Structural Evaluation of Base Layers in Concrete Pavement Systems

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A theoretically sound and practical approach is described for determining maximum responses in concrete pavement systems that incorporate a base layer. Equations are presented that may be used with either an elastic solid or a dense liquid foundation under any of the three fundamental loading conditions. These formulas are extensions of available closed-form solutions and account for the compressions in the two placed layers that are ignored by plate theory. The proposed methodology may be easily implemented in a personal computer spreadsheet or on a programmable calculator. Research activities for its full verification and refinement are continuing at this time. It is anticipated that such theoretically based investigations will encourage the elimination of theoretically questionable empirical concepts, such as that of deriving a composite "top-of-the-base" subgrade modulus.

Conventional analysis and mechanistic-based design procedures for portland cement concrete (PCC) pavement systems use closed-form analytical solutions that have been developed over the last 75 years on the basis of quite restrictive assumptions. The idealizations that led to the formulation of the well-known Westergaard equations (*1*) for a slab on a dense liquid foundation and of the less often quoted expressions by Losberg (*2*) and Ioannides (*3*) for the corresponding slab on an elastic solid subgrade treat a pavement system that

1. Considers a slab of infinite dimensions (no slab size effects),
2. Consists of only one slab panel (no load transfer),
3. Includes only one placed layer (no base or subbase),
4. Employs a semiinfinite foundation (no rigid bottom),
5. Is acted upon by a single tire print (no multiple wheel loads), and
6. Experiences no curling or warping (i.e., flat slab, no temperature or moisture differential condition).

Each of these restrictions is violated in actual concrete pavement construction, which dictates that analytical considerations be adjusted or "calibrated" before a reasonable engineering design can be made. In particular, the treatment of the concrete pavement as incorporating only one placed layer, namely, the PCC slab, has been a pervasive obstacle in the effort to arrive at a mechanistic design that would permit comparisons with alternative designs involving asphalt concrete. The inability of conventional plate theory solutions to accommodate multiple placed layers is often cited as one of the primary reasons calling for its abandonment in favor of a unified analysis and design procedure based on layered elastic theory (*4*).

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Use of layered elastic theory (*5*) in addressing the single-placed-layer (SPL) limitation of conventional plate theory solutions is not new. In fact, it is the oldest of at least three main approaches to the problem posed by bases underneath concrete pavement slabs. Even before the development of computer codes allowing the analysis of multilayered axisymmetric pavement systems, layered elastic theory was suggested by Odemark (*6*)—in the form of his celebrated method of equivalent thicknesses—as a theoretically sound methodology for extending (not replacing) plate theory applications. The reason for not calling for the outright elimination of plate theory as an analytical tool for concrete pavement systems was a recognition by early investigators of the reciprocal inability of layered elastic theory to consider the all-important phenomena pertaining to the edges and corners of concrete slabs.

With the advent of sophisticated finite-element codes, a second approach to the SPL problem emerged exemplified in the treatment of a concrete pavement system as a two-layered composite plate resting on an elastic foundation. This approach was implemented in computer programs such as ILLI-SLAB (*7*). Although treating both placed layers (slab and base) as plates does address the noted SPL shortcoming—particularly in the case of cement-treated (stiff) bases—predictions on the basis of plate theory are often found to incorporate a significant error. This error arises from the neglected compression experienced by the two layers (especially when softer, granular bases are employed).

The third and most predominant means for accounting for the presence of a base, however, has been by increasing the value of the subgrade modulus, k. Thus, in contrast to the finite-element formulation that considers the base as a structural element reinforcing the upper placed layer, namely, the PCC slab, the more conventional approach has been to regard the base as contributing exclusively to the stiffness of the subgrade. It may be argued that the philosophical basis for this approach is to be found in the work by Odemark (*6*), who suggested increasing the subgrade modulus of elasticity, E_s, to account for the contribution of the base. It appears, however, that the popularity of this approach is due more to the practical expediency and ease of solution it offers than to its theoretical merits. A literature survey conducted at the outset of this investigation identified at least 12 different ways of "bumping the k-value," or defining a composite or "top-of-the-base" subgrade modulus. According to a review of current methods for determining the composite modulus of subgrade reaction conducted by Uzan and Witczak (*8*), "the equivalent k_{comp}-values for granular bases [obtained by different methods] are essentially the same," but for "stabilized materials. . . the k-values can vary within a factor of two."

Of the three approaches to dealing with the SPL assumption outlined above, the process of increasing the value of the subgrade modulus depending on the type and thickness of the base is the least attractive from a theoretical viewpoint. Its origins may be traced to tests conducted in the 1950s by the Portland Cement Association (PCA) and by the Corps of Engineers (*9*). At that time, the "bump the k-value" approach appeared as a minor extrapolation of Methods 2 and 3 described by Teller and Sutherland (*10*) for the determination of the subgrade modulus. It should be remembered, however, that both of these methods (namely, the volumetric approach and the backcalculation approach) aimed at defining a property of the natural subgrade, just as did the plate load test (Teller and Sutherland's Method 1), rather than the property of an "equivalent" supporting medium. It is precisely the development of computerized backcalculation procedures based on matching theoretical and observed deflection basins (by determining the area or volume of the basin) that has revealed the extent of errors that may be committed through the use of top-of-the-base k-values. Such composite values are quite often much higher than those reported in earlier literature (*11*), so much so that the definition of the medium they purport to describe as a dense liquid is brought into question.

This paper offers a theoretically sound yet practical solution to the problem posed by the SPL assumption. Simple equations are presented that may easily be implemented on a personal computer or hand-held calculator and that may be used to calculate with sufficient accuracy maximum responses in concrete pavement systems incorporating a base layer. The formulas presented have been obtained through application of dimensional analysis concepts in interpreting a data base of numerical results from two computer codes, one based on plate theory (ILLI-SLAB) and one using layered elastic analysis [BISAR (*12*)]. It is hoped that such solutions, which are essentially extensions of well-known analytical equations, will eliminate the need to use empirical and theoretically questionable concepts, such as that of the composite top-of-the-base subgrade modulus, thereby preventing any associated errors and miscalculations in the future.

SCOPE OF INVESTIGATION

The proposed analysis procedure for three-layer concrete pavements begins by considering the two placed layers as a composite plate and postulating that there exists an imaginary, homogeneous "effective" plate resting on the same elastic foundation that deforms in the same manner as the real two-layer plate. The purpose of the analysis presented below is

1. To verify the existence of the "effective" plate, that is, ascertain that it is possible to define its properties in terms of the corresponding properties of the two layers in the original composite plate;
2. To obtain elastic solutions for the response of the "effective" plate using available analytical equations;
3. To relate the "effective" plate responses determined in this manner to the corresponding unknown composite plate responses; and
4. To extend the applicability of the formulas developed in Item 3 to the case of a three-layer concrete pavement system of any arbitrary stiffnesses, subject only to the assumption that one of the two placed layers is much stiffer than the foundation.

CLOSED-FORM SOLUTION FOR THREE-LAYER SYSTEM WITH UNBONDED LAYERS

Plate Theory Solution

According to medium-thick plate theory (*13*), when a flat plate of uniform cross section is subjected to elastic bending, the following moment-curvature relationships apply, expressed in polar coordinates (r,ϕ):

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}\right)\right]$$

$$M_\phi = -D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2} + \mu\frac{\partial^2 w}{\partial r^2}\right)$$

$$M_{r\phi} = (1-\mu)D\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \phi} - \frac{1}{r^2}\frac{\partial w}{\partial \phi}\right) \qquad (1)$$

in which $w(r,\phi)$ denotes the vertical displacement from the originally horizontal neutral axis of the plate. The flexural stiffness of the plate, D, is defined by

$$D = \frac{Eh^3}{12(1-\mu^2)} \qquad (2)$$

where E, μ, and h are the Young's modulus, Poisson's ratio, and thickness of the plate, respectively.

Equations 1 may be rewritten in a compact form as

$$\{M\} = -D\{L(\mu)\}\,[w(r,\phi)] \qquad (3)$$

in which $\{L(\mu)\}$ is a vector operator depending on the value of μ.

Consider now a composite plate consisting of two dissimilar plate layers resting on an elastic foundation, for example, a dense liquid or an elastic solid. Assuming that during bending the two plate layers do not experience any separation, their respective deflected shapes will be identical; that is,

$$w_1(r,\phi) = w_2(r,\phi) = w_e(r,\phi) \qquad (4)$$

Subscripts 1 and 2 denote here plate layers 1 and 2, respectively, in the original composite two-layer plate, and subscript e denotes an imaginary, homogeneous "effective" plate resting on the same elastic foundation. The "effective" plate is required to deform in the same manner as the real composite plate.

At this point, the assumption that the two plate layers in the composite plate act independently, that is, that their interface is unbonded and free of shear stress, is introduced.

Application of Equations 3 and 4 to each of these plate layers yields the following moment expressions:

$$\{M_1\} = -D_1\{L(\mu_1)\}[w_e(r, \phi)] \tag{5}$$

$$\{M_2\} = -D_2\{L(\mu_2)\}[w_e(r, \phi)] \tag{6}$$

The corresponding equation for the moment acting on the "effective" plate is

$$\{M_e\} = -D_e\{L(\mu_e)\}[w_e(r, \phi)] \tag{7}$$

The assumption is also introduced that

$$\mu_1 = \mu_2 = \mu_e \tag{8}$$

and it is noted that the composite as well as the "effective" plates are subjected to the same applied loads, experience the same deflections, and therefore are acted upon by the same foundation reactions. Thus, it is evident that

$$\{M_T\} = \{M_1\} + \{M_2\} = \{M_e\} \tag{9}$$

where $\{M_T\}$ denotes the total moment acting on the composite plate. Equation 9 yields upon substitution from Equations 5 and 6:

$$\{M_e\} = -(D_1 + D_2\}\{L(\mu_e)\}[w_e(r, \phi)] \tag{10}$$

Comparison of Equation 10 with Equation 7 results in

$$D_e = D_1 + D_2 \tag{11}$$

Equation 11 verifies that the "effective" plate postulated by Equation 4 exists and that its structural parameters can be defined in terms of the corresponding parameters of layers 1 and 2 in the original composite plate. Furthermore, it follows from Equations 7 and 11 that

$$\{M_e\} = \left(1 + \frac{D_2}{D_1}\right)\{M_1\} \tag{12}$$

Thus, the generalized stresses, $\{\sigma_e\}$, in the "effective" plate are written in terms of the corresponding stresses, $\{\sigma_1\}$, in plate layer 1 as

$$\{\sigma_e\} = \frac{6}{h_e^2}\left(1 + \frac{D_2}{D_1}\right)\{M_1\} \tag{13}$$

or

$$\{\sigma_e\} = \frac{h_1^2}{h_e^2}\left(1 + \frac{D_2}{D_1}\right)\{\sigma_1\} \tag{14}$$

$$= \frac{h_1^2}{h_e^2}\left(\frac{E_1h_1^3 + E_2h_2^3}{E_1h_1^3}\right)\{\sigma_1\} \tag{15}$$

It follows from Equations 8 and 11 that

$$E_eh_e^3 = E_1h_1^3 + E_2h_2^3 \tag{16}$$

Substituting Equation 16 into Equation 15 leads to

$$\{\sigma_e\} = \frac{h_1^2}{h_e^2}\frac{E_eh_e^3}{E_1h_1^3}\{\sigma_1\} \tag{17}$$

whence

$$\{\sigma_e\} = \frac{h_e}{h_1}\frac{E_e}{E_1}\{\sigma_1\} \tag{18}$$

Setting $E_e = E_1$, this yields

$$\{\sigma_e\} = \frac{h_e}{h_1}\{\sigma_1\} \tag{19}$$

$$\{\sigma_1\} = \frac{h_1}{h_e}\{\sigma_e\} \tag{20}$$

Furthermore, the thickness of the "effective" plate, h_e, is obtained from Equation 16 as

$$h_e = \left[\left(h_1^3 + \frac{E_2}{E_1}h_2^3\right)\right]^{1/3} \tag{21}$$

Note that Equation 20 implies, in particular, that the maximum bending stress, σ_1, developing at the bottom of plate layer 1 in the composite plate may be obtained by multiplying the corresponding maximum bending stress, σ_e, arising at the bottom of the imaginary, homogeneous "effective" plate (of modulus $E_e = E_1$) by the thickness ratio (h_1/h_e), with h_e defined by Equation 21. For example, considering the case of an elastic solid foundation, Equation 20 implies that

$$\sigma_1 = \frac{h_1}{h_e}\sigma_e = \frac{h_1}{h_e}\sigma(h_e, E_1, E_s) \tag{22}$$

The corresponding expression for a dense liquid foundation is

$$\sigma_1 = \frac{h_1}{h_e}\sigma_e = \frac{h_1}{h_e}\sigma(h_e, E_1, k) \tag{22a}$$

The notation $\sigma(h_i, E_j, F)$ in Equations 22 and 22a denotes the maximum bending stress predicted by plate theory at the bottom of a plate of thickness h_i and modulus E_j resting on a subgrade characterized by generalized stiffness parameter F, that is, Young's modulus, E_s, for an elastic solid foundation or modulus of subgrade reaction, k, for a dense liquid foundation. Furthermore, Equation 20 implies that the thickness ratio (h_1/h_e) is the constant that relates the bending stress at any point (r, ϕ) in layer 1 of the composite plate to the bending stress arising at the corresponding point in the "effective plate." Having thus obtained σ_1, the maximum bending stress at the bottom of plate layer 2 may also be calculated using plate theory as follows, subject to the assumption of Equation 8:

$$\sigma_2 = \sigma_1\frac{E_2}{E_1}\frac{h_2}{h_1} \tag{23}$$

The value of $\sigma_e = \sigma(h_e, E_1, F)$ in Equations 22 and 22a may be obtained using available closed-form analytical solutions pertaining to the particular foundation type and loading condition of interest. An equation for the maximum bending stress arising at the bottom of a homogeneous infinite plate on an elastic solid foundation loaded by an interior load has been presented by Losberg (*2*). More recently, Ioannides (*3*) considered the edge and corner loading conditions for the same plate-foundation system and provided simple formulas for the calculation of the maximum bending stress pertaining to these loading conditions as well. The corresponding equations for a plate on dense liquid foundation were given by Westergaard (*1*) for all three fundamental loading conditions.

With respect to deflections, it is noted that Equation 4 implies that the maximum deflection in the two-layer composite plate is equal to the maximum deflection experienced by the "effective" plate. The latter may be calculated using the pertinent formulas given in the publications cited above. Similarly, the maximum subgrade stress under the composite plate is equal to the corresponding stress under the "effective" plate.

Elimination of Plate Theory Restrictions

To extend the applicability of the proposed approach to layers of any arbitrary stiffness—subject only to the assumption that one of the two layers is much stiffer than the foundation—responses calculated must be adjusted for the compression that occurs within the two layers of the original composite plate and that is ignored by plate theory. To illustrate how such a corrective may be applied, the case of the maximum bending stress, σ_{1L}, occurring at the bottom of layer 1 in a three-layer system of any arbitrary stiffnesses will be considered. This response may be written in the following form:

$$\sigma_{1L} = \sigma_1 + [\vartheta\ \Delta\sigma] \qquad (24)$$

where σ_1 is the corresponding stress according to plate theory given by Equation 22 and $[\vartheta\ \Delta\sigma]$ is a "correction increment." The contribution to this increment of the compression of the second layer is usually of overriding importance. In a typical pavement system, the second layer has a lower modulus than the first layer and may therefore be expected to diverge from plate behavior (no compression) more significantly than the first layer. For this reason, an expression for $\Delta\sigma$ accounting only for the compression in the second layer is derived first (i.e., $\vartheta = 1$). Considering the case of an elastic solid foundation, the following assumption is introduced at this point:

$$\frac{\partial(\Delta\sigma)}{\partial(E_s)} = 0 \qquad (25)$$

That is, it is assumed that $\Delta\sigma$ (as well as the compression of the second layer) is largely insensitive to changes in the subgrade modulus, E_s. This assumption is a reasonable approximation for material moduli in the range of those typically encountered in concrete pavements, for which E_1 is much higher than E_s. If the assumption of Equation 25 is accepted, the case may be considered in which $E_s = E_2$, which reduces the three-layer system to a two-layer system. Then, for this case,

$$\Delta\sigma(h_1, h_2, E_1, E_2, E_s) = \Delta\sigma(h_1, h_2, E_1, E_2, E_2) \qquad (26)$$

in which the parameters listed in parentheses define the properties of the layered system considered in determining $\Delta\sigma$. By referring to Equations 24 and 22, the following expression may be written:

$$\Delta\sigma(h_1, h_2, E_1, E_2, E_2) = \sigma_{1L}(h_1, h_2, E_1, E_2, E_2) - \frac{h_1}{h_e}\sigma(h_e, E_1, E_2) \qquad (27)$$

If $E_1 >> E_2$, σ_{1L} may be evaluated according to plate theory, or

$$\sigma_{1L}(h_1, h_2, E_1, E_2, E_2) = \sigma(h_1, E_1, E_2) \qquad (28)$$

It is noted that in writing this equation, the effect of an unbonded surface at depth h_2 into the elastic half-space of modulus E_2 is assumed to be negligible. Thus,

$$\Delta\sigma(h_1, h_2, E_1, E_2, E_2) = \sigma(h_1, E_1, E_2) - \frac{h_1}{h_e}\sigma(h_e, E_1, E_2) \qquad (29)$$

That is, the correction increment $\Delta\sigma$ may be calculated as the difference between two stresses, each of which is evaluated using available closed-form solutions, such as those by Losberg (*2*) or Ioannides (*3*), for the parameters indicated by Equation 29.

The value of $\Delta\sigma$ obtained as explained above accounts only for the compression of the second layer; that is, it applies when $E_2 << E_1$. This would be the case, for example, of a PCC slab placed on a soft base. As E_2 tends to E_1, Equation 28 becomes increasingly inaccurate. Noting that for such pavements plate theory would apply without the need for corrections (since in this case both E_1 and E_2 are much higher than E_s), the correction increment should tend to zero. In addition, when $E_2 > E_1$, the correction increment must be negative, reflecting the effect of the compression in the first layer. This corresponds to the case, for example, of an asphalt concrete overlay on a PCC slab. For these reasons, therefore, the value of $\Delta\sigma$ obtained above is multiplied by a factor ϑ. Considering the interior loading condition, the following formula was developed for ϑ on the basis of comparisons of the proposed closed-form solution with the results of several three-layer runs of the BISAR computer program:

$$\vartheta = 1 - \exp\left[\frac{1}{3}\left(1 - \frac{E_1}{E_2}\right)\right] \qquad (30)$$

Thus, substituting σ_1 from Equation 22 and $\Delta\sigma$ from Equation 29 into Equation 24, the general solution for the maximum bending stress, σ_{1L}, arising at the bottom of the upper layer in an arbitrary three-layer system may be written as

$$\sigma_{1L} = \frac{h_1}{h_e}\sigma(h_e, E_1, E_s) + \vartheta\left[\sigma(h_1, E_1, E_2) - \frac{h_1}{h_e}\sigma(h_e, E_1, E_2)\right] \quad (31)$$

with h_e as defined by Equation 21 and ϑ as given by Equation 30. Each of the three bending stresses $\sigma(h_i, E_j, E_s)$ in Equation 31 may be calculated using Losberg's formula for the interior load-elastic foundation case, as follows:

$$\sigma = \frac{-6P(1+\mu)}{h_i^2} \times \left[-0.0490 + 0.1833\log_{10}\left(\frac{a}{l_e}\right) - 0.0120\left(\frac{a}{l_e}\right)^2\right] \quad (32)$$

where

$$l_e = \left[\frac{E_j h_i^3(1-\mu_s^2)}{6E_s(1-\mu_j^2)}\right]^{1/3} \quad (33)$$

μ_j, μ_s = Poisson's ratios for the plate and foundation, respectively,
P = total applied load, and
a = radius of applied load.

The proposed procedure for calculating σ_{1L} is well suited for incorporation into a personal computer spreadsheet and may be used to assess the effect on the maximum bending stress of the introduction of a base under a PCC slab.

GRAPHICAL SOLUTION FOR THREE-LAYER SYSTEM WITH UNBONDED LAYERS

Plate Theory Solution

An alternative graphical solution was also developed in this study. Its derivation proceeds from Equation 22, which may be rewritten as

$$\sigma_1 = 6\frac{M_e}{\eta_e^2} \quad (34)$$

where

$$\eta_e^2 = \frac{h_e^3}{h_1} = h_1^2 + h_2^2\left(\frac{E_2h_2}{E_1h_1}\right) \quad (35)$$

Noting that as η_e^2 tends to h_1^2, M_e tends to M_1 and σ_1 tends to $\sigma(h_1, E_1, F_1)$—the latter being the plate theory prediction for the maximum bending stress in the PCC slab resting directly on the subgrade—it may be expected that the stress ratio $[\sigma_1/\sigma(h_1, E_1, F)]$ diverges from unity as the ratio (η_e^2/h_1^2) decreases. This assertion has recently been verified by Salsilli (*14*), who considered the results of plate theory for a small factorial of unbonded, three-layer, edge-loading cases using the WINKLER option in ILLI-SLAB (*15*). He demonstrated that the relationship between the two dimensionless parameters defined above shows little sensitivity to the dimensionless load size ratio (a/l) and provided the following best-fit equation for its description:

$$\frac{\sigma_1}{\sigma(h_1, E_1, k)} = 0.0477629 + 0.265264\left(\frac{a}{l}\right) + 0.953195\left(\frac{\eta_e}{h_1}\right)^{-2} - 0.26083\left(\frac{a}{l}\right)\left(\frac{\eta_e}{h_1}\right)^{-2} \quad (36)$$

In this expression l denotes the radius of relative stiffness of the slab-dense liquid system (h_1, E_1, k), which is defined by

$$l = \left(\frac{E_1h_1^3}{12(1-\mu_1^2)k}\right)^{1/4} \quad (37)$$

The value predicted by the Westergaard edge-loading equation (*1*) may be substituted for $\sigma(h_1, E_1, k)$ in routine applications of Equation 36.

Elimination of Plate Theory Restrictions

A drastically different picture is obtained when the compressions in the two placed layers are accounted for. Interior loading results from computer program BISAR were used to establish the relationship between dimensionless ratios $[\sigma_{1L}/\sigma(h_1, E_1, E_s)]$ and (η_e^2/h_1^2) without plate theory restrictions. Interpretation of these numerical results on the basis of the principles of dimensional analysis showed that for a wide range of practical applied load radius values, the relationship between the two ratios could be defined uniquely for each value of (E_1/E_2). Thus, Figure 1 was prepared. This can

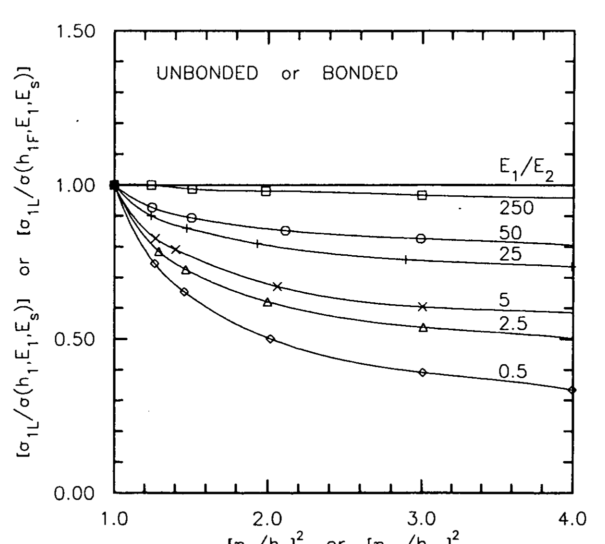

FIGURE 1 Reduction factor for determining maximum bending stress in a three-layer concrete pavement system (elastic solid foundation).

be used to obtain a reduction factor that when multiplied by the available analytical slab-on-grade solution for $\sigma(h_1, E_1, E_s)$ given by Losberg (*2*) provides an estimate for the maximum interior loading bending stress at the bottom of the top layer in a three-layer system.

SOLUTIONS FOR THREE-LAYER SYSTEM WITH BONDED LAYERS

Analytical Solution

The closed-form solution derived above for unbonded layers may be applied, with relatively few modifications, to the case of bonded layers as well. The most significant change is in the definition of the "effective" thickness, h_e (cf. Equation 21). Recall that for unbonded layers, h_e was defined using the condition of equality between flexural stiffnesses of the original composite two-layer plate and of the imaginary, homogeneous "effective" plate. Equation 16, however, applies only to unbonded layers. In the case of bonded layers, the flexural stiffness of the original composite plate may be determined using the parallel axes theorem. This results in the following alternative condition to Equation 16:

$$\frac{E_e h_e^3}{12} = \frac{E_1 h_1^3}{12} + E_1 h_1 \left(x - \frac{h_1}{2}\right)^2 + \frac{E_2 h_2}{12} + E_2 h_2 \left(h_1 - x + \frac{h_2}{2}\right)^2 \tag{39}$$

Equation 39 assumes that the neural axis of the composite system lies within layer 1 at a distance x from the top of layer 1, but the same expression is obtained if the neutral axis is assumed to lie within layer 2 (x is still measured from the top of layer 1). As done for the unbonded layers, it is assumed here that $E_e = E_1$ and that $\mu_e = \mu_1 = \mu_2$, which leads to the following expression for the thickness of the "effective" plate for the case of bonded layers:

$$h_e = \left\{h_1^3 + \frac{E_2}{E_1} h_2^3 + 12\left[\left(x - \frac{h_1}{2}\right)^2 h_1 + \frac{E_2}{E_1}\left(h_1 - x + \frac{h_2}{2}\right)^2 h_2\right]\right\}^{1/3} \tag{40}$$

The depth to the neutral axis, x, is determined by considering the first moment of area of the original composite plate, as follows:

$$x = \frac{E_1 h_1 \frac{h_1}{2} + E_2 h_2 \left(h_1 + \frac{h_2}{2}\right)}{E_1 h_1 + E_2 h_2} \tag{41}$$

Noting that the derivation of Equations 39 through 41 follows the same reasoning as that used by Tabatabaie et al. (*7*), Equation 40 may be rewritten as

$$h_e = \left(h_{1F}{}^3 + \frac{E_2}{E_1} h_{2F}{}^3\right)^{1/3} \tag{42}$$

where h_{1F} and h_{2F} are defined by

$$h_{1F} = \left(h_1^3 + 12\beta^2 h_1\right)^{1/3} \tag{43}$$

$$h_{2F} = \left(h_2^3 + 12\alpha^2 h_2\right)^{1/3} \tag{44}$$

with

$$\alpha = \left(h_1 + \frac{h_2}{2} - x\right) \tag{45}$$

and

$$\beta = \left(x - \frac{h_1}{2}\right) = \left(\frac{h_1 + h_2}{2}\right) - \alpha \tag{46}$$

It is observed that Equation 42 is identical to the corresponding Equation 21 for h_e for unbonded systems, the only substitution necessary being the introduction of the "fictitious" thicknesses, h_{1F} and h_{2F}, which are somewhat higher than the original thicknesses h_1 and h_2. It is also clear that the flexural stiffness of the original composite two-layer bonded plate is equal to the stiffness of an unbonded two-layer plate in which the plate layers retain the moduli E_1 and E_2 but are assigned "fictitious" thicknesses h_{1F} and h_{2F}. The fact that $h_{1F} > h_1$ and that $h_{2F} > h_2$ counterbalances the effect of "removing" the bond between the two plate layers.

A relationship between the bending stress at the bottom of the "effective" plate, σ_e, and that acting at the bottom of layer 1 of the original composite two-layer plate, σ_1, is then sought. This is obtained with reference to the geometry of the stress distribution diagrams pertaining to the two systems and recognition of their common slope above the neutral axis. As indicated in Figure 2,

$$\frac{\sigma_e}{\sigma_1} = \frac{h_e}{2y} \tag{47}$$

but

$$y = (h_1 - x) \tag{48}$$

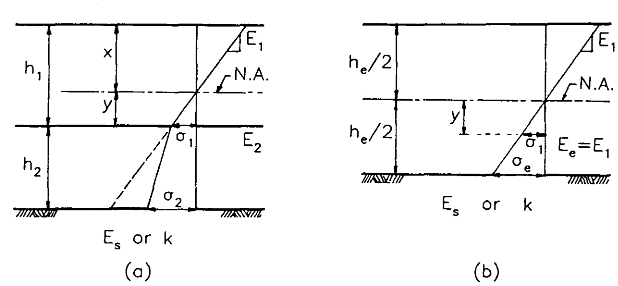

FIGURE 2 Stress distribution in bonded plate on elastic foundation system: (a) original composite two-layer plate; (b) "effective" homogeneous plate.

where

$$\sigma_1 = \sigma_e \frac{2(h_1 - x)}{h_e} \quad (49)$$

This formula is similar in form to the corresponding Equation 22 valid for unbonded systems, with the term $2(h_1 - x)/h_e$ replacing (h_1/h_e). The stress σ_e may be evaluated using the available plate theory solutions pertaining to the loading condition and foundation type of interest. Since plate theory ignores the compression in each of the two layers, σ_1 should be corrected as indicated in Equation 24. For the interior loading-elastic solid case, the necessary corrections are given by Equations 29 and 30. Note that the substitution of (h_1/h_e) by $2(h_1 - x)/h_e$ is also performed in Equation 29. Thus, the following expression is obtained for σ_{1L}, corresponding to Equation 31:

$$\sigma_{1L} = \frac{2(h_1 - x)}{h_e} \sigma(h_e, E_1, E_s) + \vartheta\left[\sigma(h_1, E_1, E_2) - \frac{2(h_1 - x)}{h_e} \sigma(h_e, E_1, E_2)\right] \quad (50)$$

with h_e as defined by Equation 42.

Graphical Solution

An alternative graphical solution is also possible. Using h_{1F} and h_{2F} instead of h_1 and h_2, the ratio $[\eta_{eF}^2/h_{1F}^2]$ may be calculated from Equation 35. Thus, Figure 1 may be used to calculate σ_{1L} in terms of $\sigma(h_{1F}, E_1, E_s)$.

VERIFICATION AND IMPLICATIONS OF PROPOSED APPROACH

The applicability of the proposed closed-form and graphical solutions for the maximum bending stress, σ_{1L}, occurring at the bottom of the top layer in a three-layer system was verified by comparing predicted values with the corresponding bending stresses from numerous analyses using computer program BISAR. It is noted that these verification runs were different from those included in the derivation of Equation 30 and of Figure 1. The predictions of the proposed procedures were also compared with the results of Odemark's method of equivalent thicknesses. These comparisons are shown in Figures 3 and 4 for the unbonded and bonded cases, respectively. It is observed that predictions are generally more reliable for the unbonded rather than the bonded cases. Furthermore, the proposed closed-form approach exhibits somewhat less scatter than the graphical approach, especially for the bonded cases. Both proposed procedures lead to improved estimates of σ_{1L} compared with Odemark's approach, verifying the wisdom of treating the base as primarily reinforcing the PCC slab rather than the subgrade. Odemark's solution leads to stresses that generally compare more favorably with BISAR stresses assuming unbonded layers.

It is noted that in writing Equation 31, no assumptions were made that would restrict it to the interior loading condition

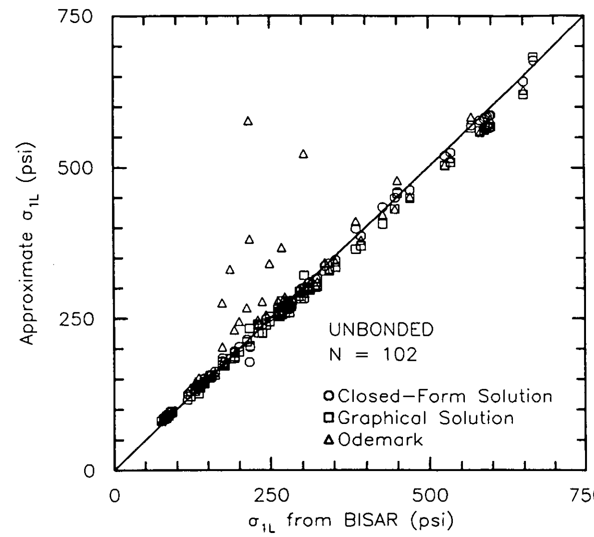

FIGURE 3 Validation of proposed procedures for σ_{1L} under interior loading (elastic solid foundation: unbonded layers).

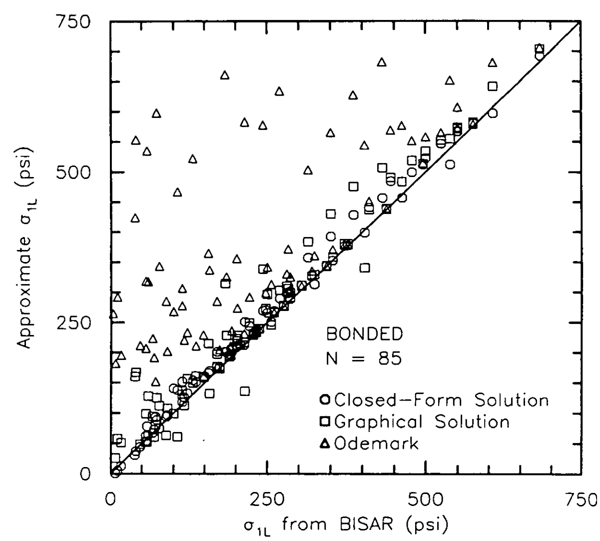

FIGURE 4 Validation of proposed procedures for σ_{1L} under interior loading (elastic solid foundation: bonded layers).

alone, with the exception of the fact that the axisymmetric program BISAR was used in the development of Equation 30 for the factor ϑ. Since, however, Equation 30 is valid for both unbonded and bonded layers, it is reasonable to assume that it is also applicable to edge and corner loading. Thus, it is suggested that Equation 31 may be used in the analysis of three-layer concrete pavement systems under these loading conditions as well. For this purpose, the plate theory expressions given by Ioannides (*3*) may be employed. Verification of this proposal would require the execution of a three-dimensional finite-element code (*16*).

Furthermore, it may be argued that it is possible to interpret the assumption of Equation 25 as also implying that the correction increment is independent of the *nature* of the foundation as well. Thus, Equation 31 may be applied to the case of a dense liquid foundation, the only necessary change being in the calculation of the plate theory stress using $\sigma(h_e, E_1, k)$ instead of $\sigma(h_e, E_1, E_s)$, where k is the modulus of subgrade reaction. The correction increment, $\Delta\sigma$, is calculated using Losberg's equation as before. This proposal dispenses with the need to define a "top-of-the-base" k-value, a procedure that often leads to erroneous conclusions (*17*). It is possible to examine the accuracy of the proposed procedure for the dense liquid-interior loading case using a general purpose two-dimensional finite-element code such as FINITE (*18*). That effort is continuing at this time. Verification of this proposal for the edge and loading conditions would require three-dimensional finite-element analysis.

Some evidence for the validity of the proposals pertaining to unbonded layers is provided by a comparison of plate theory maximum responses to finite-element results from the WINKLER option in ILLI-SLAB. For this purpose, a data base of 41 "typical" three-layer interior loading runs was assembled. It was found that Equation 22 yields the same stress as that calculated using ILLI-SLAB if $\mu_1 = \mu_2$; if $\mu_1 = \mu_2$, the predicted stress is about 5 percent lower. Furthermore, it was verified that these plate theory results can be predicted with sufficient accuracy by Equation 36, which was developed by Salsilli (*14*) for edge loading. This supports the assertion that the proposals above are applicable to all three fundamental loading conditions for both elastic solid and dense liquid foundations. Verification of the plate theory proposals for the elastic solid foundation is possible using the BOUSSINESQ option in ILLI-SLAB (*15*).

The maximum deflection calculated using ILLI-SLAB was found to be the same as that predicted by plate theory considering the "effective" plate (thickness, h_e; modulus, E_1). It is therefore suggested that the maximum deflection be taken as equal to the value computed using plate theory for any of the three fundamental loading conditions and for both elastic solid and dense liquid foundations. These plate theory predictions should be corrected for the compression of the two placed layers. An effort in this direction is also under way.

CONCLUSION

Analysis and design of concrete pavement systems have long been hampered by the restrictive assumption of available analytical solutions that the PCC slab rests directly on an elastic foundation. In reality—more often than not—concrete pavement slabs are placed on prepared bases, which are sometimes granular and sometimes bound. A number of approaches have been used in the last 40 years to overcome the "one placed layer" limitation. Most notable among these have been

1. Analyzing multilayered concrete pavement systems using Burmister's layered elastic analysis for axisymmetric conditions; such applications include Odemark's method of equivalent thicknesses and, more recently, computerized techniques as implemented, for example, in the program BISAR;
2. Analyzing three-layer concrete pavement systems using a finite-element program based exclusively on plate theory, for example, ILLI-SLAB;
3. Assigning an increased top-of-the-base subgrade modulus purporting to reflect the structural contribution of the base layer and analyzing three-layer concrete pavement systems using the available analytical or numerical procedures.

Such methodologies invariably suffer from considerable shortcomings and may in several cases lead to wrong conclusions. To remedy this situation, practical solutions to the problem posed by a three-layer concrete pavement system are presented in this paper based on sound theoretical precepts and interpretation of numerical results using dimensional analysis. The main difference of the proposed procedure from the popular "bump-the-k-value" approach is that the base layer is treated as a placed layer whose major structural contribution is to reinforce the upper placed layer—the PCC slab—rather than the natural supporting subgrade. The proposed closed-form and graphical solutions allow the calculation of maximum responses in concrete pavement systems—namely, deflection, bending stress, and subgrade stress—for all three fundamental loading conditions and for both dense liquid and elastic solid foundations. It is shown that responses obtained on the basis of plate theory alone must be corrected for the compression experienced by the two placed layers. The implications of the proposed approach with respect to current analysis and design methodologies are far reaching. Research activities for its full verification and refinement are continuing at this time. Most noteworthy among these efforts are those focusing on the development of a computerized model for a mutlilayered system supported by a dense liquid foundation (*19*).

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REFERENCES

1. H. M. Westergaard. New Formulas for Stresses in Concrete Pavements of Airfields. *Transactions*, ASCE, Vol. 113, 1948, pp. 425–439.
2. A. Losberg. *Structurally Reinforced Concrete Pavements*. Doktorsavhandlingar Vid Chalmers Tekniska Högskola, Göteborg, Sweden, 1960.
3. A. M. Ioannides. The Problem of a Slab on an Elastic Solid Foundation in the Light of the Finite Element Method. *Proceedings, 6th International Conference on Numerical Methods in Geomechanics*, Paper No. 408, April 11–15, 1988, Innsbruck, Austria, pp. 1059–1064.
4. W. R. Barker and C. R. Gonzalez. Pavement Design by Elastic Layer Theory. *Proceedings, Specialty Conference Aircraft/Pavement Interaction—An Integrated System*, (P. T. Foxworthy, ed.), ASCE/ATD/APC, Kansas City, Mo., Sept. 4–6, 1991, pp. 21–43.
5. D. M. Burmister. The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Run-

ways. *Proc.* HRB, Vol. 23, 1943, pp. 126–144; Discussion, pp. 144–148.
6. N. Odemark. *Investigations as to the Elastic Properties of Soils and Design of Pavements According to the Theory of Elasticity* (in Swedish). Meddelande 77, Statens Väginstitut, Stockholm, Sweden, 1949. [English translation, M. A. Hibbs and J. Silfwerbrand (A. M. Ioannides, ed.), 1990.]
7. A. M. Tabatabaie, E. J. Barenberg, and R. E. Smith. *Longitudinal Joint Systems in Slip-Formed Rigid Pavements, Vol. II: Analysis of Load Transfer Systems for Concrete Pavements*. Report DOT/FAA/RD-79/4, II. FAA, U.S. Department of Transportation, Nov. 1979.
8. J. Uzan and M. W. Witczak. Composite Subgrade Modulus for Airfield Pavement Design Based Upon Multilayer Theory. *Proceedings, Third International Conference on Concrete Pavement Design and Rehabilitation*, Purdue University, April 23–25, 1985, pp. 157–166.
9. E. J. Yoder. *Principles of Pavement Design*. John Wiley & Sons, Inc., New York, 1959, 569 pp.
10. L. W. Teller and E. C. Sutherland. The Structural Design of Concrete Pavements: Part 5. *Public Roads*, Vol. 23, No. 8, April, May, June, 1943, pp. 167–212.
11. A. J. Weissmann, B. F. McCullough, and W. R. Hudson. Back-Calculated Material Properties of Continuously Reinforced Concrete Pavements: A Comparison Between Two Approaches. Presented at the 69th Annual Meeting of the Transportation Research Board, Washington, D.C., 1990.
12. M. G. F. Peutz, H. P. M. Van Kempen, and A. Jones. Layered Systems Under Normal Surface Loads. In *Highway Research Record 228*, HRB, National Research Council, Washington, D.C., 1968, pp. 34–45.
13. S. P. Timoshenko and S. Woinowsky-Krieger. *Theory of Plates and Shells* 2nd ed. McGraw-Hill, New York, 1973, 580 pp.
14. R. A. Salsilli. *Development of Mechanistic Based Design Procedure for Concrete Pavements*. Ph.D. thesis. University of Illinois, Urbana, 1991.
15. A. M. Ioannides, M. R. Thompson, and E. J. Barenberg. Finite Element Analysis of Slabs-On-Grade Using a Variety of Support Models. *Proceedings, Third International Conference on Concrete Pavement Design and Rehabilitation*, Purdue University, April 23–25, 1985, pp. 309–324.
16. A. M. Ioannides and J. P. Donnelly. Three-Dimensional Analysis of Slab on Stress-Dependent Foundation. In *Transportation Research Record 1196*, TRB, National Research Council, Washington, D.C., 1988, pp. 72–84.
17. A. Ioannides. Subgrade Characterization for Portland Cement Concrete Pavement Systems. *Proceedings, International Conference on Geotechnical Engineering for Coastal Development Theory and Practice*, Port and Harbour Institute, Japan Ministry of Transport, Yokohama, Sept. 3–6, 1991.
18. L. A. Lopez. FINITE: An Approach to Structural Mechanics Systems. *International Journal for Numerical Methods in Engineering*, Vol. 11, 1977, pp. 851–866.
19. F. Van Cauwelaert. "Westergaard's Equations" for Thick Elastic Plates. *Proceedings, 2nd International Symposium on the Theoretical Design of Concrete Pavements*, Sigüenza, Spain, Oct. 4–5, 1990, pp. 165–175.