Statistical Properties of Vehicle Time Headways

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The properties of vehicle time headways are fundamental in many traffic engineering applications. The shape of the empirical headway distributions is described by density estimates, coefficients of variation, skewness, and kurtosis. The hypothesis of exponential tail is tested by Monte Carlo methods. The independence of consecutive headways is tested using autocorrelation analysis, runs tests, and goodness of fit tests for geometric bunch size distribution. The power of these tests is enhanced by calculating combined significance probabilities. The variation of significance over flow rates is described by "moving probabilities." It is shown that speed limit and road category have a considerable effect on the statistical properties of vehicle headways. The results also suggest that the renewal hypothesis should not be accepted under all traffic conditions.

Vehicle time headways play an important role in many traffic engineering applications, such as vehicle-actuated traffic signals, gap availability, and pedestrian delay. Mathematical analysis and simulation of these systems are usually based on theoretical models. The models should be verified against the properties of real world headways. This paper presents some statistical properties of vehicle headways on Finnish two-way, two-lane roads.

The properties of headways have been extensively studied, especially in the 1960s. Some of the earlier work is reviewed and compared with recent data. More powerful statistical techniques are also presented.

DATA COLLECTION AND PRELIMINARY ANALYSIS

Data Collection

The data were collected in 1984 and 1988 by the Laboratory of Traffic and Transportation Engineering at Helsinki University of Technology. A traffic analyzer with two inductive loops on both lanes recorded for each passing vehicle its serial number, time headway (time from front bumper to front bumper in units of 1/100 sec), net time headway (time from back bumper to front bumper in units of 1/100 sec), speed (in units of 1 km/hr) and length (in units of 1/10 m).

The study sites had speed limits of 50, 60, 70, 80, and 100 km/hr. The roads with lower speed limits had a lower overall standard, but all the road sections were reasonably level and straight, and no steep hills, intersections, or traffic signals were near. The samples were classified into two road categories with speed limits 50 to 70 km/hr and 80 to 100 km/hr.

The observations from high-speed (80 to 100 km/hr) roads were measured in 1984. These data have been previously analyzed by Pursula and Sainio (*1*) and Pursula and Enberg (*2*). Because the data were collected for capacity studies, the observations are concentrated in high volumes. Two-way volumes, also, are higher on high-speed roads. On low-speed roads the observations are more concentrated in low volumes.

More than 73,000 headways were recorded on 19 locations. Speed data were corrected according to radar measurements. Data sets with more than 1 percent overtakers were discarded. The samples were analyzed for trend using the method described later. Samples with more than 10 percent heavy vehicles (length > 6 m) were excluded from further analysis. Sixty-four trendless samples were chosen for further analysis.

The data that passed the preliminary phase consist of 64 samples and 16,570 observations (75 to 900 observations per sample). The flow rates vary from 140 to 1840 veh/hr.

Trend Analysis

The temporal variation of traffic is due to deterministic and random factors. To get generally applicable results about headway characteristics, it is necessary to consider stationary conditions. All nonrandom variation should be removed from the measurements as far as possible. Two approaches have been commonly used to overcome the problem of nonrandom variation: the samples are collected either as fixed time slices or using trend analysis.

In the first method the measurements are investigated in fixed time slices of length short enough to exclude any significant trend, typically 30 sec to 10 min (*1,3,4*). The number of headways in one sample is usually too small for statistical analysis, so it is necessary to group samples having nearly equal means. This may cause distortion in the empirical headway distribution because of inappropriate distribution of the sample means.

Because of these problems the second sampling method was chosen. Trend analysis was performed with a computer program (TRENDANA) showing graphically each headway, 15-point moving average, cumulative vehicle count, and the speed of each vehicle. The data were analyzed sequentially using trend tests. The sample size was incremented by 50 until the test reported trend at 5 percent level of significance. The sample was then decremented until the level of significance for trend was 30 to 70 percent with sample size more than 100 and sampling period between 5 and 40 min. Under low volumes the sample size or period length condition had to be relaxed sometimes. If a satisfactory sample was not found,

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the first observations that apparently caused the trend were removed and the process was repeated.

The program supports three trend tests: weighted sign (WS) test (*5*), Kendall's rank correlation (RC) test (*6*), and exponential ordered scores (EOS) test (*7*).

Empirical power curves of these tests were evaluated using Monte Carlo methods. The EOS test was found most powerful with RC test close behind (*8*). Because of greater computational effort, the EOS test was used only in more detailed analysis. Preliminary analysis was performed using faster tests.

SHAPE OF HEADWAY DISTRIBUTION

Density Estimates

The density function of the headway distribution is estimated by the histogram method with origin at 0 and bin width 1. Figure 1 shows a surface plot of density estimates on low-speed roads at different flow levels. Headway 0 is assigned Frequency 0. Only samples having more than 100 observations are included.

The distribution is skewed to the right. The proportion of headways less than 1 sec is small. The mode is rather constant (1.5 sec) under all speed limits and flow rates. Because the distribution is unimodal and skewed to the right, the measures of location occur in the following order: mode, median, mean (*9–11*).

Peak heights of the empirical distributions are shown in Figure 2. The peak rises as the flow increases. On high-speed roads the peak value is higher than on low-speed roads. On low-speed roads the peak rises steeply under medium and low flow rates. Under high flow rates the speed limit looses its significance.

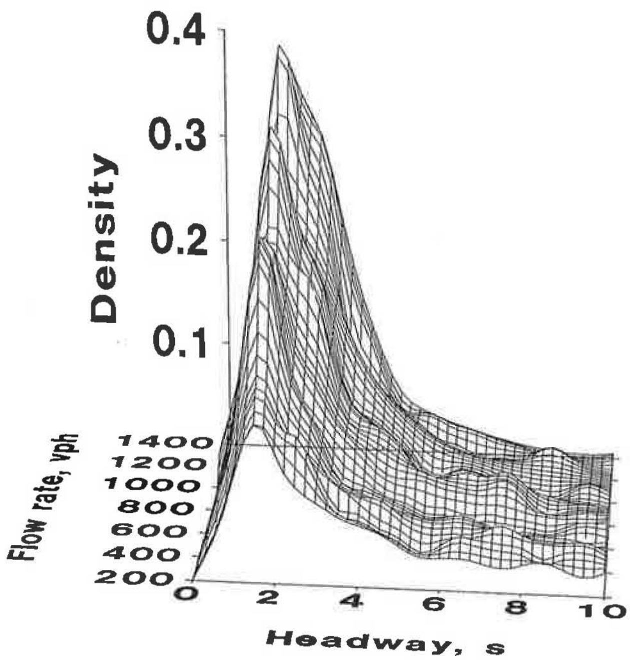

FIGURE 1 Surface plot of headway density estimates on low-speed (50 to 70 km/hr) roads.

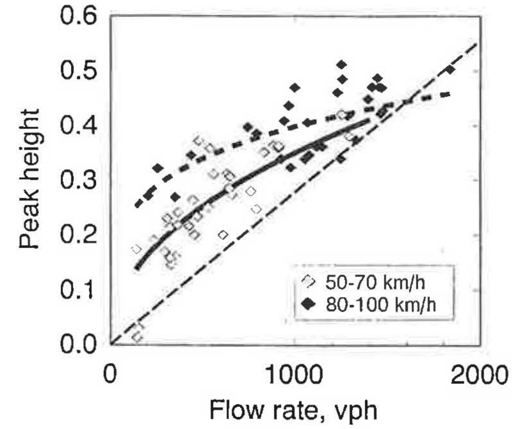

FIGURE 2 Peak height of empirical headway distributions for speed limits 50 to 70 km/hr (solid curve) and 80 to 100 km/hr (dashed curve). Thin dashed line is the peak height of the exponential distribution.

Coefficient of Variation

The sample coefficient of variation (CV) is the proportion of sample standard deviation to sample mean:

$$CV = s_T/\mu_T \tag{1}$$

In distribution functions CV is the proportion of standard deviation to expectation. The negative exponential distribution has CV equal to 1.

Polynomial curves have been fit (Figure 3) to the data for high-speed and low-speed roads. The curves are forced to 1 at flow rate $q = 0$. This is based on the assumption of Poisson tendency in low density traffic.

Some basic properties of CV can be observed in Figure 3:

1. Under heavy traffic the proportion of freely moving vehicles is small. The variance of headways is accordingly small. This phenomenon is reflected in the figure by $CV < 1$ at high flow levels.

2. The Poisson tendency of low density traffic has a theoretical (*12*) as well as an intuitive basis: under light traffic

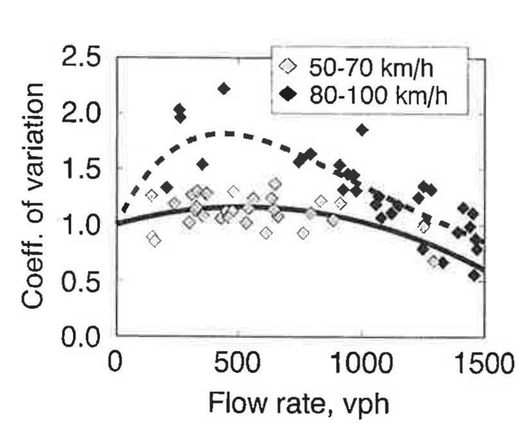

FIGURE 3 Coefficient of variation of the headway data for speed limits 50 to 70 km/hr (solid curve) and 80 to 100 km/hr (dashed curve).

vehicles can move freely, and randomness of the process increases. *CV* is therefore expected to approach 1 as flow approaches 0.

3. Under medium traffic there is a mixture of leading and trailing vehicles. This increases the variance above pure random process, and *CV* rises above 1. This is in contrast to the statement by May (*11*) that *CV* approaches 1 under low flow conditions but decreases continuously as the flow rate increases.

4. High-speed roads have greater *CV* than low-speed roads. In the present data the opposite flow rate is higher on high-speed roads, thus reducing overtaking opportunities. Other explanatory factors may be higher variation of speeds and greater willingness to overtake on high-speed roads. On low-speed roads the intersections are more densely spaced. So, there are more joining and departing vehicles, and trip lengths are usually shorter.

These observations gain at least partial support from other authors, as seen in Figure 4. The figure also shows that the studies are based on quite different data. The data sets of Breiman et al. (*13*), Buckley (*3*), and May (*10*) come from a freeway lane. The data of Dunne et al. (*14*) come from a two-lane rural road. *CV* is greater than 1 in all samples of Dunne et al., less than 1 in all samples of May, and near 1 in the samples of Buckley. The samples of Buckley have values similar to the present data from low-speed roads. Finnish studies (*1*) suggest that coefficient of variation on freeways, especially on the first lane, is lower than on two-lane highways.

Skewness and Kurtosis

The proportion of the first two moments was discussed earlier. The third and fourth moments about the mean, skewness and kurtosis, give more information about the shape of the distribution. Skewness is a measure of symmetry. Symmetric distributions have null skewness. If the data are more concentrated on the low values, as in headway distributions, skewness is positive. Kurtosis is a measure of how "heavy" the tails of a distribution are.

Figure 5 shows the sample kurtosis against the square of sample skewness. This relationship is sometimes used as a guide in selecting theoretical distributions (*15*). There is a strong linear relationship, which suggests the usefulness of this measure in model selection.

Points and lines of some theoretical distributions are shown for comparison. As skewness grows, kurtosis increases more slowly than in either the gamma or the lognormal distributions. The gamma distribution is closer to observed values than the lognormal distribution, even though the lognormal distribution is a better model for a headway distribution. The exponential distribution reduces to a point and totally loses the variety in the empirical headway distributions.

Exponential Tail Hypothesis

Several headway models are combinations of two distributions: one for leaders and the other for followers (*3,16–20*). The assumption is usually made that the leaders' headway distribution is exponential.

The tails (large headways) of empirical headway distributions were tested for exponentiality. Goodness of fit tests are based on the Anderson-Darling statistic. Such tests are more powerful than the better-known Kolmogorov-Smirnov and chi-squared tests (*21*). Because the parameters of the distribution are estimated from the sample, nonparametric tests give too conservative results (*22,23*). So, the significance of the tests was estimated using parametric tests and Monte Carlo methods (*24*). The number of replications was 10,000. The tests were performed using threshold values (t_0) from 0 to 14.5 in increments of 0.5.

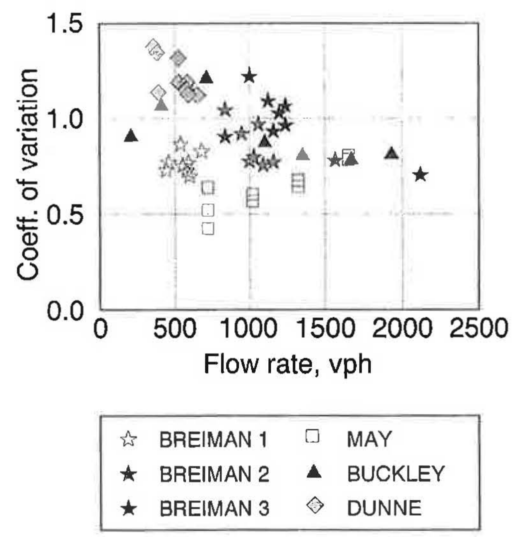

FIGURE 4 Coefficient of variation of headway distributions from different sources. (The number after "Breiman" stands for the freeway lane.)

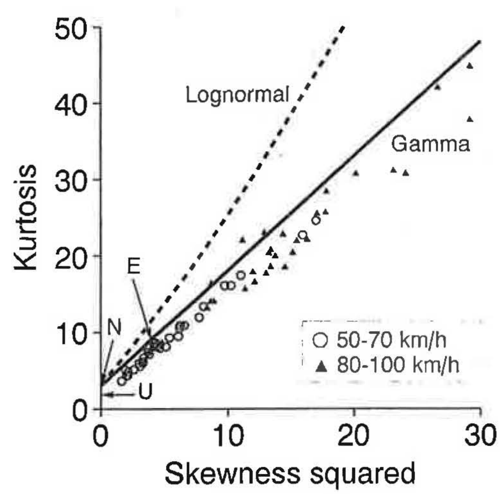

FIGURE 5 Kurtosis and squared skewness for the headway samples and some theoretical distributions. Exponential (E), normal (N), and uniform (U) distributions reduce to points shown by arrows.

To get a more powerful test, the significance probabilities from several samples were combined using the method proposed by Fisher (*25*). If the null hypothesis is true in all samples and the samples are independent, the probabilities are uniformly U(0,1) distributed. If probabilities p_i are U(0,1), the statistic

$$z = -2 \sum_{i=1}^{n} \ln p_i \tag{2}$$

has chi-squared distribution with $2n$ degrees of freedom. So, the combined significance (P) is the probability that a variable Z having chi-squared distribution with $2n$ degrees of freedom is greater than z:

$$P = Pr\{Z > z\} = 1 - F_{\text{chi2}}(z; 2n) \tag{3}$$

Figure 6 shows the combined significance levels against threshold values (t_0) at low- and high-speed roads and at different flow levels. Wasielewski (*4*) found no departures from the exponential distribution at threshold values greater than or equal to 4 sec on freeways. On the basis of Figure 6 this value appears too low on two-lane roads. The threshold value for not rejecting the hypothesis of exponential tail is about 8 sec. On low-speed roads lower threshold values may be possible. Miller (*16*) also found $t_0 = 8$ sec appropriate. Because of large t_0, a headway distribution that has positive skewness (such as gamma and lognormal distributions) should be considered for the followers.

Another indicator of the threshold is the influence of the speed of a vehicle on the speed of the trailing vehicle. A driver considering himself as a follower adjusts his speed to the speed of the vehicle ahead. This speed adjustment decreases the variation of relative speeds (speed differences) among successive vehicles. At some time distance the interaction of speeds disappears, and variation of relative speeds remains constant among vehicles having larger headways than the threshold. Figure 7 shows the standard deviation (s_v) of relative speeds against headway. Headways are combined in 1-sec intervals ($t-1$, t). At short headways s_v is small and increases as the headway increases. At large headways s_v is rather constant, but has greater variation because of smaller sample sizes. High-speed roads have larger s_v than low-speed roads.

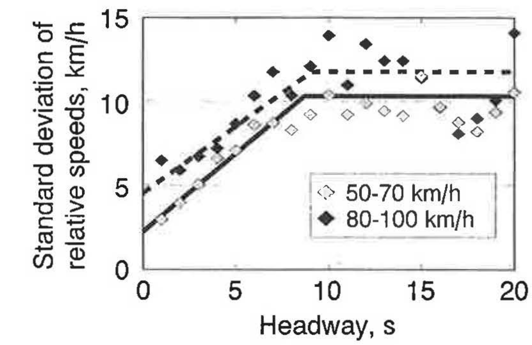

FIGURE 7 Standard deviation of relative speeds against time headways on low-speed (solid line) and high-speed (dashed line) roads.

A piecewise linear model was fit to the data—rising slope for headways less than the threshold and constant value for headways greater than the threshold. The s_v values were weighted by the number of observations in the interval. On both low- and high-speed roads the threshold value of about 9 sec was obtained. The original *Highway Capacity Manual* (*26*) applies similar methods with the same result. Similar analysis of motorway data by Branston (*27*) gives values of 4.5 sec and 3.75 sec for nearside and offside lanes, respectively.

Because all vehicles having headways ≤ 8 sec are not followers, the distribution of their relative speeds is a mixture of free speeds and constrained speeds. By fitting a mixed normal distribution to the relative speed data (Figure 8) the proportion of free-flowing vehicles among headways ≤ 8 sec is estimated to be 20 percent on high-speed roads and 15 percent on low-speed roads. The proportion of trailing vehicles is then estimated to be approximately the same as the proportion of headways ≤ 3.1 sec and ≤ 5.0 sec, respectively.

In the present data the flow rate at which the headway coefficient of variation (Figure 3) reaches its maximum has about 60 percent trailing vehicles. Yet, more extensive data sets and more accurate measuring equipment are needed for

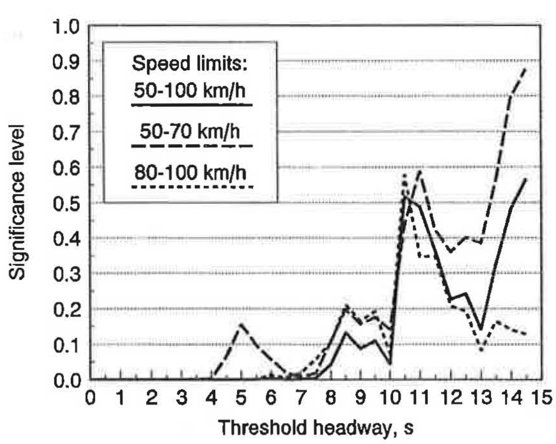

FIGURE 6 Goodness-of-fit tests for exponential tail of headway distributions.

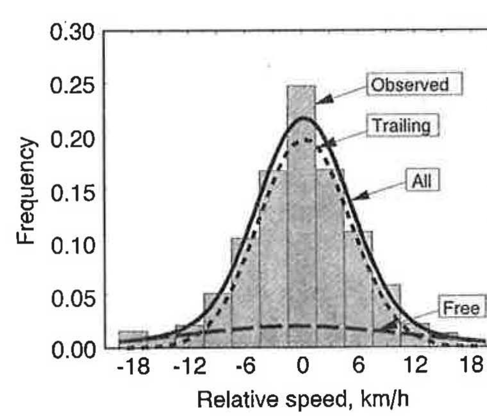

FIGURE 8 Relative speed distribution for headways ≤ 8 sec on high-speed roads as a mixed normal distribution.

further analysis, especially because relative speeds have larger measurement errors than absolute speeds.

RENEWAL HYPOTHESIS

Autocorrelation

The shape of the headway distribution describes the frequency of headways of different length. One step further in the statistical analysis is to examine the order in which the headways take place. A common assumption is the renewal hypothesis, which states that headways are independent and identically distributed. This hypothesis makes many theoretical analyses much easier. On the other hand, correlation between consecutive headways could give additional information for adaptive traffic control systems.

The autocorrelation coefficient is a measure of correlation between observations at given distances (lags) apart. In a sample of n observations the estimate of the autocorrelation coefficient at Lag k is

$$\hat{r}_k = \frac{\sum_{j=1}^{n-k} (T_j - \mu_T)(T_{j+k} - \mu_T)}{\sum_{j=1}^{n} (T_j - \mu_T)^2} \tag{4}$$

where

$$\mu_T = 1/n \sum_{j=1}^{n} T_j \tag{5}$$

The coefficient estimates are asymptotically N(0, $1/n$) distributed.

Autocorrelation coefficients indicate whether the observations are from a renewal process ($r_k = 0, k = 1, 2, \ldots$). The most important coefficient in this respect is r_1. The test is

$$H_0: r_1 \leq 0$$

$$H_1: r_1 > 0 \tag{6}$$

This is a one-sided test in contrast to the two-sided tests normally used (*14,20,28*). The tests for negative autocorrelation gave clearly nonsignificant values.

The variation of significance probabilities (p_i) over flow rates (q_i) is described by "moving probability" (Figure 9). The k-point moving probability at flow rate Q_j is

$$P_k(Q_j) = 1 - F_{\text{chi2}}(z_j; 2k) \tag{7}$$

where

$$z_j = -2 \sum_{i=j}^{j+k-1} \ln p_i \tag{8}$$

$$Q_j = 1/k \sum_{i=j}^{j+k-1} q_i \qquad j \in \{1, \ldots, n - k + 1\}$$

On high-speed roads there is significant positive autocorrelation among consecutive headways. There is a spike in the curve, for which no explanation except random variation is found. (A similar although lower spike is in Figures 10 and 11.) On low-speed roads there is no significant autocorrelation, at least under low flow rates. The moving probability curve, however, goes down to significant values near flow rate q = 1,000 veh/hr. The combined significance for high-speed samples is about $3 \cdot 10^{-22}$ and for low-speed samples 0.04.

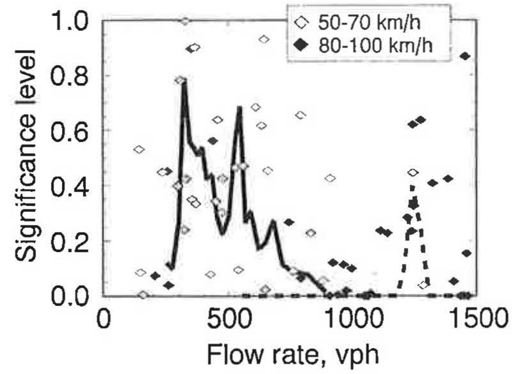

FIGURE 9 Significance of sample autocorrelation coefficients at Lag 1. Nine-point moving probabilities for low-speed (solid curve) and high-speed (dashed curve) roads.

These results suggest that the renewal hypothesis should be rejected on high-speed roads, especially under flow rates above 500 veh/hr. On low-speed roads the possibility of autocorrelation should be considered at least under flow rates greater than 1,000 veh/hr.

Dunne et al. (*14*) also studied the autocorrelation of trend-free samples. The combined probability of their nine data sets (Lag 1) is 0.702 (one-sided test), which is consistent with the renewal hypothesis. The two-sided test gives 0.284, which is also nonsignificant. Breiman et al. (*28*) found in one of eight data sets (three-lane unidirectional section of John Lodge Expressway in Detroit) significant autocorrelation (Lag 1) at the 0.05 level. The hypothesis of independent intervals was not rejected. The combined significance is 0.23. Testing for positive autocorrelation only (one-sided test), the combined significance is 0.05, suggesting possible positive autocorrelation. Cowan (*19*) studied 1,324 successive headways. The re-

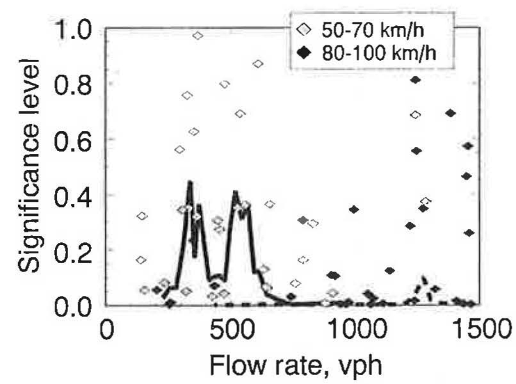

FIGURE 10 Significance of runs tests. Probability of fewer runs above or below median. Seven-point moving probabilities for low-speed (solid curve) and high-speed (dashed curve) roads.

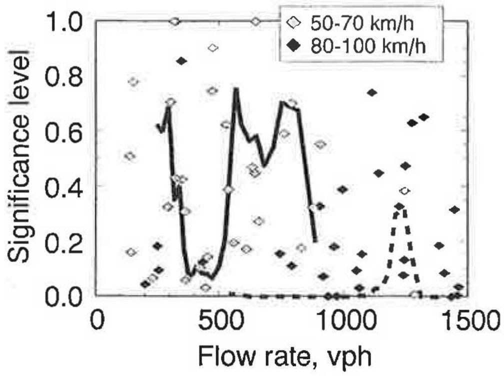

FIGURE 11 Significance of bunch size tests for geometric distribution. Nine-point moving probabilities for low-speed (solid curve) and high-speed (dashed curve) roads.

newal hypothesis was not rejected. Chrissikopoulos et al. (*20*) studied six samples. The result of their unspecified tests was that the headways are independently distributed. However, the combined significance level of their data sets (Lag 1) is 0.012 using two-sided test and 0.0015 when testing for positive autocorrelation. This result disagrees with their conclusion. (Combining further the three combined probabilities above yields 0.008 for one-sided and 0.027 for two-sided tests.) Breiman et al. (*13*) allow for the possibility of small positive autocorrelation in freeway traffic.

Previous studies have so far supported the renewal hypothesis. Further analysis of this material has now cast some doubt on the conclusion. Also, the new data presented here show that the possibility of positive autocorrelation between consecutive headways should be taken seriously, although the magnitude of autocorrelation is small (about 0.1).

Randomness

Randomness of the headway data was tested using the Wald and Wolfowitz (*29*) runs test. The test is performed to determine whether long and short headways are randomly distributed or whether short headways are clustered. Testing runs above and below the median is appropriate here (*28*). Clustering reduces the number of runs. The number of runs is also reduced by trends in the data. Therefore, it is important to have trendless data.

The number of runs is assumed to be normally distributed with mean and variance equal to

$$\text{mean} = 2r(n - r)/n + 1$$

$$\text{variance} = 2r(n - r)\,[2r(n - r) - n]/[n^2(n - 1)] \tag{9}$$

where n is the total number of observations and r is the number of observations below the median (*29*). Observations equal to the median are ignored. One-sided test is used to find the probability of fewer runs.

Figure 10 shows the moving probabilities for high- and low-speed roads. On high-speed roads the test gives significant values nearly everywhere. On low-speed roads nonrandomness is significant under flow rates > 700 veh/hr. The combined significance for high-speed roads is $1.6 \cdot 10^{-20}$ and for low-speed roads is 0.001. These results suggest that the arrival process in road traffic is not totally random, but clustered.

Breiman et al. (*28*) found only one significant value in eight runs tests on their data sets. The combined probability (0.49) is also nonsignificant. This is in contrast to the preceding results.

Bunching

Vehicle i is a follower (0) if it has headway at most s, otherwise it is a leader (1). The status of a vehicle is accordingly defined as

$$X_i = \begin{cases} 0 & \text{if } T_i \le s \\ 1 & \text{if } T_i > s \end{cases} \tag{10}$$

The difference in speed is ignored. If the headways are independent and identically distributed (i.i.d.), the probability of Vehicle i being a follower is

$$p = Pr\{T_i \le s\} \tag{11}$$

The number of vehicles in a bunch is the number of consecutive headways $\le s$ (followers) plus 1 (leader). The bunch is of size n if $X_1 = 1, X_2 = 0, \ldots, X_n = 0, X_{n+1} = 1$. If the headways are i.i.d., the bunch sizes are geometrically distributed (*30*) and the probability of bunch size k is

$$p_k = p^{k-1}(1 - p) \qquad k \ge 1 \tag{12}$$

Now the renewal hypothesis (i.i.d. headways) can be tested using the null hypothesis

$$H_0\colon p_k = p^{k-1}(1 - p) \tag{13}$$

against

$$H_1\colon p_k \ne p^{k-1}(1 - p) \tag{14}$$

The chi-squared test was performed with $m-2$ degrees of freedom, where m is the number of classes (different bunch sizes) in the sample. One degree of freedom was lost, because p was estimated from the sample. The threshold for leaders was set to $s = 5$ sec. Bunch sizes 1, 2, . . . , 20 and > 20 were separated into distinct classes. They were combined so that the expectation for each class was ≥ 5, except for the last, which was ≥ 1. On high-speed roads two samples (having flow rates of 1,837 and 1,457 veh/hr) were left out of the analysis, because after combining classes there were no degrees of freedom left.

Figure 11 shows the results of the chi-squared tests. The combined probability is 0.13 on low-speed roads. On high-speed roads the combined probability is $1.9 \cdot 10^{-9}$. So, the hypothesis of geometric bunch size distribution should be rejected on high-speed roads, at least under flow rates above 500 veh/hr. On low-speed roads the hypothesis of geometric distribution cannot be rejected. Chrissikopoulos et al. (*20*) and Taylor et al. (*31*) discard the geometric distribution as a bunching model.

CONCLUSIONS

There is a considerable difference between high-standard and low-standard roads. The headway distributions on high-standard roads have higher peak values and higher coefficient of variation. That is, at a given flow level there are more small headways on high-standard roads. The vehicles are also more clustered and there is a small positive autocorrelation between consecutive headways. On low-standard roads there is some indication of possible positive autocorrelation under high flow rates. On high-standard roads the autocorrelation is statistically significant but too small to be helpful, for example, in traffic control applications. In simulation studies and bunching models the stochastic structure of the arrival process should be fully considered.

The examination of relative speeds and the tail of the headway distribution supports the view that drivers become affected by the vehicle ahead when the headway is less than 8 to 9 sec. At larger headways the standard deviation of relative speeds is rather constant and headways are exponentially distributed. At smaller headways the standard deviation of relative speeds decreases and the hypothesis of exponentiality must be rejected. The proportion of trailing vehicles on high- and low-speed roads is approximately the same as the proportion of headways $\leq$ 3.1 sec and $\leq$ 5.0 sec, respectively.

Local conditions, such as road category, speed limit, and flow rate, have a considerable effect on the statistical properties of headways. The effect of opposing traffic, especially, deserves further research. But the statistical analysis of vehicle headways requires very extensive data sets and the application of powerful statistical techniques.

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