Left-Turn Adjustment Factors for Saturation Flow Rates of Shared Permissive Left-Turn Lanes

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For capacity analysis of signalized intersections, a left-turn adjustment factor is used in the 1985 *Highway Capacity Manual* (HCM) to account for the effect of left turns on saturation flow rates. When shared permissive left-turn lanes are the subject of analysis, the HCM uses a theoretical model to determine the related adjustment factors. Questions concerning the reliability of this model have been raised, and a recent study sponsored by the Federal Highway Administration has suggested that the HCM model be replaced by a set of new models. The new models, however, have serious flaws. An improved model that provides logical explanations of the causal relationships between the left-turn adjustment factor and its contributing factors is described. The contributing factors include opposing flow rate, number of opposing lanes, flow rate in the lane adjacent to the shared lane, proportion of left turns in the shared lane, proportion of left turns in opposing flow, proportion of opposing vehicles arriving in red interval, cycle length, green interval, and change interval. A numerical example is given to illustrate the applications of the improved model.

In the capacity analysis of signalized intersection, the 1985 *Highway Capacity Manual* (HCM) (*1*) requires that the saturation flow rate of a lane or a lane group be determined from the following formula:

$$S = S_o N F f_{LT} \tag{1}$$

where

- S = saturation flow rate (vphg) (vehicles per hour of effective green interval);
- S_o = ideal saturation flow rate, taken to be 1,800 vphg per lane;
- N = number of lanes in a lane group;
- F = the product of seven adjustment factors related respectively to lane width, heavy vehicles, approach grade, parking, blocking effects of local buses, area type, and right turns; and
- f_{LT} = adjustment factor for left turns.

When an analysis involves shared permissive left-turn lanes, the determination of the left-turn adjustment factor becomes a rather difficult problem.

A shared permissive left-turn lane refers to a lane from which opposed left-turn vehicles and vehicles of other directional movements can move into the intersection in a permissive left-turn signal phase. The presence of opposed left turns in such a lane disrupts vehicular movements and complicates the determination of f_{LT}. The 1985 HCM relies on a theoretical model to deal with this problem. The HCM concept in estimating f_{LT} has been used by Levinson (*2*) to develop a model for estimating the capacity of shared left-turn lanes. Questions concerning the reliability of the HCM model have been raised, however, and a set of new models was recommended in a recent study sponsored by FHWA (*3*). The new models were developed from regression analysis of field data. They are easy to use but do not properly account for the causal relationships between f_{LT} and its contributing factors. For example, f_{LT} can be expected to vary with opposing flow rate, yet the model recommended for single lane approach assumes implicitly that f_{LT} is independent of opposing flow rate. This flow may be partially responsible for the fact that the estimates obtained on the basis of that model have little correlation with observed values (*3*).

To provide an alternative, this paper describes an improved model that explains logically the causal relationships between f_{LT} and its contributing factors. This model deals with the left-turn adjustment factors for shared lane only (i.e., $N = 1$); it is developed on the basis of theoretical reasonings, field data, and computer simulation. A numerical example is provided to illustrate the applications of the model.

RESEARCH APPROACH

Equation 1 indicates that, if the saturation flow rate for a given F can be determined under a wide range of conditions, then a data base can be established for modeling f_{LT}. Because the vehicular movements in a shared left-turn lane can be interrupted by blocked left-turn vehicles, their saturation headway cannot be meaningfully defined and measured in the field to estimate the corresponding saturation flow rate. This problem can be overcome by taking advantage of the following relationship for $N = 1$ and for conditions represented by $F = 1.0$:

$$f_{LT} = \frac{S}{S_o} = \frac{CQ_{\max}}{G_e S_o} \tag{2}$$

where

- $Q_{\max}$ = capacity of a shared lane for $F = 1.0$ (level, 12-ft-wide approach lane and ideal conditions for other factors related to F),
- C = cycle length (sec), and
- G_e = effective green (sec).

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For a given combination of C and G_e, Equation 2 shows that finding f_{LT} is the same as finding Q_{max}. Q_{max} can be determined by estimating the number of vehicles per cycle that can move out of an intersection. This expected number of departures includes the following components: early left turns, M_1; unblocked straight-through departures, M_2; departures in leftover green, M_3; and departures after green interval, M_4. Details of these components will be discussed later.

The modeling of Q_{max} is divided into two parts. The first part concerns a basic flow pattern as shown in Figure 1 (*top*). The notations used in this figure are defined as follows: Q_{01} = flow rate in the inside opposing lane (vph), Q_{02} = flow rate in the outside opposing lane (vph), and Q_a = flow rate in the lane adjacent to the shared lane (vph). The opposing lanes of the basic pattern do not contain left-turn vehicles. Such a situation may exist at a T-intersection or at a four-leg intersection where left turns from the inside opposing lane are prohibited. The maximum number of opposing lanes considered in this study is two.

The second part of the modeling effort involves the development of a mechanism to transform a flow pattern that contains left turns in Q_{01} into an equivalent basic flow pattern. This transformation involves the conversion of Q_{01} into an equivalent straight-through opposing flow $(Q_{01})_e$ for the estimation of the capacity of a shared lane. The transformation process is shown in Figure 1 (*bottom*). In this figure, P_o represents the proportion of left turns in the inside opposing lane.

Q_{max} is considered to be a function of such variables as Q_{01}, Q_{02}, Q_a, P_o, cycle length C, green interval G, signal change interval Y, proportion of left turns in shared lane P_s, and so forth. This study employs a combination of theoretical considerations, field data, and computer simulation to identify the causal relationships between Q_{max} and its contributing variables. The simulation model used in this study is a microscopic model developed at Clarkson University. This model can realistically simulate the stochastic movements of vehicles at intersections controlled by a variety of traffic signals. Details of this model are described elsewhere (*4*). An example comparison of the simulation outputs and field observations is given in Table 1.

One concern in modeling Q_{max} and its related f_{LT} is whether the effects of signal coordination should also be considered.

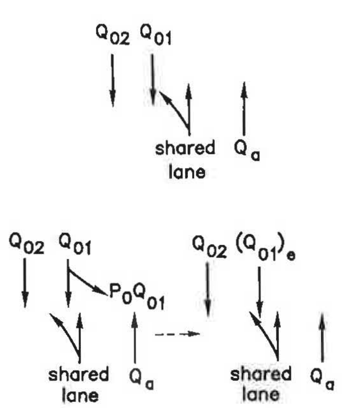

FIGURE 1 Basic flow pattern (*top*) and pattern transformation (*bottom*).

A simulation analysis reveals that the capacity of a shared left-turn lane may be affected by the presence of prominent cyclic platoons in the opposing lanes as a result of signal coordination. Nevertheless, as shown in Figure 2, the capacities of shared left-turn lanes when the arrivals are random tend to lie mostly within 50 vph of the values for coordinated signal operations. Such discrepancies are not alarmingly large in the context of capacity analysis. To avoid unnecessary complications, the arrivals will be assumed to be random for the purpose of modeling Q_{max} and f_{LT}. To account partially for the fact that arrivals are not necessarily random, the proportion of arrivals in red interval is included as a variable in the modeling process.

MODEL FOR BASIC FLOW PATTERN

Given the four components of departures per cycle, M_1, M_2, M_3, and M_4, the capacity of a shared lane for F = 1.0 can be determined as

$$Q_{max} = \left(\sum_{n=1}^{4} M_n\right) \frac{3{,}600}{C} \tag{3}$$

The modeling of each of the departure components is discussed.

Early Left Turns, M_1

Early left turns refer to those leading left-turn vehicles in various cycles that turn in front of the leading opposing vehicle shortly after green onset. Given the proportion of left turns in a shared left-turn lane (P_s) and the probability of early left turn α for a leading left-turn vehicle, the expected number of early left turns per cycle can be estimated as

$$M_1 = \alpha P_s \tag{4}$$

The values of α are often less than 0.5. Therefore, when P_s is much smaller than 1.0, M_1 becomes negligibly small.

Unblocked Straight-Through Departures, M_2

After the green onset, those straight-through vehicles ahead of the first left-turn vehicle can move out without facing the possibility of being blocked. To facilitate the estimation of such departures, the directional movements of the vehicles in a shared left-turn lane are classified into a series of events as shown in Figure 3. In this figure, K_2 represents the expected maximum number of straight-through vehicles that can move into the intersection before the green interval G expires. The value of K_2 can be determined as

$$K_2 = \frac{G - L_s}{H_s} \tag{5}$$

where L_s is lost time due to starting delays (sec), and H_s is saturation headway when only straight-through vehicles are present (sec).

TABLE 1 Comparison of Observed and Simulated Signal Operations

		Average Green, sec				Average Stopped Delay, sec/veh	
		Observed		Simulated			
Case	Phase	Mean	S.D.[1]	Mean	S.D.	Observed	Simulated
A	1 2	6.5 32.4	2.6 25.2	5.1 30.3	2.1 24.6	7.7[2]	6.6
B	1 2 3	33.8 5.4 24.0	18.2 2.1 0.0	31.9 5.1 24.0	17.6 2.3 0.0	11.1[3]	10.7
C	1 2	27.8 12.4	12.8 6.3	27.6 13.0	12.1 6.3	14.7[3]	14.9
D	1 2	18.8 10.7	9.2 5.8	17.5 9.5	8.1 5.2	Not available	Not available
E	1 2	29.3 20.5	7.6 0.7	30.2 20.2	9.1 0.9	42.3[4]	43.7
F	1 2 3	12.9 9.2 33.6	3.9 4.1 7.3	12.6 9.4 32.1	3.5 4.0 6.7	14.0[3] 3.1[5]	13.5 2.7
G	1 2 3	10.8 30.0 19.1	6.0 0.8 0.5	12.1 30.0 19.2	3.7 0.0 0.0	25.1[6]	25.6
H	1 2 3	13.4 30.0 19.1	3.4 0.2 0.7	13.4 30.0 19.9	3.0 0.1 2.0	41.9[7]	41.6

[1]S.D. = Standard deviation
[2]single-lane flow with right turns and left turns
[3]exclusive left-turn flow
[4]shared-permissive left-turn flow (85% left turns)
[5]exclusive right-turn flow with right-turn-on-red
[6]shared-permissive left-turn flow (93% left turns)
[7]shared-permissive left-turn flow (95% left turns)

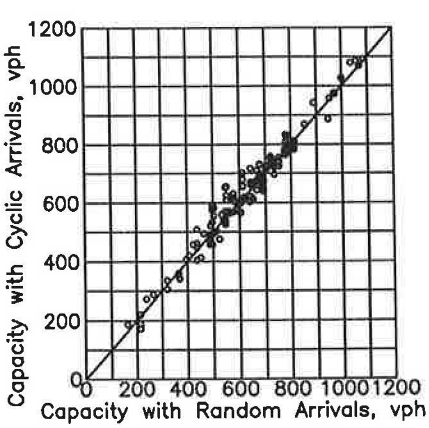

FIGURE 2 Capacities with cyclic platoon arrivals in opposing lanes versus capacities with random arrivals.

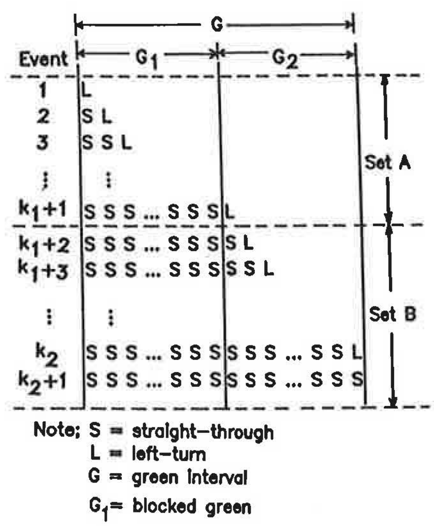

FIGURE 3 Classification of arrival sequences.

The expected number of unblocked straight-through departures per cycle can be estimated as

$$M_2 = \begin{cases} K_2 \quad \text{if } P_s = 0 & (6a) \\ \sum_{n=0}^{K_2-1} n(1 - P_s)^n P_s + K_2 (1 - P_s)^{K_2} = \\ [1 - P_s - (1 - P_s)^{K_2}]/P_s \quad \text{if } P_s > 0 & (6b) \end{cases}$$

Departures in Leftover Green, M_3

In Figure 3, the green interval G is divided into two components: G_1 and G_2. G_1 is the average portion of the green interval consumed by the queueing vehicles in the opposing lanes before these vehicles cross the conflicting point. The conflicting point can be considered to be a representative location at which a leading left-turn vehicle that is blocked would come to a stop to wait for a suitable gap in the opposing flow. On the basis of G_1, the events shown in the figure are grouped into Set A and Set B. Set A events allow a maximum of K_1 straight-through vehicles to move into the intersection before a left-turn vehicle becomes the leading vehicle in the shared lane. The number of straight-through vehicles ahead of the first left-turn vehicle in Set B ranges from $K_1 + 1$ to K_2. The value of K_2 is determined from Equation 5, and K_1 can be determined in a similar manner as

$$K_1 = \frac{G_1 - L_s}{H_s} \geq 0 \tag{7}$$

After the departures of unblocked straight-through vehicles, a mix of left-turn and straight-through vehicles can move out by using leftover green intervals. For each of the Set A events, the leftover green interval is G_2. For the Set B events, the leftover green intervals depend on the portion of the green interval already consumed by the unblocked straight-through vehicles.

To facilitate the estimation of G_1 and the leftover green intervals, let us define the following additional variables:

- i = inside opposing lane ($i = 1$) or outside opposing lane ($i = 2$),
- S_{0i} = saturation flow rate of the ith opposing lane (vph),
- x_i = number of queueing vehicles in the ith opposing lane at green onset,
- m_{0i} = average number of queueing vehicles in the ith opposing lane at green onset,
- q_{0i} = arrival rate in the ith opposing lane during green and signal change intervals (vph),
- q_{12} = $q_{01} + q_{02}$ = sum of arrival rates q_{01} and q_{02} (vph),
- R_o = proportion of arrivals in red in opposing lanes, and
- β = time required for queueing vehicles to go from the stop line until they clear the conflicting point (sec).

On the basis of these definitions, m_{0i} and q_{0i} can be determined as

$$m_{0i} = \frac{Q_{0i} R_o C}{3{,}600} \tag{8}$$

and

$$q_{0i} = \frac{Q_{0i}(1 - R_o)\, C}{G + Y} \tag{9}$$

If there are x_i queueing vehicles in the ith opposing lane at the green onset, the portion of the green interval t_i consumed by these and subsequent queueing vehicles before they all cross the conflicting point may be estimated as

$$t_i = \begin{cases} 0 \quad \text{if } x_i = 0 & (10a) \\ \dfrac{3{,}600 x_i + L_s q_{0i}}{S_{0i} - q_{0i}} + L_s + \beta \leq G \quad \text{if } x_i > 0 & (10b) \end{cases}$$

Therefore, if $t_2 \leq t_1$, the queuing vehicles in the inside opposing lane would govern the time required to discharge all queueing vehicles in a given cycle. Otherwise, the queueing vehicles in the outside lane would govern. In other words, the queuing vehicles in the inside lane would govern if the following inequality holds:

$$\frac{3{,}600 x_1 + L_s q_{01}}{S_{01} - q_{01}} \geq \frac{3{,}600 x_2 + L_s q_{02}}{S_{02} - q_{02}} \tag{11}$$

This inequality can be rewritten as

$$x_2 \leq \frac{1}{3{,}600}\left[\frac{3{,}600 x_1 + L_s q_{01}}{S_{01} - q_{01}}(S_{02} - q_{02}) - L_s q_{02}\right] \tag{12}$$

Let X be the largest integer of x_2 that satisfies Equation 12 and $P(x_i)$ be the probability of having x_i queueing vehicles in Lane i at the green onset. For a given x_1, the portion of the green interval consumed by opposing queueing vehicles would be t_1 if x_2 has a value equal to or less than X. On the other hand, the portion of the green interval consumed by opposing queueing vehicles would be t_2 if x_2 has a value larger than X. Therefore, the expected value of G_1 can be estimated as

$$G_1 = \sum_{x_1=0}^{\infty} P(x_1)\left[t_1 \sum_{x_2=0}^{X} P(x_2) + \sum_{x_2=X+1}^{\infty} t_2 P(x_2)\right] \tag{13a}$$

If the arrivals in red are random, the $P(x_i)$ in Equation 13a can be determined from the following Poisson distribution:

$$P(x_i) = \frac{m_{0i}^{x_i} e^{-m_{0i}}}{x_i!} \tag{13b}$$

When only one opposing lane is present and arrivals are random, Equation 13a can be reduced to

$$G_1 = \frac{3{,}600 m_{01}}{S_{01} - q_{01}} + \left(\frac{L_s q_{01}}{S_{01} - q_{01}} + L_s + \beta\right)(1 - e^{-m_{01}}) \leq G \tag{14}$$

If two opposing lanes are present, Equation 13a can be approximated by a simplified equation. Let Q_H be the larger one of Q_{01} and Q_{02}. And, if $q_{01}/S_{01} \geq q_{02}/S_{02}$, let $m_H = m_{01}$,

$q_H = q_{01}$, $S_H = S_{01}$, $S_L = S_{02}$, and $q_L = q_{02}$. Otherwise, let $m_H = m_{02}$, $q_H = q_{02}$, $S_H = S_{02}$, $S_L = S_{01}$, and $q_L = q_{01}$. Then the simplified equation can be written as

$$G_1 = \frac{3{,}600 m_H}{S_H - q_H} + \left(\frac{L_s q_H}{S_H - q_H} + L_S + \beta\right)(1 - e^{-m_H}) + 2\gamma_1 e^{\gamma_2 - \gamma_3(1-\gamma_1)} \leq G \tag{15a}$$

where

$$\gamma_1 = \frac{q_L\, S_H}{q_H\, S_L} \tag{15b}$$

$$\gamma_2 = \frac{(0.042 + 0.01R_o)Q_H C}{3{,}600} \tag{15c}$$

and

$$\gamma_3 = e^{\frac{.08 Q_H C}{3{,}600}} - 1 \tag{15d}$$

Equation 15a is the same as Equation 14 when only one opposing lane is present.

Twelve samples of field data were collected from three intersections to test Equations 13a and 15a. The observed values of G_1 and the estimates obtained from these equations are given in Table 2. The discrepancies between the observed and the estimated values were small.

Two other models have been recommended for estimating G_1 in the HCM and in the FHWA study (2). For comparison, the estimated values of G_1 obtained from these models are also given in Table 2. The model recommended in the FHWA study measures G_1 in terms of the time required for the front end of the last queueing vehicle to reach the stop line after green onset. Therefore, the estimates obtained from this model are increased by an amount equal to β. The reference line used in the model given in the HCM is unknown, and therefore no adjustments are made.

Given G_1, the leftover green interval for Set A events is $G_2 = G - G_1$. To facilitate further analysis, this leftover green interval is modified into an effective leftover green T_a, where

$$T_a = \begin{cases} G - G_1 & \text{if } G_1 \geq L_s \quad (16a) \\ G - G_1 - L_s\left(1 - \dfrac{G_1}{L_s}\right) & \text{if } G_1 < L_s \quad (16b) \end{cases}$$

This equation implies that if G_1 is greater than or equal to the lost time L_s, there is no need to adjust for starting delays because the lost time is completely accounted for by G_1. When G_1 is smaller than L_s, however, G_1 may not fully account for the starting delays associated with the vehicles in the shared lane. Equation 16b provides an adjustment for such a situation.

For the determination of the average leftover green interval for Set B events, one may proceed with the determination of the probability of an event being in Set B. This probability is $(1 - P_s)^{K_1+1}$, where P_s is the proportion of left turns in the shared lane. Therefore, for the Set B events and $0 < P_s < 1$, the average number of straight-through vehicles $\overline{K}_b$ that can move out before a left-turn vehicle becomes the leading vehicle in the shared lane can be determined as

$$\overline{K}_b = \frac{1}{(1 - P_s)^{K_1+1}} \left\{ \left[\sum_{n=K_1+1}^{K_2-1} n(1 - P_s)^n P_s \right] + K_2(1 - P_s)^{K_2} \right\} = \frac{1}{P_s}\left[1 + K_1 P_s - (1 - P_s)^{K_2 - K_1}\right] \tag{17}$$

TABLE 2 Observed and Estimated Values of G_1

						G_1				
Case	Q_{01}	Q_{02}	C	G	R_o	Actual	Eq. 13a	Eq. 15a	HCM	FHWA
1	323	266	81.8	23.0	.79	20.0	20.0	20.2	13.0	23.0
2	278	246	81.8	23.0	.88	19.3	19.2	20.4	11.3	23.0
3	350	360	81.8	23.0	.75	20.4	21.6	23.0	16.4	23.0
4	388	0	70.6	30.0	.67	19.1	-	18.9	11.2	19.2
5	415	0	72.2	30.0	.68	19.4	-	20.6	12.6	20.3
6	502	0	71.2	30.0	.62	22.4	-	23.5	15.9	21.0
7	494	0	72.3	30.0	.71	24.7	-	24.4	16.0	24.6
8	440	0	68.9	30.0	.52	19.9	-	18.4	12.6	16.2
9	356	0	80.0	22.4	.80	21.5	-	21.0	14.2	22.2
10	274	0	103.7	19.2	.92	19.2	-	19.2	15.2	19.2
11	274	0	82.8	17.2	.72	17.2	-	17.2	11.8	17.2
12	218	0	84.0	18.2	.78	13.4	-	14.8	9.1	16.4

Note: L_s = 2.0 sec (assumed value)
β = 2.2 to 3.8 sec
Saturation flow rate: 1500 to 1750 vphg
Lost time per phase = 4 sec (assumed for HCM model)

The corresponding values of $\overline{K}_b$ for $P_s = 0$ and $P_s = 1$ are

$$\overline{K}_b = \begin{cases} K_2 & \text{if } P_s = 0 \quad (18a) \\ 0 & \text{if } P_s = 1 \quad (18b) \end{cases}$$

The average portion of the green interval consumed by these $\overline{K}_b$ vehicles is approximately $\overline{K}_b H_s + L_s$. The corresponding effective leftover green interval T_b for the Set B events becomes

$$T_b = G - \overline{K}_b H_s - L_s \tag{19}$$

With the exception of the last event shown in Figure 3, a left-turn vehicle becomes the leading vehicle in the shared lane after unblocked straight-through vehicles have moved out. This leading vehicle has to use the gaps in the opposing flow to move out. Assuming that a waiting left-turn driver will only accept those gaps longer than τ sec, the average number of gaps J that will be rejected before a gap is accepted can be estimated as

$$J = \sum_{n=0}^{\infty} nZ(h \leq \tau)^n[1 - Z(h \leq \tau)] \tag{20}$$

where $Z(h \leq \tau)$ is the probability that a gap h in the opposing flow is less than or equal to τ.

It can be shown that, for random arrivals, the average number of rejected gaps can be approximated as

$$J = e^{\frac{q_{12}\tau}{3{,}600}} - 1 \tag{21}$$

The value of τ can be considered to be equal to the median of the lengths of accepted gaps. Typical values of τ are between 4.5 and 5.5 sec. For such values of τ, the average length of each rejected gap can be approximated as $\tau/2$ without incurring significant errors in estimating the capacity of a shared left-turn lane. On the basis of this approximation, a waiting left-turn driver will wait an average of $J\tau/2$ sec before accepting a gap. After a decision is made to accept a gap, it will take an additional δ sec for the left-turn vehicle to cross the conflicting point and for the next vehicle to move up. Typical values of δ are between 2.0 and 2.5 sec.

Let H_x represent the expected portion of the green interval consumed by the first left-turn vehicle. Then, H_x can be determined as

$$H_x = \frac{\tau}{2}\left(e^{\frac{q_{12}\tau}{3{,}600}} - 1\right) + \delta \tag{22}$$

After the first left-turn vehicle has moved out, the vehicle following can be either a straight-through or a left-turn vehicle. The expected time H_y needed by either of such vehicles to move out is not amenable to simple analytical modeling. Nevertheless, when the opposing flow does not exist, the saturation headway H_o of the vehicles in the shared lane can realistically be estimated as

$$H_o = (1 - P_s)H_s + P_s H_e \tag{23}$$

where H_e is saturation headway only when unopposed left-turn vehicles are present and H_s is saturation headway when only straight-through vehicles are present.

When opposed left turns exist, the average departure headway can be expected to exceed H_o and to increase with the proportion of left turns in the shared lane P_s and the opposing flow rate q_{12}. A logical model characterizing this relationship between H_y, P_s, and q_{12} is

$$H_y = H_o e^{A\left(\frac{q_{12}}{100}\right)^B} \tag{24a}$$

For random arrivals, simulation reveals that the coefficients A and B in this equation can be estimated as

$$A = 0.18P_s^{0.68} \tag{24b}$$

and

$$B = 1.02P_s^{-0.15} \tag{24c}$$

The first left-turn vehicle in a Set A event consumes H_x sec of the effective leftover green T_a, and the subsequent vehicles consume an average of H_y sec each. Thus, the expected number of departures W_a related to Set A events is

$$W_a = \begin{cases} \dfrac{T_a}{H_x} & \text{if } T_a \leq H_x \quad (25a) \\ 1.0 + \dfrac{T_a - H_x}{H_y} & \text{if } T_a > H_x \quad (25b) \end{cases}$$

Similarly, the expected number of departures W_b related to Set B events is

$$W_b = \begin{cases} \dfrac{T_b}{H_x} & \text{if } T_b \leq H_x \quad (26a) \\ 1.0 + \dfrac{T_b - H_x}{H_y} & \text{if } T_b > H_x \quad (26b) \end{cases}$$

Set A events and Set B events account for a total probability of $1 - (1 - P_s)^{K_1+1}$ and $(1 - P_s)^{K_1+1}$, respectively. Therefore, the total expected departures during the effective leftover green in a cycle is

$$M_3 = W_a[1 - (1 - P_s)^{K_1+1}] + W_b(1 - P_s)^{K_1+1} \tag{27}$$

Departures After Green Interval, M_4

For capacity analysis of signalized intersections, the 1985 HCM assumes implicitly that two blocked vehicles can move out of the intersection after the green interval expires. On the basis of this assumed condition, simulation data generated in this study show that the following equation can provide reasonable estimates of M_4:

$$M_4 = 1.3 + 0.0033\, e^{-0.007GP_s^{0.2}} q_{12} \leq 2.0 \tag{28}$$

This equation implies that M_4 varies from 1.3 to 2.0 vehicles. The value of M_4 is 1.3 vehicles when opposed left turns do not exist (P_s = 0.0 or q_{12} = 0.0), and it reaches 2.0 vehicles when blocked vehicles are present in virtually every cycle after the green interval expires.

STRAIGHT-THROUGH EQUIVALENT OF Q_{01}

The present of left-turn vehicles in the inside opposing lane can increase the number and the size of the gaps usable to the drivers in a shared left-turn lane. Therefore, when the proportion of the left-turn vehicles in the opposing lane increases, the capacity of a shared left-turn lane also increases. The increase in capacity can be affected by several other factors. This phenomenon is shown in Figure 4 on the basis of simulation data. By comparing such data as shown in this figure with data for an inside opposing flow Q_{01} that contains no left turns, one can determine the straight-through equivalent $(Q_{01})_e$ of Q_{01}. An example of $(Q_{01})_e$ expressed as a ratio to Q_{01} is shown in Figure 5.

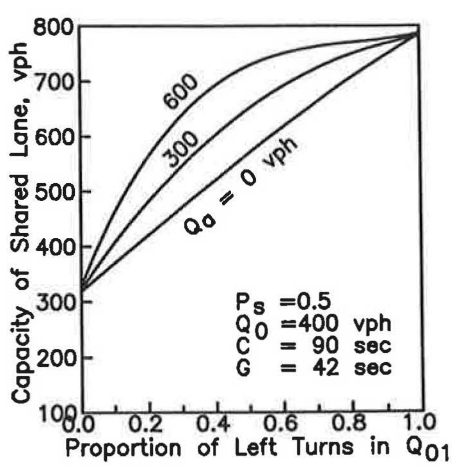

FIGURE 4 Effects of left turns in Q_{01} on capacity of shared left-turn lanes.

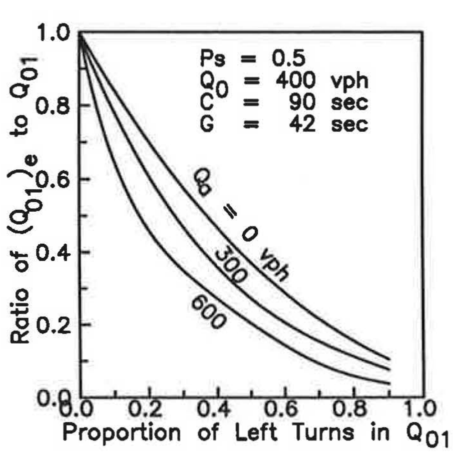

FIGURE 5 Characteristic relationships between Q_{01} and its straight-through equivalent $(Q_{01})_e$.

In general, the relationship between $(Q_{01})_e$ and Q_{01} can be represented by

$$(Q_{01})_e = Q_{01}(1 - \beta_1 P_o)e^{-\beta_2 P_o} \tag{29a}$$

where β_1 can be treated as a constant coefficient with a value of 0.97 and β_2 is a function of several variables.

To identify the relationship between β_2 and the influencing variables, a very large number of simulation runs were performed to develop a data base. On the basis of these simulation data, an analysis was carried out to isolate the effects of each variable on β_2. This effort produced a set of equations for determining β_2.

To facilitate the determination of β_2, let us define two functions, δ_1 and δ_2, as follows:

$$\delta_1 = -\left(e^{1.39\frac{G+Y}{C}} - 1\right)P_s \tag{29b}$$

and

$$\delta_2 = \left(0.0006 + 0.00233\frac{G+Y}{C} + 0.0021P_s\right)Q_a \tag{29c}$$

Then, for $Q_{01} \leq 400$ vph, the value of β_2 is

$$\beta_2 = 1.5\, e^{-2.7P_s} + \frac{0.9Q_{01}}{400}\, e^{\delta_1+\delta_2} \tag{29d}$$

And, for $Q_{01} > 400$ vph, the value of β_2 is

$$\beta_2 = 1.5\, e^{-2.7P_s} + 0.9\, e^{\delta_1+\delta_2} + \frac{Q_{01}-400}{400}\left(4.5 - 3.6\frac{G+Y}{C} - 0.5\, P_s\right) \tag{29e}$$

The straight-through equivalent of each vehicle in Q_{01} (i.e., $(Q_{01})_e/Q_{01}$ as determined from Equation 29a through 29e) has several characteristics that are worth noting. First, larger Q_a and P_o increase the chance of an opposing left-turn vehicle being blocked and, thus, allow more vehicles in a shared left-turn lane to move out. Under such conditions each opposing vehicle becomes less of a factor affecting the capacity of a shared left-turn lane. This is the reason why, as shown in Figure 5, $(Q_{01})_e/Q_{01}$ decreases with Q_a and P_o. An increase in the opposing flow Q_{01} has similar effects. In contrast, $(Q_{01})_e/Q_{01}$ increases with P_s and $(G + Y)/C$.

APPLICATIONS

In general, the applications of the analytical model described involves the transformation of Q_{01} into a straight-through equivalent, the determination of Q_{max} for the basic flow pattern resulting from the transformation, and the use of Equation 2 to determine f_{LT}. A numerical example is given in Table 3 to illustrate the applications of the model. The example involves a flow pattern that has Q_a = 400 vph, Q_{01} = 200 vph with P_o = 0.2, Q_{02} = 350 vph, R_o = 0.32, and P_s = 0.8. The related signal control has a cycle length C of 50 sec, a green interval G of 30 sec for the permissive left-turn phase,

TABLE 3 Estimation of $Q_{\max}$ and f_{LT}—An Example

A. Transformation of Q_{01} into $(Q_{01})_e$

β_1 = 0.97 (in Eq. 29a); δ_1 = -1.26 (Eq. 29b); δ_2 = 1.55 (Eq. 29c);
β_2 = 0.77 (Eq. 29d); $(Q_{01})_e$ = 138 vph (Eq. 29a)

B. Estimation of Q_{max} [set Q_{01} to $(Q_{01})_e$ = 138 vph]

Determination of M_1
M_1 = 0.16 veh/cycle (Eq. 4)

Determination of M_2
K_2 = 14 (Eq. 5); M_2 = 0.25 veh/cycle (Eq. 6b)

Determination of M_3
m_{01} = 0.61 (Eq. 8); m_{02} = 1.56 (Eq. 8); q_{01} = 138 (Eq. 9);
q_{02} = 350 (Eq. 9); q_{12} = 488; In Eqs. 15a through 15d, $S_{01} = S_{02}$ = 1,800;
m_H = 1.56; $S_H = S_L$ = 1,800; $q_H = q_{02}$ = 350; $q_L = q_{01}$ = 138; $Q_H = Q_{02}$ = 350;
γ_1 = 0.39; γ_2 = 0.22; γ_3 = 0.475; G_1 = 8.5 (Eq. 15a); T_a = 21.5 (Eq. 16a);
K_1 = 3.3 (Eq. 7); $\bar{K}_b$ = 4.6 (Eq. 17); T_b = 18.8 (Eq. 19); H_x = 5.5 (Eq. 22);
H_o = 2.08 (Eq. 23); A = 0.155 (Eq. 24b); B = 1.055 (Eq. 24c);
H_y = 4.7 (Eq. 24a); W_a = 4.40 (Eq. 25b); W_b = 3.83 (Eq. 26b);
M_3 = 4.40 veh/cycle (Eq. 27)

Determination of M_4
M_4 = 2.6 > 2.0 (Eq. 28); set M_4 = 2.0 veh/cycle

Determination of Q_{max}
Q_{max} = 490 vph (Eq. 3)

C. Estimation of f_{LT}

f_{LT} = 0.45 (Eq. 2 with G_e = G = 30 sec)

and a change interval of 4 sec. The saturation flows for straight-through movements and unopposed left turns are, respectively, 1,800 vphg (H_s = 2.0 sec) and 1,700 vphg (H_e = 2.1 sec). In addition, the following parameters are used: τ = 5.5 sec, δ = 2.5 sec, β = 2.5 sec, α = 0.2, and L_s = 2.0 sec. The model gives an estimated $Q_{\max}$ of 490 vph and a f_{LT} of 0.45. In comparison, direct simulation gives a $Q_{\max}$ of 442 vph.

Over a wide range of conditions, the values of $Q_{\max}$ estimated from the analytical model and those determined from direct simulation are mostly within 50 vph of each other. This characteristic is shown in Figure 6. Figure 7 further shows the ability of the model to estimate $Q_{\max}$ and the related f_{LT} when a lane is changed from a straight-through-only lane (P_s = 0.0) to a shared left-turn lane (0.0 < P_s < 1.0) and, finally, to an exclusive left-turn lane (P_s = 1.0).

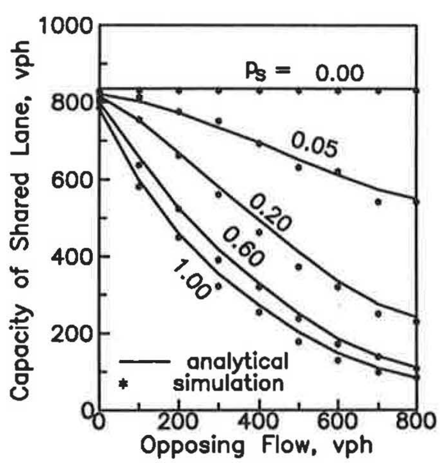

FIGURE 6 Capacities estimated from analytical model versus capacities determined from direct simulation.

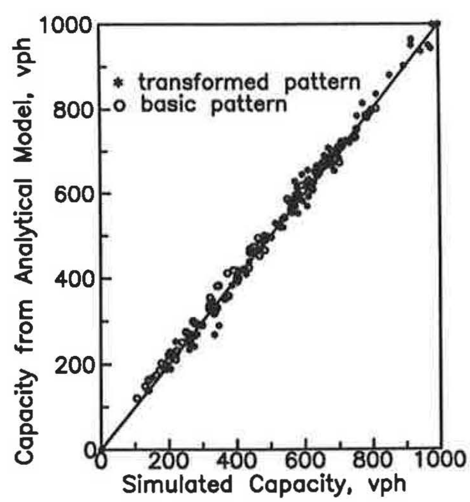

FIGURE 7 Simulated capacities and estimates obtained from analytical model for basic flow patterns with 90-sec cycle and 42-sec green interval.

CONCLUSIONS

The left-turn adjustment factor for a shared left-turn lane is a complex function of a number of variables. Reflecting this complexity, the analytical model developed in this study is much more complicated than the HCM and FHWA models. The added sophistication enables the resulting model to provide better explanations of the causal relationships between the adjustment factor and its contributing factors.

In comparison with elaborate microscopic simulation, the analytical model developed in this study can yield equally realistic estimates. Field data may be collected in future studies to test and modify the constant coefficients associated with the model.

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