Methodology To Analyze Driver Decision Environment During Signal Change Intervals: Application of Fuzzy Set Theory

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During a signal change interval, most drivers must decide whether to stop or go based on uncertain information on speed, the remaining yellow time, and distance from the intersection. The decision process under fuzzy information, such as this case, is suited for analysis by fuzzy set theory. Fuzzy set theory and its logic has been used to analyze the driver's decision between "stop" and "clear" during the signal change interval. Fuzzy sets of "not safe stopping" and "not safe clearing" are defined along the approach roadway. Depending on the way the two sets intersect, the dilemma, indecision, or option zones can be represented. Possibility and necessity measures of "safe stopping" and "safe clearing" were defined, and they were assumed to represent the decision criteria of aggressive and conservative drivers, respectively. Based on these measures, a new approach to determine signal change intervals is suggested. It states that an interval should be such that, at any point along the approach, the possibility of "safe stopping" or "safe clearing" are always one, and the necessity measure of the two sets should exceed a certain minimum value.

The signal change interval has been a topic of traffic engineering studies since its formulation by Gazis et al. (*1*) in the early 1960s. Traditionally, this interval is computed to eliminate the dilemma zone. The dilemma zone is known to exist if the signal change interval is such that the driver cannot stop or go without violating the traffic rules. If a driver is in the dilemma zone at the onset of the yellow phase and decides to proceed, the vehicle will still be in the intersection when the all-red period ends. Conversely, if the driver decides to stop, the vehicle will not be able to stop without entering the intersection. In the traditional formula, signal change intervals are computed based on the idea that, at any point along the approach, at least one of the actions (stop or clear) is possible.

If all the characteristics of the approaching vehicles and drivers are the same as the assumed input values, and if the driver knows his exact location at the time of signal change, the signal change intervals set by the traditional formula should be adequate to allow time for the driver to complete one of the actions. Hence, at the end of a signal change interval, the intersection should be cleared for crossing traffic.

However, because the information available to a driver concerning location, the amount of yellow time remaining, approach speed, and other parameters is fuzzy, the result is driver indecision when the yellow indication appears. Furthermore, because not all drivers are approaching at the same speed and their decision criteria are different, some drivers always experience a certain degree of indecision of dilemma. Thus, the traditional formula is not applicable for the analysis of the decision process of individual drivers. Rather, it is suited for diagnostic analysis of the signal timing as stated by Mahalel and Prashker (*2*).

In this paper, we recognize that some of the information available to the driver is fuzzy; in other words, the values known to the driver are only approximate. We then attempt to analyze the decision-making environment using the logic of fuzzy set theory. Two basic fuzzy sets are defined: one representing the minimum distance from which stopping is possible and the other the maximum distance from which the "clearing" maneuver is possible. Both distances are measured from the intersection stop line. Knowing these fuzzy distances, we determine the possibility and necessity measures of completing the stopping and clearing maneuvers safely at any point along the approach roadway. The possibility and necessity measures are believed to represent the decisions of two types of drivers: aggressive (or risk-taking) and conservative (or risk-averting), respectively. We then introduce a fuzzy set that represents the approximate location as perceived by a driver and evaluate the driver's likelihood of completing each of the actions using the possibility and necessity measures. The level of the driver's anxiety and the zones where the dilemma and indecision occur can also be illustrated using the intersection of the membership functions of these fuzzy sets.

Analyses similar to the authors' are found in prior papers dealing with the stopping probability function. Among them are Olson and Rothery (*3*) and Sheffi and Mahmassani (*4*). The probability function represents the observed frequency of stopping decisions along the approach. It shows the final action of the drivers in an aggregated form, but it does not differentiate the circumstances in which individual drivers made the choice. At a given location, the same value of stopping probability may be obtained when the driver is in the dilemma zone or in the option zone. The existence of the probability function itself, however, shows that every driver's judgment and decision pattern is different and that each reacts differently to the given information. The previous work has helped develop our idea for using fuzzy sets to analyze the effect of fuzziness of information on individual drivers' decisions of stopping and clearing. Another work related to this study is one by May (*5*), in which extensive field measurements of driver behavior and risks were compiled and analyzed. What we propose as possibility and necessity measures in

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this paper are perhaps related to May's observation of risk measurements.

The authors suggest two criteria in determining signal change intervals: one, the possibilities of both the stopping or going actions at any point along the approach should be one; two, the necessity measures of stopping and clearing should be greater than a given minimum level at any point along the approach roadway. Further, it is suggested that the use of possibility and necessity measures provides the theoretical basis for explaining the regions of dilemma, indecision, and option.

TERMINOLOGY AND CONVENTION OF SYMBOLS

The following terminology is used to indicate the condition of the driver at the onset of the yellow indication: dilemma, indecision, option, or imperative.

Dilemma is the situation in which the driver can complete neither the stopping nor clearing action safely. Two types of dilemma can exist. Type 1 dilemma is the situation in which the driver can perform neither action at all. Type 2 dilemma is the situation in which the driver can perform one or both of the actions but with difficulty. The dilemma zone is the area in which the driver faces a dilemma of either type.

Indecision is the situation in which the driver can complete one of the actions safely and the other action with some level of difficulty.

Option is the situation in which the driver can complete both actions safely.

Imperative is the condition in which the driver must choose either stopping or clearing; if the driver is far from the intersection, he or she must stop, and if the driver is very close to the intersection, he or she must clear it.

A bold letter indicates a fuzzy number set. The following are symbols of sets and parameters:

1. **SD:** fuzzy stopping distance (distance from the intersection) when the approach speed is fuzzy;
2. **CD:** fuzzy clearing distance (distance from the intersection) when approach speed (V) and the remaining signal change interval (t) are fuzzy;
3. **V:** fuzzy set of approximate speed;
4. **S:** fuzzy set of safe stopping distances;
5. **C:** fuzzy set of safe clearing distances;
6. **NS:** fuzzy set of distances from which the vehicle is not able to stop safely;
7. **NC:** fuzzy set of distances from which the vehicle is not able to clear safely;
8. **L:** fuzzy set of the location of the driver at the onset of the yellow indication;
9. **t:** fuzzy signal interval perceived by the driver; and
10. **E:** universal set.

All distances are measured from the intersection stop line.

TRADITIONAL MODEL OF SIGNAL CHANGE INTERVALS

The traditional model of signal change intervals is based on equating two distances that are both measured from the intersection stop line: one to stop (stopping distance, D_s), and the other to clear the intersection during the signal change interval (clearing distance D_g). Finding the signal change interval by equating the two distances means that, at any point along the approach roadway, either the stopping or the clearing action can be completed within the signal change interval.

The stopping distance and the clearing distance are, respectively,

$$D_s = V^2/2b + Vd \tag{1}$$

and

$$D_g = Vt - (w + \ell) + a(t - d)^2/2 \tag{2}$$

where

d = driver perception and reaction time,
V = approach speed,
b = deceleration rate,
w = intersection width,
ℓ = vehicle length, and
a = acceleration rate.

If it is assumed that the vehicle will not accelerate upon seeing the yellow light,

$$t = d + (V/2b) + (w + \ell)/V \tag{3}$$

If it is assumed that the vehicle will accelerate upon seeing the yellow light,

$$t = d + [-V + \sqrt{V^2 + 2a(V^2/2b + w + \ell)}]/a \tag{4}$$

Equation 3 is the most commonly quoted expression, and it is recommended by ITE (*6*). [A recently recommended alternative from the ITE Technical Council includes the impact of grade (*7*).]

If t in Equation 3 is set at $D_g > D_s$, both the stopping and going actions are possible at a particular speed. If, on the other hand, t is set at $D_s > D_g$, a dilemma zone is said to exist, because for section ($D_s - D_g$) neither the stopping nor clearing action is possible.

ANALYSIS USING FUZZY SET THEORY

The use of Equation 3 (or 4) assumes that all the parameters are known to the driver as "crisp" values and that the driver makes the correct decision depending on location. In reality, the driver's understanding of the situation is not clear. He can only approximate the following:

- The remaining amber and all-red time,
- The location of the vehicle relative to the boundary between the stop zone and clearing zone (relative to D_s and D_g),
- The speed of the vehicle,
- The vehicle's acceleration and deceleration capabilities, and
- The width of the intersection (if the driver is not familiar with it).

The length of the driver's perception/reaction time depends on how precise the information is to him or her. An interesting analysis of driver perception/reaction time using a computer simulation model is presented by Chan and Liao (*8*). The model allows the analyst to test driver reaction while watching a vehicle approaching the intersection on the screen.

The quantities of the parameters above are normally judged by the driver based on his or her experience, intuition, and familiarity with the intersection and signal. Furthermore, the driver can control the values of some of the parameters (for example, the braking and acceleration force to be applied). Thus, the values of the variables are not completely random; rather, they are subjectively judged numbers, given a myriad of factors such as the driver's personality, physical condition, vehicle characteristics, the environment and geometric design of the intersection approach, relationship to other vehicles in the traffic stream, and so forth. Yet, in many cases, after stopping or clearing, the driver still wonders if he or she has made the best decision. Detailed discussions of how some of the parameters influence the driver's decision are presented in the work of Cheng et al. (*9*). In this paper, we treat these parameters as fuzzy, and introduce fuzzy set theory to analyze the decision-making process.

Analysis Procedure

The parameters considered fuzzy here are those perceived by drivers. Among them, we assume the following three parameters as fuzzy quantities: the approach speed, the remaining signal change interval, and the driver's location at the time of signal change. These three parameters are selected only to simplify the presentation; fuzziness of other parameters can be incorporated without compromising the generality.

When the driver's knowledge of speed (**V**) and signal change interval (**t**) is approximate, the stopping distance and the clearing distance can be defined as fuzzy quantities with fuzzy sets called "stopping distance" (**SD**) and "clearing distance" (**CD**), respectively.

Next, we determine a set that represents the distances greater than **SD** and call it a set of safe stopping distances. Similarly, we determine a set that represents the distances smaller than **CD** and call it a set of safe clearing distances. If the vehicle is within the safe stopping distance, it can stop safely; if it is within the safe clearing distance, it can clear the intersection safely.

Determining whether a driver can stop or clear safely from a particular distance involves comparing the distance with the fuzzy sets of safe stopping distance and safe clearing distance. A comparison of a crisp number with a fuzzy number requires the introduction of possibility and necessity measures, because the term greater or smaller (than a fuzzy number) can only be stated by a degree. In this case, the possibility measure represents the optimistic judgment and the necessity measure represents the pessimistic judgment when comparing two numbers.

First we develop possibility distributions of "safe stopping" and "safe clearing" with respect to the distance from the intersection. These define the approach area where a stopping maneuver is possible and the area where a clearing maneuver is possible, respectively. Second, we define the necessity measures of the two sets again in terms of the distance from the intersection. Thus, these possibility and necessity measures can represent the judgment of aggressive and conservative drivers, respectively, because of their criteria for comparing two numbers. It must be noted that the types of drivers represent a state of mind; thus, the same driver can be in an aggressive or conservative state depending on the circumstances.

Because the driver's knowledge of a location is also fuzzy, we compute how much the driver's approximate location belongs to each of the sets ("safe stopping" and "safe clearing"). As a result, we should be able to measure the possibility and the necessity of completing each action based on the approximate information.

Stopping Maneuver

Fuzzy Set of Stopping Distance

Given an approximate approach speed (**V**) in Equation 1, the approximate stopping distance, **SD**, is computed, where **SD** is a fuzzy set. The membership function **SD** is denoted as $h_{SD}(x)$. A possible shape of $h_{SD}(x)$ is shown in Figure 1a. This function shows the degree that a given distance x belongs to the set "stopping distance." In other words, it shows the possibility distribution of stopping distance when the approach speed is approximately V.

Possibility of Safe Stopping

The possibility of stopping safely from a distance x is determined by comparing x with **SD**. If x is greater than **SD**, the vehicle can stop safely. The set of numbers which is "possibly" greater than **SD** is called "safe stopping distance," and it is denoted by **S**. Because **SD** is a fuzzy number, the distance greater than **SD** must also form a fuzzy set. The membership function of set **S** is

$$h_S(x) = \max_{x \geq z} h_{SD}(z) \tag{5}$$

or

$$h_S(x) = \begin{cases} h_{SD}(x) & x_1 \leq x \\ 1 & x > x_1 \end{cases}$$

where x_1 is the value of distance where $h_{SD}(x)$ becomes 1.

This membership indicates the possibility that the vehicle can stop safely from distance x. Thus, it is denoted

$$\text{Poss}\ (x \in \mathbf{S}) = h_S(x) \tag{6}$$

The shape of $h_S(x)$ is similar to a cumulative distribution of $h_{SD}(x)$ as seen by the solid line in Figure 1b. It shows the degree (in numbers between 0 and 1) that the vehicle can stop safely along the approach. For a distance close to the intersection, the possibility is 0, whereas for a distance much farther from the intersection, it is 1. This possibility measure should represent the judgment of an aggressive, risk-taking, or optimistic driver because it accounts for any evidence that may indicate that his location is greater than **SD**. For the definition of possibility measure, refer to Klir and Folger (*10*) and Zimmermann (*11*).

Necessity of Safe Stopping

The necessity measure of safe stopping distance (a degree of "necessarily greater" than **SD**) is derived from

$$\mathrm{Nec}(x \in \mathbf{S}) = 1 - \mathrm{Poss}(x \in \mathbf{NS}) \qquad (7)$$

where **NS** is the complement of **S**; thus, Nec($x \in$ **S**) is 1 minus the possibility of "not being able to stop safely" (or a set of "unsafe stopping" distances) from distance x. Because Poss($x \in$ **NS**) can be derived from the membership function that represents a number possibly less than **SD** (as shown by the dashed line in Figure 1b), Nec($x \in$ **S**) is derived by Equation 7 and is shown in Figure 1c.

In contrast to Poss($x \in$ **S**), it can represent the judgment of a conservative or risk-averting driver because it takes only the sure evidence to justify that x is less than **SD**. For an explanation of necessity measures, refer to Klir and Folger (*10*).

From location p on the approach, for example, a risk-averting driver may not feel it is safe to stop, whereas a risk-taking driver may feel it is safe to stop, as seen by the comparison of values of the possibility and necessity measures of safe stopping in Figure 1c. The fact that the necessity distribution of safe stopping is located to the right of the possibility distribution in Figure 1c indicates that a risk-taking driver would feel the need to stop before the risk-averting driver as each approaches the intersection. The difference between the values of possibility and necessity measures originates from the lack of accurate information available to the driver. If sufficient information is available, and if the driver is normative, the possibility and necessity measures should be equal and the decision becomes crisp as would result from the traditional equation.

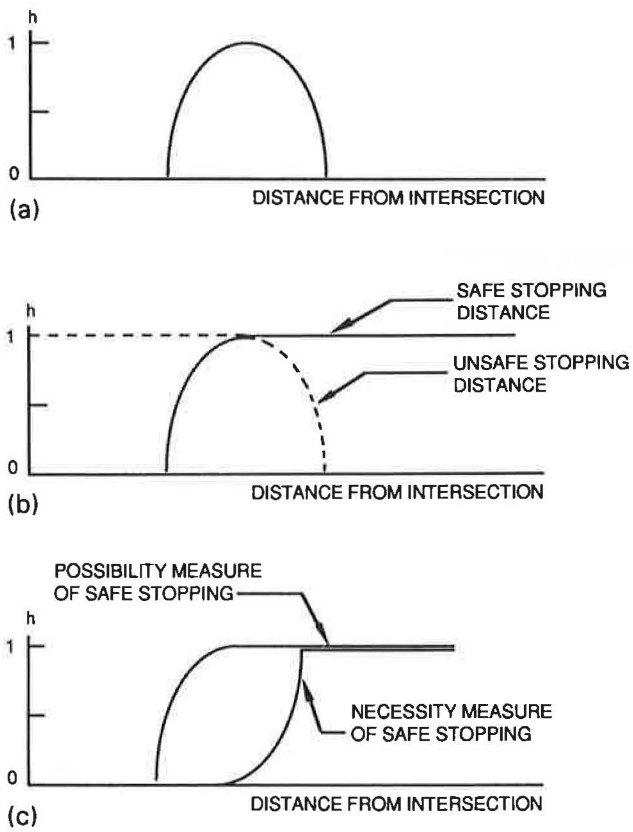


FIGURE 1 Stopping maneuvers considered: (*a*) fuzzy set representing stopping distance, (*b*) possibility distribution of safe stopping distance and unsafe stopping distance, and (*c*) possibility and necessity distributions of safe stopping distance.

Clearing Maneuver

Fuzzy Set of Clearing Distance

Given the approximate values of approach speed and signal change interval, the fuzzy clearing distance is derived from Equation 2 and is denoted **CD**. The membership function of the set is denoted $h_{\mathrm{CD}}(x)$ and its hypothetical shape is shown in Figure 2a.

Possibility of Safe Clearing

The possibility of clearing the intersection from distance x is determined by examining whether x is smaller than **CD**. The membership function of the set of numbers that is possibly smaller than **CD** is defined by

$$h_{\mathrm{C}}(x) = \max_{x \le z} h_{\mathrm{CD}}(z)$$

or

$$h_{\mathrm{C}}(x) = \begin{cases} 1 & x \le x_1 \\ h_{\mathrm{CD}}(x) & x > x_1 \end{cases} \qquad (8)$$

where x_1 is the value of x where $h_{\mathrm{CD}}(x)$ takes the maximum value (which is 1), and **C** denotes the set of clearing distances.

The possibility of clearing safely from distance x is now presented by the possibility measure and shown by the dashed line in Figure 2b:

$$\mathrm{Poss}(x \in \mathbf{C}) = h_{\mathrm{C}}(x) \qquad (9)$$

Similar to the case of the stopping maneuver, this function is believed to represent the judgment of a risk-taking or optimistic driver.

Necessity of Safe Clearing

The corresponding necessity is derived from its definition and shown in Figure 2c:

$$\mathrm{Nec}(x \in \mathbf{C}) = 1 - \mathrm{Poss}(x \in \mathbf{NC}) \qquad (10)$$

where Poss($x \in$ **NC**) represents the possibility of "not able to clear" from distance x, in other words, the possibility that

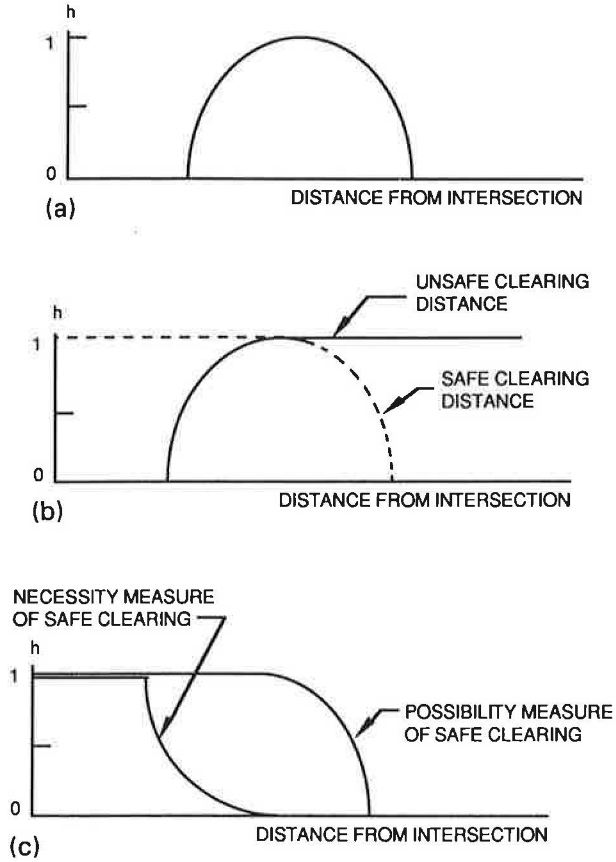


FIGURE 2 Clearing maneuvers considered: (*a*) fuzzy set representing clearing distance, (*b*) possibility distribution of safe clearing distance and unsafe clearing distance, and (*c*) possibility and necessity distributions of safe clearing distance.

x is greater than **CD**, which is shown by the solid line in Figure 2b.

Again, the necessity measure is believed to represent the judgment of a conservative or risk-averting driver. The fact that the possibility distribution of safe clearing extends to the right of the necessity distribution in Figure 2c indicates that a risk-taking driver perceives that it is safe to clear before the risk-averting driver does as each approaches the intersection.

Dilemma Zone, Indecision Zone, and Option Zone

The dilemma, indecision, and option zones can be illustrated by the way in which **S** and **C** intersect. The zones are defined based on the intersection of Poss($x \in$ **S**) and Poss($x \in$ **C**), and also on Nec($x \in$ **S**) and Nec($x \in$ **C**), separately. Respectively, they represent the decision-making environment for risk-taking and risk-averting drivers.

Based on Possibility Measure (for Risk-Taking Drivers)

When the possibility measures of safe stopping and clearing are superimposed, the possible patterns of overlaps are shown in Figures 3a, 3b, and 3c. In Figure 3a, there is a section (D_1) where neither safe stopping nor safe clearing is possible. This section is the Type 1 dilemma zone. In Figures 3a and 3b, there are zones (D_2) where the possibilities of both safe stopping and safe clearing are less than 1. These zones correspond to the Type 2 dilemma. Also in Figure 3b, at I, one of the actions is possible but the other is not completely possible. This zone corresponds to the indecision zone. In Figure 3c, both the safe stopping and clearing actions are possible in Section O. This is the option zone. The option zone, however, is located between indecision zones.

Based on Necessity Measure (for Risk-Averting Drivers)

Similarly, the intersection of two necessity measures of safe stopping and clearing are shown in Figures 3d, 3e, and 3f. Similar to the cases of Figures 3a, 3b, and 3c, the dilemma, indecision, and option zones of risk-averting drivers can be identified using the necessity measures.

It is clearly seen by comparing Figures 3a and 3d that the total area of dilemma is greater when the necessity measures are used to describe it. This indicates that risk-averting drivers would experience a greater level of uncertainty than the risk-taking drivers. When the information is assumed to be crisp, as in the traditional signal change interval formula, the value of the measure changes from 0 to 1 abruptly. Thus, the zones of indecision and Type 2 dilemma could not be identified for the two types of drivers.

Degrees of Dilemma

In the Type 2 dilemma zone, the degree of dilemma the driver experiences can be expressed by the intersections of the sets "cannot safely stop" (**NS**) and "cannot safely clear" (**NC**). **NS** and **NC** are the complements of **S** and **C**. The degrees of dilemma based on possibility and necessity measures, respectively, may be described by the height of the intersection of Poss($x \in$ **NS**) and Poss($x \in$ **NC**), or Nec($x \in$ **NS**) and Nec($x \in$ **NC**).

Fuzziness of Vehicle Location and Its Impact on Dilemma, Indecision, and Option

Next, we incorporate the fact that the driver's knowledge of his location is usually fuzzy at the onset of the yellow indication. The fuzzy set of this location is denoted by a membership function $h_L(x)$. This function represents a fuzzy set that states "the driver's location is approximately L feet from the intersection." Given $h_L(x)$, the state of the driver's decision process can be examined from the intersections of Poss($x \in$ **S**) and Poss($x \in$ **C**) with $h_L(x)$. The intersection indicates the degree that the approximate distance L belongs to the safe stopping set or the safe clearing set.

The possibility and necessity measures of safe stopping and safe clearing from approximate distance L are computed as

$$\text{Poss}(L \in \mathbf{S}) = \max\{\min(h_L(x), \text{Poss}(x \in \mathbf{S}))\} \quad \text{for all } x \qquad (11)$$

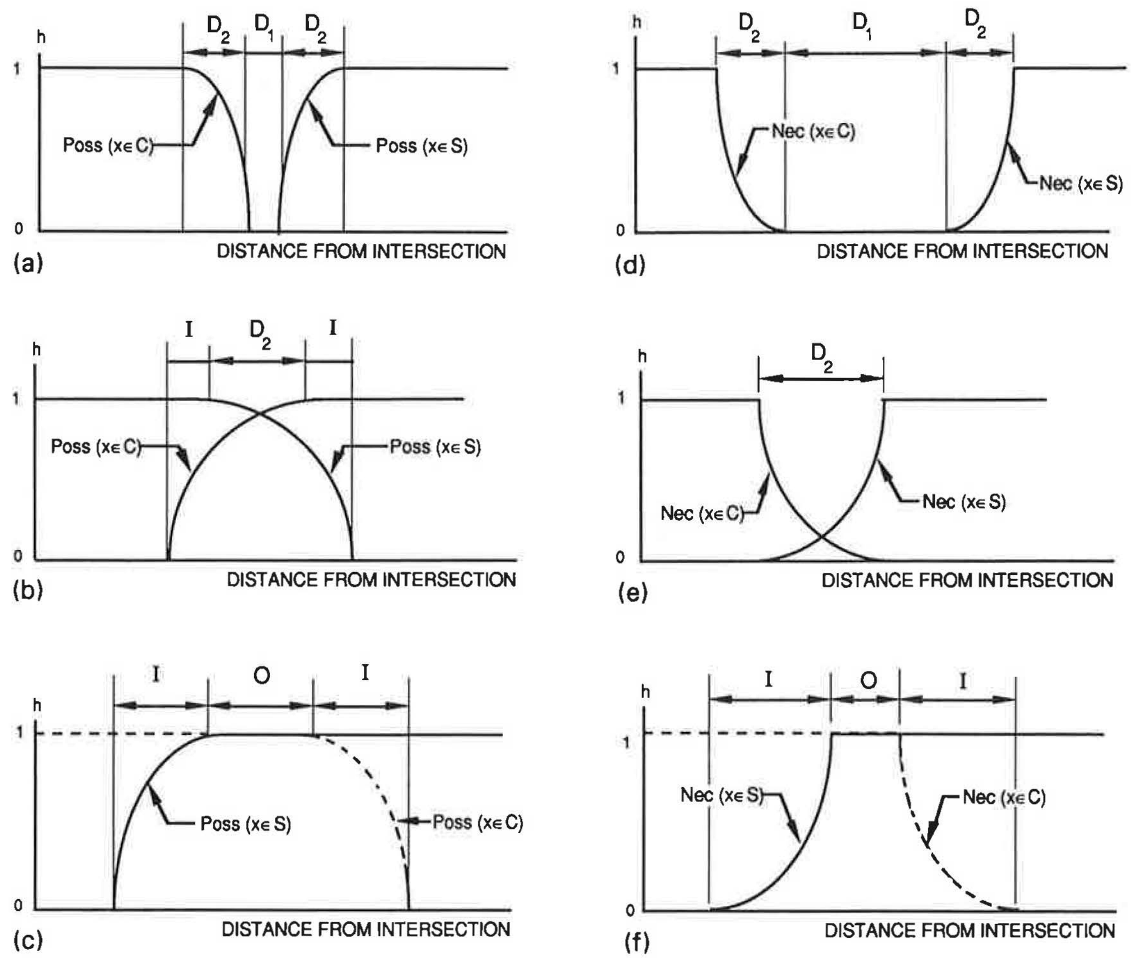


FIGURE 3 Dilemma, option, and indecision zones considered: (*a*) Type 1 and 2 dilemma zones based on possibility distribution, (*b*) Type 2 dilemma zone based on possibility distribution, (*c*) option and indecision zones based on necessity distribution, (*d*) Type 1 and 2 dilemma zones based on necessity distribution, (*e*) Type 2 dilemma zone based on necessity distribution, and (*f*) option and indecision zones based on necessity distributions.

$$\mathrm{Nec}(L \in \mathbf{S}) = 1 - \mathrm{Poss}(L \in \mathbf{NS}) \tag{12}$$

$$\mathrm{Poss}(L \in \mathbf{C}) = \max\{\min(h_L(x), \mathrm{Poss}\ (x \in \mathbf{C}))\} \quad \text{for all } x \tag{13}$$

$$\mathrm{Nec}(L \in \mathbf{C}) = 1 - \mathrm{Poss}(L \in \mathbf{NC}) \tag{14}$$

For an approximate distance L, the possibilities of safe stopping and safe clearing according to Equations 11 and 13 are illustrated in Figures 4a and 4b. They are given by the heights of a and b, respectively, in the figures.

Criteria for Determining Signal Change Intervals

Determination of signal change intervals should account for the fuzziness of perceived values of the parameters and the difference in decision criteria of different types of drivers. The following criteria may be suggested to determine the signal change intervals:

1. The possibility of taking at least one action safely must be guaranteed at any point x along the approach:

$$\mathrm{Min}\ (\mathrm{Poss}(x \in \mathbf{S}), \mathrm{Poss}(x \in \mathbf{C})) = 1 \tag{15}$$

This criterion accounts for safe completion of actions by aggressive drivers.

2. The necessity measures of taking one of the two actions must be greater than a given level α at any point along the approach:

$$\mathrm{Min}\ (\mathrm{Nec}(x \in \mathbf{S}), \mathrm{Nec}(x \in \mathbf{C})) \geq \alpha \tag{16}$$

This criterion guarantees the minimum level of safe completion of action. It is an attempt to offer a certainty level that a risk-averting driver can take at least one of the actions safely.

The signal change interval derived from the traditional formula satisfies the first criterion so that at least one of the actions is possible along the approach. It does not allow for the maximum certainty of safe stopping or safe clearing when measured on the basis of necessity.

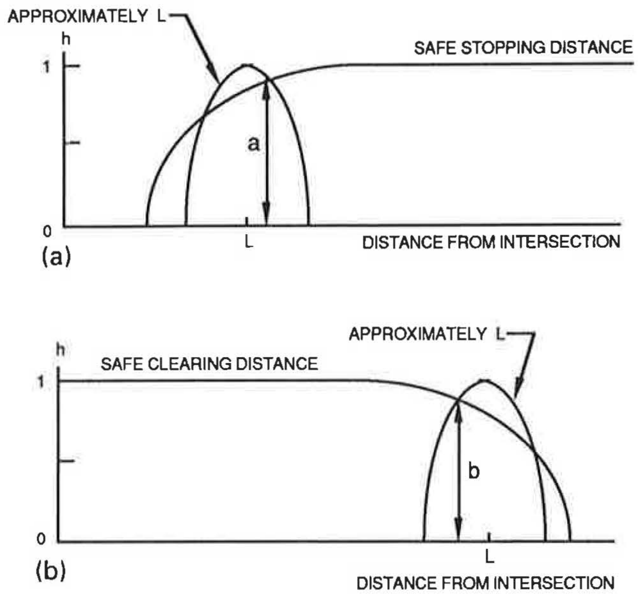


FIGURE 4 Possibility measures for safe stopping and clearing from a distance of approximately *L*: (*a*) possibility of safe stopping and (*b*) possibility of safe clearing.

EXAMPLE ANALYSIS

In this section, using a set of example values, we present two procedures: one to identify the dilemma and indecision zones and a second to develop signal change intervals based on the method just presented.

The following values are used for the examples: approach speed (V) = 40 mph, deceleration rate (b) = 9 ft/sec^2, intersection width (w) = 50 ft, vehicle length (ℓ) = 20 ft, and perception/reaction time (d) = 1.5 sec. The signal change interval computed with the traditional formula Equation 3 is 5.3 sec.

Dilemma and Indecision Zones

We now analyze the dilemma and indecision zones for the same intersection, assuming that the perceived approach speed and signal change interval are approximately 40 mph and approximately 5 sec, respectively. The fuzzy sets for the approach speed (V) and perceived signal change interval (t) are defined using a triangular fuzzy number (TFN) of the form (f1, f2, f3), which respectively represent minimum, most likely, and maximum values for the approximate number. "Approximately 40 mph" can be represented by (30, 40, 50) mph and the "approximately 5 sec" by (4, 5, 6) sec. This assumption of a TFN for approximately 40 mph is reasonable when compared to the observed distribution of approach speed presented by Olson and Rothery (*3*). Introducing these TFNs for **V** and **t**, we compute the fuzzy sets for "stopping distance" and "clearing distance" as **SD** = (174, 282, 406) ft and **CD** = (138, 282, 447) ft using Equations 1 and 3. These fuzzy sets are shown in Figures 5a and 5b. For the arithmetic operations of the fuzzy numbers, refer to Kaufmann and Gupta (*12*). The corresponding possibility and necessity measures of "safe stopping" and "safe clearing" are computed in Equations 5, 7, 8, and 10 and shown in Figures 5c and 5d.

The indecision and dilemma zones are presented in Figures 5e and 5f. Figure 5e shows that the possibility measure of at least one of the actions is 1 along the approach; thus, at least one action is possible. This is expected, because the values of f2 for **SD** and **CD** are the same as the original crisp values. The necessity measures of the two sets, illustrated in Figure 5f, show Type 2 dilemma zones. The value of the necessity measure is less than 1. These two figures indicate that, for a 5-sec signal change interval, no dilemma exists for risk-taking drivers, but a Type 2 dilemma exists for risk-averting drivers.

The length and the location of the indecision zone or Type 2 dilemma zone in Figures 5e and 5f are compared with the observed and derived stopping probabilities presented by Olson and Rothery (*3*), Sheffi and Mahmassani (*4*), and Zegeer and Deen (*13*). Our example shows that the Type 2 dilemma zone lies between 138 and 447 ft from the intersection as seen in Figure 5f. The observations reported by Olson and Rothery (*3*) show the range in which stopping probability is between 0 and 1 as 200 to 380 ft at approach speed 50 mph, and 80 to 200 ft at 30 mph speed; Zegeer and Deen (*12*) show 100 to 300 ft at approach speed 40 mph; Sheffi and Mahmassani's (*4*) derived probability shows approximately 60 to 350 ft at 40 mph.

Figure 6 compares their stopping probability functions with our possibility and necessity measures of "safe stopping" and "safe clearing." The shaded area is bounded by the possibility of "safe stopping" and the necessity of "not safe to clear," the former representing the aggressive driver's stopping criterion and the latter the conservative driver's clearing criterion. Lines 1 and 2 represent the stopping probability curves shown by Sheffi and Mahmassani (*4*) and Zegeer and Deen (*13*), respectively. Lines 3 and 4 and Lines 5 and 6 are the observed stopping probability frequencies for approach speed of approximately 30 mph and 50 mph, respectively, reported by Olson and Rothery (*3*). The shaded area is very close to Lines 1 and 2, and it also lies between line pairs of 30 mph approach speed and 50 mph. Line 7 shows the necessity measure of safe stopping—a risk-averting driver's stopping criterion. The characteristics of the intersections presented in the previous papers are probably not identical. However, the lines derived by the fuzzy measure closely match with the ones that were surveyed or mathematically derived previously. This suggests that the fuzzy measures can be an alternative method to identify the zones of dilemma and indecision, and to examine the adequacy of the signal change interval.

Signal Change Interval

The authors next used the criteria presented earlier to suggest a signal change interval that accounts for the driver's fuzzy perception of V and t. Because the first criterion is satisfied by the 5.3 sec, the interval computed at the beginning of this section, the signal change interval that satisfies the second criterion is computed. This is accomplished by determining t so that the slope of the decreasing section of Nec($x \in \mathbf{C}$) intersects with Nec($x \in \mathbf{S}$) in Figure 7 at a height greater than α. In other words, the following condition must be satisfied at the intersection of the Nec($x \in \mathbf{C}$) and Nec($x \in \mathbf{S}$) lines:

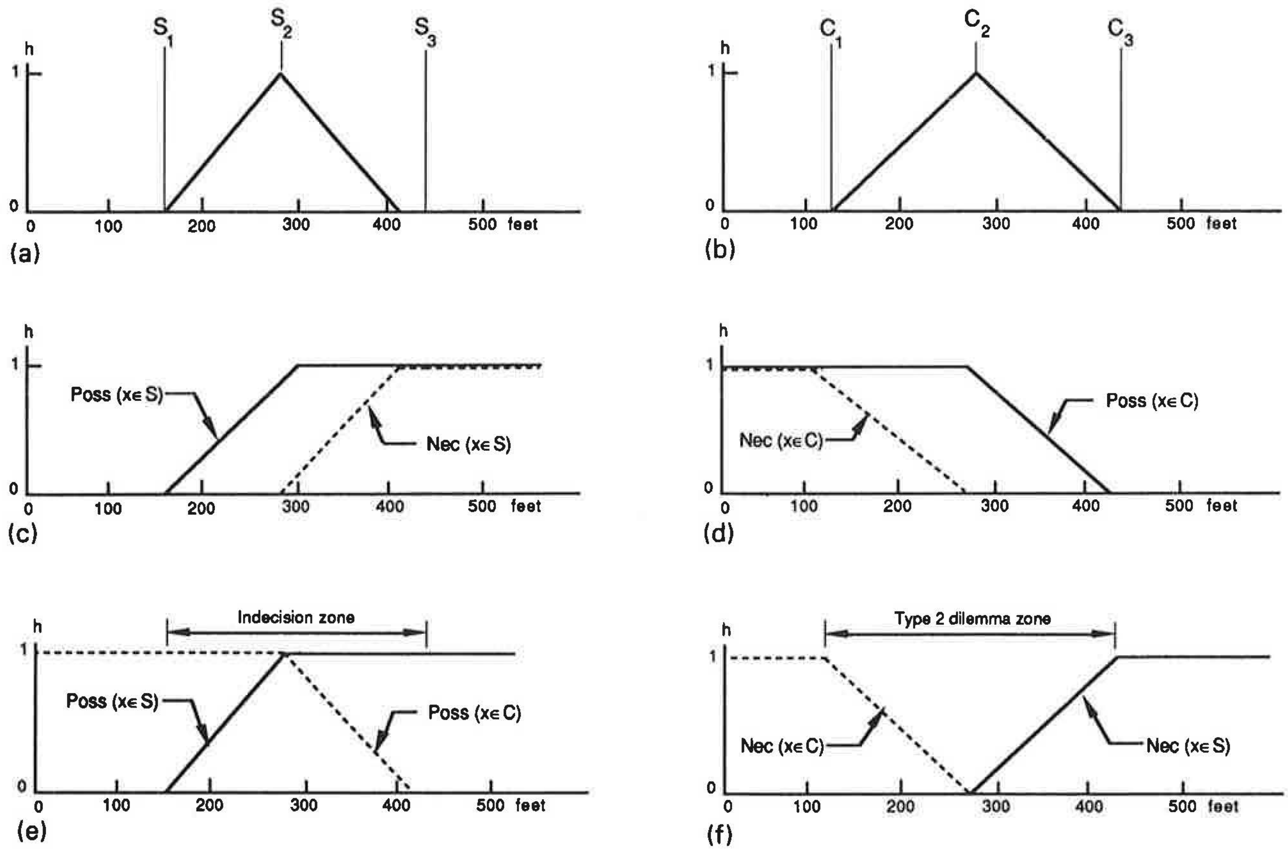
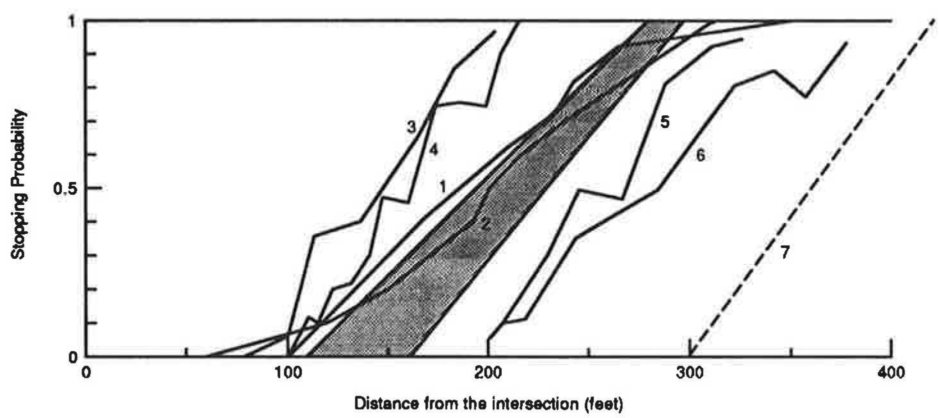


FIGURE 5 Fuzzy sets and possibility and necessity measures for example problem: (*a*) fuzzy stopping distance (SD), (*b*) fuzzy clearing distance (CD), (*c*) safe stopping distance (S), (*d*) safe clearing distance (C), (*e*) indecision zone, and (*f*) Type 2 dilemma zone.



Notes — Shaded area: likelihood of stopping derived from the fuzzy measures Nec (x∈NC) and Poss (x∈S) at approach speed = 40 mph;

Line 1: Stopping probability (approach speed = 40 mph) by Zegeer and Deen (13);

Line 2: Stopping probability (approach speed = 40 mph) by Sheffi and Mahmassani (4);

Lines 3 and 4: Stopping probability (approach speed = 30 mph) by Olson and Rothery (3);

Lines 5 and 6: Stopping probability (approach speed = 50 mph) by Olson and Rothery (3);

Line 7: Necessity measure of stopping (Nec (x∈S) at approach speed = 40 mph.

FIGURE 6 Comparison of stopping probabilities and likelihood of stopping derived from fuzzy measures.

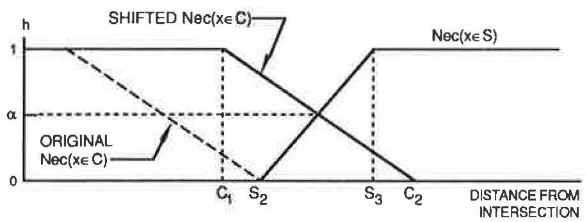


FIGURE 7 Determination of C_1 and C_2 for given α.

$$(1 - \alpha)C_2 + \alpha C_1 = (1 - \alpha)S_2 + \alpha S_3 \quad (17)$$

where C_1, C_2, S_2, and S_3 are as shown in Figure 7. Because C_1 and C_2 are functions of t, once they are determined by Equation 17, t can be obtained.

For an α value of 0.5, for example, t becomes 7.8 sec. This is 2.5 sec longer than the value computed by Equation 3. However, the 7.8 sec of yellow and all-red time for the given condition is still within the range of maximum signal change interval demand surveyed by Lin and Vijaykumar (*14*) for similar conditions.

CONCLUSIONS

Recognizing the fact that drivers must decide whether to stop or clear based on fuzzy information, the authors propose the use of fuzzy set theory for the analysis of the driver's decision-making environment at signalized intersections.

Using fuzzy measures of the safe stopping and safe clearing sets, the dilemma, indecision, and option zones are defined along the approach. The possibility and necessity measures of the two sets identify these zones for risk-taking and risk-averting drivers, respectively. The intersection of the complements of the two sets identifies the level of dilemma for the two types of drivers. Criteria for setting signal change intervals are also suggested when the information available to the driver on speed, location, and the remaining time of the signal change interval are vague.

The difference between the possibility and necessity measures narrows as more accurate information becomes available to the driver. Eventually, if the information is totally crisp to the driver, the two measures coincide. Under this environment, the traditional equation for the signal change interval is justified.

Providing accurate information to the driver reduces dilemma and indecision and, therefore, helps to reduce the signal change interval. Any measures resulting in more accurate driver-perceived information is essential to reduce driver indecision and shorten the signal change interval.

The stopping probability functions studied by many in the past represent only the consequences of decisions made. The possibility and necessity measures that we propose, on the other hand, can indicate the availability of the choices and the degree of safety for completing them for two extreme types of drivers (risk-taking and risk-averting). The normative driver's behavior perhaps lies between the two extreme types of drivers; thus the proposed method can identify the ranges of the dilemma and indecision zones. The approach presented here could be extended to the analysis of other risk-measurement problems in traffic engineering.

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