Oversaturation Delay Estimates with Consideration of Peaking

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A deterministic oversaturation queueing model that uses a generalization of the peak hour factor concept of the U.S. *Highway Capacity Manual* (HCM) as a simple variable demand model is described. The model is used to explore several issues related to oversaturation models. In particular, the relationship between the delay measurement methods (queue sampling and path trace) and the delay definitions used in the corresponding analytical delay models is investigated with a view to level of service assessment and performance prediction. The differences in delay definitions and delay measurement methods are negligible for undersaturated conditions (low to medium v/c ratios). However, as flows approach capacity (high v/c ratios below capacity) and exceed capacity (v/c ratio greater than 1), the selection of the duration of the flow period, delay definition, and delay measurement method affects delay estimates significantly. Substantial differences in delay and queue estimates are found between the cases of peak flow and maximum delay periods regardless of the delay measurement method. The use of the average delay experienced by individual vehicles in a maximum delay period creates problems in system performance analysis. A delay definition based on a maximum delay period reveals an inconsistency in relation to delays measured in the field. Whereas the HCM recommends that field delays be measured in the peak flow period, the maximum delay period does not coincide with the peak flow period. It is therefore important that the delay definition implied by the present HCM delay formula for signalized intersections be clarified.

The U.S. *Highway Capacity Manual* (HCM) (*1*) qualifies the signalized intersection delay equation given in Chapter 9 as:

> The delay equation may be used with caution for up to (a degree of saturation of) 1.2, but delay estimates for higher values are not recommended. Oversaturation, i.e. $x > 1.0$, is an undesirable condition that should be ameliorated if possible.

However, from a congestion management viewpoint, it is desirable to be able to predict oversaturation delays without any limitation.

A paper by Akçelik (*2*), which discussed the x^2 factor in the second (random plus oversaturation) term of the HCM delay model, is related to this issue. For background information on delay models in general, and the HCM delay equation in particular, the reader is referred to McShane and Roess (*3*).

Messer (*4*) analyzed oversaturation delays in relation to the justification of the x^2 factor. Through subsequent private communication with Messer, it is understood that the x^2 factor is intended to convert the delay in the peak flow period (15 min in the HCM) to a peak delay value that is the maximum delay observed sometime during or after the peak flow period.

This paper explores the issues related to oversaturation models by highlighting differences between various delay definitions (delay during the peak flow period versus a maximum delay period, and delay measured in the specified period versus delay experienced by all vehicles arriving during the specified period) and the corresponding delay measurement methods (queue sampling and path trace).

A deterministic (nonrandom) oversaturation queueing model is presented that uses a generalization of the peak hour factor concept of the HCM as a simple variable demand model. Consideration is given to the choice of the duration of the peak flow period and to the average flow rates and degrees of saturation in the peak and nonpeak flow periods.

A numerical example is given to demonstrate the effects of different delay definitions and the choice of the peak flow period on estimates of delay and queue statistics.

NOTATION

Symbol	*Definition*
T	Duration of the total flow period
T_p	Duration of the peak flow period in the total flow period ($0 < T_p \leq T$)
T_o	Duration of the oversaturation period (time from the start of the peak flow period until the queues clear), $T_o = (1 - \alpha)x_pT_p/(1 - \alpha x_p)$
q_p	Average flow rate in the peak flow period (during T_p)
q_n	Average flow rate in the nonpeak flow period (during $T - T_p$)
q_a	Average flow rate during the total flow period (during T)
α	The ratio of nonpeak and peak flow rates, $\alpha = q_n/q_p$
c_p	Peak period capacity throughout the oversaturation period (T_o)
c_n	Nonpeak period capacity outside the oversaturation period ($T - T_o$)
x_p	Peak period degree of saturation (v/c ratio), $x_p = q_p/c_p$
αx_p	Degree of saturation during the remainder of the oversaturation period following the peak flow period ($T_o - T_p$), $\alpha x_p = q_n/c_p$
x_n	Nonpeak period degree of saturation outside the oversaturation period ($T - T_o$), $x_n = q_n/c_n$
PFF	Peak flow factor: the ratio of average flow rates in the total and peak flow periods, PFF $= q_a/q_p$
PHF	Peak hour factor: special case of PFF where the total flow period T is 1 hr, PHF $= q_a/q_p$
PTF	Peak time factor: the ratio of durations of the peak and total flow periods, PTF $= T_p/T$
y	A variable that defines the delay period. This is the time from the start of the peak flow period to the start of a floating delay period of duration T_p in T (for $y = 0$, the

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Symbol	Definition
	delay period is the peak flow period, and $y = y_m$ corresponds to the maximum delay period)
y_m	The value of y that gives the maximum value of delay (total or average) for any floating delay period of duration T_p in T
D_y	Total oversaturation delay for the delay period as defined by variable y
d_y	Average oversaturation delay for the delay period as defined by variable y
N_{sy}	Queue size at the start of the delay period as defined by variable y
N_{ey}	Queue size at the end of the delay period as defined by variable y
N_{ay}	Average overflow queue size for the delay period as defined by variable y

Flow and capacity (q, c) are in vehicles per hour (vehicles per second), total delay (D) is in vehicle-hours (vehicle-seconds), average delay (d) is in hours (seconds) per vehicle, and queue size (N) is in vehicles.

ISSUES AND DEFINITIONS

When demand flow of traffic in a lane (or lane group) at an intersection exceeds the capacity by a large margin as represented by a high degree of saturation (volume/capacity ratio $x >> 1.0$), overflow queues develop and persist over a considerable period of time. In such oversaturated conditions, stochastic variations in demand flows have minimal influence on the system operation, and a simple deterministic input-output queueing model is adequate for representing the resulting queueing phenomenon.

Deterministic oversaturation models are key predictors of delays and queues under highly congested conditions. Such models are also important in defining continuum models that allow for stochastic variations in demand flows, have time-dependent characteristics, and apply to undersaturated as well as oversaturated conditions.

A continuum model (i.e., an entire delay or queue length curve) with time-dependent characteristics can be obtained by means of the coordinate transformation method (*5,6*). This process essentially shifts the steady-state (stochastic) queueing model from its vertical asymptote to a time-dependent deterministic asymptote as shown in Figure 1. The resulting time-dependent model incorporates both random and oversaturation delays for high degrees of saturation. Therefore, the positioning of the time-dependent asymptote has profound implications for delay and queue estimation around capacity, which are the most relevant operating conditions in an intersection design context (*7*).

The deterministic oversaturation delay function provides a lower bound of delay for oversaturated conditions, which should apply independent of traffic control (e.g., signalized or unsignalized) or arrival characteristics (e.g., random or platooned).

Several definitional issues arise in deriving equations that express delay and queue statistics in an oversaturation queueing model: (*a*) Is the delay measurement method used in the field consistent with the delay models used in operational analysis? (*b*) Which combination of time period and delay measurement method should be used for level of service (LOS) assessment?

With regard to the first point, two basic methods can be identified. The HCM recommends that field delays be measured using a periodic queue sampling process (at 10- to 20-sec intervals). Total delay is then estimated as the area under the queue profile. Average delay is computed by dividing the total delay by the number of vehicle arrivals during the study interval. On the other hand, the path-trace method measures individual vehicle delays from arrival to departure time, even if the latter occurred beyond the observation period. Delay models used in Australia (*2,6,8*) are consistent with the path-trace method. The queue sampling and path-trace methods of delay measurement are compared in Figures 2a and 2b.

The second and key issue is concerned with the selection of a combination of time period and delay measurement method for the purpose of LOS assessment. Messer (*4*) points out that the maximum vehicle delays in an oversaturated period

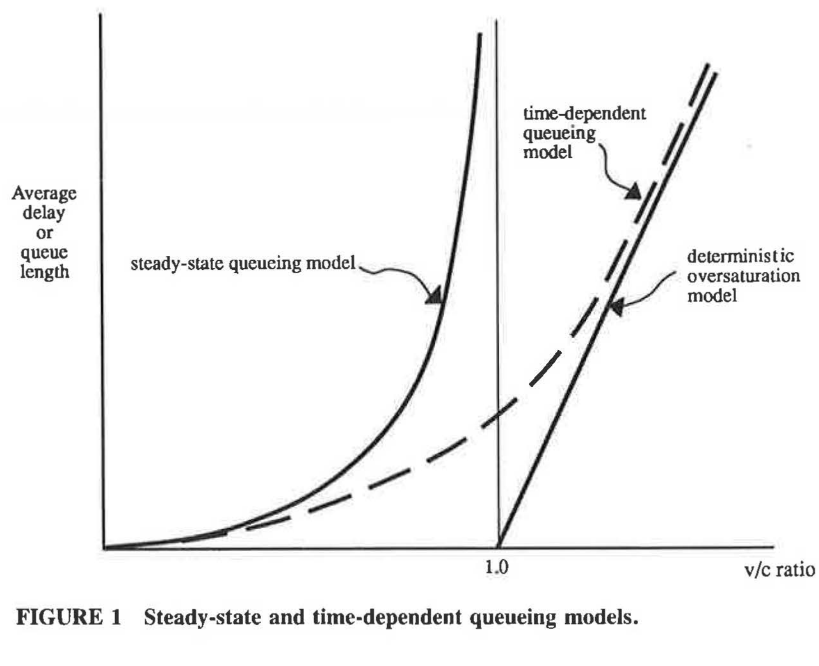

FIGURE 1 Steady-state and time-dependent queueing models.

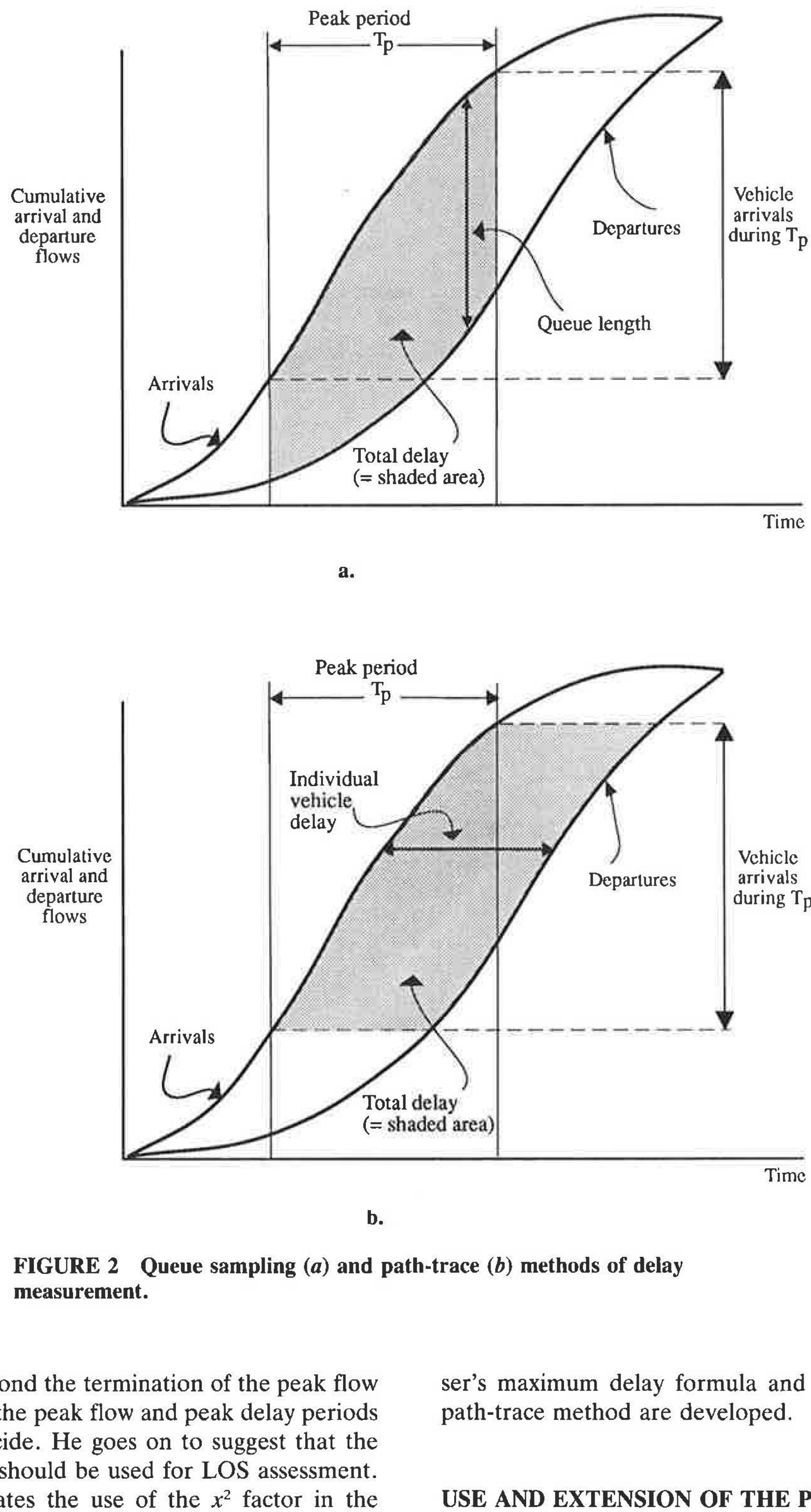

FIGURE 2 Queue sampling (*a*) and path-trace (*b*) methods of delay measurement.

typically occur at or beyond the termination of the peak flow period; in other words, the peak flow and peak delay periods do not necessarily coincide. He goes on to suggest that the maximum delay period should be used for LOS assessment. In that context, he relates the use of the x^2 factor in the incremental delay term in the HCM delay formula to the estimation of the peak delay in any floating 15-min period within the peak hour.

The formula given by Messer (*4*) to calculate the peak oversaturation delay corresponds to the queue sampling method. However, it is suggested that a delay formula corresponding to the path-trace method is more relevant to the LOS concept because this represents the delay experienced by individual vehicles. In this paper, a generalization of Messer's maximum delay formula and a formula based on the path-trace method are developed.

USE AND EXTENSION OF THE PEAK HOUR FACTOR CONCEPT

For the identification and evaluation of the period when the maximum delay occurs, a detailed analysis of the demand flow profile around the peak flow conditions is needed. This process, however, may be too complex and cumbersome to be applied in a simple operational analysis context as in the HCM since signal timing and capacity analysis for each time interval in the peak period is required. However, a simple representation of the peaking characteristics can be gained through the use of the peak hour factor (PHF) parameter. For this purpose, a simplified demand profile is described with average flow rates q_p and q_n in the peak flow period (T_p) and the nonpeak flow period ($T - T_p$), where T is the total flow period ($0 < T_p \leq T$).

The PHF parameter (see Figure 3) characterizes the peaking of demand flows by relating the average flow rate q_a in the peak hour (T = 1 hr) and the average flow rate q_p in the peak flow period ($T_p \leq$ 1 hr) through

$$\text{PHF} = q_a/q_p \tag{1}$$

By the principle of conservation of vehicles,

$$q_a T = q_p T_p + q_n(T - T_p) \tag{2}$$

From Equations 1 and 2 with T = 1 hr, the nonpeak flow rate q_n is expressed as

$$q_n = q_p \frac{(\text{PHF} - T_p)}{(1 - T_p)} \tag{3}$$

Note that PHF $\leq$ 1.0 and $q_n \geq 0$ since, by definition, $0 < T_p \leq 1.0$. When $T_p = T = 1$ hr, $q_a = q_p$ and $q_n = 0$, therefore PHF = 1.0. This corresponds to a constant demand rate during the total flow period.

The PHF parameter may be generalized by specifying a general value for the total flow period T (instead of T = 1 hr) during which the peak flow period T_p occurs ($0 < T_p \leq T$). For the generalized parameter, let us use the term peak flow factor, PFF, instead of PHF, and let us define a new parameter called the peak time factor, PTF:

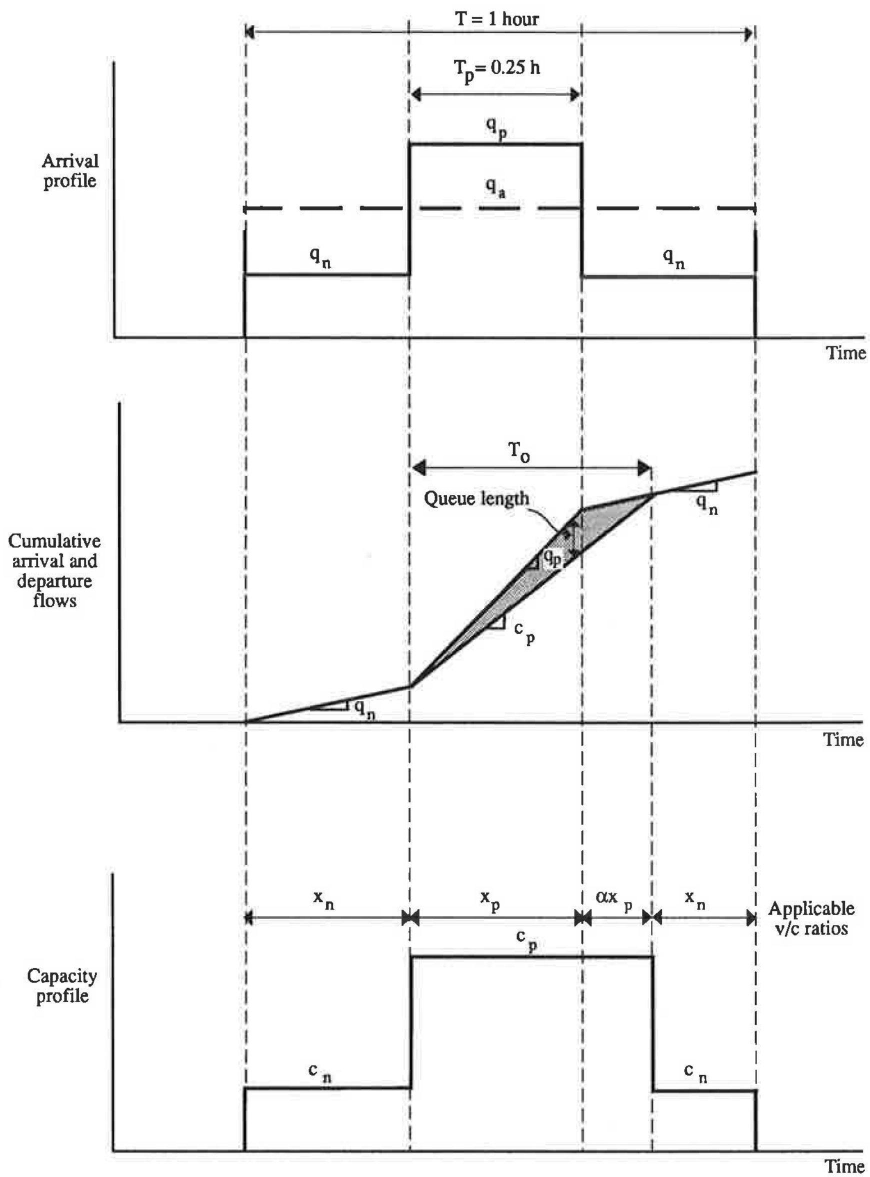

FIGURE 3 Demand, capacity, and queue profiles using the peak hour factor concept.

$$\mathrm{PFF} = q_a/q_p \tag{4a}$$

$$\mathrm{PTF} = T_p/T \tag{4b}$$

It can be shown that PTF $\leq$ PFF $\leq$ 1.0. In the HCM, T_p = 0.25 hr and T = 1 hr are used, yielding 0.25 $\leq$ PFF $\leq$ 1.0.

Rewriting Equation 3 for the general case gives

$$q_n = q_p \frac{(\mathrm{PFF} - \mathrm{PTF})}{(1 - \mathrm{PTF})} \tag{5}$$

Defining parameter $\alpha = q_n/q_p$ $(0 \leq \alpha \leq 1)$, Equation 5 can be expressed as

$$q_n = \alpha\, q_p \tag{6a}$$

where

$$\alpha = \frac{(\mathrm{PFF} - \mathrm{PTF})}{(1 - \mathrm{PTF})} \tag{6b}$$

The peak period capacity, c_p, is considered to apply as long as oversaturation persists, since this situation represents heavy demand conditions (e.g., leading to maximum green times at traffic signals). The oversaturation period (i.e., the time from the start of the peak flow period until the time the oversaturation queue clears) is given by

$$T_o = \frac{(1 - \alpha)x_p T_p}{1 - \alpha x_p} \tag{7}$$

where $x_p = q_p/c_p$.

The validity of the oversaturation queueing model given in this paper is predicated on the assumption that the peak period queues must not grow after the termination of the peak flow period so that the oversaturation period is not indefinite (Equation 7). This constraint is expressed as

$$\alpha x_p < 1.0 \tag{8a}$$

This is equivalent to

$$x_p < 1.0/\alpha \quad \text{or} \quad q_n < c_p \tag{8b}$$

For example, applying the HCM values T = 1 hr, T_p = 0.25 hr, PFF = PHF, PTF = 0.25, Equation 6b gives

$$\alpha = \frac{\mathrm{PHF} - 0.25}{0.75} \tag{9}$$

and from Equation 8b, the condition for the oversaturation queues to clear is

$$x_p \leq \frac{0.75}{\mathrm{PHF} - 0.25} \tag{10}$$

For example, when PHF = 0.9, x_p must not exceed 1.15 for the oversaturation queues to clear after the peak flow period.

DEVELOPMENT OF AN OVERSATURATION QUEUEING MODEL

An oversaturation queueing model is given here that extends Messer's original formulation (*4*) as follows:

- The model is used to derive the delay and queue statistics for either the peak flow period or a maximum delay period of duration T_p.
- Separate equations are given for estimating delays in accordance with the queue sampling and path-trace methods of measuring delays.

In Figure 2, cumulative arrival and departure patterns and resulting queues during an oversaturation period are shown. The specific queueing model used in this paper is depicted in Figures 3, 4a, and 4b. The dual flow rate approach used in this model is consistent with the PHF concept in the HCM. The model differs from the so-called low-definition approach used in the United Kingdom (*5*) in dividing the total flow period into peak and nonpeak periods with constant flow rates rather than using a constant average flow rate throughout the total flow period. It also differs from the low definition approach by assuming that the peak period capacity applies throughout the oversaturation period (i.e., the nonpeak capacity applies only after the oversaturation queues have cleared).

Let us define a variable (floating) delay period that starts at time y after the start of the peak flow period, terminates before the end of the oversaturation period $(0 \leq y \leq T_o - T_p)$, and is of the same duration as the peak flow period (T_p). For $y = 0$, the delay period is identical to the peak flow period, and $y = y_m$ defines a maximum delay period. The delay and queue statistics for various combinations of delay period definition and delay measurement method are given as follows.

Delay and Queue Statistics Using the Queue Sampling Method of Measuring Delay

The total oversaturation delay measured by the queue sampling method as incurred in a floating delay period of duration T_p starting at time y after the onset of the peak flow period (see Figure 4a) is given by

$$D_y = 0.5c_p[(x_p - 1)(T_p^2 + 2T_p y - y^2) - y^2(1 - \alpha x_p)] \tag{11}$$

The number of vehicles experiencing the total delay given by Equation 11 is

$$N_y = q_p(T_p - y) + q_n y = q_p[T_p - y(1 - \alpha)] \tag{12}$$

Therefore, the average delay corresponding to Equation 11 is

$$d_y = D_y/N_y \tag{13}$$

The start and end overflow queue lengths for the delay period are

$$N_{sy} = c_p y(x_p - 1) \tag{14}$$

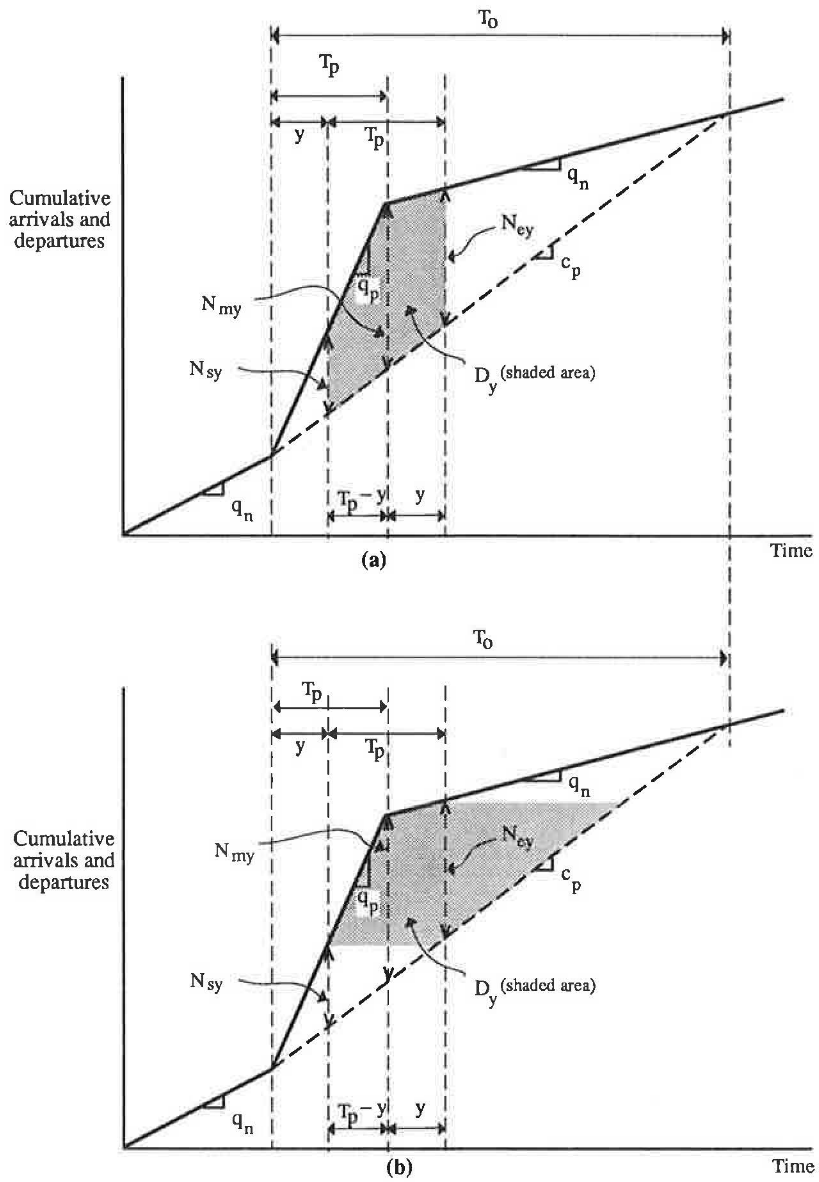

FIGURE 4 Oversaturation models with *(a)* queue sampling and *(b)* path-trace methods of delay measurement.

$$N_{ey} = c_p[T_p(x_p - 1) - y(1 - \alpha x_p)] \tag{15}$$

The average overflow queue length for the delay period of duration T_p is

$$N_{ay} = D_y/T_p \tag{16}$$

Application of the general equations for the queue sampling method of delay measurement to the maximum total delay and peak flow periods is given in the following subsections.

Delay and Queue Statistics for the Maximum Delay Period

The value of y that gives the maximum value of the total delay from Equation 11, y_m, can be determined by setting the derivative of the total delay with respect to y to zero:

$$y_m = \frac{T_p(x_p - 1)}{x_p(1 - \alpha)} \tag{17}$$

Note that Equation 17 always satisfies $y_m \leq T_o - T_p$ (i.e., the maximum delay period ends before the end of the oversaturation period). From Equations 11 and 17, the maximum total delay is

$$D_m = 0.5T_p^2c_p(x_p - 1)\left[1 + \frac{(x_p - 1)}{x_p(1 - \alpha)}\right] \tag{18}$$

The number of vehicles experiencing the maximum total delay given by Equation 18 is

$$N_m = c_pT_p \tag{19}$$

The corresponding value of the average delay is

$$d_m = 0.5T_p(x_p - 1)\left[1 + \frac{(x_p - 1)}{x_p(1 - \alpha)}\right] \tag{20}$$

The equation given by Messer (*4*) corresponds to Equation 20. Thus, Messer's formula gives the maximum delay for the queue sampling method of measurement. It should be noted that there is an inconsistency in the definition of the delay factor (k) used by Messer (bracketed term in Equation 20) since he calculated it as the ratio of the maximum delay with the queue sampling method (Equation 20) to the delay to individual vehicles arriving during the peak flow period, which implies the path-trace method of delay measurement (Equation 34). Furthermore, Equation 20 does not necessarily give the maximum value of the average delay experienced by individual vehicles since it is based on maximum total delay.

From Equations 14, 15, and 17, the start and end overflow queue lengths in the maximum delay period can be shown to be equal and have the value

$$N_{sm} = N_{em} = \frac{T_p c_p (x_p - 1)^2}{x_p (1 - \alpha)} \tag{21}$$

From Equations 16 and 17, the average overflow queue length in the maximum delay period is

$$N_{am} = 0.5 T_p c_p (x_p - 1) \left[1 + \frac{(x_p - 1)}{x_p (1 - \alpha)} \right] \tag{22}$$

Delay and Queue Statistics for the Peak Flow Period

Delay and queue statistics for the peak flow period with the queue sampling method of measurement can be derived by setting $y = 0$ in Equations 11 to 16. The nonpeak flow rate is not relevant to estimating queues and delays in this case (see Figure 4a). Therefore, the total delay in the peak flow period is

$$D_o = 0.5 T_p^2 c_p (x_p - 1) \tag{23}$$

The number of vehicles experiencing the total delay given by Equation 17 is

$$N_o = q_p T_p \tag{24}$$

The average delay measured in the peak flow period is

$$d = D_o / N_o = \frac{0.5 T_p (x_p - 1)}{x_p} \tag{25}$$

The start and end overflow queue lengths in the peak flow period are

$$N_{so} = 0 \tag{26}$$

$$N_{eo} = T_p c_p (x_p - 1) \tag{27}$$

and the average overflow queue length in the peak flow period is

$$N_{ao} = 0.5 T_p c_p (x_p - 1) \tag{28}$$

Delay and Queue Statistics Using the Path-Trace Method of Measuring Delay

The path-trace method measures delays experienced by individual vehicles, which is more relevant to the LOS concept than the queue sampling method, which relates to a system concept. Therefore, this method considers delays to vehicles arriving in that period regardless of departure times (see Figures 2b and 4b).

The total oversaturation delay measured by the path-trace method that is incurred in a floating delay period of duration T_p starting at time y after the onset of the peak flow period is given by

$$D_y = 0.5 c_p x_p \{(x_p - 1)(T_p^2 - y^2) + \alpha y[2T_p(x_p - 1) - y(1 - \alpha x_p)]\} \tag{29}$$

The average delay corresponding to Equation 29 can be calculated from the general relationship described by Equation 13. This includes individual vehicle delays experienced beyond the delay period (i.e., after time $y + T_p$). However, all queue statistics are equivalent to those derived for the queue sampling method (Equations 14 to 16).

Application of the general equations for the path-trace method of delay measurement to the maximum average delay and peak flow periods is given in the following subsections.

Delay and Queue Statistics for the Maximum Delay Period

For LOS assessment purposes, the period maximizing the average delay rather than the total delay should be used. The value of y that gives the maximum value of the average delay from Equations 13 and 29, y_m, can be determined by setting the derivative of the average delay with respect to y to zero:

$$y_m = \frac{T_p}{1 - \alpha} \left[1 - \sqrt{1 - \frac{(x_p - 1)(1 - \alpha^2)}{\alpha (1 - \alpha x_p) + (x_p - 1)}} \right] \tag{30}$$

subject to $y_m \leq T_o - T_p$. The upper bound on y_m ensures that the maximum delay period ends before the end of the oversaturation period.

The maximum value of the average delay is obtained from $d_m = D_m / N_m$ using the total delay, D_y, from Equation 29 and the number of vehicles experiencing that total delay, N_y, from Equation 12 with $y = y_m$:

$$d_m = 0.5 \frac{(x_p - 1)(T_p^2 - y_m^2) + \alpha y_m [2T_p(x_p - 1) - y_m(1 - \alpha x_p)]}{T_p - y_m(1 - \alpha)} \tag{31}$$

The start, end, and average overflow queue lengths for the delay period maximizing the average delay can be obtained by substituting $y = y_m$ in Equations 14 to 16.

Delay and Queue Statistics for the Peak Flow Period

Delay and queue statistics for the peak flow period with the path-trace method of measurement can be derived by setting

$y = 0$ in Equation 29. The resulting equations are equivalent to the equations given by Akçelik (*6*). In this case, the non-peak flow rate is not relevant to estimating queues and delays to vehicles arriving in the peak flow period (see Figure 2b). Therefore, the total delay in the peak flow period is

$$D_o = 0.5T_p^2c_px_p(x_p - 1) \tag{32}$$

The number of vehicles experiencing the total delay given by Equation 22 is

$$N_o = q_pT_p \tag{33}$$

Therefore, the average delay experienced by vehicles arriving in the peak period is

$$d_o = 0.5T_p(x_p - 1) \tag{34}$$

The start, end, and average overflow queue lengths in the peak flow period (N_{so}, N_{eo}, and N_{ao}) are identical to those obtained by the queue sampling method (see Equations 26 to 28).

NUMERICAL EXAMPLE

A numerical example illustrating the use of the equations given in the preceding section for estimating queue and delay statistics for the peak flow and maximum delay (total or average) periods is given in this section. In particular, the following points are explored:

- Effect of flow profile aggregation (i.e., the choice of the duration of peak flow period) on delay and queue estimates,
- Effect of the delay measurement method (queue sampling or path trace) on predicted oversaturation delay, and
- The relationship between the average oversaturation delay incurred in the peak flow period and that incurred in the maximum delay period given the method of delay measurement.

In this example, demand flow data are assumed to be collected in eight 15-min intervals within a 2-hr peak (T = 2 hr). The observed flow profile is shown in Figure 5a. The demand profiles synthesized according to selected durations of the peak flow period (T_p) are shown in Figure 5b.

The example relates to a traffic stream in a signalized intersection approach lane. As a rough way of emulating the

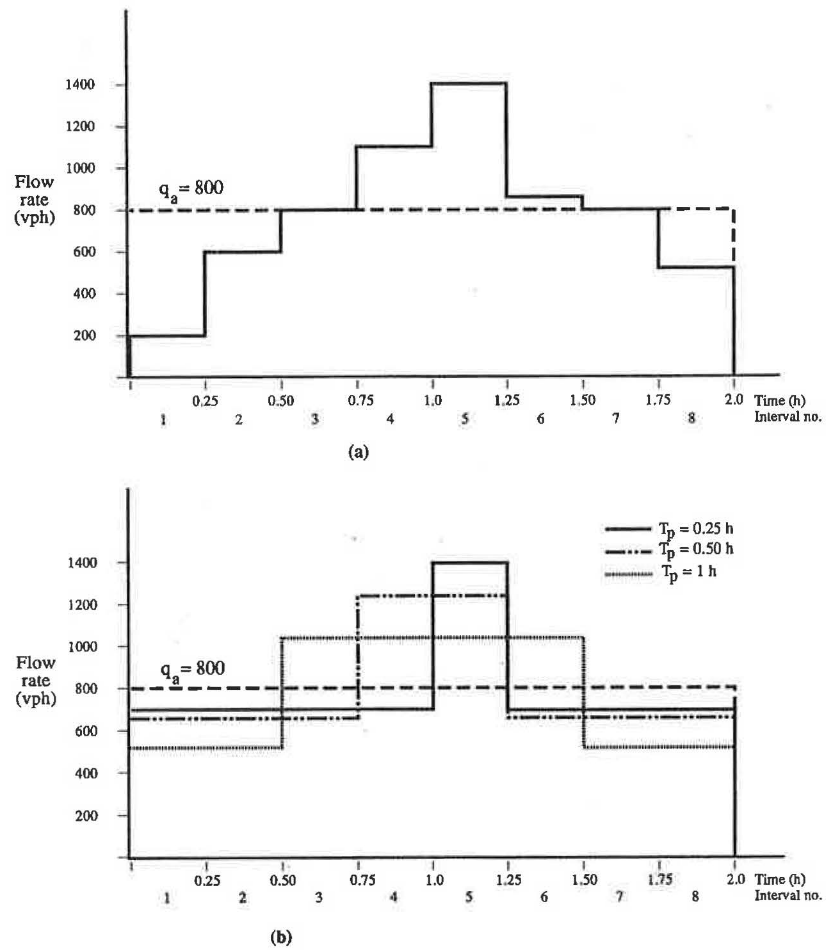

FIGURE 5 Numerical example (T = 2 hr): (*a*) actual demand profile; (*b*) synthesized demand profiles for indicated T_p.

operation of a vehicle-actuated signal controller, the capacity function is set at

$$c = \min\{c_m, q/x_d\} \tag{35}$$

where

c = capacity (veh/hr) for a given demand level q,
c_m = maximum movement capacity attained during the oversaturation period, that is, as long as overflow queues exist during and after the peak flow period (the maximum is a result of limitations on maximum green time, cycle length, etc.), and
x_d = design (or practical) degree of saturation at demand level q.

In this example, c_m = 1,000 veh/hr and x_d = 0.90 are used. The average flow rate over the 2-hr period is set at q_a = 800 veh/hr.

The capacity model given in Equation 35 yields higher capacities with increasing flow levels up to a maximum value. In some cases, the signalized intersection capacities may decrease as flow levels increase [e.g., where such factors as opposed turns, short lanes, and shared lane blockages are dominant (8)]. At roundabouts and other signalized intersections, capacities always decrease with increasing flow levels because of the underlying gap acceptance process. The results from the model based on Equation 35 should therefore not be generalized.

The results are summarized in Tables 1 to 5 as explained below. In all tables, flow rates are given in vehicles per hour, times in hours, queue lengths in vehicles, total delays in vehicle-hours, and average delays in seconds per vehicle.

TABLE 1 Flow and Capacity Parameters for Selected Peak Flow Period Lengths (c_p = 1,000 veh/hr)

Selected T_p	PTF (Eqn 4b)	Average q_p	PFF (Eqn 4a)	α (Eqn 6b)	q_n (Eqn 6a)	x_p	αx_p	T_o (Eqn 7)
0.25	0.1250	1400	0.571	0.510	714	1.400	0.714	0.600
0.50	0.250	1250	0.640	0.520	650	1.250	0.650	0.857
0.75	0.375	1133	0.705	0.528	598	1.133	0.598	0.998
1.00	0.500	1050	0.762	0.524	550	1.050	0.550	1.111

TABLE 2 Delay and Queue Statistics Using the Queue Sampling Method: Maximum Delay Period

T_p	x_p	αx_p	y_m	D_m	d_m	N_{sm}	N_{em}	N_{am}
0.25	1.400	0.714	0.146	19.79	284.9	58.3	58.3	79.2
0.50	1.250	0.650	0.208	44.27	318.8	52.1	52.1	88.6
0.75	1.133	0.598	0.187	46.85	224.9	24.9	24.9	62.5
1.00	1.050	0.550	0.100	27.50	99.0	5.0	5.0	27.5

TABLE 3 Delay and Queue Statistics Using the Queue Sampling Method: Peak Flow Period

T_p	x_p	αx_p	y	D_o	d_o	N_{so}	N_{eo}	N_{ao}
0.25	1.400	0.714	0.0	12.50	128.6	0.0	100.0	50.0
0.50	1.250	0.650	0.0	31.25	180.0	0.0	125.0	62.5
0.75	1.133	0.598	0.0	37.41	158.8	0.0	100.0	50.0
1.00	1.050	0.550	0.0	25.00	85.7	0.0	50.0	25.0

TABLE 4 Delay and Queue Statistics Using the Path-Trace Method: Maximum Delay Period

T_p	x_p	αx_p	y_m	D_m	d_m	N_{sm}	N_{em}	N_{am}
0.25	1.400	0.714	0.165	18.88	286.9	66.0	52.8	78.7
0.50	1.250	0.650	0.250	42.50	322.0	62.5	37.5	87.5
0.75	1.133	0.598	0.239	45.48	226.8	31.8	3.8	61.3
1.00	1.050	0.550	0.111	27.45	99.4	5.6	0.0	27.5

TABLE 5 Delay and Queue Statistics Using the Path-Trace Method: Peak Flow Period

T_p	x_p	αx_p	y	D_o	d_o	N_{so}	N_{eo}	N_{ao}
0.25	1.400	0.714	0.0	17.50	180.0	0.0	100.0	50.0
0.50	1.250	0.650	0.0	39.06	225.0	0.0	125.0	62.5
0.75	1.133	0.598	0.0	42.50	180.0	0.0	100.0	50.0
1.00	1.050	0.550	0.0	26.30	90.0	0.0	50.0	25.0

1. Table 1 gives the demand and capacity flow parameters in the peak and nonpeak periods. Capacity for the peak flow period is derived using the peak flow rate (q_p) in Equation 35. Since all peak flows are above 1,000 veh/hr, the peak period capacity is the same for all T_p cases ($c_p = c_m$ = 1,000 veh/hr). Table 1 also gives the oversaturation period (T_o) during which the peak period capacity (c_p) applies. The nonpeak period capacities (c_n) are not given in Table 1 since they are not used in the deterministic oversaturation model (Figure 4).
2. Tables 2 and 3 provide comprehensive delay and queue statistics for the maximum delay and peak flow periods obtained using the queue sampling method.
3. Finally, Tables 4 and 5 give comparable delay and queue statistics corresponding to the path-trace method.

A study of the results given in Tables 1 to 5 indicates a strong correlation between the duration of the selected peak interval and the corresponding delay and queue statistics. This is explained by the fact that the peak and nonpeak flow rates are averages within and outside the peak period. A peak flow period with longer duration (larger T_p) implies a longer peak, but a smaller peak flow rate. This trade-off is evident when comparing the average delay and queue length values within each table for different T_p values. In most cases, a 30-min interval yielded the highest delay. Thus, the blanket use of a fixed peak and analysis periods as in the HCM (T_p = 0.25 hr and T = 1 hr) is not supported by this example. In fact, such blanket definitions may not be consistent with the intended use of the delay models, which are meant to simulate the performance of a (possibly) saturated peak within an undersaturated total flow period.

The differences in queue and delay statistics obtained from the queue sampling and path-trace methods of delay measurement can be seen by comparing the results in Tables 2 and 4 for maximum delays or Tables 3 and 5 for average delays in the peak flow period. Maximum delays estimated by the two methods are similar. The path-trace method gives higher delay for the case of analysis for the peak flow period, which is due to the allowance for oversaturation delays experienced after the peak flow period.

As expected, substantial differences in delay and queue statistics were observed in the cases of peak flow and maximum delay periods regardless of the delay measurement method (comparing the results in Tables 2 and 3 or those in Tables 4 and 5). The level of difference is also seen to be affected by the choice of the duration of the peak flow period.

CONCLUSION

This paper has presented a deterministic oversaturation queueing model, which generalizes the peak hour factor concept of the U.S. HCM (*1*). Using this simple variable demand model, several issues related to oversaturation models have been explored. In particular, consistency of delay definitions and delay measurement methods has been investigated. A numerical example has been used to illustrate the application of the model. For the discussion of a full time-dependent model allowing for both random and oversaturation delays, the user is referred to Akçelik and Rouphail (*7*).

The queue and delay estimates are highly sensitive to the selected peak flow period duration irrespective of the delay definition or the delay measurement method. In fact, variations caused by the choice of the peak flow period duration are as significant as those resulting from the use of a different delay definition or delay measurement method.

For the example analyzed, the ability to vary the duration of the peak flow period revealed that a 30-min peak period was more critical in terms of resulting delays and queues than the 15-min peak duration specified in the HCM.

As expected, substantial differences in delay and queue estimates are observed between the cases of peak flow and maximum delay periods regardless of the delay measurement method. In the numerical example, the maximum delays estimated by the queue sampling and path-trace methods are similar, but the path-trace method produces higher delays for the peak flow period.

The use of the average delay experienced by individual vehicles in a maximum delay period appears to have some merit in terms of LOS. However, this creates several problems in system performance analysis (including estimation of operating cost, fuel consumption, and pollutant emission).

First, the number of vehicles experiencing this delay is smaller than the number of vehicles arriving in the peak flow period. By applying this delay to the peak flow period, the total delay would be overestimated. Therefore, the use of this delay should be restricted to LOS assessment purposes only. For the purpose of system performance design and evaluation, total oversaturation delay should be used.

Second, a delay definition based on a maximum delay period reveals an inconsistency in relation to delays measured in the field. Simply stated, whereas the HCM recommends that field delays be measured in the peak flow period, the maximum delay period does not coincide with the peak flow period. Thus, the two delays are not comparable.

Furthermore, if the use of maximum delay is adopted, the path-trace rather than the queue sampling method should be used, since the former is more relevant to LOS assessment in reflecting the delays experienced by individual vehicles.

It is therefore important that the delay definition implied by the present HCM delay formula for signalized intersections be clarified in view of the comments presented in this paper. Specifically, if the x^2 factor in the incremental delay term of the HCM delay formula is intended to produce a maximum delay estimate for oversaturated conditions as put forward by Messer (*4*), the delay estimates from the HCM delay formula should not be expected to correspond to delays measured in the 15-min peak flow period by the queue sampling method specified in the HCM.

Thus, it would be advisable to consider two distinct delay models for system performance and LOS assessment purposes. In the former, delays incurred throughout the oversaturation period would be considered for estimating total delay, operating cost, fuel consumption, and pollutant emissions. The latter should strictly apply to the maximum delay period using the path-trace method for LOS assessment.

The differences in delay definitions and delay measurement methods that have been emphasized in this paper are relevant to oversaturated conditions only. For undersaturated conditions represented by low to medium v/c ratios, the effect of the time-dependence of demand flows (i.e., the duration of the peak flow period) on delays and queues is negligible, and therefore the delay definitions used in the delay formulas have little effect on delay estimates. Similarly, the queue sampling and path-trace methods of delay measurement should yield similar delays under low to medium v/c ratios.

However, as flows approach capacity (undersaturated but high v/c ratios near capacity) and exceed capacity (v/c ratio greater than 1), the selection of the duration of the flow period, delay definition, and delay measurement method affect delay estimates significantly. Because of the dual nature of the delay-flow functions, the use of a factor that applies to all flow conditions [such as the x^2 factor in the incremental (random plus oversaturation) term of the HCM delay equation or the progression factor that multiplies both terms of the equation] is not appropriate for modeling oversaturation effects (*2,9*). A time-dependent continuum model satisfying these requirements is described in a follow-up paper (*7*).

It is realized that the dual flow model presented in this paper is still a simplification of the variable demand model case. Nevertheless, the concept builds on flow data that are gathered routinely as part of intersection operational analysis studies. The end user must realize, however, that some prior investigation is needed to select sensible durations for the peak and total flow periods. As a guide, a 15-min peak flow period appears to be the smallest aggregation period for which volumes can be assumed uniform. The total flow period is more difficult to ascertain except that it is advisable that no overflow queues should be present at either its start or its termination. Determining the critical duration of the peak period interval (in multiples of 15-min) should take into account the level of peaking. Short peak flow periods are required in high peaking cases (low PHF or PFF values) to allow for long oversaturation periods resulting from a high v/c ratio in the peak flow period. In the numerical example, a 30-min peak flow period produced the largest delay and queue estimates. Further work is required on the effect of the choice of the location and duration of the peak flow period in terms of the total system performance in the total flow period.

The practical difficulty of measuring the true demand profile, which requires measuring arrival flows at the end of the queue, should also be recognized. Volume counts at the stop line cannot identify oversaturation since the stop line flows can never exceed the capacity (in the example shown in Figure

5, Intervals 4 and 5 have demands that exceed the capacity of 1,000 veh/hr, but the stop line counts would yield an apparent demand of 1,000 veh/hr). On the other hand, the stop line method would count the excess demand in subsequent intervals. This would indicate less peaking (a higher PHF or PFF value) than the real demand profile. Stop line volume counts supplemented by queue counts (*10*) could be used to estimate the true demand for oversaturated conditions.

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