Modeling Queued Driver Behavior at Signalized Junctions

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Some of the findings from a recent study of the queue discharge headway process are summarized. One outcome of the study was the development of a model of discharge headway at signalized junctions. The model is based on vehicle and driver capabilities, including driver reaction time, driver acceleration, and vehicle speed. To calibrate the model, data were collected at five signalized junctions. The discharge headway model developed in this research indicates that the minimum discharge headway of a traffic movement is not reached until the eighth or higher queue positions. Application of the model suggests that the minimum discharge headway of a traffic movement under ideal conditions may be shorter than 2.0 sec/veh and that its corresponding startup lost time may be longer than 2.0 sec.

Some of the findings from a recent study of the queue discharge headway process at single-point urban interchanges (SPUIs) (1) are summarized. One outcome of the study was the development of a model of discharge headway at signalized junctions. This model is based on vehicle and driver capabilities such as driver reaction time, driver acceleration, and vehicle speed.

BACKGROUND

Discharge Headway

Average vehicle headways by queue position have been the subject of several past studies (2–5). The headways reported in these studies for passenger car through movements are shown in Figure 1, which indicates that the discharge rate varies during the initial portion of the green interval. The variation reflects the reaction time of the first driver responding to the change in signal indication and the steady acceleration of the first few vehicles in queue. Eventually, the headways stabilize at a relatively constant value, which is called the minimum discharge headway. In recognition of this trend toward convergence after the first few vehicles, the 1985 Highway Capacity Manual (HCM) (6, Chapter 9) recommends that the headways of the fifth and subsequent queued vehicles be averaged to estimate the minimum discharge headway.

Under ideal operating conditions (i.e., 12-ft lanes, all through vehicles, all passenger cars, no parking, flat grade, and no pedestrian activity), the 1985 HCM recommends 1,800 vphgpl as the saturation flow rate of a traffic lane at a signalized intersection. This value corresponds to a minimum discharge headway of 2.0 sec/veh. More recent research, such as that by Lee and Chen (5) and Zegeer (7), suggests that the ideal minimum discharge headway may be shorter than 2.0 sec/veh. Although Lee and Chen do not specifically calculate a minimum discharge headway for their data set, the average of the 5th through 10th headways that they reported is 1.97 sec/veh. Similarly, Zegeer (7) found an ideal minimum discharge headway of 1.92 sec/veh.

Headway Models

The discharge headway between successive vehicles has been described by Drew (8) in terms of a time-space diagram, as shown in Figure 2. The curved lines in Figure 2 represent the trajectories of individual vehicles as they travel through the intersection. The curved portions of each trajectory represent the acceleration or deceleration of the individual vehicles. As each successive vehicle crosses the stop line, its speed increases and its headway decreases. At a point after the fourth or fifth vehicle, the speed of each vehicle crossing the stop line becomes constant and, as a result, so do the headways between vehicles.

A deterministic model of the headway process based on the trajectories shown in Figure 2 has been described by Briggs (9). His model, which is based on the assumption that queued vehicles accelerate at a constant rate, has the following form.

\[ h_n = T + \sqrt{\frac{2 + d + n}{A}} - \sqrt{\frac{2 + d + (n - 1)}{A}} \]  

\[ \text{Discharge Headway (sec)} \]

\[ \text{Queue Position} \]

FIGURE 1 Comparison of past studies of queue discharge headway.
\[ h_n = T + \frac{d}{V_q} \]
\[ d_{\text{max}} = \frac{V_q^2}{2 \cdot A} \]

where
- \( h_n \) = headway of the \( n \)th queued vehicle (sec),
- \( n \) = queue position (\( n = 1, 2, 3, \ldots \)),
- \( d \) = distance between vehicles in a stopped queue (ft),
- \( V_q \) = desired speed of queued traffic (fps),
- \( d_{\text{max}} \) = distance traveled to reach speed \( V_q \) (ft),
- \( T \) = driver starting response time (sec), and
- \( A \) = constant acceleration of queued vehicles (fps).

Briggs calibrated his model using data from five previous studies conducted in the United States and Germany by other researchers. The parameters yielding the best fit were \( T = 1.22 \) sec, \( A = 3.67 \) fps, \( d = 19.65 \) ft for each queued vehicle, and \( V_q = 29.4 \) fps.

The model has two parts. The part to use depends on whether the vehicle speed at the stop line has reached the desired speed \( (V_q) \). For the first few queue positions, vehicle speed is less than \( V_q \) and headway is a function of acceleration and queue position. However, after vehicles reach the desired speed (i.e., \( n \cdot d \geq d_{\text{max}} \)), headways become dependent only on driver response time and desired speed. Thus, this model indicates that headways become essentially constant after the desired speed is reached.

The predictive ability of Briggs's headway model is compared with the results of previous headway studies in Figure 1. As this figure indicates, Briggs's model yields a relatively good fit to the data and appears to explain the trend toward decreasing headways with queue position.

### Starting Response Time and Distance Between Queued Vehicles

Driver starting response time and the distance between vehicles in a stopped queue at signalized intersections have been the subject of several previous studies \( (10-12) \). Messer and Fambro \( (10) \) found that driver response was fairly constant at \( 1.0 \) sec, regardless of queue position. The only exception was with the driver in the first queue position, who had an additional delay of \( 2.0 \) sec. The shorter response time of the second and subsequent queued drivers is probably due to their ability to anticipate the time to initiate motion by seeing the signal change or the movement of vehicles ahead, or both. Messer and Fambro also found that the average length of roadway occupied by each queue position is about \( 25 \) ft.

Another study of driver response time was conducted by George and Heroy \( (11) \). They found driver response to be relatively constant at about \( 1.3 \) sec for all queue positions. However, further examination of their data suggests that the first driver's response time was slightly longer, at about \( 1.5 \) to \( 2.0 \) sec.

Response times in the preceding studies were all measured at the start of vehicle motion. Herman et al. \( (12) \) found that
driver response to disturbance (including the start of motion) remained fairly constant as the platoon of queued vehicles increased its speed. In particular, they found that the speed of propagation of the response wave was relatively constant at about 26 fps up to platoon speeds of 30 fps. Beyond this speed, the response wave began to slow down as speeds neared the final cruising speed. A constant speed of propagation implies that all vehicles in the queue have the same trajectory, which supports a fundamental premise of Briggs's headway model. The authors also found the average distance between stopped vehicles to be 25.9 ft. Using this value, the starting response time can be calculated as 1.0 sec ($\approx 25.9/26$).

**MODEL DEVELOPMENT**

The most restrictive assumption in Briggs’s model is that of constant acceleration. Experience suggests that drivers vary their acceleration as they increase their speed. Thus, the objective of this section is to develop an alternative headway model based on a nonconstant acceleration behavior.

**Driver Acceleration Model**

An early study of driver acceleration characteristics on freeway on-ramps was conducted by Buhr et al. (13). On the basis of their study of passenger cars undergoing “normal” acceleration from a stopped condition at the ramp entrance, the authors determined that acceleration decreased linearly with increasing speed. The model they proposed was

$$a = A_{\text{max}} \left(1 - \frac{V}{V_{\text{max}}}\right)$$  \hspace{1cm} (4)

where

- $a$ = instantaneous acceleration (fps$^2$),
- $V$ = velocity of vehicle (fps),
- $A_{\text{max}}$ = maximum acceleration (fps$^2$), and
- $V_{\text{max}}$ = maximum speed corresponding to zero acceleration (fps).

For on-ramps on level terrain, the authors found that parameter values of $A_{\text{max}}$ equal to 15 fps$^2$ and $V_{\text{max}}$ equal to 60 fps would describe the acceleration behavior of ramp drivers.

A study of the acceleration and speed characteristics of queued drivers departing from a stop line was conducted by Evans and Rothery (14). The average speed and acceleration for each queue position as observed by Evans and Rothery are shown in Figure 3. As the figure indicates, the average speed found for each queue position increases exponentially to an average “desired” speed of about 50 fps.

The apparent desired speed of 50 fps is somewhat lower than the reported speed limit of 45 mph (66 fps). The reason for this difference is not clear from the authors’ paper; however, it is probably attributable to the constraining effect of queued flow conditions. Under this assumption, higher speeds would be reached only as vehicles begin to increase their spatial separation and transition from a queued flow regime to a free flow regime downstream of the intersection.

![Figure 3 Acceleration and speed of queued vehicles at an intersection as observed by Evans and Rothery.](image)

![Figure 4 Acceleration versus speed relationship of queued vehicles as observed by Evans and Rothery.](image)

The relationship between acceleration and speed for each queue position is shown in Figure 4, which indicates a strong linear relationship between acceleration and speed. The negative accelerations (i.e., decelerations) accompanying the higher speeds are probably a manifestation of a speed correction effect that stems from queued flow behavior, as observed by Herman et al. in an earlier study (12).

The acceleration model in Equation 4 can be rewritten in terms of a differential equation of motion to derive other relationships between speed, acceleration, distance, and time. In particular, integral calculus can be used to determine the following speed-time and distance-time relationships:

$$a = \frac{\Delta V}{\Delta t} = A_{\text{max}} \left(1 - \frac{V}{V_{\text{max}}}\right) = A_{\text{max}} - B \cdot V$$  \hspace{1cm} (5)

$$V = \frac{A_{\text{max}}}{B} \left(1 - e^{-Bt}\right) + V_0 \cdot e^{-Bt}$$  \hspace{1cm} (6)

$$x = \frac{A_{\text{max}}}{B} \cdot t - \frac{A_{\text{max}}}{B^2} \cdot (1 - e^{-Bt})$$

$$+ \frac{V_0}{B} \cdot (1 - e^{-Bt})$$  \hspace{1cm} (7)
where
\[ a = \text{instantaneous acceleration (fps/s)} , \]
\[ V = \text{velocity of vehicle (fps)} , \]
\[ t = \text{time of acceleration (sec)} , \]
\[ A_{\text{max}} = \text{maximum acceleration (fps/s)} , \]
\[ V_{\text{max}} = \text{maximum speed (fps)} , \]
\[ B = A_{\text{max}}/V_{\text{max}} \]
\[ V_0 = \text{initial velocity at time } t = 0 \text{ (fps)} , \]
\[ x = \text{acceleration distance (ft)} . \]

Discharge Headway Model

In general, a vehicle's time of arrival at the stop line is composed of two time increments. The first increment is the time measured from the beginning of the signal phase to the instant when the driver first begins motion. This increment will be called the cumulative starting response time, and, as previous studies have indicated, it can be estimated for each queue position \( n \) as \( \tau + n \cdot T \), where \( T \) is the starting response time for an individual driver and \( \tau \) is the additional response time of the first driver.

The second increment represents the time needed to accelerate over the distance between the stopped vehicle and the stop line \( (t_a) \). As in the approach of Briggs, the distance occupied by each queued vehicle is assumed equal to \( d \) ft. Thus, the back axle of the first vehicle is \( d \) ft back from the stop line, the second axle is \( 2 \cdot d \) ft back, and the \( n \)th vehicle is \( n \cdot d \) ft back. The time for the \( n \)th queued vehicle to reach the stop line once it starts (i.e., \( V_0 = 0 \)) can then be expressed using Equation 7 as

\[ x_n = n \cdot d = \frac{A_{\text{max}}}{B} \cdot t_{a(n)} - \frac{A_{\text{max}}}{B^2} \cdot (1 - e^{-Bt_{a(n)}}) \]  

(8)

Examination of Equation 8 indicates that a closed-form solution for \( t_{a(n)} \) is not obtainable. However, substitution of Equation 6 into Equation 8 will replace the exponential term with a term representing stop line speed \( (V_s) \) and thereby yield a solution for \( t_{a(n)} \):

\[ t_{a(n)} = \frac{n \cdot d}{V_{\text{max}}} + \frac{V_{s(n)}}{A_{\text{max}}} \]  

(9)

The discharge headway \( (h) \) between the \( n \)th and \( (n - 1) \)th vehicles can be calculated as the difference between their stop line arrival times:

\[ h_n = \tau \cdot N_1 + (n \cdot T + t_{a(n)}) \]

\[ - [(n - 1) \cdot T + t_{a(n-1)}] \]  

(10)

Finally, substituting Equation 9 into Equation 10 and simplifying leads to the proposed headway model:

\[ h_n = \tau \cdot N_1 + T + \frac{d}{V_{\text{max}}} + \frac{V_{s(n)} - V_{s(n-1)}}{A_{\text{max}}} \]  

(11)

Stop Line Speed Model

As discussed previously, Equation 6 can be used to relate stop line speed to the time of acceleration. Recognizing that time spent accelerating to the stop line \( (t_a) \) is dependent on queue position, a more useful model of stop line speed can be derived by substituting \( n \) for \( t_{a(n)} \) and setting \( V_0 = 0 \), yielding the following result:

\[ V_{s(n)} = V_{\text{max}} \cdot (1 - e^{-n \cdot \tau}) \]  

(12)

where
\[ \tau = \text{additional response time of the first queued driver (sec)} , \]
\[ T = \text{driver starting response time (sec)} , \]
\[ N_1 = 1 \text{ if } n = 1 \text{ or } 0 \text{ if } n > 1 \]
\[ d = \text{distance between vehicles in a stopped queue (ft)} , \]
\[ V_{s(n)} = \text{stop line speed of the } n \text{th queued vehicle (fps)} , \]
\[ V_{\text{max}} = \text{maximum speed (fps)} , \]
\[ A_{\text{max}} = \text{maximum acceleration (fps/s)} . \]

Comparing this model to that proposed by Briggs (Equations 1-3) reveals an important similarity. That is, as the speed of queued vehicles becomes constant \( (n \to \infty) \), the time headway between successive vehicles converges to the minimum discharge headway \( H = T + d/V \), where \( V \) represents the ultimate speed of queued flow for both models. This similarity supports the argument that the \( V_{\text{max}} \) parameter of the acceleration model represents the average queued driver's desired speed.

Start-Up Lost Time Model

As indicated by Figure 1, the first few vehicles in a traffic queue experience headways in excess of the minimum discharge headway. Any discharge time in excess of the minimum headway is essentially unused, or lost, time. Moreover, the sum of the lost times for the first few queue positions is called start-up lost time \( (K_s) \), which can be calculated as

\[ K_s = \sum_{n=1}^{N} (h_n - H) \]  

(13)
where \( N \) is the number of vehicles crossing the stop line that have speeds less than \( V_{\text{max}} \). Substitution of Equation 11 for \( h_n \) and \( T + d/V_{\text{max}} \) for \( H \) and elimination of the summation yield the proposed start-up lost time model:

\[
K_s = \tau + \frac{V_{\text{max}}}{A_{\text{max}}} \tag{14}
\]

The start-up lost time predicted by Equation 14 represents a limiting value as \( N \to \infty \). Because the queue theoretically never reaches \( V_{\text{max}} \), this suggests that all queue positions incur some lost time. This is in contrast to the method suggested by the 1985 HCM, wherein only the first four vehicles are assumed to have headways in excess of the minimum discharge headway. Thus, Equation 14 is not limited by an a priori assumption as to which queue position first achieves the minimum discharge headway.

**EXPERIMENTAL DESIGN**

**Study Sites**

Three SPUIs and two at-grade intersections (AGIs) were identified as candidate study sites. All three SPUIs are located on an 8-mi section of US-19 in the Tampa, Florida, area. Two of the SPUIs have been in operation for more than 15 years each, and the third has been open to traffic for about 6 months (at the time of the study).

Because one objective of this research was to identify the effect of SPUI geometry on selected traffic characteristics, it was decided that the field study should also include two typical, high-type AGIs for statistical control. One of the two AGIs selected for study is located about 1 mi from the other SPUIs (at SR-60 and Belcher Road). The second AGI selected for study is located on Wellborn Road in College Station, Texas.

All four Florida sites have actuated control, whereas the Texas site has pretimed timing plans downloaded from a centralized computer by time of day. Cycle lengths at the SR-60 SPUI, SR-694 SPUI, and the Wellborn AGI ranged from 90 to 100 sec; cycle lengths at the SR-686 SPUI and the Becher AGI ranged from 120 to 140 sec. Traffic demands were well below capacity, as indicated by the volume-to-capacity ratios, which ranged from 0.30 to 0.70.

**Data Collection System**

The computerized data collection system used for the field studies relied on a series of sensors located around the junction. One type of sensor used to monitor vehicular motion was the tapeswitch. Another type of sensor used was the photocell. Photocells were connected to the load switch LEDs inside the traffic signal controller cabinet and used to monitor the status of the signal indications.

These sensors were monitored by an Environmental Computer (EC) manufactured by the Golden River Corporation. The EC has the capability to check the status of each sensor every 1/600 sec.

**Field Study Procedure**

The data collection system was used to record discharge headway, speed, and acceleration data for the cross road through movement at the SPUIs and for the major road through movement at the AGIs. Left-turn movements were also studied for this research (1); however, only the findings for the through movements will be discussed here.

To collect the headway data for this study, a tapeswitch was located just past the point on the interchange approach where vehicles most frequently stopped (usually the stop line). A second tapeswitch was installed a known distance from the stop line tapeswitch along the study movement's travel path. The distance to this switch ranged from 50 to 150 ft, depending on the particular site.

**Data Reduction Procedure**

The technique used to calculate stop line speed and acceleration was based on a procedure described by Evans and Rothery (14). In general, this technique assumes that a vehicle's acceleration is constant between the two tapeswitches, which form a "trap" of known length (D). The assumption of a constant acceleration allows the vehicle trajectory to be modeled by a second-order equation of distance (x) as a function of time (t). In fact, this assumption allows two equations to be written, one for each axle of the vehicle. This relationship is shown in Figure 5 in terms of the vehicle's trajectory in time and space.

As shown in Figure 5, the trajectories of each axle are identical but separated by a distance equal to the vehicle wheelbase (L). When a vehicle crosses the trap, it causes four event times to be recorded: \( t_1 \), the time the first axle hits the first switch; \( t_2 \), the time the first axle hits the second switch; \( t_3 \), the time the second axle hits the first switch; and \( t_4 \), the time the second axle hits the second switch. Subtracting \( t_1 \) from each of these event times yields the relative travel time

![FIGURE 5 Time-space trajectory of a vehicle as it traverses the tapeswitch trap.](image-url)
of the front and back axles through the trap. By setting \( x = 0 \) when \( t = 0 \) for the transformed data, the trajectory of the front and back axles can be described by the following second-order equation:

\[
x = b_1 \cdot t + b_2 \cdot t^2
\]

(15)

where \( x \) and \( t \) correspond to the remaining three time-event pairs: \( (x = D, t = t_1) \), \( (x = L, t = t_2) \), and \( (x = D + L, t = t_3) \). Using these values of \( x \) and \( t \), three equations can be written to solve for the three unknowns \( b_1, b_2, \) and \( L \). Solving the three equations for \( b_1, b_2, \) and \( L \) yields

\[
b_2 = \frac{D \cdot (t_4 - t_3 + t_2)}{t_2 \cdot (t_4 - t_3) \cdot (t_2 + t_3 - t_2)}
\]

(16)

\[
b_1 = \frac{D}{t_2} - b_2 \cdot t_2
\]

(17)

\[
L = b_1 \cdot t_3 + b_2 \cdot t_2^2
\]

(18)

Finally, differentiating the second-order equation gives the stop line speed and acceleration as \( V_a = b_1 \) and \( A = 2 \cdot b_2 \), respectively.

**STATISTICAL ANALYSIS AND MODEL CALIBRATION**

Regression techniques were used to calibrate the proposed models. For the acceleration, stop line speed, start-up lost time, and discharge headway models, sufficient data were collected to use half for model calibration and the other half for validation. The regression models were based on the models developed in preceding section; however, covariates found to be significant from an analysis of variance (ANOVA) were also included in the final model form. The statistical analysis was conducted using the SAS system (15).

**Driver Acceleration Model**

To calibrate the acceleration model, passenger car acceleration and speed data were collected for one through movement at each of the five study sites. All total, acceleration and speed data were collected for 4,820 through vehicles. The relationship between speed and acceleration is shown in Figure 6. The data points represent the observed acceleration and speed averaged by queue position.

As Figure 6 indicates, a relatively strong linear relationship exists between acceleration and speed. The strength of the linear relationship between speed and acceleration was tested using least-squares linear regression techniques (1/). This analysis also indicated that each site had a maximum acceleration \((A_{max})\) in the relatively narrow range of 6.0 to 8.0 fps. Further statistical analysis indicated that these maximum accelerations were not significantly different from their average value of 6.63 fps (\( p = 0.10 \)). This average value is similar to the 6.0 fps found by Evans and Rothery (14) (see Figure 4).

**Stop Line Speed Model**

The relationship between speed and queue position is shown in Figure 7, which indicates that the speeds of the first few queue positions increase rapidly but tend to reach a maximum at later positions. This trend is consistent with the exponential form of the stop line speed model (Equation 12). The maximum speed obtained from this analysis represents the best estimate of the common desired speed of queued traffic \((V_{max})\), as required by the discharge headway model.

Closer examination of the desired speeds in Figure 7 indicates a relatively constant value for four of the five through movements. In particular, the desired speed at these four sites is in the relatively narrow range of 46.7 to 51.0 fps with a median value of about 49 fps. The desired speed of 39.9 fps found at the Wellborn AGI is well below this range. A possible explanation for the lower desired speed at this AGI is the driver's awareness of the relatively near (about 1,000 ft) downstream signalized intersection and of the likelihood of encountering stopped, turning, or weaving vehicles before reaching a higher speed. The nearest downstream intersection...
at the other study sites was much more distant than at the Wellborn AGI.

Although intuition suggests that there should be some correlation between speed limit and the desired speed of through traffic, a significant relationship was not found at the five study sites (p = 0.32). The lack of significance found in this analysis may be partly attributed to other influential factors (e.g., downstream effects) and to the relatively small sample size. As a result, it cannot be concluded from this analysis that desired speed is unaffected by speed limit at all intersections. This analysis will only support the use of 49 fps as the best estimate of the desired speed of a through movement. However, \( V_{\text{max}} \) was found to vary as a function of radius for the left-turn movements that were also studied for this research (1).

As a result of the calibration process, the stop line speed model is modified slightly from that originally described in Equation 12. The revised, calibrated model is

\[
V_{s(n)} = V_{\text{max}} \cdot (1 - e^{-\frac{V_{\text{max}}}{N}})
\]  \hspace{1cm} (19)

where

- \( V_{s(n)} \): stop line speed of the \( n \)th queued vehicle (fps),
- \( k = -0.290 + 24.0/V_{\text{max}} \)
- \( n \): queue position (\( n = 1, 2, 3, \ldots \)), and
- \( V_{\text{max}} \): common desired speed of queued traffic (fps).

The ability of this model to predict the observed speeds (as averaged by queue position) is shown in Figure 7. As the figure indicates, the model was a relatively good predictor of stop line speed \((R^2 = 0.9) (1)\).

### Discharge Headway Model

To calibrate the discharge headway model, passenger car headways were collected for one through movement at each of the five study sites. Headways were collected for 12,053 vehicles at the five study sites. The average headway for each site, movement studied, and queue position is provided elsewhere (1).

Calibration of the discharge headway model was based on a linear regression of discharge headways averaged by site and queue position. Because the number of headways recorded varied widely among these factors, the headway data were averaged to remove the bias that an unequal sample size would have on model parameters. Because of this technique, the statistics used to assess model fit to the data (i.e., standard deviation and \( R^2 \)) do not reflect the total variability in individual driver headways. Rather, the statistics indicate the ability of the model to predict the average discharge headway by queue position.

The calibrated discharge headway model for through movements is

\[
h_n = \tau' \cdot N_1 + T' + d' + b_o \cdot \left( \frac{V_{s(n)} - V_{s(n-1)}}{A_{\text{max}}} \right) + b_1 \cdot N_1 + b_2 \cdot V + b_3 \cdot N_1 \cdot A_{\text{max}} \]  \hspace{1cm} (20)

The model parameters are

- \( \tau' \): regressed additional response time of the first queued driver (sec),
- \( T' \): regressed driver starting response time (sec),
- \( d' \): regressed distance between vehicles in a stopped queue (ft),
- \( V_{s(n)} \): stop line speed of the \( n \)th queued vehicle (fps),
- \( V_{\text{max}} \): common desired speed of queued traffic (fps), and
- \( A_{\text{max}} \): maximum acceleration (fps).

The model variables are

- \( h_n \): headway of the \( n \)th queued vehicle (sec),
- \( n \): queue position (\( N = 1, 2, 3, \ldots \)),
- \( v \): traffic pressure (veh/cycle/lane),
- \( N_1 \): indicator variable (1 for first queue position; 0 for all others), and
- \( AGI \): indicator variable (1 for AGI; 0 for SPUI).

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Observations: 164
Std. Deviation: 0.16
\( R^2 \): 0.88

In general, the parameter values for \( b_0, b_1, \) and \( b_2 \) are consistent with the definitions of the theoretical model parameters to which they correspond (i.e., \( \tau, T, \) and \( d \), respectively). As these values do not, however, represent actual measurements, a prime symbol ('') has been added to each model parameter to denote that its value was established using regression analysis and that the relationship between this value and its definition may be distorted.

The ANOVA of individual headways revealed that traffic pressure, as measured by lane volume per cycle, was significant in reducing discharge headway \((p = 0.001)\). In using this component of the headway model, a one-to-one relationship between the duration of the volume average and the predicted average headway must be maintained. In other words, a total lane volume representing a 1-hr average should be used in the regression model to predict discharge headways during the same 1-hr period.

In general, the significance of the calibrated parameter values combined with the theoretical basis of the discharge headway model suggests that the model adequately describes the headway process of queued vehicles. This ability is demonstrated in Figure 8, in which the calibrated model is compared with the data for two movements.

The exponential form of the stop line speed model implies that \( V_{\text{max}} \) (and the minimum discharge headway) will, theoretically, never be reached. An examination of Figure 8, how-
ever, indicates that the predicted discharge headway does reach a relatively constant, minimum value after the eighth or ninth queue position. The likelihood that a minimum value is not reached until at least the eighth or ninth queue position suggests that the 1985 HCM's method for estimating the minimum discharge headway (i.e., average headways of the fifth through last queue positions) may be biased because it includes queue positions that have not achieved a minimum headway.

This potential bias-by-queue-position in the 1985 HCM method has implications for capacity evaluation and statistical analysis of cause and effect. Application of the HCM method will probably result in an estimate of minimum headway that is longer than that ultimately achieved by the traffic queue. Thus, the capacity of high-demand movements may be underestimated using the HCM method. More important, the trend in headway studies toward observing more headways for the lower queue positions tends to magnify any bias-by-queue-position. Thus, a statistical analysis of cause and effect (e.g., lane width, grade) may be clouded, and perhaps misdirected, by the added variance introduced by unequal numbers of headways observed at each queue position among the study sites.

Minimum Discharge Headway Model

According to the headway model (Equation 11), a minimum discharge headway \( H \) is not reached until the queue reaches its desired speed \( V_{\text{max}} \). At this point, the difference in stop line speed of successive vehicles is zero and the minimum discharge headway becomes

\[
H = T + \frac{d}{V_{\text{max}}} \tag{21}
\]

On the basis of the regression results, the calibrated minimum discharge headway model is

\[
H = 1.57 + \frac{25.25}{V_{\text{max}}} - 0.0086 \cdot \nu - 0.23 \cdot \text{AGI} \tag{22}
\]

where

- \( H \) = minimum discharge headway for a through movement (sec/veh),
- \( V_{\text{max}} \) = common desired speed of queued traffic (fps),
- \( \nu \) = traffic pressure (veh/cycle/lane), and
- \( \text{AGI} \) = 1 if the movement is at an AGI and 0 if it is at a SPUI.

Equation 22 implies that an AGI with a common desired speed of 49 fps and a nominal traffic pressure of 5.0 veh/cycle/lane would have a minimum discharge headway of 1.81 sec/veh. This value suggests that the ideal minimum headway may actually be shorter than the 2.0 sec/veh recommended by the 1985 HCM.

Start-Up Lost Time Model

The theoretical start-up lost time is calculated from the calibrated minimum discharge headway model and Equation 14 as

\[
K_s = 1.03 + 0.357 \cdot \frac{V_{\text{max}}}{A_{\text{max}}} \tag{23}
\]

where \( K_s \) is start-up lost time for a through movement (sec/phase) and \( A_{\text{max}} \) is maximum acceleration (equal to 6.63 fps/s). The start-up lost time for a typical through movement with \( V_{\text{max}} \) of 49 fps and \( A_{\text{max}} \) of 6.63 fps/s is 3.67 sec. This value suggests that start-up lost time for lengthy traffic queues may be greater than the 2.0 sec suggested by the 1985 HCM (6, Chapter 2).

In theory, the values for \( H \) and \( K_s \) from Equations 22 and 23, respectively, represent limiting values for infinitely large queues. Alternative forms of these equations could be derived to predict the average headway and lost time for queues of a more practical size. However, the comparative use of Equation 20 \((h_n)\) versus Equations 22 \((H)\) and 23 \((K_s)\) to predict service time \((T_N)\) (i.e., \( T_N = \Sigma h_n \), versus \( T_N = N \cdot H + K_s \)) suggests that this added rigor is not necessary. In general, using \( N \cdot H + K_s \) yields service times that are only about 0.7 sec (at \( N = 6 \)) and 0.2 sec (at \( N = 12 \)) longer than would be determined by summing the individual headways. Thus, it appears that there is little to be gained by using more complicated model forms, such as \( T_N = \Sigma h_n \), to predict service time or phase capacity.

CONCLUSIONS

The examination of driver acceleration indicated a linear trend toward decreasing acceleration with increasing vehicle speed. Both the initial acceleration and ultimate desired speed of these drivers were essentially constant among sites (i.e., \( A_{\text{max}} = 6.63 \) fps/s and \( V_{\text{max}} = 49 \) fps) for through traffic movements.

A linear relationship between speed and acceleration implies an exponential increase in vehicle speed with time. For this research, the exponential speed-time relationship was extended to the modeling of queued vehicle speed at the stop line. On the basis of the results of this research, the speed of
queued vehicles was found to agree closely with the exponential model form. The calibrated stop line speed model indicated that drivers do not reach a practical maximum speed until the eighth or higher queue positions (although, theoretically, the exponential form implies that the maximum speed is never attained).

The minimum discharge headway of a traffic movement is a complex process that is dependent on driver response time, desired speed, and traffic pressure. The discharge headway model developed in this research indicates that practical values of the minimum discharge headway of a traffic movement are not reached until the eighth or higher queue positions. Application of this model suggests that the minimum discharge headway of a traffic movement under ideal conditions may be shorter than 2.0 scc/veh and that its corresponding start-up lost time may be longer than 2.0 scc.

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