Development of an Improved High-Order Continuum Traffic Flow Model

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Widespread use of continuum traffic models in practical applications has not been realized since their introduction in the early 1960s and 1970s. This is because some improvements are necessary for wide acceptance of these models, especially in congested and interrupted flow situations. A new high-order formulation capable of describing traffic dynamics under these conditions is introduced and implemented. The formulation does not contain an equilibrium speed-density relationship and therefore requires less calibration effort in field applications. Satisfactory results are obtained when the model is tested using field data representing flows at pipeline freeway and freeway junctions with entrance and exit ramps.

Advanced traffic management and control schemes as well as simulation require reasonably accurate description of flow dynamics, especially in congested situations. Conventional input/output models in general are adequate for simulating traffic in a coarse sense and can be used for planning purposes. However, they are not sufficiently sophisticated for use in the development of high-performance freeway surveillance and control systems or real-time applications. Continuum models are more suitable for representing the short-term traffic behavior because they include both time and space in the state equations and take compressibility into account. The most widely known continuum formulations can be characterized as being either simple order (*1*) or high-order (*2–4*).

The simple continuum formulation is based only on the mass conservation equation, which is supplemented by an equilibrium equation of state. In the high-order continuum formulation, a momentum equation is added to the mass conservation to achieve conservation of momentum as well. In spite of the conceptual appeal of continuum models, they have not been widely used, partly because of our inexperience in implementing them in practical situations and partly because of some needed improvements in their formulation. For instance, the simple continuum model is known for its simplicity, but in principle it is not suitable for describing nonequilibrium traffic dynamics because it does not take into account acceleration and inertia effects. However, it is not clear how important these effects really are, especially in congested conditions. On the other hand, although the existing high-order continuum models consider acceleration and inertia, they appear problematic at congested and interrupted flows (*5–7*). Motivated by these considerations, in this paper we introduce a new formulation based on the existing high-order continuum models. This formulation does not contain an equilibrium speed-density relationship as in most existing continuum models and thus is more attractive in field applications. In addition, it includes a friction term to address the effect of ramp flows especially at congested flows. The proposed model is implemented numerically through finite difference methods. A number of such methods have been tried, and in this paper only the most successful one is presented. A stability analysis has also been performed to determine appropriate mesh sizes in space and time and to bound the parameter values of the model. Subsequently, both qualitative and quantitative tests have been performed to test and validate the model. Whereas the qualitative tests are focused on its physical behavior at congested flows, the quantitative tests of the model evaluate its ability to estimate real-world traffic at congested and interrupted flows. To understand the effect of different discretizations, a number of mesh sizes have been applied (within stability limit), and the findings are discussed. Test results from a simple continuum model are also presented and compared with those from the proposed model.

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SIMPLE AND HIGH-ORDER CONTINUUM FORMULATIONS

Continuum models are needed not only for better understanding the collective behavior of traffic, but also for analyzing flow conditions in a dynamic fashion in devising efficient control strategies, simulation, and assessing the effects of geometric improvements. According to the simple continuum model, flow can be described by the conservation equation, which has the following general form:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = g(x, t) \tag{1}$$

where

$q = q(k) = uk$ is the flow rate of the traffic stream (veh/hr);
k = density (veh/mile);
u = speed (mi/hr);
t = time;
x = space; and
$g(x, t)$ = generation rate, which is equal to zero in freeway sections without entrances or exits.

In Equation 1 speed is related to density through an equilibrium relationship:

$$u = u_e(k) \tag{2}$$

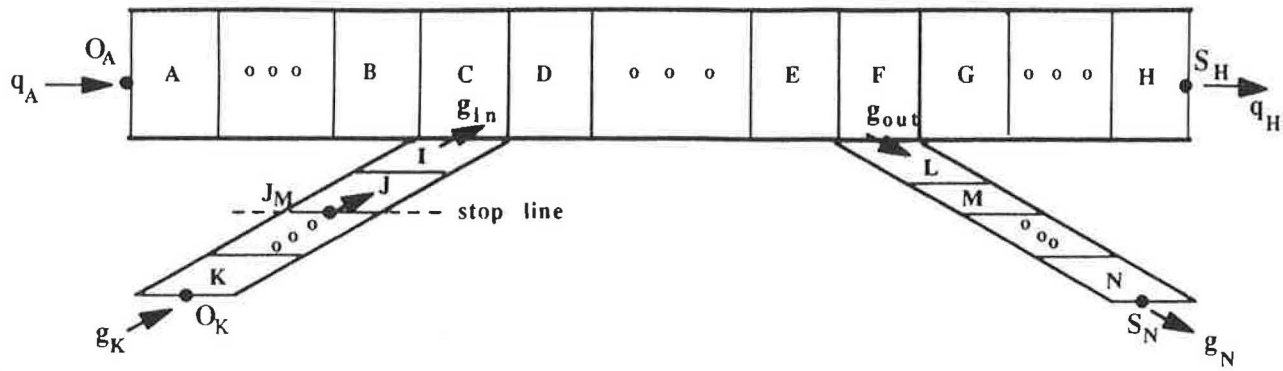

FIGURE 1 Space discretization of a typical freeway section.

metering rate, Link J is treated the same as the other links in Item 2; when g_K^{n+1} > metering rate, Link J is treated as an internal boundary and g_J^{n+1} = metering rate.

If the entrance ramp is not metered, Link J is treated the same as the links in Item 2.

4. Link C: Link C represents a freeway junction with an entrance ramp. Since there is a merging flow, Equations 6 through 11 need to be modified. A generation term g_{in} will be added to the right-hand side of Equations 6 and 9. $g_{in} = q_{in}/\Delta x_c$, where $q_{in} > 0$ is the merging flow from Link I to Link C. In addition, the viscosity term in Equation 13 will be added to the right-hand side of Equations 7 and 10.

Determination of g_{in} is explained in Item 5.

Equations 8 and 11 remain unchanged.

5. Link I: Link I connects the ramp to the freeway. On one hand, this link provides ramp volume q_{in} to Link C. On the other, it serves as the downstream boundary of the ramp being considered as a pipeline. In general merging volume q_{in} is not equal to ramp demand q_K, especially when freeway is congested and there is a waiting queue at the ramp. Since we consider freeway and ramp as being connected at a single point, it is not necessary to use the same treatment of merging dynamics used in our earlier work (*12*), in which Link I and Link C are further discretized into small Δx's. However, in seeking the relationship of the maximum merging flow and the freeway mainline flow, the fundamental rule that merging value is governed by the gap availability on mainline freeway is still followed. Figure 2 shows the merging volume versus the Lane 1 (the rightmost lane) volume (q_1) of freeway from our field data collected at eight entrance ramps (on two- and three-lane freeways) in 14 peak periods. The volume is measured by loop detectors (installed after the metering stop line) when its corresponding occupancy is so high that it suggests that there are excessive cars at Links J and I waiting to merge pending the gaps in the mainline flow. In order to curve-fit the merging capacity C_g with the freeway Lane 1 volume, we have modified one of the merging capacity equations by Adams (*13,14*) and applied the least-square technique to fit the upper half of the curve in Figure 2, which applies to uncongested mainline flow situations. When freeway becomes congested, density increases, resulting in reduction of mainline flow rate and available space for vehicles to merge from ramps. Merging capacity therefore decreases with the reduction of mainline flow. Unfortunately, we have not been able to obtain enough data to estimate the relationship of merging

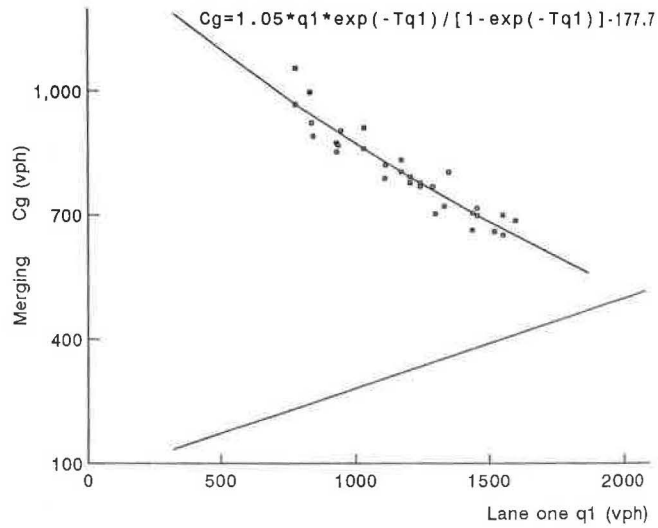

FIGURE 2 Merging capacity at Link I.

capacity and merging volume at congested flows. Experimentally, a straight line is used as an approximation to this relationship on the basis of traffic operation practices of the Traffic Management Center of the Minnesota Department of Transportation. Consequently, q_{in} is determined as follows:

$$q_{in} = \min \{q_1, C_g, (\mathrm{Cap}_C - q_C)\}$$

where q_{in} is the flow rate on Link I and C_g is determined from the two empirical curves in Figure 2. Cap_C is the capacity of Link C and q_C is the total mainline flow before merging takes place.

6. Links L and F: Similar to Link I, Link L receives diverging volumes from freeway to surface streets, and it serves as the upstream boundary of the exit ramp. All exiting demand is assumed to leave freeway through Link L.

The exiting volume from Link F to Link L is in general not greater than the capacity at the downstream end of the exit ramp. Specifically,

$$q_{out}^{n+1} = \min \{q_{exit}^{n+1}, q_M^n, \mathrm{Cap}_L\}$$

where q_{exit}^{n+1} is the exiting demand, g_M^n is the flow rate on Link M at the nth time step, and Cap_L is the capacity of Link L.

Congestion may spill back from Link L to Link F when $q_{exit} > q_{out}$. In this case, the through capacity of Link F is reduced. First, the cumulative exiting demand at mainline freeway is determined as follows:

$$(ST)^{n+1} = (ST)^n + (g_{exit}^{n+1} - g_{out}^{n+1}) * \Delta t/3{,}600 \qquad (ST)^{n+1} \geq 0$$

where ST is the cumulative exiting demand remaining on the mainline freeway. Second, the through capacity TC of Link F becomes

$$TC_F = Cap_F * (LN_F - 1)/LN_F$$

where Cap_F is the capacity of Link F under normal conditions and LN_F is the number of lanes at Link F.

Once q_{out}^{n+1} is obtained, Link L is treated as a pipeline section and the associated equations discussed before for pipeline freeways will be used. Link F is treated the same as Link C (except now $g_{out} < 0$) if there is no congestion spillback and is treated as a pipeline section with reduced capacity for the through traffic if such a spillback prevails.

The modeling presented allows simultaneous treatment of the freeway and its ramps in an integrated fashion. This feature is especially important in future development of this model. For example, in modeling a freeway corridor, both surface streets and the freeway, which are connected through the ramps, will be processed in a uniform and integrated manner. The numerical scheme used to implement the model provides a stable difference approximation with first order accuracy in both Δx and Δt. The model does not include a $u_e(k)$-k relationship, and the arrival and departure patterns are the only inputs to the model that can be in any form, including stochastic ones.

TESTING AND FIELD VALIDATION

To evaluate the effectiveness of the model implementation, we present the results of our model in both qualitative and quantitative tests. Whereas the former is done by applying the model to a hypothetical situation, the latter is based on field data involving basic freeway segments and entrance and exit ramps.

Qualitative Testing

An effort was first made to examine whether the model is physically reasonable in representing traffic dynamics, especially at congested flows. The test scheme involves a 2-mi, three-lane pipeline freeway with demand at upstream boundary and capacity at downstream shown in Figure 3. Starting from $t = T_a = 4$ min 0 sec, downstream capacity is reduced by about one-half because of an incident. A bottleneck is therefore created and remained until $t = T_b = 7$ min 0 sec, when the incident is cleared and the initial capacity at the downstream boundary is restored. Figure 4 shows the density distribution given by the model at four time instants during and after the incident. It can be observed that congestion

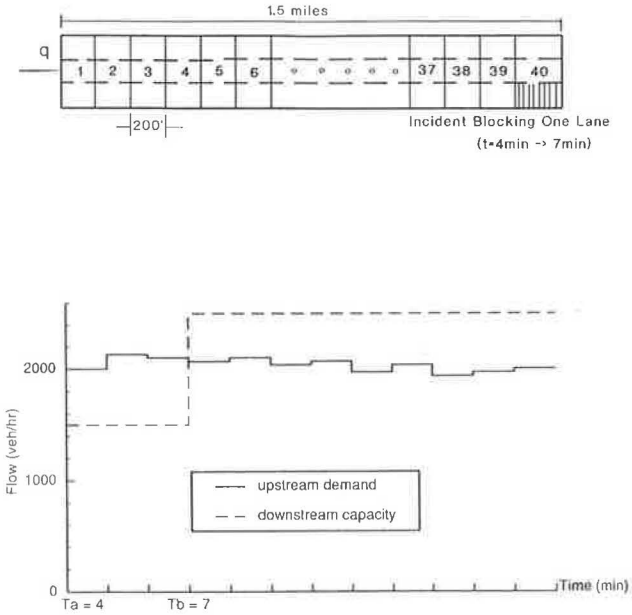

FIGURE 3 Geometry and boundary conditions for qualitative testing.

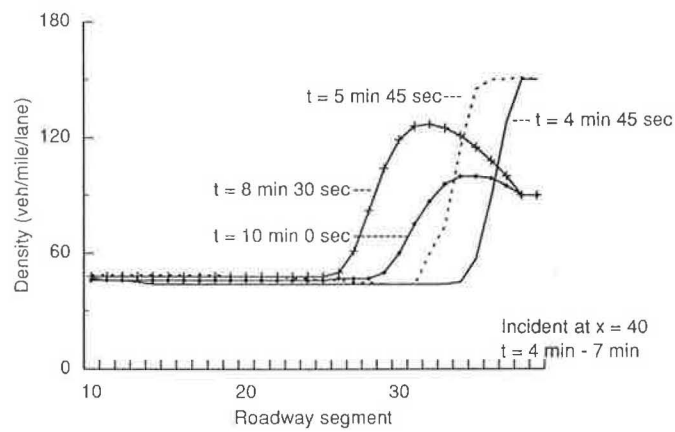

FIGURE 4 Queue propagation (during incident) and dissipation (after incident).

propagates to the upstream freeway at $t_1 = 4$ min 45 sec and $t_2 = 5$ min 45 sec, and is gradually smoothed out toward the downstream freeway at $t_3 = 8$ min 30 sec and $t_4 = 10$ min 0 sec in the dissipation process.

Testing on Pipeline Freeway

Field validation of the model is done by using data collected from the I-35W in Minneapolis. In a recent research project funded by the Minnesota Department of Transportation, an 8-mi section of the freeway was selected that connects downtown Minneapolis to the southern suburban areas and contains a variety of geometric types, such as entrances, exits, weaving areas, and so forth. The test scenario (Case 1) involves a four-lane freeway close to downtown Minneapolis

that carries southbound traffic from 4:00 to 6:00 p.m. Congestion starts at 4:10 p.m. at the downstream boundary and reaches the upstream boundary by 4:15 p.m. The freeway remains congested until 6:00 p.m., when congestion dissipates through the downstream boundary. The arrival and departure traffic patterns (boundary conditions) are shown in Figure 5. The initial traffic condition of the system is obtained from data at the time interval before the start of simulation. There are three mainline detection stations at the test site. Two stations at the upstream and downstream boundaries provide boundary conditions at every 5-min interval. The remaining station ("check station" in Figure 5) is located between the boundaries, and it provides measurements to compare with the simulation results.

To evaluate the quantitative effectiveness, we included in the testing three models [simple continuum, Payne, and Papageorgiou (*15*)] together with our proposed model (upwind version), and they are referred to as Models 1, 2, 3, and 4, respectively. Whereas the simple continuum model was implemented through the Lax (*16*) scheme, Models 2 and 3 were discretized by using the Euler (*3,15*) scheme as recommended by the original authors. A different implementation of our model (Lax version) was also included. Our model was properly calibrated by using field data, and in both versions $\Delta x = 200$ ft and $\Delta t = 1$ sec were used. On the basis of the deviations of the estimated results from the field observations, the following statistics are calculated:

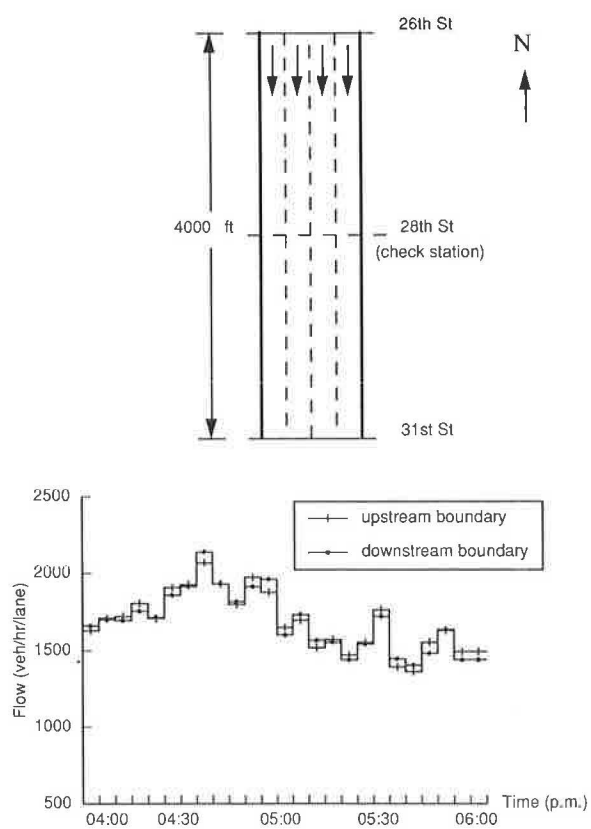

FIGURE 5 Geometry and arrival and departure patterns for Case 1.

$$\text{Mean absolute error (MAE)} = (\sum_1^N |\text{observed} - \text{estimated}|)/N$$

$$\text{MAE (\%)} = \sum_1^N (|\text{observed} - \text{estimated}|/\text{observed})/N$$

$$\text{Mean square error (MSE)} = [\sum_1^N (\text{observed} - \text{estimated})^2]/N$$

$$\text{St. deviation} = \{[\sum_1^N (\text{observed} - \text{estimated})^2]/(N - 1)\}^{1/2}$$

where N is the number of observations.

Computational efficiency is also compared among all the included models. Computer execution time was measured from an IBM PC (results could be machine dependent), and an index (Comp Index) was used to indicate the ratio of computation time with respect to Model 1.

Test results from Case 1 are summarized in Table 1, where only the error indices for volumes are presented since speed data at the check station were not available. Table 1 indicates the following:

1. When there is downstream congestion, all high-order models included performed substantially more accurately than the simple continuum model.
2. Model 3 was more accurate than Model 2, both of which are implemented with the same method (Euler).
3. Model 4 (upwind version) was the best overall in terms of accuracy, and it was faster than Models 2 and 3 by 20 to 25 percent and faster than Model 1 by 14 percent.
4. The Lax version of Model 4 was not as good as its upwind version because it not only produced larger errors but also required more computation time.
5. Model 4 is faster than Model 1 even for the same numerical method (Lax), mainly because of the absence of a u_e-k table to look up.

Testing with Entrance/Exit Ramps

Two additional test cases are presented in this section that involve merging/diverging flows at freeway junctions with cn-

TABLE 1 Error Indices and Computational Efficiency for Case 1

Models	1	2	3	4 (upwind)	4 (Lax)
MAE (a)	82	59	46	23	34
MAE(%)	15.26	8.10	7.84	4.46	6.43
MSE (b)	5871	4662	3725	808	1470
Std. Dev.	90.02	74.80	66.85	31.15	42.00
Comp Index(c) (Time Improv)	1.00	1.07 (-7%)	1.10 (-10%)	0.86 (+14%)	0.97 (+3%)

(a) MAE, veh/5 minutes;
(b) MSE, veh²/5 minutes; Std. Deviation, veh/5 minutes;
(c) Computer execution time index, compared with Model 1 and based on IBM-PC 386-25Mhz machine.

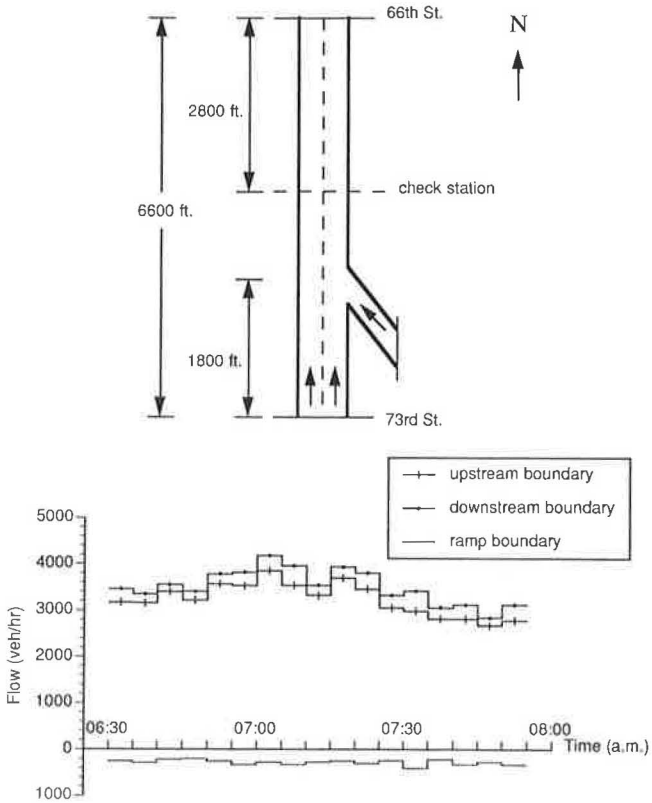

FIGURE 6 Geometry and arrival and departure patterns for Case 2.

trance/exit ramps. Case 2 is based on a two-lane freeway with morning traffic from 6:30 to 8:00 a.m. Case 3 is also a two-lane freeway with an exit ramp carrying northbound traffic from 7:00 p.m. to 8:20 a.m. The roadway geometry and traffic patterns at boundaries and the demand at the entrance ramp for Case 2 are shown in Figure 6.

Since it is not clear how other high-order models are implemented at freeway ramp junctions, we have included in addition only the simple continuum model in the testing. Furthermore, since similar error statistics are obtained for Cases 2 and 3, only those from Case 2 are summarized in Table 2 because of space limitation. To understand the effect of variation in the mesh size, results from the application of several Δx and Δt values have been presented. The results are as follows:

1. In Cases 2 and 3, the high-order model is able to estimate traffic volumes at a relatively high level of accuracy. Among the volume errors, 2.03 to 2.05 percent was produced in Case 2 and 3.51 to 3.54 percent in Case 3.
2. The simple continuum model does not have the same accuracy as the high-order model: 9.7 to 11.9 percent of volume error was produced in Case 2 and 10.4 to 13.5 percent in Case 3.
3. Speed estimations in both models produced larger errors. Whereas the high-order model generated 3.7 percent of errors in Case 2 and 4.09 to 4.1 percent in Case 3, the simple continuum yielded 9.3 to 10.3 percent in Case 2 and 13.3 to 13.6 percent in Case 3.
4. The simple continuum model seems to be sensitive to the changes in Δx and Δt sizes. With the increase of Δx, the errors in traffic volumes (MAE, MSE, and Std. Dev.) increase.
5. For the high-order model, changes in Δx and Δt sizes have no obvious effect on the errors in both volumes and speeds (when $\Delta x \leq 500$ ft). All MAE, MSE, and Std. Dev. remained about the same. This feature produces flexibility to choose the level of discretization according to the purpose of the application while operating the model at about the same error level.

CONCLUSIONS

An improved high-order continuum model was proposed and implemented with very encouraging results. A traffic friction term is included to handle the effect of vehicular interactions at ramp junctions. The use of an explicit $u_e(k)$-k relationship is not needed. This feature saves a significant amount of effort for acquiring such a relationship, making our model more practical for field applications. Qualitative testing showed good capability of describing queue propagation and dissipation properties.

For a pipeline situation our model was compared with a simple continuum model and other high-order models and produced lower error. In general, high-order models seem to be significantly better than the simple continuum model in

TABLE 2 Error Indices for Case 2

Tests(a)	dx=100, dt=1		dx=200, dt=1		dx=300, dt=1		dx=400, dt=2		dx=500, dt=3	
	vol	spd	vol	spd	vol	spd	vol	spd	vol	spd
MAE(b)	6 (28)	2.1 (5.6)	6 (33)	2.1 (5.0)	6 (34)	2.1 (5.0)	6	2	6	2
MAE (%)	2.03 (9.7)	3.72 (10.3)	2.04 (11.7)	3.72 (9.3)	2.05 (11.9)	3.72 (9.3)	2.05	3.72	2.04	3.71
MSE(c)	46 (1061)	5.9 (34.7)	46 (1455)	5.9 (29.1)	47 (1482)	5.9 (29.2)	46	5.9	46	5.9
Std. Dev.(d)	7.01 (33.6)	2.51 (6.1)	7.00 (39.4)	2.51 (5.6)	7.04 (39.8)	2.51 (5.6)	7.02	2.51	6.98	2.51

(a) Numbers in brackets are from simple continuum model.
(b) MAE, veh/5 minutes for volumes, mile/hr for speeds.
(c) MSE, veh^2/5 minutes for volumes, (mile/hr)^2 for speeds.
(d) Std. Dev. veh/5 minutes for volumes, mile/hr for speeds.

congested situations, although further testing is needed before such a generalized conclusion can be made. Finally, the choice of the numerical method (e.g., upwind versus Lax) seems to have substantial impact on accuracy and efficiency.

A simplified modeling methodology suitable for freeway pipeline plus entrance/exit ramps was used. Two test cases were shown, and our model produced lower errors compared with the simple continuum model.

The overall performance of our model was very promising. However, more field testing/validation is needed to improve our model on more complicated geometrics and by using more detailed traffic measurements (shorter time and space increments). As a future possibility, traffic data can be collected from AUTOSCOPE (*17*), which is currently being developed at the University of Minnesota and will include 38 video detectors by 1993 along a 2.5-mi section of the I-394 freeway in Minneapolis (*17*). This freeway section will serve as a laboratory for collecting and studying traffic characteristics and testing and validating traffic flow models, including the one presented in this paper.

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REFERENCES

1. M. H. Lighthill and G. B. Whitham. On Kinematic Waves: II. A Theory of Traffic Flow on Long Crowded Roads. *Proc. R. Soc. London*, Ser. A. 229, 1955, pp. 317–345.
2. H. J. Payne. Models of Freeway Traffic and Control. In G. A. Bekey, Mathematical Models of Public Systems, Simulation Counc. Proc. Ser., 1, 1971, pp. 51–61.
3. H J. Payne. FREFLO: A Macroscopic Simulation Model of Freeway Traffic. In *Transportation Research Record 772*, TRB, National Research Council, Washington, D.C., 1979, pp. 68–75.
4. W. F. Phillips. *A New Continuum Model for Traffic Flow*. Report DOT-RC-82018. Utah State University, Logan, 1979.
5. N. A. Derzko, A. J. Ugge, and E. R. Case. Evaluation of a Dynamic Freeway Model Using Field Data. In *Transportation Research Record 905*, TRB, National Research Council, Washington, D.C., 1983, pp. 52–60.
6. A. K. Rathi, E. B. Lieberman, and M. Yedlin. Enhanced FREFLO Program: Simulation of Congested Environments. In *Transportation Research Record 1112*, TRB, National Research Council, Washington, D.C., 1987, pp. 61–71.
7. P. Ross. Traffic Dynamics. *Transp. Res. B*, Vol. 22B, No. 6, 1988, pp. 421–435.
8. P. G. Michalopoulos, P. Yi, D. E. Beskos, and A. S. Lyrintzis. Continuum Modelling of Traffic Dynamics. *Proc., Second International Conference on Applications of Advanced Technologies in Transportation Engineering*, ASCE, Minneapolis, Minn., 1991.
9. G. J. Forbes and F. L. Hall. The Applicability of Catastrophe Theory in Modelling Freeway Traffic Operations. *Transpn. Res. A*. Vol. 24A, No. 5, 1990.
10. C. Hirch. *Numerical Computation of Internal and External Flows*. Vol. 1. John Wiley and Sons, 1990.
11. I. Prigogine and R. Herman. *Kinetic Theory of Vehicular Traffic*. American Elsevier, New York, 1971.
12. P. G. Michalopoulos, E. Kwon, and J. G. Kang. Enhancement and Field Testing of a Dynamic Freeway Simulation Program. In *Transportation Research Record 1320*, TRB, National Research Council, Washington, D.C., 1991.
13. W. F. Adams. Road Traffic Considered as a Random Series. *J. Inst. Civil Eng.*, Vol. 4, 1936, pp. 121–130.
14. D. L. Gerlough and M. J. Huber. *Special Rept 165: Traffic Flow Theory*. TRB, National Research Council, Washington, D.C., 1975.
15. M. Papageorgiou, J. M. Blosseville, and H. Hadj-Salem. Macroscopic Modelling of Traffic Flow on the Boulevard Peripherique in Paris. *Transpn. Res. B*, Vol. 23B, 1989, pp. 29–47.
16. P. D. Lax. Weak Solution of Non-Linear Hyperbolic Equations and Their Numerical Computations. *Communs. Pure Appl. Math.*, Vol. 7, 1954, pp. 159–173.
17. P. G. Michalopoulos, B. Wolf, and R. Benke. Testing and Field Implementation of the Minnesota Video Detection System. In *Transportation Research Record 1287*, TRB, National Research Council, Washington, D.C., 1990.

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