGROUND ON FERETS AND MARTIN DIAMETERS**

Measuring the diameters of an irregular particle is a difficult problem (3). Consequently, the measurement of the projected area of particle onto a plane is a difficult problem. Figure 2 has important relevance to the following discussion.

One way to measure the projected area of an irregular particle is to project the silhouette of the particle onto a piece of cross-hatched paper and then count squares comprising the projection. Alternatively, the projection onto the paper can be cut out, then weighed. Knowing the density and thickness of the paper allows the area of the projection to be computed from the weight.

Alternatives to these two methods involve the use of Ferets or Martin diameters (shown in Figure 2). The Martin statistical diameter bisects the image area (projected silhouette) of the particle, where the direction in which the bisect is measured is the same for all particles. The Ferets diameter is the distance between two opposite sides of the particle's projection, where the distance is measured parallel to a fixed direction (such as the horizontal direction shown in Figure 2). Once either diameter is measured, the surface area is computed as that of a circle having the computed diameter (i.e., the projected area is equated to that of a circle having the same area).

As seen from this discussion, Ferets or Martin diameters are measured somewhat arbitrarily. Moreover, the area of the projection of a particle is equated to that of an ideal circle. What is desired, though, is a measure of surface area; this is a notion that differs from the measure of the area of the particle's projection. However, preparing projections of several perspectives of a particle leads to an approximation of surface area by summing the areas of the different projections.

**TABLE 1** Attempts To Measure the Length of an Arbitrary Section of the West Coast of Great Britain

<table>
<thead>
<tr>
<th>$y$ (km)</th>
<th>$N + f/y$</th>
<th>Length: $(N + f/y) y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>4.55</td>
<td>910 km</td>
</tr>
<tr>
<td>100</td>
<td>10.75</td>
<td>1075 km</td>
</tr>
<tr>
<td>50</td>
<td>28.25</td>
<td>1412 km</td>
</tr>
</tbody>
</table>

**FIGURE 1** Plot of $\log(N + f/y)$ versus $\log(y)$ using the results presented in Table 1. Similar plots (not shown) are used to determine fractal dimensions for the results given in Tables 2, 3, and 4.

**ESTIMATION OF SURFACE AREA OF PARTICLES OF ISOTROPIC SHAPE**

It is evident from the foregoing discussion that the use of Ferets or Martin diameters leads to an approximate measure of surface area. As a proposed alternative, fractal dimensions of the perimeters of silhouettes of particles are used to generalize the formulas for the calculation of surface area for some regular volumes, such as spheres and cubes.

**FIGURE 2** Explanation of the Ferets and Martin diameters.
Isotropy in shape is exemplified by spheres and cubes. The surface area of a sphere is calculated as

$$\text{Area} = 4\pi r^2$$  \hfill (6)

where $r$ is the radius of the sphere. A slice through the sphere, passing through the center, is a circle with a circumference $C$ equal to

$$C = 2\pi r$$  \hfill (7)

If a sphere has a fractal surface area, a photograph of the sphere in silhouette is taken, from which the fractal dimension of the profile of the circumference is determined as previously described for the west coast of Great Britain. The length of the profile is calculated using Equation 3.

Setting Equation 7 equal to Equation 3 gives

$$C = 2\pi r = \left( N + \frac{f}{y} \right) y^D$$  \hfill (8)

which yields an equivalent radius for the sphere:

$$r = \frac{\left( N + \frac{f}{y} \right) y^D}{2\pi}$$  \hfill (9)

Substituting this value for $r$ into the equation for surface area of the sphere gives

$$\text{Area} = 4\pi r^2 = 4\pi \left[ \frac{\left( N + \frac{f}{y} \right) y^D}{2\pi} \right]^2$$  \hfill (10)

which simplifies to

$$\text{Area} = \frac{\left( N + \frac{f}{y} \right)^2 y^{2D}}{\pi}$$  \hfill (11)

which is the estimate of surface area for a sphere having a fractal roughness.

An extension to cubic particles follows the same logic. Given a cube having a dimension, $x$, the exposed surface area is equal to the sum of the exposed surface area of each face. Six faces are exposed, each having a surface area equal to $x^2$. Therefore, the total surface area for the cube is

$$\text{Area} = 6x^2$$  \hfill (12)

For a cube whose faces exhibit fractal roughness, the equivalent dimension, $x$, of the cube becomes

$$x = \frac{\left( N + \frac{f}{y} \right) y^D}{4}$$  \hfill (13)

because, using the photograph of the silhouette, the total length, $L$, is that of the perimeter, and $x$, the dimension of

one side of the cube, is one-fourth of the perimeter. The area of each face of the cube is

$$\text{Area of face} = x^2 = \frac{\left( N + \frac{f}{y} \right)^2 y^{2D}}{16}$$  \hfill (14)

The total surface area of the cube having a fractal surface roughness is

$$\text{Area (total)} = 6\text{(area of face)} = \frac{3\left( N + \frac{f}{y} \right)^2 y^{2D}}{8}$$  \hfill (15)

Several aggregate silhouettes are shown in Figure 3. Fractal dimensions for the profiles and subsequent surface area estimates are presented in Table 2. Also presented is the surface area for each particle estimated using Ferets and Martin diameters. There is good agreement between the two surface area estimation methods for Samples 1 and 2. For the other samples, however, the estimates derived using the fractal dimension method are much greater than those derived using

### TABLE 2 Surface Area Estimates for Aggregate Shown in Figure 3

<table>
<thead>
<tr>
<th>Sample</th>
<th>$D$</th>
<th>$y^*$</th>
<th>$(N + f/y)^*$</th>
<th>Fractal</th>
<th>Ferets/Martin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.043</td>
<td>1.1</td>
<td>8.00</td>
<td>32.87</td>
<td>36.80</td>
</tr>
<tr>
<td>2</td>
<td>1.118</td>
<td>1.1</td>
<td>6.64</td>
<td>23.83</td>
<td>21.08</td>
</tr>
<tr>
<td>3</td>
<td>1.102</td>
<td>1.1</td>
<td>7.77</td>
<td>33.57</td>
<td>18.46</td>
</tr>
<tr>
<td>4</td>
<td>1.084</td>
<td>1.1</td>
<td>9.23</td>
<td>45.99</td>
<td>33.21</td>
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<tr>
<td>5</td>
<td>1.131</td>
<td>1.1</td>
<td>6.55</td>
<td>46.62</td>
<td>22.75</td>
</tr>
<tr>
<td>6</td>
<td>1.082</td>
<td>1.1</td>
<td>8.64</td>
<td>43.84</td>
<td>31.70</td>
</tr>
</tbody>
</table>

*Because fractal dimension, $D$, is listed, all that is required to estimate surface area is one of the rulers, $y$, and corresponding $(N + f/y)$ value; $y$ has units of cm.
Ferets and Martin diameters. Samples 1 and 2 conform reasonably well to an isotropic spherical shape (albeit with distortion), but the other samples are strongly anisotropic in shape. A generic procedure for surface area estimation using the fractal dimension is needed.

**ESTIMATION OF SURFACE AREA FOR AGGREGATE OF ARBITRARY SHAPE**

The foregoing discussion assumes aggregate pieces conform closely to a spherical or cubic form. Few aggregate pieces, except by accident, conform to such shapes. One failing of the foregoing section is the lack of accountability for anisotropic shapes. Photographing the silhouette for only one perspective of a particle is a sufficient characterization of an isotropic form. Two or three mutually orthogonal perspectives are required for anisotropic forms (see Figure 4).

Two of the aggregate pieces shown in Figure 3 are again photographed in silhouette and shown in Figures 5 through 8 to emphasize their different shapes for two orthogonal directions (sufficient characterization for these particles). This set of photographs explains, in part, the discrepancies noted in Table 2 between the two surface area estimation methods; the one based on fractal dimension assumed equidimension for all directions, but a clear anisotropy is noted in the suite of photographs, Figures 5 through 8. This explains why surface area was overestimated by the fractal dimension method.

In Figures 5 and 6, two orthogonal perspectives are shown in silhouette for one aggregate sample. Table 3 gives the fractal dimensions for the two silhouettes. That for the profile featured in Figure 5 is 1.107. The length of this profile is calculated using Equation 3. Using the results shown in Table 3, the length is 11.8 cm. The average thickness, t, of the aggregate particle perpendicular to this profile (this average thickness is determined using Figure 6) is 1.5 cm. The surface area for that portion of the aggregate particle orthogonal to the silhouette shown in Figure 5 is equal to Lt, which, for this sample, is 11.8 cm \( \times \) 1.5 cm = 17.7 cm\(^2\).

In Figure 6, the silhouette of an orthogonal perspective is shown with a fractal dimension of 1.0 for the left face and 1.0 for the right face. The fractal length of each face is determined from Table 3 to be

\[
L(\text{left face}) = \left( N + \frac{t}{y} \right) y^D = 4.5 \text{ cm}
\]

(16)

**FIGURE 5** Silhouette of one of the aggregate pieces shown in Figure 3 (this piece is Number 4 in Figure 3).

**FIGURE 6** Silhouette of the side view of the aggregate piece shown in Figure 4. This aggregate piece has anisotropic shape and is roughly modeled as a fractal disk.
FIGURE 7 Silhouette of one of the aggregate pieces shown in Figure 3 (this piece is Number 3 in Figure 3).

\[ L_{(rt \ face)} = \left( N + \frac{f}{y} \right) y^D = 4.0 \text{ cm} \]  \hspace{1cm} (17)

The average dimension orthogonal to these profiles for each face, left and right, is measured from Figure 5 and is found to be 2.2 cm. The surface area for the left face is determined to be \( L_{(lft \ face)} \times t = 4.5 \text{ cm} \times 2.2 \text{ cm} = 9.9 \text{ cm}^2 \), and for the right face is determined to be \( L_{(rt \ face)} \times t = 8.8 \text{ cm}^2 \). The total surface area for the particle is now estimated to be 17.7 + 9.9 + 8.8 = 36.4 cm². This compares very well to the surface area determined using Ferets and Martin diameters, 33.2 cm², and is much less than what was estimated before (46.0 cm²; see Table 2). This emphasizes the need to account for shape anisotropy.

An alternative approach is to view the aggregate particle of Figures 5 and 6 as a fractal disk. Such a form has two faces and an edge. A coin is a rough approximation (see Figure 4). The surface area of such a disk is calculated as

\[ \text{Area} = 2\pi r^2 + 2\pi r t = 2\pi (r + t) \]  \hspace{1cm} (18)

Incorporating the fractal dimension in this equation gives:

\[ \text{Area} = \left( N + \frac{f}{y} \right) y^D \left[ \left( N + \frac{f}{y} \right) y^D \right] + t \]  \hspace{1cm} (19)

because

\[ 2\pi r = \left( N + \frac{f}{y} \right) y^D \]  \hspace{1cm} (20)

Using this equation, applied to the sample of Figures 5 and 6, an estimate of surface area is obtained:

\[ 11.8 \text{ cm} \left[ \frac{11.8 \text{ cm}}{2\pi} + 1.5 \text{ cm} \right] = 39.9 \text{ cm}^2 \]  \hspace{1cm} (21)

which compares well with that estimated above, 36.4 cm². In this case, though, only the profile of Figure 5 is used, along with the knowledge that the sample is roughly described as a disk.

Another application is reviewed using the aggregate particle shown in Figures 7 and 8. The fractal dimensions for the silhouette profiles are shown in Table 4. The total length of the profile shown in Figure 7 is 9.8 cm, calculated using Equation 3. This value is the circumferential length for this sample. Like the sample shown in Figures 5 and 6, this sample is modeled as a disk. Using the equation developed above for fractal disks, surface area is estimated to be

\[ 9.8 \text{ cm} \left[ \frac{9.8 \text{ cm}}{2\pi} + 0.6 \text{ cm} \right] = 21.2 \text{ cm}^2 \]  \hspace{1cm} (22)

which compares well to that estimated using Ferets and Martin diameters, 18.5 cm².

TABLE 3 Fractal Dimension Calculation Results for Figures 5 and 6

<table>
<thead>
<tr>
<th>( y ) (mm)</th>
<th>( (N + f/y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 5:</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9.2</td>
</tr>
<tr>
<td>5</td>
<td>21.0</td>
</tr>
<tr>
<td>2</td>
<td>47.0</td>
</tr>
<tr>
<td>D = 1.107</td>
<td></td>
</tr>
<tr>
<td>Figure 6:</td>
<td></td>
</tr>
<tr>
<td>Left face:</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>6.9</td>
</tr>
<tr>
<td>2</td>
<td>22.0</td>
</tr>
<tr>
<td>D = 1.0</td>
<td></td>
</tr>
<tr>
<td>Right face:</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>6.0</td>
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<tr>
<td>2</td>
<td>19.5</td>
</tr>
<tr>
<td>D = 1.0</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 8 Silhouette of the side view of the aggregate piece shown in Figure 6. This aggregate piece is roughly modeled as a fractal disk.
TABLE 4 Fractal Dimension Calculation Results for Figures 7 and 8

<table>
<thead>
<tr>
<th>Figure 7:</th>
<th>y (mm)</th>
<th>((N + fy))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>16.4</td>
</tr>
<tr>
<td></td>
<td>D = 1.102</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure 8:</th>
<th>Left Face:</th>
<th>y (mm)</th>
<th>((N + fy))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>14.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D = 1.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Right Face:</th>
<th>y (mm)</th>
<th>((N + fy))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>D = 1.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note on the Foregoing Examples

Tables 3 and 4 list fractal dimensions for silhouettes shown in Figures 5 through 8 and have been introduced above. Regarding Figures 6 and 8, these tables distinguish between the left faces and right faces of the silhouettes, a distinction that was not made with respect to Figures 5 and 7. This distinction was made to emphasize that the aggregate particles in these figures are considered to be fractal disks and that the top surface of the disk may have a different roughness in comparison to the bottom surface. For the particle photographed in silhouette in Figure 6, the left and right faces have the same fractal dimension, so there is no real need to distinguish between these faces for this particle. Alternatively, the particle photographed in Figure 8 is associated with different fractal dimensions for the left and right faces, hence the distinction is more important for this particle.

Error Involved in Calculating Surface Area

The foregoing examples show the error involved in the calculation of surface area when the shape of the aggregate particle is not correctly characterized. By assuming that a particle having anisotropic shape characteristics is adequately characterized as having isotropic shape, an error in estimating surface area approaching 40 percent was realized in the examples.

Aside from misjudging the conformance of a particle to an isotropic morphology, the comparison of Equation 11 to Equation 14 shows that the characterization of a spherical particle as a cube, or vice versa, results in an error of approximately 16 percent: 

\[ \frac{1}{2}(1/\pi)(3/8) = 0.84; 1.0 - 0.84 = 0.16 = 16\% \]

In this calculation, \( a \) is the major semiaxis and \( b \) is the minor semiaxis.

ADDITIONAL SHAPES USEFUL FOR CHARACTERIZING AGGREGATES

Some additional shapes are useful for characterizing aggregates. For each, the fractal dimension is incorporated.

Some aggregates are pyramidal in form. Given a pyramid having four sides, each side an equilateral triangle of dimension \( x \), the height of each triangular face is

\[ \text{height} = \sqrt{3x/2} \]  

(23)

The area of each triangular face is equal to \( xh/2 \), where \( h \) is the height; therefore

\[ \text{Area(one face)} = \sqrt{3x^2/4} \]  

(24)

Because the pyramid has four such faces, total surface area is equal to

\[ \text{Area(total)} = \sqrt{3x^2} \]  

(25)

For a pyramid having fractal roughness, the profile of the silhouette of such a form can be described using fractal dimension. The length of the profile is \( (N + fy) yD \). Therefore, \( x \) is equal to one-third of this (the profile is triangular in shape), or \( x = (N + fy) yD/3 \). Substituting this into the equation for surface area of the pyramid gives

\[ \text{Area(total)} = \frac{\sqrt{3}(N + fy)^2 yD}{9} = \frac{\left(N + fy\right)^2 yD}{3\sqrt{3}} \]  

(26)

Therefore, the area of the profile is

\[ \text{Area(profile)} = \frac{\sqrt{3}(N + fy)^2 yD}{9} = \frac{\left(N + fy\right)^2 yD}{3\sqrt{3}} \]  

(26)

A generalization of the cube is the rectangular volume having unequal dimension. For such a shape, the length of the perimeter is not that useful because two prominent directions, \( x \) and \( z \), are apparent, one smaller than the other (see Figure 4). Let \( x \) and \( z \) each be fractal lengths determined using the procedures described above. Then surface area is determined as

\[ \text{Area} = 4xz + 2x^2 = 2x(2z + x) \]  

(27)

Whereas a sphere is discussed previously as a model for aggregate shape, an ellipsoid has not been discussed. There are two types of ellipsoids, prolate (like the shape of a football) and oblate (like the shape of the earth). The surface area for a prolate ellipsoid is calculated as

\[ \text{Area} = 2\pi b^2 + 2\pi \frac{ab}{e} \sin^{-1}e \]  

(28)

where \( e \), the eccentricity, is calculated as

\[ e = \frac{\sqrt{a^2 - b^2}}{a} \]  

(29)

In this calculation, \( a \) is the major semiaxis and \( b \) is the minor semiaxis.
For a prolate spheroid, a top or side perspective looks like an ellipse, and the end perspective looks like a circle. The circumference of the elliptical perspective is herein designated as \( p \) and is calculated as
\[
p = 2\pi \sqrt{\frac{a^2 + b^2}{2}} \tag{30}
\]
and the circumference, \( c \), of the circular perspective is simply
\[
c = 2\pi b \tag{31}
\]
If \( p \) and \( c \) are fractal, then
\[
p = \left( N + \frac{\ell}{y_p} \right) y_p^p \tag{32}
\]
where the \( p \) subscript indicates that the fractal dimension pertains to the elliptical perspective. For the circular perspective,
\[
c = \left( N + \frac{\ell}{y_c} \right) y_c^p \tag{33}
\]
Once \( p \) and \( c \) are determined, the values are used to calculate equivalent values for \( a \) and \( b \) using the equations for \( p = f(a, b) \) and \( c = f(b) \) shown above. Once equivalent values for \( a \) and \( b \) are calculated, the eccentricity, \( e \), is calculated, and finally the surface area is calculated.

For an oblate spheroid, the surface area is
\[
\text{Area} = 2\pi a^2 + \pi \frac{b^2}{e} \log_e \frac{1 + e}{1 - e} \tag{34}
\]
and equivalent values of \( a \) and \( b \) are found using the fractal dimension calculation results.

A wide range of shapes are now generalized to fractal form: spheres, cubes, pyramids, rectangular volumes, disks, and ellipsoids. With respect to the disk, the circular form of the disk can be generalized to an ellipse, substituting the equation for \( p \) above for the circumference.

**PRACTICAL IMPLEMENTATION**

A procedure for calculating the surface area of aggregate based on gradation is shown elsewhere (5). The percentages of material passing various sieve sizes is shown by the gradation. Multiplying these percentages by surface area factors for each sieve size, then summing these products, yields the estimate for the surface area of the sample in m²/kg or ft²/lb.

An example (5) is shown in Table 5. The surface area factors shown in this table assume that aggregate particles are smooth and spherical in shape. This paper shows, however, that many particles are not perfectly smooth and often show a fractal roughness. One way to incorporate the fractal dimension into this procedure is as follows:

1. Determine the gradation for an aggregate sample.
2. Determine the surface area as shown in Table 5.
3. Select one particle for each sieve size and determine its fractal dimension as described earlier in this paper or by Carr et al. (2).
4. Recalculate the surface area for each sieve size as
\[
\text{Area}^D = \text{Area}^b \cdot D^D \tag{35}
\]
where \( D^D \) is the fractal dimension for particles retained on that sieve. The fifth and sixth columns of Table 5 show hypothetical fractal dimensions and associated estimates of area.

This procedure offers a very practical way to incorporate fractal dimension in the determination of surface area for aggregate as a function of the weight of the aggregate.

**TABLE 5 Calculation of Surface Area Using a Gradation (5)**

<table>
<thead>
<tr>
<th>Sieve Size</th>
<th>Percent Passing</th>
<th>Surface Area Factor</th>
<th>Surface Area&lt;sup&gt;b&lt;/sup&gt;</th>
<th>( D^D )</th>
<th>Surface Area&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4 in.</td>
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<sup>a</sup>Hypothetical fractal dimension values; Surface Area<sup>b</sup> is raised to the \( D^D \) power to yield Surface Area<sup>c</sup>.

<sup>b</sup>Surface area in ft²/lb, calculated by multiplying the percent passing the sieve by the surface area factor.

<sup>c</sup>Surface area in ft²/lb
SUMMARY

Surface area estimated using the fractal dimension approach is often larger than that estimated using Ferets and Martin diameters. This is expected. Fractal dimensions capture information related to surface irregularity. These dimensions are used herein to modify equations that determine the euclidean surface area of ideally shaped particles. Ferets and Martin diameters are determined somewhat arbitrarily. There can be considerable error, therefore, when determining surface area using these diameters. Determining surface area using fractal dimensions is not such an arbitrary process. Fractal dimensions are determined through attempts to measure the circumferential length of a particle's projection. Reference direction, while necessary for the Ferets or Martin diameters, is not required.

No method, other than estimation, is available for determining the surface area of large particles (large, in this paper, is 1 mm or larger in diameter). The difficulty of measuring the surface area of large particles is emphasized in the following:

The difficulty in stating just what is meant by the "surface area" of a solid, in one aspect, can be illustrated by considering the somewhat analogous question of what is meant by the "length" of a particular section of coastline. (6)

This paradox of coastline length, of course, is addressed by Mandelbrot (4) and serves as the basis for defining fractal geometry. That Adamson (6) equates the surface area problem to the coastline length problem is further evidence that fractals are ideal for estimating surface area.

Finally, it is important to emphasize how the accurate determinate of surface area of concrete aggregate benefits the transportation industry. An aggregate with a rough surface texture will bond better with a binder, such as asphalt, which will result in a stronger mix (5). An aggregate with a rough surface texture has a larger surface area than a similarly shaped and sized aggregate having a smooth surface texture. Hence, surface area relates to surface texture; surface texture has relevance for strength of a concrete mix design. A greater surface area (a rough surface texture), though, requires that more binder be added to a mix to aid workability. This, however, relates to the increased strength of the mix: more binder is required because the rough texture is associated with a larger surface area available for bonding. Hence more binder is required to cover the larger surface area. Because there is no established method (other than an indirect inference derived from the results of strength tests on a mix) for deter-

CONCLUSION

In lieu of chemical measures of surface area, such as nitrogen adsorption and heat of immersion, empirical methods can yield accurate estimates of surface area. For particles larger than 0.5 mm, the empirical estimates appear to be all that is available for determining surface area.

The fractal dimension is easily implemented in the procedure for determining surface area using the gradation for a sample of aggregate. Therefore, it is shown here how surface area can be computed for a single particle and can be calculated for an entire sample of aggregate by using the gradation for the sample.

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REFERENCES


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