# Characterization of Resilient Modulus of Compacted Subgrade Soils Using Resonant Column and Torsional Shear Tests

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Both resonant column (RC) and torsional shear (TS) tests were performed at small (below 0.001 percent) to intermediate (below 0.1 percent) strain levels to investigate the effects of variables such as strain amplitude, loading frequency, and number of cycles of loading on the resilient modulus ($M_R$) of compacted subgrade soils. Plasticity index (PI) was found to be an important variable in evaluating these effects, with the majority of the tests performed on cohesive subgrade soils compacted at optimum moisture content. Resilient moduli determined by RC, TS, and $M_R$ tests compare well when measurements are compared at the same strain amplitude, time of confinement, and excitation frequency. Elastic threshold strains were determined for the compacted subgrade soils. At cyclic strains below the elastic threshold strain, resilient modulus of a given soil is independent of strain amplitude and is the maximum value measured. At cyclic strains above the elastic threshold strain, $M_R$ decreases as strain amplitude increases. The elastic threshold strains of compacted cohesive soils ranged from 0.0008 to 0.0048 percent as PI varied from 4 to 52 percent. Both RC and TS tests accurately measured resilient moduli below the elastic threshold strain. On the other hand, $M_R$ equipment could not be used in this strain range because of the lack of resolution. Resilient modulus was found to increase as loading frequency increased, even below the elastic threshold. Moduli obtained from RC, TS, and $M_R$ tests agreed well at strains above about 0.01 percent once the effect of frequency was taken into account. Modulus reduction curves showing the variation in normalized modulus, resilient modulus divided by the small-strain resilient modulus, versus strain amplitude correlate well with plasticity index.

In 1986 AASHTO adopted the use of resilient modulus ($M_R$) in the design of pavement structures. In the laboratory, $M_R$ testing has been developed to determine resilient modulus of subgrade soils (*1*). In the field, nondestructuve testing methods such as the dynaflect, falling weight deflectometer, and spectral analysis of surface waves have been developed for evaluation of $M_R$ of pavement materials. However, different testing techniques are likely to have different applied stress (or strain) levels and different loading frequencies, and the response of pavement materials can be affected by these variables. Therefore, the effects of strain amplitudes (or nonlinear behavior) and loading frequencies on the stiffness of pavement materials need to be considered, and measured values should be adjusted to the strain amplitudes and frequencies where the actual system is working.

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It has long been recognized that all subgrade soils exhibit nonlinear behavior at strains above about 0.001 percent (*2*). The axial strain levels encountered in pavement subgrades generally range from small ($< 10^{-3}$ percent) to intermediate ($< 10^{-1}$ percent) levels (*3*). However, most $M_R$ testing equipment cannot accurately measure moduli at axial strains smaller than about 0.01 percent because of the limitation in resolution of transducers and the compliance of the system itself (*4,5*). Therefore, the complete stress-strain behavior of subgrade soils over a strain range from small to intermediate strains is not generally obtained in $M_R$ testing unless the soils are very stiff.

To investigate the effects of strain amplitude, loading frequency, and number of loading cycles on the resilient modulus of subgrade soils at small to intermediate strains, both resonant column (RC) and torsional shear (TS) tests were performed on the same specimen. RC tests were performed over strains between $10^{-4}$ percent and $10^{-1}$ percent, and the complete modulus-strain behavior was obtained. The effect of frequency was also investigated at two strain levels: a small-strain level of about 0.0007 percent and an intermediate strain level of about 0.007 percent. TS tests were also used in this evaluation. Finally, plasticity index (PI) was investigated as one variable in defining the effects of strain amplitude, frequency, and number of loading cycles on $M_R$ of compacted subgrade soils. Modulus reduction curves, showing Young's modulus at a given strain amplitude divided by Young's modulus at small strains ($E/E_{max}$) versus strain amplitude were developed. The trend in the modulus reduction curves with PI was also investigated. In this case, Young's modulus, E, was assumed to be equivalent to $M_R$.

## TEST EQUIPMENT AND MEASUREMENT TECHNIQUES

### Torsional Resonant Column Test

Resonant column equipment of the torsional fixed-free type was used. In the fixed-free resonant column test, the bottom end of the specimen is rigidly fixed against rotation at the base pedestal while the top (free end) is connected to a drive system that is used to excite and monitor torsional motion, as shown in Figure 1a.

The basic operational principle is to vibrate the cylindrical specimen in first-mode torsional motion. Once first mode is established, measurements of the resonant frequency and amplitude of vibration are made as shown in Figure 1b. These measurements are then combined with equipment characteristics and specimen size to calculate shear wave velocity ($V_s$), shear modulus (G), and shearing strain amplitude ($\gamma$) (*6*).

One-dimensional wave propagation in a circular rod is used to analyze the dynamic response of the specimen. The basic data-reduction equation is expressed as follows:

$$I/I_o = \omega_r L/V_S \tan(\omega_r L/V_S) \qquad (1)$$

where

- $I$ = mass moment of inertia of the specimen,
- $I_o$ = mass moment of inertia of drive system,
- $\omega_r$ = resonant circular frequency,
- $L$ = length of the specimen, and
- $V_S$ = shear wave velocity.

Once the value of shear wave velocity is determined from Equation 1, shear modulus (G) of the specimen can be calculated from

$$G = \rho V_S^2 \qquad (2)$$

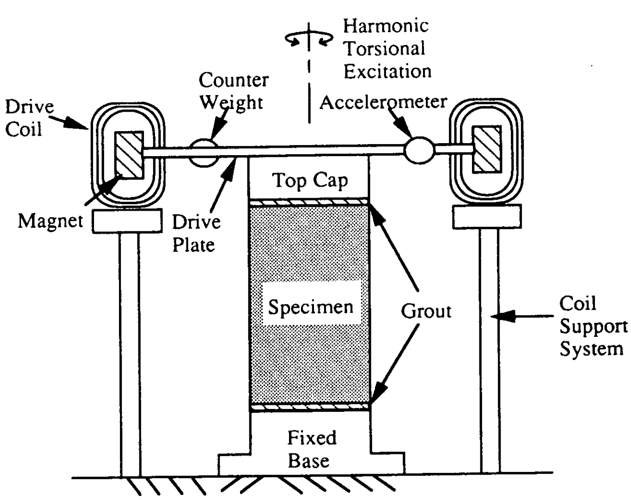
where $\rho$ is the mass density. The shearing strain ($\gamma$) is calculated from the peak rotation of the top of the specimen at 0.80 times the radius of the solid sample (*7,8*).

Standard procedures followed in relating cyclic triaxial and resonant column results were also used to relate $M_R$ and RC results. For cyclic triaxial and RC results, Young's modulus, E, and axial strain, $\varepsilon_a$, are taken to be compatible with shear modulus, G, and shearing strain, $\gamma$ (*9*) through

$$E = 2G(1 + \nu) \qquad (3)$$

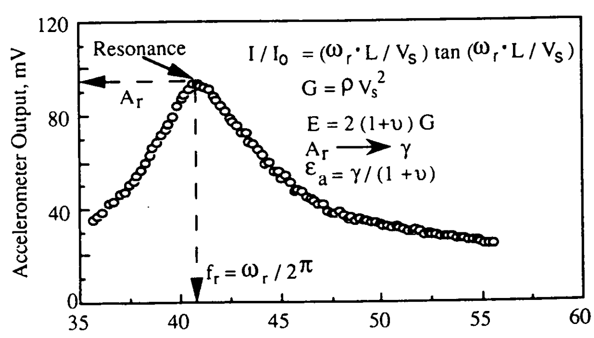
and

$$\varepsilon_a = \gamma/(1 + \nu) \qquad (4)$$



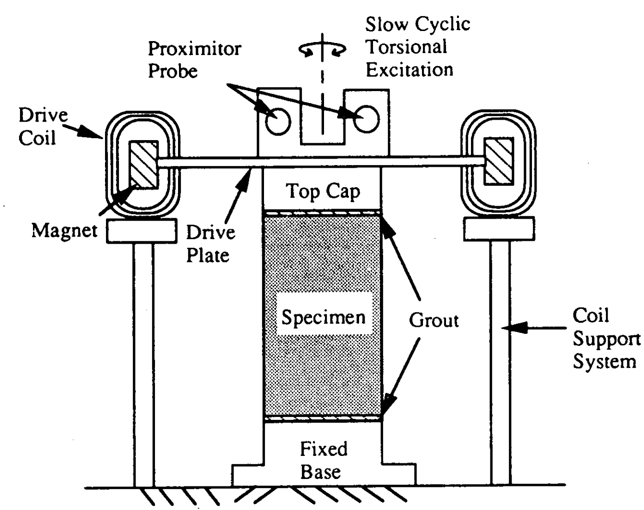

a) Specimen in the Resonant Column Apparatus

(The Resonant Column and Torsional Shear Apparatus is One Piece of Equipment Operated Either at Resonance, Resonant Column, or in Slow Cyclic Loading, Torsional Shear)



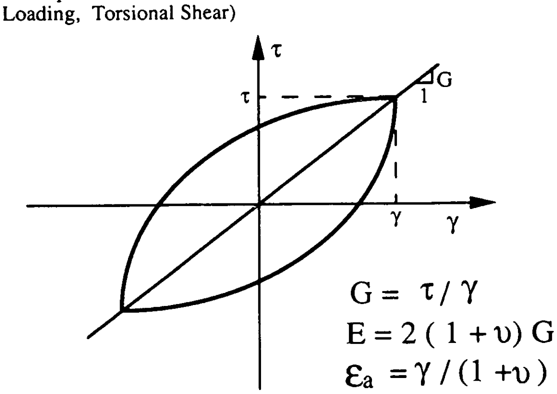

b) Typical Frequency Response Curve

**FIGURE 1 Fixed-free resonant column test and an associated frequency response curve.**




a) Specimen in the Torsional Shear Apparatus

(The Resonant Column and Torsional Shear Apparatus is One Piece of Equipment Operated Either at Resonance, Resonant Column, or in Slow Cyclic Loading, Torsional Shear)




b) Measurement of Shear Modulus from Hysteresis Loop

**FIGURE 2 Torsional shear apparatus and an associated hysteresis loop.**

where $\nu$ is Poisson's ratio. In $M_R$ testing, E and $M_R$ are assumed equal. Therefore, $M_R$ is simply inserted in Equation 3 for E. In applying these equations, the material is assumed to be homogeneous and isotropic. Values for Poisson's ratio of the compacted cohesive soils tested in this work were not measured but were assumed to be 0.45 and to be independent of excitation frequency. This value was selected because the cohesive soils were nearly saturated and Poisson's ratio most likely fell within the range of 0.40 to 0.50. As a result, the assumption of 0.45 causes less than a 5 percent error in using Equations 3 and 4.

### Torsional Shear Test

The TS test is another method of determining shear and Young's moduli with the same equipment used in resonant column testing but operating it in a different manner. In this test, a cyclic torsional force with a given frequency, generally below 10 Hz, is applied at the top of the specimen while the bottom is held fixed, as shown in Figure 2a. Instead of determining a resonant frequency, the stress-strain hysteresis loop is determined from measuring the torque-twist response to the specimen. Proximeters are used to measure the angle of twist while the voltage applied to the coils is calibrated to yield torque. Shear modulus (G) is calculated from the slope of a line through the endpoints of the hysteresis loop as shown in Figure 2b. Thus, the shear modulus is calculated from

$$G = \tau/\gamma \tag{5}$$

Shearing strain amplitude is calculated at 0.80 times the radius of the specimen, just as in resonant column tests. Once G and $\gamma$ are determined, Young's modulus and axial strain can be determined using Equations 3 and 4.

## TEST MATERIALS AND SAMPLE PREPARATION

### Test Materials

Ten disturbed subgrade soils were gathered from across the state of Texas. The Texas State Department of Highways and Public Transportation helped collect the samples from subgrades of actual pavement projects that have already been constructed and put into operation. Table 1 presents the basic properties of the test soils. Plasticity index (PI) was investigated to evaluate its importance as a correlation parameter.

**TABLE 1 Summary of Properties of Subgrade Soils Collected Around Texas**

| Soil ID | District County Highway | AASHTO Class. | Passing No. 200 (%) | Liquid Limit | Plasticity Index | Optimum Moisture Content (%) | Sample Moisture Content (%) | Total Unit Wt. (pcf) |
|---|---|---|---|---|---|---|---|---|
| 1 | 14 Travis Mopac-183 | A-7-6 | 87.3 | 56 | 29 | 19.3 | 19.3 | 112.7 |
| 2 | 21 Starr FM755 | A-4 | 34.9 | 25 | 10 | 10.6 | 10.5 | 129.8 |
| 3 | 5 Hockley US62 | A-6 | 100 | 30 | 15 | 12.7 | 12.8 | 131.4 |
| 4 | 4 Potter Spur951 | A-6 | 100 | 37 | 20 | 16.5 | 22.0 | 123.4 |
| 5 | 4 Gray SH70 | A-7-6 | 100 | 52 | 34 | 19.2 | 22.2 | 120.6 |
| 6 | 5 Lubbock FM835 | A-4 | 91 | 20 | 4 | 10.6 | 13.0 | 132.2 |
| 7 | 20 Jasper FM252 | A-7-6 | 100 | 79 | 52 | 19.9 | 20.0 | 123.1 |
| 8 | 20 Jefferson US69 | A-7-6 | 96 | 54 | 36 | 18.0 | 10.2 | 120.6 |
| 9 | 7 Tom Green US67 | A-7-6 | 98 | 58 | 40 | 20.1 | 20.0 | 125.4 |
| 10 | 8 Haskell Abilene | A-7-6 | 97 | 51 | 29 | 16.2 | 16.4 | 125.2 |

### Sample Preparation

Compacted subgrade specimens were prepared following Tex-101-E-part II "Preparation of Soil and Flexible Base Materials for Testing" (*10*). Test Method Tex-101-E is in close agreement with AASHTO Designation T 146-86 and T 87-86.

To prepare the sample, the soil was first air dried. The soil was then placed in a 20-rpm mixer and mixed with the proper amount of distilled water to create the design water content. The kneading compaction method was used to compact the samples to a diameter of 4 in. (10.2 cm) and a height of 6 in. (15.2 cm). Five layers were used, and the compaction effort specified in Test Method Tex-113-E was applied. The samples were prepared at optimum, wet of optimum, and dry of optimum moisture contents. The "dry" and "wet" samples were prepared to achieve 95 percent of the maximum dry density. However, samples compacted at dry of optimum were very difficult to trim, which resulted in samples at optimum and wet of optimum being the ones on which most testing was performed.

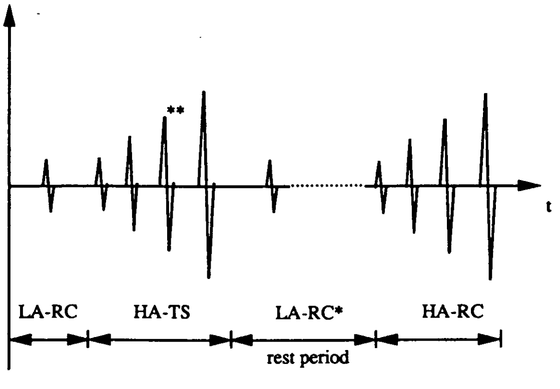
After compacting the soil specimens, they were carefully extruded out of the steel mold using the hydraulic extruder. The samples were usually trimmed by hand to 2.0 in. (5.1 cm) in diameter and 4.0 in. (10.2 cm) in height. Trimmed samples were wrapped and stored in the moisture room for about 5 days before they were removed for setup and testing.

## TESTING PROCEDURES

Before testing, each specimen was grouted to the top cap and base pedestal using a thin layer of hydrostone paste manufactured by U.S. Gypsum. Grouting had the beneficial result of achieving the fixed-free boundary condition in the RC and TS tests. An external rubber membrane was placed on the specimen immediately after grouting and sealed to the top cap and bottom pedestal with O-rings. The sample-cement connections were allowed to cure overnight before testing.

Six days after compaction, both RC and TS tests were performed on the same specimen under an isotropic confining pressure of 6 psi. Only one pressure was used because the main goal of this study was to evaluate the influence of strain amplitude and loading frequency on $M_R$. In addition, all specimens were compacted, which resulted in overconsolidating the cohesive soils. Overconsolidated cohesive soils exhibit only a small influence of confining pressure on stiffness (*2*); therefore, this effect was not studied further.

The series of RC and TS tests is shown in Figure 3. Initially the change in the low-amplitude shear modulus (at $\gamma < 0.001$ percent) with confinement time (usually 1 hr) was defined in




LA-RC = low-amplitude resonant column test (at $\gamma < 0.0001\%$).

HA-TS = high-amplitude torsional shear tests; 10 loading cycles are applied during each constant-amplitude test; amplitude of successive test is approximately doubled each time.

LA-RC* = low-amplitude resonant column test; compare modulus with value before HA-TS; HA-RC tests not started until low-amplitude modulus equals value before HA-TS tests.

HA-RC = high-amplitude resonant column tests; amplitude of successive test is approximately doubled each time.

** Check the effect of loading frequency at shearing strains of 0.001% and 0.01%.

**FIGURE 3 Typical series of cyclic tests used to investigate the resilient modulus of compacted subgrade soils at $\sigma_o$ = 6 psi.**

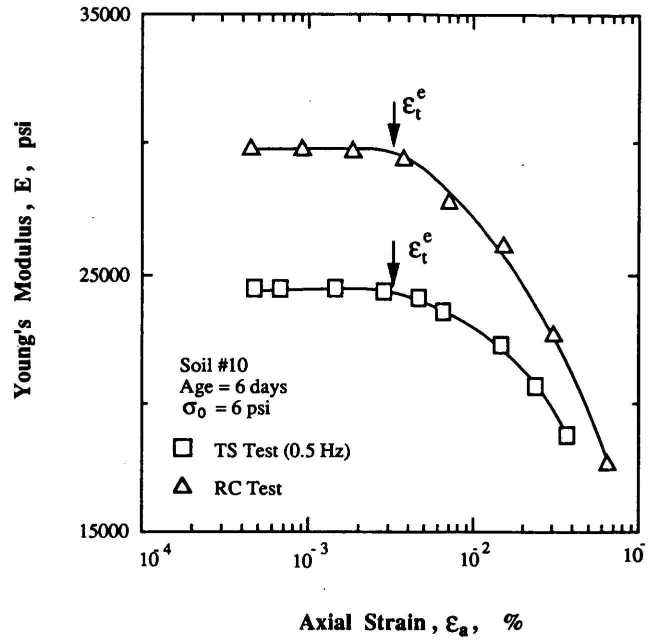



**FIGURE 4 Typical variation in Young's modulus with axial strain for a compacted subgrade as determined by resonant column and torsional shear tests.**

the RC test. A series of high-amplitude TS tests was then performed. The TS tests were conducted at 0.5 Hz. To investigate the effect of loading frequency on modulus, loading frequencies were changed at shearing strain amplitudes of about 0.001 and 0.01 percent. Typical loading frequencies used in this study were 0.05, 0.1, 0.5, 1, 5, and 10 Hz. After completion of high-amplitude TS testing, the low-amplitude shear modulus was again measured in the RC test and compared with the value before high-amplitude TS testing. Sometimes, the low-amplitude shear modulus decreased because of cyclic degradation. However, a rest period following high-amplitude testing allowed the stiffness to regain with time at constant confinement. Once the low-amplitude modulus had regained to the previous value, high-amplitude RC testing was performed with increasing strain amplitudes.

## TEST RESULTS

### Effect of Strain Amplitude on Stiffness of Compacted Subgrade Soils

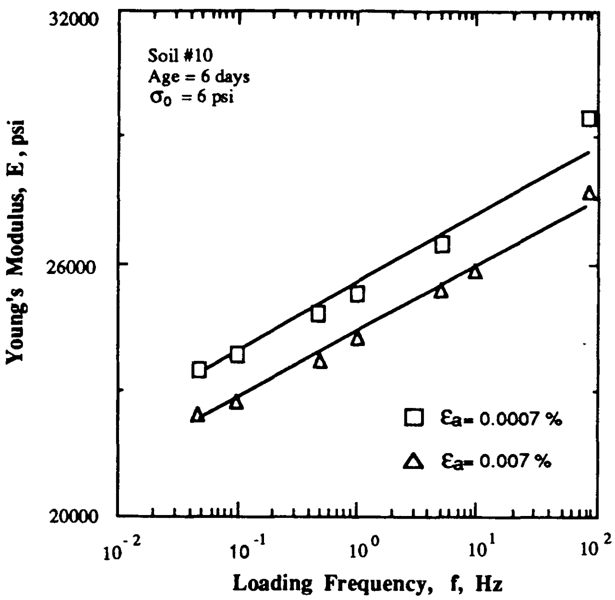
Typical variations in Young's modulus with axial strain determined by RC and TS tests are shown in Figure 4. Shear moduli and shearing strains obtained from both tests were converted to the equivalent Young's moduli and axial strains using Equations 3 and 4. For strain levels below about 0.003 percent, moduli determined by the RC and TS tests are independent of strain amplitude. In this strain range, the behavior of soil is elastic (or proportional). The upper bound strain of this range is defined as an elastic threshold strain, $\varepsilon_t^e$. Above the elastic threshold, moduli determined by both RC and TS tests decrease as strain amplitude increases. As shown in Figure 4, RC and TS tests can be performed at small to intermediate strains, resulting in a more complete picture of the modulus strain behavior from the proportional range to the nonlinear range than is presently possible with typical $M_R$ equipment.

Moduli determined by the RC test are larger than those obtained by the TS test over the whole strain range. In the RC test, moduli at small strains were measured at a loading frequency of about 90 Hz, whereas corresponding values in the TS test were obtained at 0.5 Hz. Generally, the stiffness of cohesive soil increases with increasing loading frequency. Therefore, the moduli difference between the two types of tests can be explained mainly by the difference in loading frequency (as discussed in more detail later). For these soils, number of loading cycles had very little effect on the moduli, especially for soils compacted at the optimum moisture content.

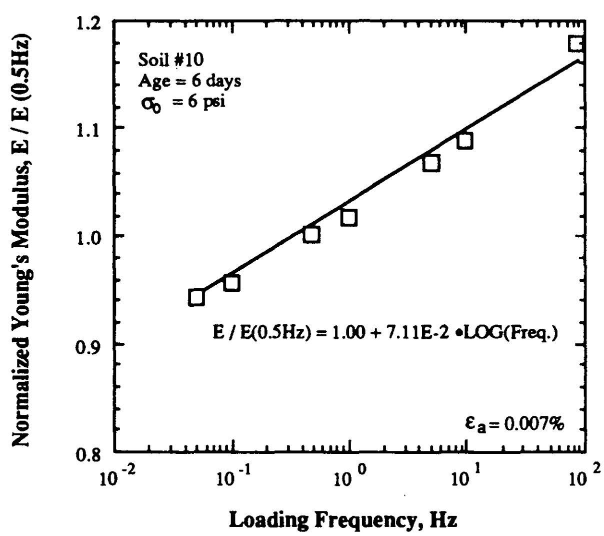
### Effect of Frequency on Stiffness of Compacted Subgrade Soils

To investigate the effect of loading frequency on stiffness, the variation in Young's modulus of compacted subgrade soil with loading frequency is plotted at axial strain amplitudes of 0.0007 and 0.007 percent in Figure 5a. Moduli from the RC tests are also included. The modulus of this compacted subgrade soil increases almost linearly as a function of the logarithm of loading frequency.

To quantify the influence of loading frequency on stiffness, Young's modulus was normalized by the value of Young's modulus at a loading frequency of 0.5 Hz. Typical normalized behavior is plotted in Figure 5b. This normalization was done




a.) Variation in Young's Modulus with frequency




b.) Variation in normalized Young's Modulus with frequency

**FIGURE 5 Typical variation in moduli with frequency for a compacted subgrade soil as determined by resonant column and torsional shear tests.**

for an axial strain amplitude of 0.007 percent. By performing least-squares curve fitting on these data, the fitting curve yields the effect of loading frequency on stiffness, which is 7.1 percent per log cycle of loading frequency.

Correlation of the effect of loading frequency on the stiffnesses of compacted subgrade soils with PI at different strain amplitudes is presented in Figure 6. The open symbols represent testing at a strain amplitude of 0.0007 percent, and the solid symbols represent the results at a strain amplitude of 0.007 percent. The best-fit curve is plotted with a solid line approximately in the middle of the data band. The effect of frequency on compacted subgrade soils increases with increasing PI and ranges from 4.5 and 8.4 percent. In addition,

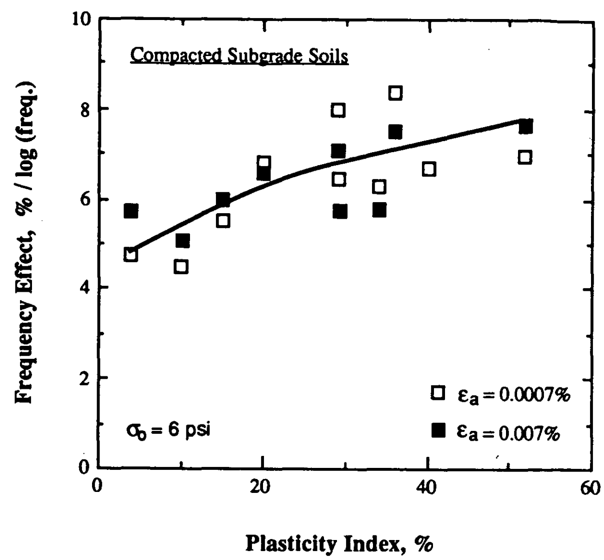



**FIGURE 6 Variation in the effect of loading frequency on the stiffness of compacted subgrade soils with PI at axial strain amplitudes of 0.0007 and 0.007 percent.**

it appears that strain amplitude has little influence on the frequency effect.

### Comparison Between $M_R$, RC, and TS Tests

To determine the capability of the testing equipment, several comparisons of results between $M_R$, RC, and TS tests were made. $M_R$ tests were performed by following the recommendations published by Strategic Highway Research Program (SHRP) Protocol P-46 (*11*). Details of the resilient modulus tests are described by Pezo (*4*). To make this comparison, moduli obtained with the RC and TS tests were converted to equivalent resilient moduli using Equations 3 and 4. In addition, the moduli were finally adjusted to an excitation frequency of 10 Hz, which is the primary loading frequency in the $M_R$ measurement. Figure 7 shows the typical variation in

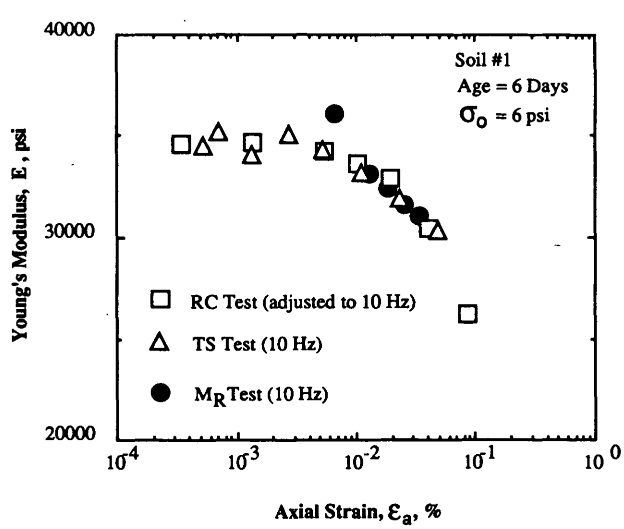



**FIGURE 7 Comparison of $M_R$ values of compacted subgrade soil determined by RC, TS, and $M_R$ tests (*5*).**

resilient modulus with axial strain as determined by the three different testing methods. Moduli obtained from the RC and TS tests overlap nicely, with values from the $M_R$ test showing that RC and TS tests can be used in determining resilient modulus of subgrade soils provided the effect of loading frequency is considered. $M_R$ testing equipment could not be used to measure moduli at axial strains smaller than about 0.01 percent, and thus the elastic threshold strain could not be defined.

### Effect of PI on Normalized Behavior of Compacted Subgrade Soils

Since Young's modulus in the small-strain region ($E_{max}$) as well as moduli at larger strains were measured, normalized modulus reduction curve, $E/E_{max}$ versus log $\varepsilon$, could be developed. The RC test is important in these measurements because it can generate higher strains at the same applied torque than the TS test because of dynamic magnification. Therefore, the RC test was used to investigate the normalized behavior of compacted subgrades over a wide range of strain amplitudes.

Normalized modulus reduction curves for the compacted subgrade soils are shown in Figure 8. Figure 8a presents the variation of normalized modulus reduction curves of subgrade soils compacted at optimum moisture content, and Figure 8b presents those results of subgrade soils compacted at wet of optimum (95 percent of maximum dry density). The range of PI of the samples is 4 to 52 percent. Soil 6 (PI = 4 percent), which is the least plastic soil, exhibits nonlinear behavior beginning at a strain amplitude of about 0.0008 percent, and Soil 7 (PI = 52 percent), the most plastic soil, exhibits nonlinear behavior beginning at strains of about 0.005 percent. These results show how the $E/E_{max}$ versus log $\varepsilon$ curve shifts toward increasing elastic threshold strains as soil plasticity increases. Therefore, it can be concluded that PI of the soil is an important variable in evaluating the nonlinear behavior of (compacted) subgraded soils. Vucetic and Dobry (*12*) have shown the small effect of PI on the location of the normalized shear modulus reduction curve, $G/G_{max}$ versus log $\gamma$, and Kokusho et al. (*13*) have demonstrated that, even for large overconsolidation ratios (OCRs), the value of OCR has practically no effect on the position of the $G/G_{max}$ versus log $\gamma$ curve.

To fit these test data, a Ramberg-Osgood (R-O) fitting method was used (*4,8*). The R-O fitting equation can be written as

$$\varepsilon = E' \cdot \varepsilon + C\,(E' \cdot \varepsilon)^R \tag{6}$$

where $E' = E/E_{max}$ = normalized Young's modulus and C and R are the R-O parameters. Equation 6 can be rewritten as

$$\varepsilon \cdot (1 - E') = C \cdot (E' \cdot \varepsilon)^R \tag{7}$$

By taking the logarithm of both sides, Equation 7 yields

$$\log\,[\varepsilon \cdot (1 - E')] = \log C + R \cdot \log\,(E' \cdot \varepsilon) \tag{8}$$

Using a least-squares curve fitting, the R-O parameter R is directly determined from the slope, and the parameter C is calculated from the intercept. In using this approach, normalized Young's moduli larger than 0.99 are deleted from the fitting process.

The R-O parameters, C and R, were determined for each sample, and the values are given in Table 2. Correlation of R-O parameters with PI for the subgrade soils compacted at optimum moisture content was determined using linear regression, and the variation of the general normalized curves with PI is plotted in Figure 9. Using the curves in Figure 9, once the small-strain modulus ($E_{max}$) of compacted subgrade soils is obtained from field seismic methods, it is possible to predict the strain-dependent behavior of subgrades using the normalized curve at the given PI.

To determine the influence of PI on the elastic threshold strain, the R-O curve-fitting method was also used. The elastic threshold strain is defined as the point where $E/E_{max}$ is 0.98, and the associated values of $\varepsilon_t^e$ are given in Table 2. The

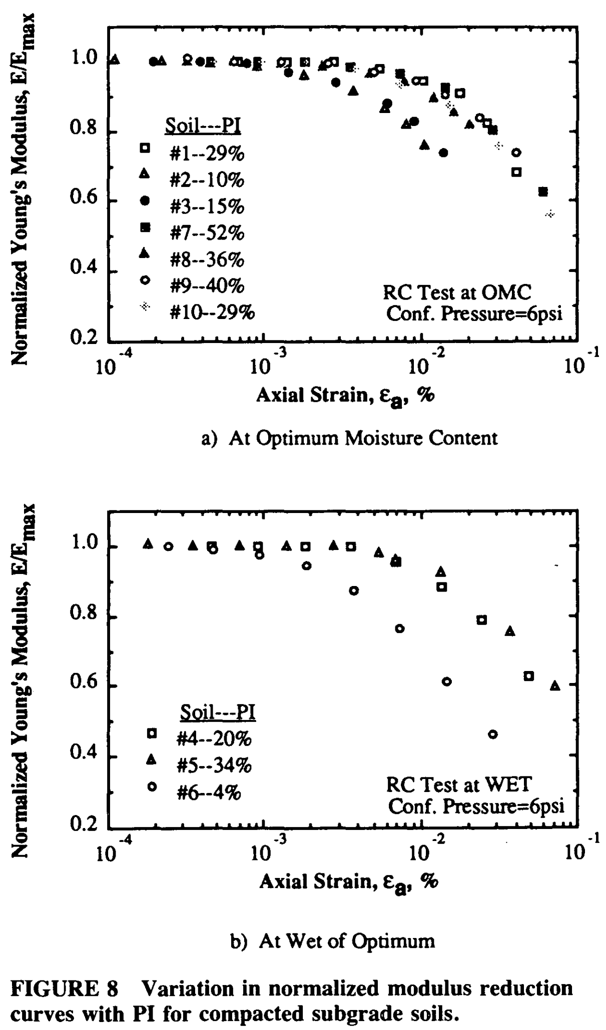



**FIGURE 8 Variation in normalized modulus reduction curves with PI for compacted subgrade soils.**

**TABLE 2 Summary of Ramberg-Osgood Parameters and Elastic Threshold Strains for Compacted Subgrade Soils#**

| Sample ID | Plasticity Index (%) | Sample Condition* | R | C | Elastic Threshold (%) |
|---|---|---|---|---|---|
| soil #1 | 29 | OMC | 2.4968 | 38530 | 0.0045 |
| soil #2 | 10 | OMC | 2.3448 | 58076 | 0.0011 |
| soil #3 | 15 | OMC | 2.4252 | 102612 | 0.0014 |
| soil #4 | 20 | WET | 2.6321 | 174100 | 0.0040 |
| soil #5 | 34 | WET | 2.5715 | 71203 | 0.0048 |
| soil #6 | 4 | WET | 2.3883 | 155131 | 0.0008 |
| soil #7 | 52 | OMC | 2.5627 | 65811 | 0.0048 |
| soil #8 | 36 | OMC | 2.3626 | 17611 | 0.0031 |
| soil #9 | 40 | OMC | 2.4771 | 34324 | 0.0043 |
| soil #10 | 29 | OMC | 2.4820 | 49534 | 0.0035 |

# All of the tests were performed at confining pressure of 6 psi.
* OMC = Compacted Optimum Moisture Content
Wet = Compacted Wet of Optimum

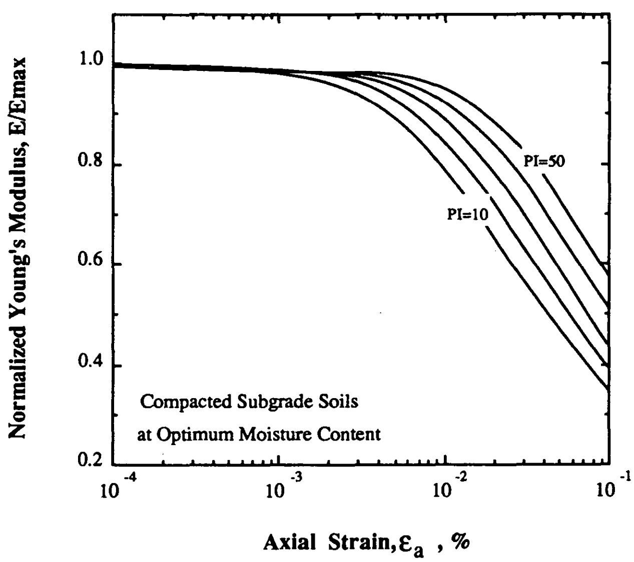



**FIGURE 9 Variation in normalized Young's modulus with PI for compacted subgrade soils at optimum moisture content.**

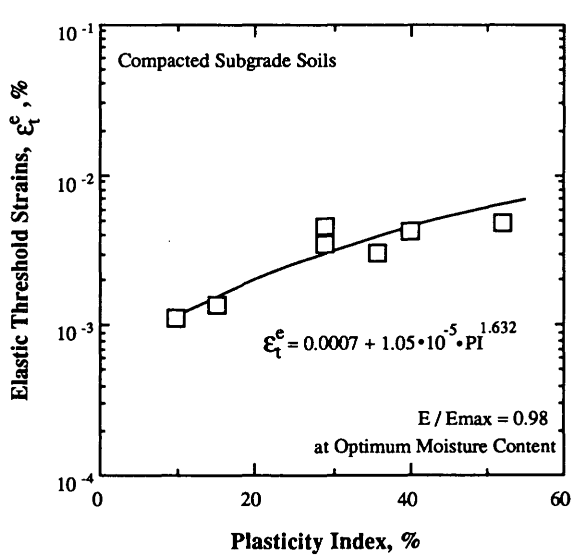



**FIGURE 10 Variation in elastic threshold strain with PI of compacted subgrade soils at optimum moisture content.**

variation in elastic threshold strain of compacted subgrade soils at the optimum moisture contents with PI is shown in Figure 10. The elastic threshold increases with increasing PI. The elastic threshold strain ranges between 0.0008 and 0.0048 percent when PI varies from 4 to 52 percent.

## CONCLUSIONS

Resilient modulus of compacted subgrade soils was investigated using RC and TS tests. The following conclusions can be drawn from this study:

1. An elastic threshold strain can be defined below which resilient moduli of compacted subgrade soils are independent of strain amplitude. Above the elastic threshold strain, moduli decrease as strain amplitude increases. The elastic threshold strain increases with increasing PI of the soil. Elastic threshold strains for the cohesive soils ranged from 0.0008 to 0.0048 percent as PI varied from 4 to 52 percent.
2. The resilient modulus of compacted subgrade soils increases linearly as a function of the logrithm of loading frequency. The effect of frequency on resilient modulus increases as PI increases. The effect of frequency ranged from 4.5 to 8.4 percent per log cycle of loading frequency as PI varied from 4 to 52 percent.
3. Resilient modulus can be measured at small (below 0.001 percent) to intermediate (below 0.1 percent) strains using RC and TS tests, whereas $M_R$ testing generally produces reliable measurements at strains above about 0.01 percent. Moduli obtained from RC, TS, and $M_R$ tests agreed well at strains above about 0.01 percent, provided the effect of frequency on stiffness was considered in the comparison. Therefore, RC and TS tests can also be used to determine resilient modulus over a wide range of strain amplitudes.
4. Normalized modulus ($E/E_{max}$) versus strain curves move to increasing strains as the PI of cohesive soils increases. Correlation curves between normalized modulus and PI for cohesive soils compacted at optimum moisture contents are presented. Once the small-strain modulus ($E_{max}$) of a given soil is obtained from field seismic measurements, it is possible to predict the strain-dependent behavior of the subgrade soil using the normalized curve at the given PI as is often done in geotechnical earthquake engineering.

## ACKNOWLEDGMENTS

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## REFERENCES

1. *Resilient Modulus of Subgrade Soils.* AASHTO T274-82. AASHTO, 1986, pp. 1157–1177.
2. B. O. Hardin and V. P. Drnevich. Shear Modulus and Damping in Soils: Measurement and Parameter Effects. *Journal of Soil Mechanics and Foundation Division*, ASCE, Vol. 98, No. SM6, June 1972, pp. 603–624.

3. D.-W. Chang, J. M. Roesset, and K. H. Stokoe II. Nonlinear Effects in Falling Weight Deflectometer Tests. Presented at 71st Annual Meeting of the Transportation Research Board, Washington, D.C., 1992.
4. R. F. Pezo. *Development of Reliable Resilient Modulus Test for Subgrade and Subbase Materials for Use in Routine Pavement Design.* Ph.D. Dissertation. The University of Texas at Austin, Austin, 1991.
5. R. F. Pezo, D. S. Kim, K. H. Stokoe II, and W. R. Hudson. Developing a Reliable Resilient Modulus Testing System. Presented at the 70th Annual Meeting of the Transportation Research Board, Washington, D.C., 1991.
6. Standard Test Methods for Modulus and Damping of Soils by the Resonant-Column Method. *1987 Annual Book of Standards,* Vol. 04.08 Soil and Rock; Building Stones. American Society for Testing and Materials, ASTM D4015-87, pp. 507-525.
7. A. T. F. Chen and K. H. Stokoe II. *Interpretation of Strain Dependent Modulus and Damping from Torsional Shear Tests.* Report USGS-GD-769-002. United States Geological Survey, 1979, 46 pp.
8. D. S. Kim. *Deformational Characteristics of Soils at Small to Intermediate Strain from Cyclic Tests.* Ph.D. Dissertation. The University of Texas at Austin, Austin, 1991, 340 pp.
9. M. L. Silver and T. K. Park. Testing Procedure Effects on Dynamic Soil Behavior. *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 101, No. GT10, Oct. 1975, pp. 1061–1083.
10. *Manual of Testing Procedures,* Texas Highway Department, Vol. 1. Jan. 1983.
11. *Resilient Modulus of Unbound Granular Base, Subbase Materials and Subgrade Soils.* Strategic Highway Research Program Protocol P-46, UG07, SS07, 1989.
12 M. Vucetic and R. Dobry. Effect of Soil Plasticity on Cyclic Response. *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 117, No. 1, Jan. 1991, pp. 89–107.
13. T. Kokusho, Y. Yoshida, and Y. Esashi. Dynamic Properties of Soft Clay for Wide Strain Range. *Soils and Foundations,* JSSMFE, Vol. 22, No. 4, Dec. 1982, pp. 1–18.


# DISCUSSION


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The authors are commended for conducting and reporting the results of acceptable and proven laboratory seismic testing techniques of the resonant column and torsional shear tests and comparing these with the resilient modulus ($M_r$) results of the repeated load triaxial using the SHRP-LTPP protocol. The paper provides new insight for laboratory characterization of resilient properties of cohesive subgrade soils and indicates the need for high-resolution instrumentation capable of generating strain-softening $M_r$ relationships similar to those shown in Figures 7 and 8 for the resonant column and torsional shear tests.

The following observations and discussion are offered with the view of encouraging the use of the developments in earthquake engineering where the in situ moduli determined from field seismic tests are corrected for strain-softening behavior, a highly recommended concept for calculating nonlinear moduli of soils (*1–4*).

1. $M_r$ results from all three types of tests agree fairly well at and above 0.01 percent strain levels, as shown in Figures 7 and 8 of the paper.

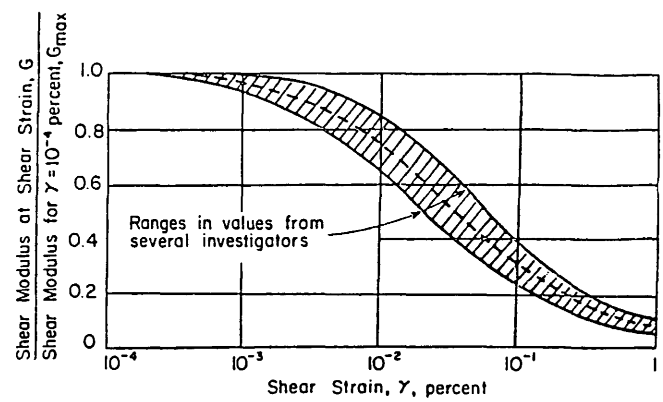


**FIGURE 11 Typical reduction of normalized shear modulus with shearing strain amplitude presented by Seed and Idriss (*2*).**

2. There is a critical threshold strain level below which the normalized modulus is independent of strain amplitude and corresponds to the maximum modulus value.

3. The authors agree with the strain softening behavior (Figure 11) used for dynamic response analysis in earthquake engineering (*2*).

4. The low amplitude strain level test results presented by the authors show that below 0.01 percent strain amplitude the $M_r$ results from the triaxial testing equipment (customarily used by the pavement community) could not be obtained because of the inadequacy of the instrumentation.

5. Low amplitude strain levels are also implied during nondestructive testing (NDT) of pavements using the Dynaflect, falling weight deflectometer (FWD), Road Rater, and spectral-analysis-of-surface-wave (SASW) method (*3–5*).

6. It is expected that the moduli calculated from the light-load Dynaflect and SASW seismic testing in the field are maximum moduli because these correspond to the elastic strain level below 0.005 percent strain amplitude or below the critical strain level, whereas the same may not be true for relatively heavy NDT devices or for truck loading where the strain amplitude is expected to be in the range of 0.001 to 1 percent.

7. Therefore, the strain-softening models, as shown in Figure 11 and presented by the authors in Figure 8, can be used to correct the NDT in situ moduli for nonlinear behavior. This behavior is exhibited by cohesive soils as well as granular material.

I first recognized the importance of the strain-softening behavior of unbound granular pavement layers and roadbed soils in early 1980s and developed an equivalent linear analysis procedure to correct the effective in situ Young's moduli of these layers backcalculated from the Dynaflect data. The equivalent linear analysis procedure used the normalized modulus versus strain amplitude relationships shown in Figure 12 and typical relationships of elastic parameters similar to Equations 3 and 4. The equivalent linear analysis is equally applicable to other NDT deflection equipment and the SASW method. My equivalent linear analysis procedure is incorporated in the FPEDD1 and RPEDD1 backcalculation programs (*3*).

On the basis of the excellent dynamic laboratory tests data presented by the authors and this discussion, there seems to be a strong need to extend this initiative in the following areas:

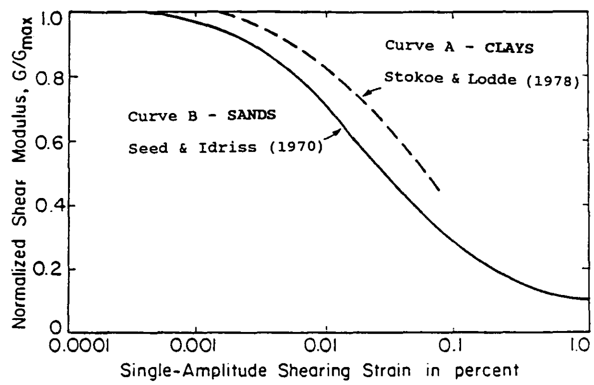


**FIGURE 12 Strain-softening models used in the FPEDD1 and RPEDD1 programs to calculate nonlinear moduli of unbound granular pavement layers and roadbed soils (*3*).**

1. Improve the $M_r$ triaxial test equipment to make accurate and precise measurement of resilient moduli in the low strain amplitude range of 0.001 percent and below.
2. Conduct a series of tests on different soil types using the resonant column techniques and SHRP-LTPP $M_r$ protocol at varying confining pressures and develop the strain-softening relationships that can be used to derive typical curves for both unbound granular base/subbase materials and roadbed soils.
3. Conduct the preceding test program for undisturbed and disturbed samples extracted from a few SHRP-LTPP test sections located in different environmental regions of the United States. This will provide strain-softening models for use in calculating nonlinear FWD moduli.

## REFERENCES

1. B. O. Hardin and V. P. Drnevich. Shear Modulus and Damping in Soils: Design Equations and Curves. *Journal of SMF Div.*, ASCE, Vol. 98, No. SM7, July 1972, pp. 667–692.
2. H. B. Seed and I. M. Idriss. *Soil Moduli and Damping Factors for Dynamic Response Analysis.* Report EERC 70-10. Earthquake Engineering Research Center, University of California, Berkeley, 1970.
3. W. Uddin, A. H. Meyer, W. R. Hudson, and K. H. Stokoe II. Project-Level Structural Evaluation Based on Dynamic Deflections. In *Transportation Research Record 1007*, TRB, National Research Council, Washington, D.C., 1985, pp. 37–45.
4. W. Uddin. Recent Advances in Nondestructive Dynamic Testing and Evaluation Methods. *Proc., 8th Asian Regional Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, Kyoto, Japan, 1987.
5. S. Nazarian and K. H. Stokoe II. Use of Surface Waves in Pavement Evaluation. In *Transportation Research Record 1070*, TRB, National Research Council, Washington, D.C., 1986, pp. 132–144.

# AUTHORS' CLOSURE

We thank the discussant for his fine discussion of our paper. We agree with his comments and observations about small-strain moduli and the relationship between small-strain moduli (at strains less than about 0.001 percent) and nonlinear behavior exhibited by soils at larger strains. As the profession comes to understand the importance of small-strain moduli that can be measured in the field and in the laboratory, improvements will occur in the design, analysis, and testing of pavement systems.

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