Car-Following Model Based on Fuzzy Inference System

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Car-following theory has been receiving renewed attention for its use in the analysis of traffic flow characteristics and vehicle separation control under the IVHS. A car-following model that uses the fuzzy inference system, which consists of many straightforward natural language-based driving rules, is proposed. It predicts the reaction of the driver of the following vehicle (acceleration-deceleration rates) given the action of the leading vehicle. A range of possible reaction is predicted and expressed by the fuzzy membership function. The model is applied to the analysis of traffic stability and speed-density relationship. For traffic stability, the results are compared with those derived from the deterministic approach. The speed-density relationship derived from the model is compared with a set of actual flow data. The predicted range is found to be reasonable. The proposed fuzzy approach helps explain the scatter of the actual data as possibility rather than random variation.

For the past several decades traffic flow has been generally analyzed under the premise that all drivers behave in a similar manner and that a general law exists governing the flow characteristics in the traffic stream. On the basis of this premise, characteristics of flow have been analyzed from both the microscopic and the macroscopic standpoints. Most studies have considered that a deterministic relationship exists between the action of a vehicle and the reaction of the vehicles that follow. Whereas the existence of this cause and effect relationship is not disputable, the reactions of a driver to the actions of other drivers are perhaps not based on a deterministic one-to-one relationship, but on a set of vague driving rules developed through experience. The way in which the rules are applied may differ with different drivers, and even for the same driver, it differs with different conditions. The rules are not rigid but are natural language based. For example, if the leading vehicle (LV) decelerates, then the following vehicle (FV) should decelerate; or, if the distance between the LV and FV becomes very short, the FV should decelerate and try to increase the distance. Such a linguistic reasoning pattern is suited for an analysis using fuzzy logic and approximate reasoning techniques. Fuzzy set theory and logic allows the mathematical treatment of subjective judgment and inference, and in recent years fuzzy logic has been applied to many practical problems involving controls and decisions under the environment of the imprecise human reasoning process.

This paper proposes a fuzzy rule-based car-following model that assumes that a decision made by a driver is the result of a fuzzy reasoning process and then predicts the possibilities of the reaction of the FV.

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CAR-FOLLOWING MODELS: TRADITIONAL APPROACH

This section is divided into two subsections. The first subsection describes the car-following models developed by the General Motors research group (GM Model) and their assumptions. The second subsection discusses traffic stability and speed-density relationships in the car-following context.

The car-following theory evolved in the 1950s. Among the researchers who pioneered in the field, Pipes (*1*, pp. 164–166) developed a microscopic model that assumed that the minimum safe distance between vehicles was a function of speed. His work was followed by that of Forbes (*1*, pp. 116–167). While Pipes modeled the traffic flow assuming that drivers maintain a constant distance headway, Forbes assumed that drivers maintain a constant time headway. However, by far the largest contribution was made by the GM's research team (*2–5*). Some of the GM models are discussed here.

Models and Assumptions

The GM models were based on the premise that the reaction of the FV at time t depends on the sensitivity of the FV and the strength of the stimulus given by the LV at time $t - \Delta t$, where the strength of the stimulus is measured in terms of the relative velocity between the LV and the FV, the reaction of the FV is measured by the acceleration or deceleration rate, the time difference, Δt, is equal to the perception/reaction time, and the sensitivity term maps the unit of a stimulus to a reaction. The GM team developed five models that have the same general structure but differ from one another in the sensitivity term. The fifth model is a generalized representation of the first four models:

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{\ell,m}(\dot{x}_{n+1}(t + \Delta t))^m}{[x_n(t) - x_{n+1}(t)]^\ell} \cdot [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad (1)$$

where

$\ddot{x}_{n+1}(t + \Delta t)$ = acceleration or deceleration rate of $(n + 1)$th car at time $t + \Delta t$,
$\dot{x}_n(t)$ = speed of nth car at time t,
$x_n(t)$ = position of nth car at time t,
ℓ = parameter for sensitivity to distance $x_n(t) - x_{n+1}(t)$,

m = parameter for sensitivity to speed $\dot{x}_{n+1}(t + \Delta t)$, and

$\alpha_{\ell,m}$ = constant.

This model has the following characteristics:

1. The interaction between stimulus and reaction has a one-to-one correspondence. The notion that a driver's reaction pattern is imprecise is not fully represented. Ceder (*6*) expressed a similar concern. Representation of a human behavioral pattern may be better explained by an approximate reasoning process than a deterministic equational model.
2. The FV reacts even to minute changes in relative velocity between the LV and FV in a deterministic manner.
3. Sensitivities of the FV to the positive and negative relative velocities are the same. Equation 1 suggests that if the FV accelerates at y ft/sec^2 when the relative speed is β ft/sec, then it decelerates at $-y$ ft/sec^2 when the relative speed is $-\beta$ ft/sec. It has been observed that drivers react differently when the distance between cars is increasing or decreasing. Leutzbach (*7*) also states that "drivers pay closer attention to spacing decreases (decrements) than to spacing increases (increments) simply on the basis of their own safety."

Applications of GM Model

This section discusses two topics to which the car-following models have been applied: traffic stability and macroscopic speed-density relationships.

Traffic Stability

Traffic stability is a study of how stability is restored in the traffic flow after the leader of a platoon "destabilizes" the flow by accelerating or decelerating. The traffic stability analyses have focused on how the vehicle spacing changes with time. Two types of stability patterns have been studied in the past: local stability and asymptotic stability. Extensive analyses of local stability patterns were conducted for different car-following models (i.e., different combinations of m and ℓ in Equation 1) by Herman et al. (*5*), Chandler at al. (*2*), and Herman and Potts (*8*). Herman and Potts (*8*) present the results from three different cases: (*a*) $m = \ell = 0$, (*b*) $m = \ell = 0$ but with two values of α, and (*c*) $m = 0$ and $\ell = 1$. Only the first case ($m = \ell = 0$) has been analyzed mathematically.

While these analyses provide insight into what happens in reality, each has its own limitations. For example, the constant sensitivity case ($m = \ell = 0$) implies that the reaction to a given relative velocity is independent of the distance between LV and FV. Though an improvement over the previous one, even the reciprocal spacing ($m = 0, \ell = 1$) model has shortcomings; one of them is that no difference between the stimuli of positive and negative relative velocity is made in the sensitivity term. Another drawback of this model is that the FV's reaction is independent of the velocity of the FV. It can be argued that as velocity increases the reaction to positive relative velocity is subdued and negative relative velocity enhanced. The model represented in Equation 1, though still deterministic in nature, is the closest to reality. However, with nonzero coefficients of m and ℓ, the difference differential equation of Equation 1 becomes difficult to solve.

Asymptotic stability is concerned with how the instability introduced by the LV propagates down a line of traffic. This is an interesting topic in the sense that it may explain certain causes of accidents and congestion. Herman et al. (*5*) and Herman and Potts (*8*) have also presented results from their study on asymptotic stability, and these remain the most extensive study on this topic.

Speed-Density Relationship (u-k Relationship)

The bridge built by Gazis et al. (*4*) between the microscopic car-following model with $m = 0$ and $\ell = 1$ and Greenberg's macroscopic speed-density relationship was a significant step toward unifying the microscopic and macroscopic approaches. This effort has made it possible to show that other macroscopic speed-density models can also be derived from different assumed values of m and ℓ in Equation 1: $m = 0$ and $\ell = 2$ for Greenshields; $m = 1$ and $\ell = 2$ for Underwood; and $m = 1$ and $\ell = 3$ for Northwestern's (*1*, p. 304).

The relationship between the microscopic and macroscopic models allows the examination of the validity of the microscopic model by the observed *u-k* relationship. The facts that the observed data points in the *u-k* relationship are scattered and the observed and predicted characteristics have significant discrepancies suggest that a problem might lie in the deterministic approach.

This has been pointed out by some. For example, Ross (*9*) states, "The idea that there is deterministic relationship between speed and density, be it straight line or curve, is simply untenable. The most obvious problem is that speed-density observations always have much more scatter than can be explained by any reasonable amount of experimental error." This concern was echoed by Gilchrist and Hall (*10*): "The scatter in the traffic data is sufficient to cast doubt on the narrow linear representation of any relationship between traffic flow variables." Underwood (*11*) developed probability distributions for speed for different volumes.

These comments, combined with the belief that drivers do not behave in a rigid deterministic manner, lead us to consider a model based on a fuzzy inference system.

FUZZY RULE–BASED MODEL FOR THE CAR-FOLLOWING PROBLEM: RATIONALE

In the car-following situation, one follows a set of driving rules built over time through experience. Examples of the rules that the FV might apply are as follows: (*a*) accelerate if the LV accelerates, and (*b*) decelerate and keep longer distance if the LV decelerates and the distance between cars is short.

Each rule is built on natural language, and no exact boundary for the applicability of the rule is defined. Hence, many of the rules may be applied (or "fired") simultaneously in the mind of the driver, and the driver may not be completely certain of the appropriateness of his action. The probability approach, which has traditionally been used to analyze uncertainty, however, cannot deal with linguistic variables such as "fast" and "slow"; further, it must follow a rigid set of rules defining the properties of the probability function.

If we postulate that a driver's reaction is one of several possible actions available, the variation of the reaction pattern and the scatter of the observed u-k relationship may be explained. A fuzzy set, which will be explained later, is actually the set of elements with the possibility of being in the set of discourse (*12*). In recent years, fuzzy sets have been used to represent the approximate reasoning and decision process. This approach may offer an alternative explanation of the car-following phenomenon.

ELEMENTS OF FUZZY SET THEORY

This section presents elements of fuzzy set theory that are relevant to the construction of the proposed model. More detailed explanation of fuzzy theory can be found elsewhere (*13–15*).

Fuzzy Sets

A fuzzy set is a set for which the criterion for belonging to the set is not dichotomous. The membership of the set is defined by a grade (or degree of compatibility or degree of truth) whose value is between 0 and 1. A membership function determines the grade and is defined as

$$h_A(x): X \rightarrow [0,1] \qquad (2)$$

where A is a fuzzy set defined on the universal set X.

The notion of "high speed" or "low speed," for example, can be represented by fuzzy sets whose membership functions define the perception of high or low in terms of numerical value of speed. Similarly, an approximate integer constitutes a fuzzy set that is normal and convex. "Approximately 5" may have the following membership function:

"Approximately 5" = 3/0.4 + 4/0.8 + 5/1.0 + 6/0.6 + 7/0.4

Arithmetic operations on fuzzy numbers are defined using the extension principle. For a detailed description of fuzzy arithmetic, readers are referred to Dubois and Prade (*16*) and Kaufmann and Gupta (*17*).

Operations of Fuzzy Sets

Among the set operations relevant to the subsequent discussions are union, intersection, and complement, defined by Equations 3, 4, and 5, respectively.

$$h_{A \cup B}(x) = h_A(x) \vee h_B(x) \qquad (3)$$

$$h_{A \cap B}(x) = h_A(x) \wedge h_B(x) \qquad (4)$$

$$h_{\bar{A}}(x) = 1 - h_A(x) \qquad (5)$$

In these equations, $\wedge$ indicates the minimum and $\vee$ the maximum of the operands [$h_A(x)$ and $h_B(x)$, in this case].

Fuzzy Inference

Under fuzzy logic, the inference process includes fuzzy input and a fuzzy relationship, as follows:

Input: x is somewhat A $(x = A')$

Rule: if x is A then y is B $(\text{R}: x = A \rightarrow y = B)$

Conclusion: y is somewhat B $(y = B')$ (6)

where all or some of A, A', B, and B' are fuzzy sets, and the rule represents a fuzzy cause-and-effect relation between x and y. The first part of the rule, "x is A," is called the premise, and the second, "y is B," is called the consequence. The validity of the consequence depends on the compatibility between the input and the premise of the rule. In other words, the degree to which "y is B" is true is dictated by the degree of match between "x is somewhat A" and "x is A."

A fuzzy inference system can be composed of more than one rule with each rule consisting of more than one premise variable, as follows:

Input: $x_1 = A'$ and $x_2 = B'$

Rule 1: If $x_1 = A_1$ and $x_2 = B_1$, then $y = C_1$

Rule 2: If $x_1 = A_2$ and $x_2 = B_2$, then $y = C_2$

. . . . . .

Rule i: If $x_1 = A_i$ and $x_2 = B_i$, then $y = C_i$

Conclusion: $y = C'$ (7)

The compatibility between the input and the premise of a rule i, W_i, is examined as follows:

$$W_i = \{\vee_{x1} [h_{A'}(x_1) \wedge h_{Ai}(x_1)]\} \wedge \{\vee_{x2} [h_{B'}(x_2) \wedge h_{Bi}(x_2)]\} \qquad (8)$$

When n different rules are applied (or "fired") for the given input, the degree of compatibility between the input and the premise is computed for each rule, and then the conclusion is the average of the individual consequences, C_i's, weighted by W_i's:

$$C' = \frac{\sum W_i \cdot C_i}{\sum W_i} \qquad (9)$$

where C' is still a fuzzy number.

This operation is, in fact, an interpolation of C_i's. This method is an extension of the one proposed by Takagi and Sugeno (*18*) where their consequence C_i's are crisp numbers.

Expression 9 is computed as follows:

1. Normalize the values of W_i:

$$\eta_i = \frac{W_i}{\sum W_i} \qquad (10)$$

2. Multiply fuzzy number C_i by η_i to obtain a new fuzzy number D_i:

$$h_{Di}(\lambda) = \max_{\lambda = y \times \eta_i} h_{Ci}(y) \tag{11}$$

3. Add Di's for all i's for which the rules apply:

$$h_{C'}(y) = \max_{y = \lambda_1 + \lambda_2 + \ldots + \lambda_N} \{h_{D1}(\lambda_1), h_{D2}(\lambda_2), \ldots, h_{DN}(y - \lambda_1 - \lambda_2 - \ldots - \lambda_{N-1})\} \tag{12}$$

where $h_{C'}(y)$ is the membership function of the conclusion.

A comprehensive discussion of fuzzy logic is given by Zimmermann (*14*).

FUZZY RULE BASED CAR-FOLLOWING MODEL

The model consists of two modules: a fuzzy inference system and a system that executes the inference system.

Fuzzy Inference System

The inference system infers the reaction of the FV in acceleration (or deceleration) rates in response to the action of the LV. Using the following notation, the structure of the system is presented.

d : distance between LV and FV (in specific value)
s : relative speed between LV and FV (in specific value)
a : rate of change of speed of LV (in specific value)
DS_i : perceived distance (in fuzzy number)
RS_i : perceived relative speed (in fuzzy number)
ALV_i : perceived rate of change of speed of LV (in fuzzy number)
AFV_i : reaction of FV in acceleration (or deceleration) rate (in fuzzy number)
AFV' : predicted reaction of FV in acceleration (or deceleration) rate given the input (in fuzzy number)
Input : $x_1 = d, x_2 = s, x_3 = a$
Rule i : If $x_1 = DS_i$, and $x_2 = RS_i$, and $x_3 = ALV_i$, then $y = AFV_i$
. . . . . .
Rule n : If $x_1 = DS_i$, and $x_2 = RS_i$, and $x_3 = ALV_i$, then $y = AFV_i$
Conclusion : $y = AFV'$

In the following, input, rules (the premise, consequence, and structure), and the conclusion of the inference system of the proposed model are discussed.

Input. Since the purpose of the model is to predict the behavioral pattern of the FV when a specific condition is given, the input is a set of parameter values that would affect the FV decision. They are

- Distance between FV and LV (ft),
- Speeds of FV and LV (ft/sec) (to obtain the relative speed), and
- Acceleration or deceleration rate of the LV (ft/sec^2).

Rule: premise. The premise variables of a rule are the distance between the LV and FV (DS), relative speed of the vehicles (RS), and the acceleration (or deceleration) rate of the LV (ALV). The quantity of each of the first two variables is grouped into 6 natural language-based categories, while the last variable is grouped into 12 such categories (6 for acceleration and 6 for deceleration). Each of these categories is a fuzzy set. They are presented in Table 1.

For all categories, triangular membership functions are assumed. For categories that represent DS, the membership function varies with the speed of the FV because it is believed that the notion of safe distance is relative to the speed at which the FV is traveling.

The reason for considering acceleration and deceleration separately is based on our belief that the intensity of FV's reaction is different when the LV is accelerating and deceleration, as discussed in the second section.

Rule: consequence. The consequence of a rule is the FV's reaction in terms of acceleration or deceleration rate expressed in fuzzy quantity (AFV). Each fuzzy quantity can be represented by a natural language term such as VERY STRONG DECELERATION. The reaction of FV should be similar in nature to that of LV since FV wishes to maintain the relative speed near zero. Thus, the membership function of AFV should be similar to that of ALV, but it is modified by the categories chosen for DS and RS in the premise.

If the category of DS in a rule is ADEQUATE, the AFV (in fuzzy number) is computed as follows:

$$\{(RS_i + ALV_i \bullet \Delta t)/\gamma\} = AFV_i \tag{13}$$

where Δt is the time interval at which the rules are applied (or the time intervals at which the inference is run; in our model $\Delta t = 1$ sec); γ is the time in which FV wishes to "catch up" with LV. We choose $\gamma = 2.5$ sec, which keeps the FV's acceleration and deceleration rates within a realistic range (less than approximately 10 ft/sec^2).

The numerator of Equation 13 represents the relative speed at time $t + \Delta t$. Dividing it by γ, we obtain the rate of speed change required for FV to restore zero relative speed. RS_i, ALV_i, and AFV_i are all fuzzy numbers.

If the category of DS in Rule i is different from ADEQUATE, the value of AFV_i is modified. The modification is done by sliding the membership function of AFV_i to the right or to the left (making it larger or smaller) according to DS_i's deviation from the category ADEQUATE. For each deviation to a shorter distance category, AFV_i is reduced by -1 ft/sec^2; for each deviation to a longer distance category AFV_i

TABLE 1 Categories (Fuzzy Sets) of Premise Variables

		Distance btwn. LV and FV (DS)	Relative speed (RS)	Actions of LV (ALV)	
				Acceleration	Deceleration
Categories	(1)	very small	FV slower	strong	strong
	(2)	small	FV slightly slower	somewhat strong	somewhat strong
	(3)	adequate	near zero	normal	normal
	(4)	more than adequate	FV slightly faster	mild	mild
	(5)	large	FV quite faster	very mild	very mild
	(6)	very large	FV faster	none	none

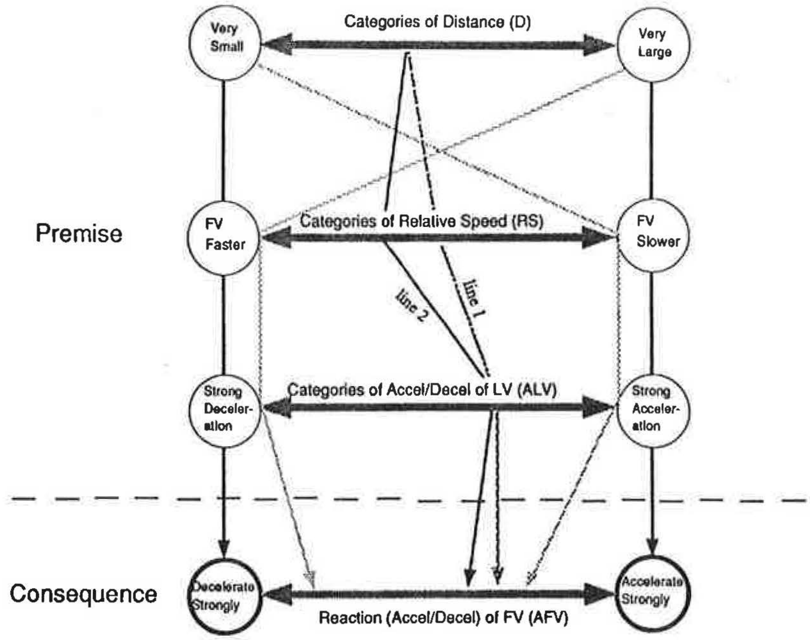


FIGURE 1 Formation of rules.

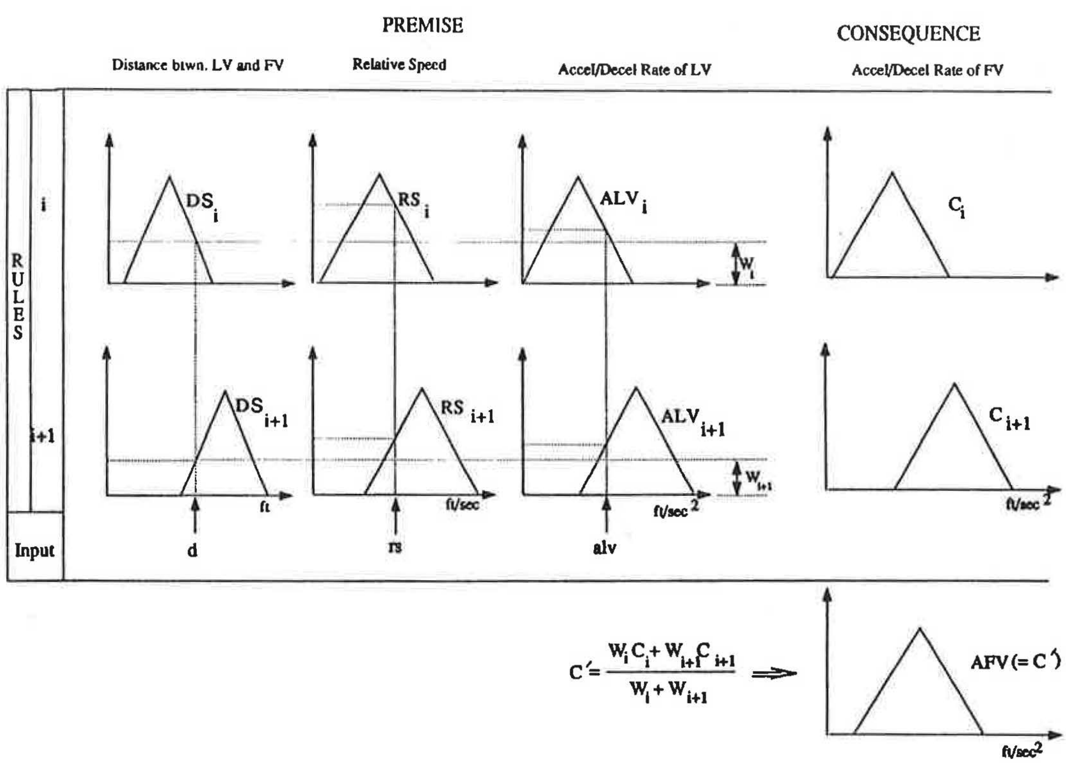


FIGURE 2 Execution of the fuzzy inference system.

is increased by +1 ft/sec². That is, AFV_i is determined by

$$\{(RS_i + ALV_i \bullet \Delta t)/\gamma\} + \beta_{DSi} \bullet \phi = AFV_i \quad (14)$$

where β_{DSi} is the number of categories for which DS_i deviates from ADEQUATE (it can be a positive or negative number depending on whether the deviation is to a longer distance or a shorter distance, respectively), and ϕ in this case is 1 ft/sec².

Rule: structure. Each rule is a conditional statement in the sense that, given a set of conditions represented by the premise variables, the consequence is predicted. The following is an example:

If Distance (DS): ADEQUATE,
Relative Speed (RS): NEAR ZERO, and
Acceleration of LV (ALV): MILD,
then FV should accelerate MILDLY.

The selection of categories of the premise variables and consequences are based on the method discussed previously. Figure 1 shows how a combination of the categories of the premise variables results in a particular consequence (which should lie between STRONG ACCELERATION and STRONG DECELERATION). For instance, the rule in the example could be represented by Line 1 in the figure. It is interesting to observe that a line connecting the upper circles of the premise leads to the very large acceleration of FV, and the line connecting the bottom circles leads to very large deceleration, thus setting the two extreme cases.

The conclusion. The level of compatibility between the input and premise of a rule i, W_i, is determined by the operation shown in Equation 8, except that in this case there are three premise variables. For all the rules for which the value of W_i is greater than zero, the conclusion is computed according to Equation 9 (or Equations 10, 11, and 12) where C's are the FV's acceleration (or deceleration) rate expressed in fuzzy number. The process of deriving the conclusion according to Equation 9 is shown in Figure 2 for the case in which two rules are applied to the same input. (In our example, on the average, three to four rules were fired for the same input).

Execution of the Model

The model executes the inference system at small time increments (1 sec in our example). At each time increment, the action of LV can be changed; for example, in one time increment, it accelerates at a given rate; at the next time interval, it accelerates at another rate. The speed and position of the FV relative to the LV are then updated after each time increment. The time delay between the actions of the LV and FV due to the perception and reaction process is assumed to be 1 sec in our example.

ANALYSIS: TRAFFIC STABILITY AND SPEED-DENSITY RELATIONSHIP

This section applies the model to the analyses of traffic stability and speed-density relationships, representing application to microscopic and macroscopic analyses of traffic flow, respectively. The output of the model is a fuzzy number. The lines that will be shown as model output in Figures 3, 4, 5, and 6 represent the values whose membership grade is 1. Lines A and B in Figure 7 correspond to the value at a membership grade of 0.2.

Traffic Stability

Traffic stability is examined from the local and asymptotic stability standpoint.

Local Stability

After an initial disturbance, the distance between LV and FV stabilizes into a pattern; this pattern is examined for different input conditions.

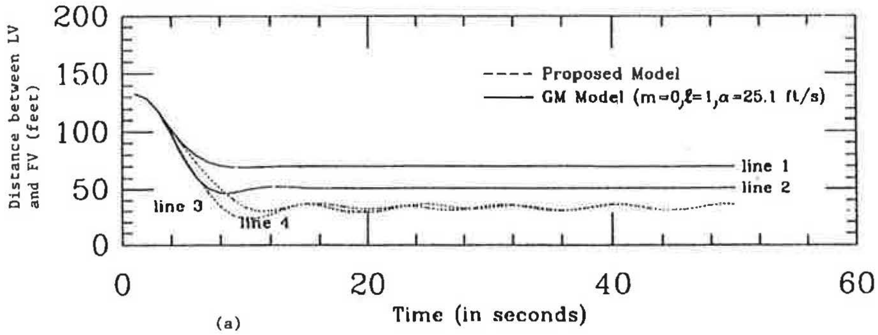
Figure 3 compares traffic stability obtained from the GM model (for $m = 0$, $\ell = 1$) with the one obtained from the proposed model. The two models are compared under the following conditions.

- Initial distance between LV and FV, 133 ft;
- Speed change of LV: Case 1, LV decelerates from 44.1 to 28.1 ft/sec in 2 sec and remains constant; Case 2, LV decelerates from 52.1 to 28.1 ft/sec in 3 sec and remains constant.

Case 1 corresponds to the example presented by Herman and Potts (*8*). Figures 3a and 3b differ in the assumed value of α in the GM model: $\alpha = 25.1$ ft/sec in Figure 3a, $\alpha = 17$ ft/sec in Figure 3b. Lines 1 and 2 represent the results of the GM model for Cases 1 and 2, and Lines 3 and 4 represent the results of the proposed model for Cases 1 and 2, respectively. Line 1 of Figure 3a is actually the same as the one presented by Herman and Potts (*8*, Figure 15).

Since the final speeds are the same in Cases 1 and 2, the results of Cases 1 and 2 should converge as time increases. This is the case in the proposed model (Lines 3 and 4 eventually merge). However, in the GM model, Lines 1 and 2 remain separate both in Figures 3a and 3b. Figure 3b shows that, for an arbitrarily chosen value of $\alpha = 17$ ft/sec in the GM model, the result of Case 2 is almost identical to the one derived from the proposed model. As seen in the forthcoming figures in this section, for the same final speed the proposed model yields the same stable distance between LV and FV regardless of the initial condition.

Figure 4 shows how the speed change of LV affects the distance D between LV and FV in the proposed model. LV changes its speed from 60 to 50 ft/sec in 2 sec, changes back to 60 ft/sec in 2 sec, and thereafter continues to travel at 60 ft/sec. Each of the four lines represents a different initial distance between LV and FV (100, 120, 150, and 180 ft). It is seen that D settles to approximately 125 ft regardless of the initial distance. However, the way D settles to 125 ft differs with the initial distance. When the initial distance is near 125 ft (final stable distance), D fluctuates more before settling to the stable distance. This suggests that the model can represent the susceptibility of the FV to the action of LV.



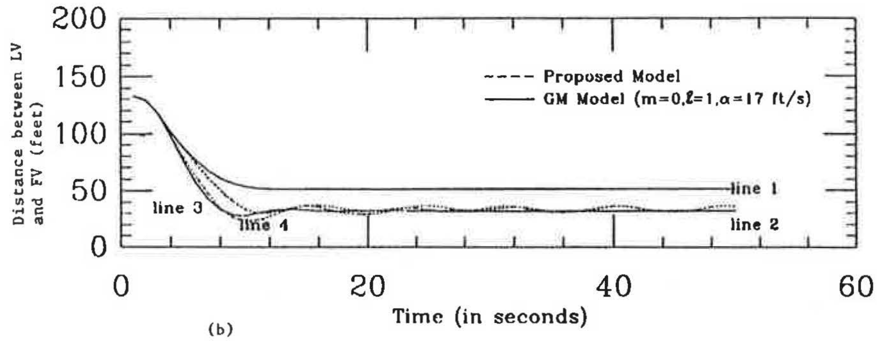


FIGURE 3 Comparison of the traditional car-following model with the proposed model.

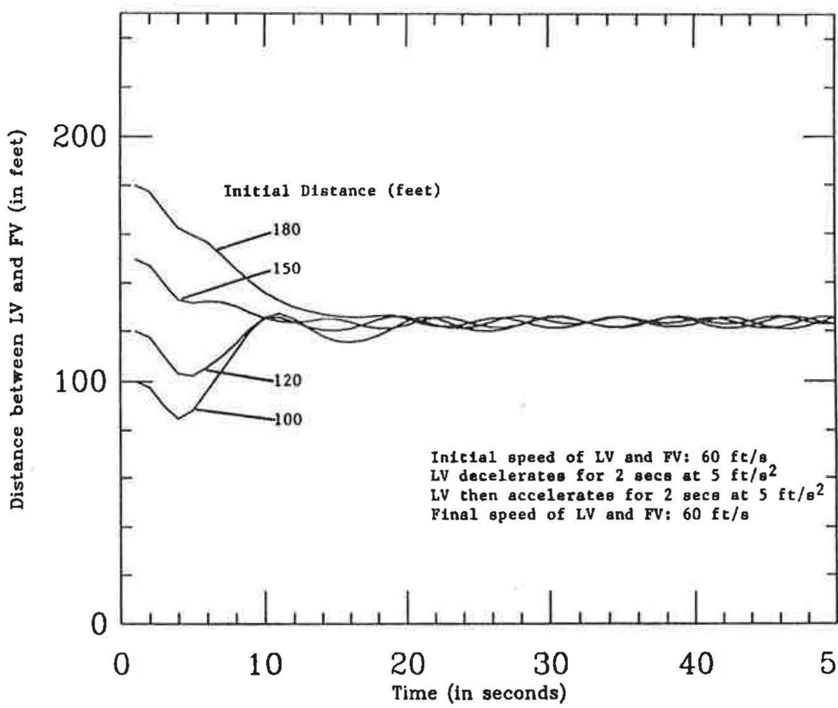


FIGURE 4 Local traffic stability: different initial distances between LV and FV.

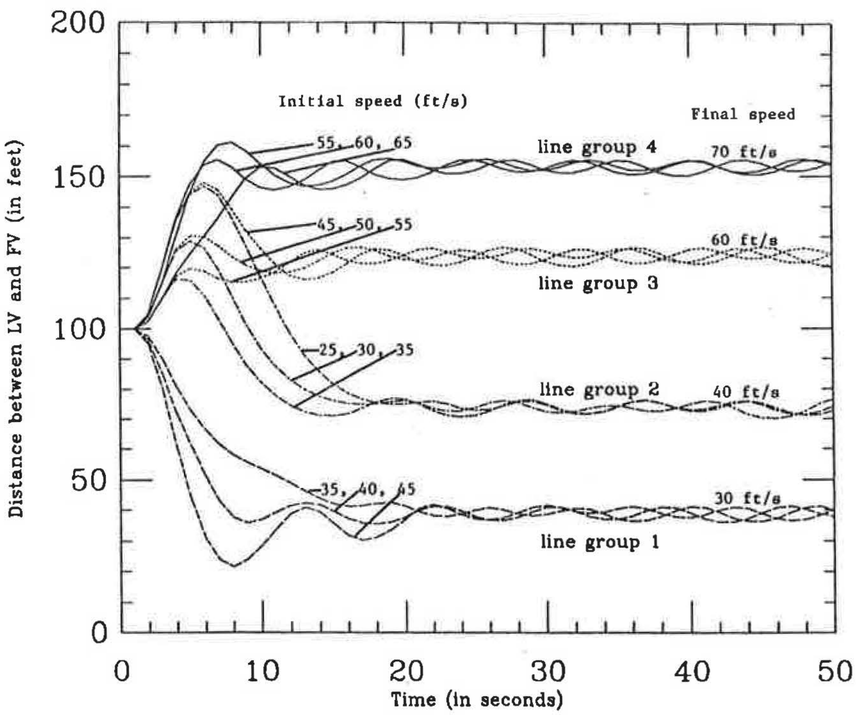


FIGURE 5 Local traffic stability: different initial and final speeds.

Figure 5 shows how D, the distance between LV and FV, depends on the LV's final speed using the proposed model. In all cases, the initial distance is 100 ft. Line Groups 1, 2, 3, and 4 present the final speeds of 30, 40, 60, and 70 ft/sec, respectively. The three lines in each group represent different initial speeds, as noted in the figure. For all cases, the LV is assumed to attain the final speed in 3 sec. Regardless of the initial speed, D approaches a higher constant value for a higher final speed.

Asymptotic Stability

Figure 6 shows how the distance between individual cars in a platoon can vary when the first car in the platoon decelerates and then accelerates under the proposed model. In this example, the platoon consists of five cars. The first car decelerates from 50 to 40 ft/sec in 2 sec, then accelerates back to 50 ft/sec in 2 sec, and thereafter travels at a constant speed of 50 ft/sec. Line 1 represents the variation in distance between the first and second cars, Line 2 between the second and third cars, and so forth. It is interesting to note that the pattern of variation in distance between the third and fourth car (Line 3) is different from the rest.

Speed-Density Relationship

After the distance between LV and FV settles to a stable value, D^*, the speed-density (u-k) relationship is analyzed. This was performed for different final stable speeds. Density is computed in the number of vehicles per mile. Since the value of D^* obtained from the model is a fuzzy number, the density obtained from D^* is also a fuzzy number, and thus the predicted u-k relationship is a fuzzy relationship. The computed fuzzy u-k relationship is compared with the plot of observed data in Figure 7.

In the figure the band formed by Lines A and B is the range of possible densities for the speed. The range corresponds to the density whose membership grade is 0.2 or greater. The value 0.2 is chosen only for the purpose of reference in this paper. Line C corresponds to the locus of the density whose membership grade is 1.

When density is high, the vehicles are expected to travel in the car-following pattern. Therefore, a reasonable match between the predicted and observed u-k relationships is expected. This notion is supported by the figure for the range where density is greater than approximately 40 vehicles per mile (vpm).

When density is low, the vehicles are expected to travel independent of one another. Therefore the car-following pattern (stimulus-response interaction) is not likely to be sustained, and hence, the proposed model would not be valid; thus the lines A, B, and C are shown only for density values greater than 40 vpm, the region where flow is reasonably dense.

The data points in the previous figure are derived from the speed-occupancy data obtained from Queen Elizabeth Way, Ontario, Canada (courtesy of Fred Hall). The conversion from the occupancy measure to the density measure is performed on the basis of an average vehicle length of 20 ft.

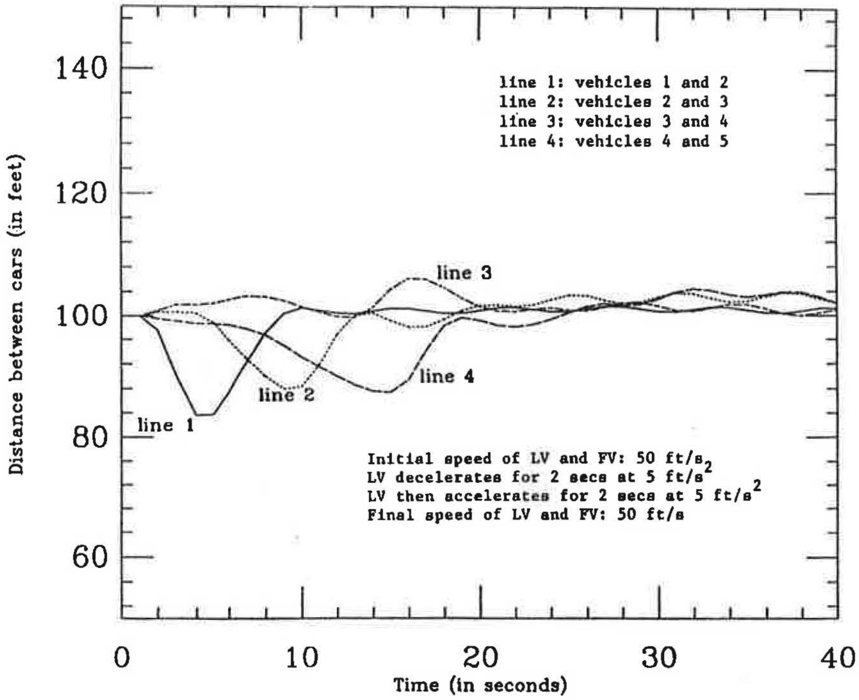


FIGURE 6 Asymptotic traffic stability for five-car platoon.

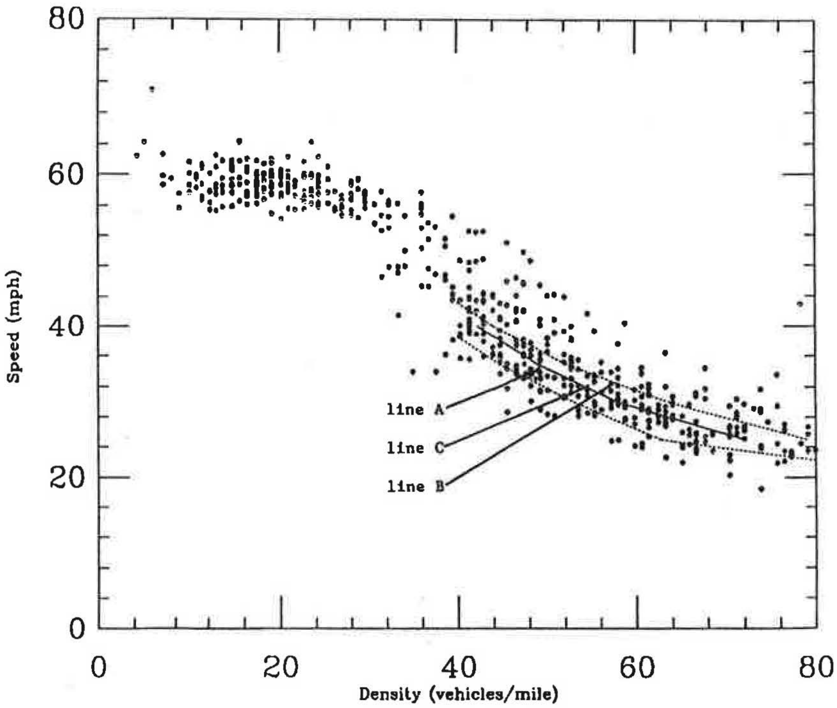


FIGURE 7 Speed-density relationship: predicted relationship and observations.

CONCLUSIONS

Decisions and actions of a driver are believed to follow a reasoning process based on vague logic. The model proposed in this paper applies the fuzzy inference system and simulates the car-following phenomenon. The output of the inference system is the FV's reaction (acceleration or deceleration rate) in fuzzy number in small time increments. By integrating this output over time, the movements of FV relative to LV are simulated. The purpose of the paper is to present the methodology of building the model. The shapes of specific membership functions used in the model must still be verified through field data collection.

The proposed model is a response to the concern that drivers do not exercise the dichotomous decision criteria assumed in the traditional deterministic car-following models. The model proposed has the following characteristics:

1. Driver's decision criteria are handled by fuzzy inference logic, which allows several decision rules to fire at the same time for a given set of input. As a result, the final output incorporates the ambiguity of the decision process.
2. The inference rules are a collection of natural language–based straightforward driving rules. The number of rules can be adjusted, and each rule can be independently modified to suit the decision criteria.
3. The output is a fuzzy number that represents a range of possible acceleration (or deceleration) rates of the FV. Thus, it captures the characteristics of traffic flow as the conglomeration of an individual driver's possible actions. Under the deterministic models, the variation of data points is viewed as random variation from a norm.
4. The result is realistic and consistent with the general expectation from a car-following model: for the same final speed, the distance between LV and FV eventually converges to the same value regardless of the initial condition. The "drift," oscillation of the distance between LV and FV, can also be captured.

The proposed approach to the car-following problem should have a number of applications, including control of vehicle separation under the IVHS. For the traffic flow analysis, the model can be extended to derive a possibility-based speed-volume relationship. Such a relationship would allow us to analyze the capacity as a fuzzy number and to recognize the level of service as the fuzzy measure of traffic conditions, instead of as the traditional rigidly bounded measure.

ACKNOWLEDGMENT

The authors are grateful to Fred L. Hall of McMaster University, Ontario, Canada, for providing the traffic flow data used in this study.

REFERENCES

1. A. D. May. *Traffic Flow Fundamentals*. Prentice-Hall, Englewood Cliffs, N.J., 1990.
2. R. E. Chandler, R. Herman, and E. W. Montroll. Traffic Dynamics: Studies in Car-Following. *Operations Research*, Vol. 6, 1958, pp. 165–184.
3. D. C. Gazis, R. Herman, and R. B. Potts. Car-Following Theory of Steady State Traffic Flow. *Operations Research*, Vol. 7, 1959, pp. 499–505.
4. D. C. Gazis, R. Herman, and R. W. Rothery. Non-Linear Follow the Leader Models of Traffic Flow. *Operations Research*, Vol. 9, 1960, pp. 545–567.
5. R. Herman, E. W. Montroll, R. B. Potts, and R. W. Rothery. Traffic Dynamics: Analysis of Stability in Car-Following. *Operations Research*, Vol. 7, 1959, pp. 86–106.
6. A. Ceder. A Deterministic Traffic Flow Model for the Two-Regime Approach. In *Transportation Research Record 567*, TRB, National Research Council, Washington, D.C., 1976, pp. 16–30.
7. W. Leutzbach. *Introduction to the Theory of Traffic Flow*. Springer-Verlag, 1988.
8. R. Herman and R. B. Potts. Single-Lane Traffic Theory and Experiment. *Proc., Symposium on the Theory of Traffic Flow*, Dec. 1959, pp. 120–146.
9. P. Ross. Modelling Traffic Flow. *Public Roads*, Dec. 1987, pp. 90–96.
10. R. S. Gilchrist and F. L. Hall. Three-Dimensional Relationships Among Traffic Flow Theory Variables. In *Transportation Research Record 1225*, TRB, National Research Council, Washington, D.C., 1989, pp. 99–108.
11. R. T. Underwood. *Speed, Volume and Density Relationships*. Thesis. Bureau of Highway Traffic, Yale University, New Haven, Conn., 1960.
12. L. A. Zadeh. Fuzzy Sets as a Basis for a Theory of Possibility. *Fuzzy Sets and Systems*, Vol. 1, 1978, pp. 3–28.
13. G. J. Klir and T. A. Folger. *Fuzzy Sets, Uncertainty, and Information*. Prentice-Hall, Englewood Cliffs, N.J., 1988.
14. H.-J. Zimmerman. *Fuzzy Set Theory and Its Applications* (2nd ed.). Kluwer Academic Publishers, 1990.
15. B. Kosko. *Neural Networks and Fuzzy Systems*. Prentice-Hall, Englewood Cliffs, N.J., 1991.
16. D. Dubois and H. Prade. *Fuzzy Sets and Systems, Theory and Applications*. Academic Press, New York, 1980.
17. A. Kaufmann and M. M. Gupta. *Introduction to Fuzzy Arithmetic, Theory and Applications*. Van Nostrand Reinhold Company, New York, 1985.
18. T. Takagi and M. Sugeno. Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 15, No. 1, 1985, pp. 116–132.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.