Planning of Parallel Pier Airport Terminals with Automated People Mover Systems Under Constrained Conditions

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Automated people mover (APM) systems are used in large airport terminals to reduce passenger walking and to improve terminal operation. However, there is a trade-off between passenger convenience and APM cost. If a terminal geometry is selected without the explicit consideration of both factors, it can result in needless passenger walking or increased expenditure for the APM. A method is proposed to determine an optimum geometry for a parallel pier/APM airport terminal with certain constraints. It is capable of restricting the number and the lengths of remote piers to satisfy airline and space requirements. The terminal geometry in terms of the number of piers and their sizes is obtained by minimizing the total cost of the system, which includes the disutility of walking, disutility of using the APM system (riding and access, egress, and waiting time) and the relevant capital and operating costs of the APM, subject to the constraints and the number of aircraft gates. Two case studies, the new Denver and the Atlanta Hartsfield airports, are presented to demonstrate the application of the proposed method. It is shown that the optimum terminal geometry is sensitive to the ratio of the cost of walking per unit time to the cost of riding per unit time, which can be interpreted as the relative disutility of walking. Further, the optimal geometries for the two airports are compared and contrasted with the design geometries.

The increased demand for air transport and specially the increase in hub and spoke operations has resulted in a need for large airport terminals. Some of the larger airports, such as Atlanta and Dallas–Fort Worth, have used automated people mover (APM) systems to reduce walking, especially to improve the level of service for transferring passengers. In a number of recent terminal designs, pier-type terminals with APMs have been considered when the terminal has a high percentage of transfer passengers (e.g., new airports in Denver and Seoul). The best arrangement for a pier-type terminal with an APM is to connect the terminal block to the centers of piers, located parallel to each other, by a below-grade concourse along which the APM is operated (e.g., Atlanta; see Figure 1). This arrangement is preferable because passenger walking distances between piers, and between piers and the terminal block, are essentially eliminated. The operation of the APM vehicles is along a simple linear route, in all-stop mode, at stations centered on each pier. This configuration is also preferable with respect to the aircraft taxiing distances if the terminal is located between two runways. Other advantages are the potential for expansion (number and length of piers) and the potential for easy transfer between a rail system (for airport access) and the APM.

Interest in the configurations and geometries of airport terminals has been renewed in recent years. Geometries (i.e., the arrangement of gates and piers) that minimize walking for arriving, departing, hub, and nonhub transfer passengers have been proposed by Bandara (1) and Bandara and Wirasinghe (2) for satellite and pier-finger terminals, respectively. Robusté (3) undertook a similar analysis for arriving, departing, and hub transfer passengers for centralized pier-finger (remote and attached) and certain other configurations; the remote piers were found to decrease in length with increasing distance from the terminal block. Bandara and Wirasinghe presented guidelines for choosing among satellite, pier-finger, and pier-satellite configurations for nonhub, moderate-hub, and all-hub (wayport) terminal concepts (4). Shen incorporated the effects of an APM in a terminal by setting the distance traveled using the APM equal to zero (5). McKelvey and Sproule compared different intra-airport transportation systems, for two basic unit terminals with 8 and 16 gates and their combinations, taking into account the capital, operating, and maintenance costs and related travel times and walking distances (6).

If a parallel pier–type terminal with an APM is to be considered along with other terminal configurations, it is necessary during the early planning stages of an airport to consider the best geometry for each configuration in the comparison. The high cost of APM systems and high disutility of walking makes it essential that a utility-maximizing geometry be chosen for a terminal with an APM, even if a comparison with other configurations is not being made.

Wirasinghe and Bandara have proposed a method to determine the unconstrained geometry for a parallel pier terminal with an APM (Figure 1) that minimizes the sum of the disutilities associated with passenger walking, as well as waiting for and riding the APM system, and the relevant APM capital and operating costs (7). The terminal type considered consists of uniformly spaced remote parallel piers (not necessarily of equal length) and a pier attached to the terminal block. Only the number of gates is prespecified. However, in practice, it may not be possible to implement such a geometry when the number of piers and their lengths are constrained by airline requirements and space availability.

The main objective of this paper is to determine the geometry taking into account any constraint due to airline requirements or space availability. A secondary objective is to
analyze the geometries of an existing parallel pier/APM terminal (Atlanta) and one under construction (the new airport in Denver, or New Denver).

**UNCONSTRAINED GEOMETRY**

Wirasinghe and Bandara have considered the optimum unconstrained geometry for a centralized terminal with parallel remote piers and a terminal pier, as shown in Figure 1 (7). It is assumed that the size of the terminal in terms of the planned number of gate positions, G, is known and that all gates are identical and evenly spaced. Gates are located on both sides of all the piers and along the airside of the terminal block. The width of the section of the terminal block at which aircraft are parked on the airside, S, and the spacing between gates, y, are known. Piers are arranged parallel to each other at a uniform spacing, S, and the below-grade APM system that connects the piers to the terminal block runs through the middle of each pier. The total airside frontage available for gates is

\[
2L = GS_y = 4y + b + 2 \sum_{i=1}^{n} x_i
\]  

where the \(x_i\)'s represent the lengths of the remote piers \(i = 1, \ldots, n\), and \(y\) represents the lengths of the half-piers attached to the terminal block (Figure 1).

Passengers are assumed to be uniformly distributed among gates over the life of the terminal and divided into two major groups: those arriving and departing and those transferring. The fraction of transfers with respect to the total number of passengers is defined as \(P\). Transferring passengers are divided into two groups: nonhub and hub, depending on whether they are required to visit the terminal block before departure. The fraction of hub transfers with respect to the total transfers is defined as \(Q\). Hub transfers are further divided into two groups for which a fraction, \(r\), of hub transfers is assumed to depart from a gate in their arrival pier and the remaining fraction, \(1 - r\), is assumed to have an equal probability of departing from any gate in the terminal, including the arrival pier.

It is assumed that the APM stations are identical and are located at the middle of each remote pier. APM vehicles operate at a known uniform headway. The running time between stations is known, and all the passengers, other than transfers within a pier, will use the APM system.

The objective is to determine the geometry that minimizes the total disutility associated with the terminal/APM system. The cost components related to the total disutility are divided into user costs and operator costs: user costs include the utilities associated with walking, level changes, and waiting for and riding the APM; operator costs consist of the relevant APM capital, operating, and maintenance costs. Only the relevant mandatory walking distances within the terminal, which include the walking distance between two gate positions or the walking distance between a gate and an APM station, are taken into account.

The mean disutility of walking is obtained by multiplying the mean walking distance by the perceived mean cost of walking a unit distance, \(\gamma_w\). The disutility associated with travel by APM consists of two components: disutility of riding and disutility of access, egress, and waiting for the APM system. The mean disutility of riding is obtained by multiplying the mean riding distance by the mean cost of riding a unit distance, \(\gamma_R\). The value of \(\gamma_R\) is obtained by dividing the mean cost of riding the APM system per unit time (value of time) by the mean operating speed of an APM vehicle. The passengers who use the APM system will experience the disutility associated with access, egress, and waiting only once during their trips irrespective of the riding distance. The access and egress disutilities are those related to extra walking and level changes (usually using escalators) to get to and from a station. If \(\gamma_A\) represents the perceived mean cost associated with access, egress, and waiting per passenger, the mean disutility of access, egress, and waiting per passenger is obtained by multiplying the disutility of access and waiting by the probability that a passenger will use the APM system.

The components of the capital cost—station, line, and fleet costs—are functions of the number of remote piers, \(n\). The costs of the stations at the terminal block and the costs of the piers are excluded because they are essentially common to any terminal geometry. As the operating cost (including maintenance cost) of the APM system can also be expressed as
function of the number of remote piers, the total APM cost per passenger, \( \gamma_{x} \), is expressed as a function of the number of remote piers.

The geometry of the terminal is defined by the number and lengths of remote piers and the length of the pier connected to the terminal block. The optimum geometry minimizes the total disutility of intraterminal travel. The unknowns are the number of remote piers, \( n \); the lengths of the remote piers, \( x_{i} \) for \( i = 1 \) to \( n \); and the half length of the terminal pier, \( y \).

The trade-off between the user costs and the operator costs indicates that there will be a minimum-disutility solution. If \( n \) is assumed to be given, the optimum pier lengths can be obtained by minimizing an objective function (see Appendix A, Equation 2) consisting of the user and operator costs. The optimum geometry for a given configuration can be obtained by comparing the total cost for the optimum geometries for each integer value of \( n \) between the lower and the upper bounds.

It is shown that, in general, the optimum geometry consists of a nonuniform set of piers with longer piers toward the terminal block. The optimum geometry is sensitive to the ratio of the cost of walking to the cost of riding per unit time, which can be interpreted as the relative disutility of walking.

### CONSTRAINED GEOMETRY

In practice, some major airlines may require their gates to be in a single exclusive pier or want to keep the maximum walking distance below an acceptable limit. Each can be accomplished by fixing certain pier lengths. However, it may not be possible to accomplish both together. Furthermore, land availability or the orientation of runways could govern the number of remote piers and their lengths. The method proposed by Wirasinghe and Bandara is extended here to account for constraints (7).

The number of gates (or equivalently the pier length) for the pier attached to the terminal block and the gates in up to \( n - 1 \) consecutive remote piers starting from the one closest to the terminal block (Pier 1) can be prespecified. The search for the optimum solution can be restricted to a specified number of remote piers.

Several parallel pier configurations as shown in Figure 2 are analyzed. The differences among the three configurations are found essentially in the variations of the gate arrangement on the airside of the terminal block and the attached terminal pier.

### Parallel Pier Terminal

A parallel pier terminal (Figure 2a) has at least one remote parallel pier and gates along the airside of the terminal block. Furthermore, gates are located on both sides of a terminal block pier.

### Modified Parallel Pier Terminal

A modified parallel pier terminal (Figure 2b) is similar to a parallel pier terminal with one exception: gates are located only on the airside of the terminal block pier. The cause is usually the proximity of terminal block pier. This is essentially the Atlanta configuration.

### Remote Parallel Pier Terminal

A remote parallel pier terminal (Figure 2c) has no pier or gates attached to the terminal block. The spacing between the first remote pier and the terminal block is reduced in comparison to the parallel pier and modified parallel pier configurations. This is similar to the New Denver configuration.

Let

- \( y \) = length of half-piers attached to terminal block,
- \( b_{1} \) = width of terminal block, and
- \( x_{j} \) = length of remote pier that is prespecified, where \( j = 1, \ldots, m \) for \( m \leq n \).

When there are gates at the airside of the terminal block, the number of gates should be specified so that the airside frontage at the terminal block available for gates, \( b \), used in Equation 1, can be calculated. When there are no gates at the terminal block, \( b \) becomes zero; otherwise \( b \) is equal to \( b_{1} \).

The terminal configuration in which there is no terminal block pier (Figure 2c) can be obtained by setting the value of \( y \) to zero. The configuration in which the terminal block pier has gates only on one side (Figure 2b) can be obtained by specifying the entire pier length as the terminal block width. The proposed model is also applicable when the spacing between the terminal block and Pier 1 is different from the uniform spacing, \( s \), between remote piers for all the configurations discussed. Let \( S_{0} \) be the spacing between the terminal block and Pier 1 and let \( S = S_{0} - S \). The objective function that represents the total (user and operator) disutility of the system for a constrained configuration is obtained by modifying the objective function for the unconstrained configuration (see Appendix A, Equation 7).

It can be shown that the geometry for \( Q = 0 \) (no hub transfers) can be considered as the lower bound for the optimum solution. When \( Q = 0 \), it is also possible to determine the values of \( y \) and the remote pier lengths \( x_{j} \). The pier lengths should always be positive, so the maximum number of remote piers, \( n_{\text{max}} \), for a given number of gates can also be calculated. The optimum geometry that represents the lower bound is obtained by comparing the total cost for the solutions for each integer value of \( n \) between 1 and \( n_{\text{max}} \). When the lower bound is known, the optimum geometry is obtained by comparing the total cost for each integer value of \( n \) between the lower bound and the value of \( n \) that ensures that all optimal remote pier lengths are positive.

The optimum solution for a given \( n \) is obtained by solving \( n + 1 \) nonlinear simultaneous equations that represent the partial derivatives of the objective function with respect to each of the pier lengths. These equations are solved numerically using Zeidel's method of iteration [Zuguskin (8)]. A computer program (PPAPM) has been developed to determine the optimum constrained or unconstrained geometries.
CASE STUDIES

Two case studies that represent Atlanta Hartsfield and New Denver airports are considered. In the following section, variations in the optimal terminal geometry with respect to user costs and imposed geometrical constraints are discussed.

Cost Components

The average value of time of an air traveler is considered to be $0.75/min in 1990 dollars (7). Assuming that walking will require an additional effort, ranges of values are considered to represent the walk/ride cost ratio with respect to time and to distance. The disutility of walking is considered to be linearly related to the walking distance. It is assumed that riding will be five times faster than walking. Allowing for boarding and alighting at stations, an average APM travel time of 2.4 min/km is considered. Waiting and access cost is calculated.
on the basis of an average waiting time of 1 min (2-min APM headway) and a $0.25/passenger access and egress cost. The capital and operating costs of the APM systems are calculated on the basis of available information on the New Denver airport APM (N. D. Witteveen, personal correspondence, 1990) and the cost values reported by McKelvey and Sproule approximately adjusted to 1990 dollars (6). Table 1 shows the unit cost values that were used.

Table 2 shows the input parameters required for the PPAPM program and cost ratios used for the two case studies. The walk and ride cost values for the program should be given per unit distance per passenger. For example, let ride cost be $1.50/km/passenger and the disutility of walking be twice the disutility of riding with respect to time. Then, the walk cost that should be entered into the program is equal to $15.00/km/passenger if it is assumed that riding will be five times faster than walking. However, all cost values in the objective function can be expressed as ratios between the particular cost value and the ride cost per unit distance. There will be no change in the optimum geometry as long as these cost ratios remain the same irrespective of the monetary value of the value of time.

### Terminal Characteristics

Two terminals with 138 and 107 gates are considered to represent the Atlanta and New Denver airports, respectively. A uniform spacing of 40 m between gates is considered for both cases. Table 2 gives the spacing between remote piers, spacing between terminal block and first remote pier, and the terminal block widths.

To represent the Atlanta terminal, a basic configuration as shown in Figure 3 that consists of a terminal block with six gates along the airside is considered. This configuration is similar to the one shown in Figure 2b. There is no pier extending from the terminal block. This basic configuration is slightly different from the existing Atlanta terminal. In the existing terminal the six gates attached to the terminal block are located in a pier that extends from one side of the terminal block, whereas here the gates are distributed symmetrically.

The basic configuration that is selected to represent the New Denver terminal does not have a pier connected to the terminal block, and there are no gates along the terminal block airside (Figure 4). The spacing between the terminal block and the first remote pier is 170 m. An average spacing

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### Table 1: Unit Cost Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Cost (1990 Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk Cost</td>
<td>/km/passenger</td>
<td>9.00 - 36.00</td>
</tr>
<tr>
<td>Ride Cost</td>
<td>/km/passenger</td>
<td>1.80</td>
</tr>
<tr>
<td>Wait Cost</td>
<td>/passenger</td>
<td>1.80</td>
</tr>
<tr>
<td>APM Capital Cost</td>
<td>/section/passenger</td>
<td>0.10 - 0.20</td>
</tr>
<tr>
<td>APM Operating Cost</td>
<td>/km/passenger</td>
<td>0.20 - 0.06</td>
</tr>
</tbody>
</table>

### Table 2: Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Atlanta</th>
<th>New Denver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gates (G)</td>
<td>138</td>
<td>107</td>
</tr>
<tr>
<td>Gate Spacing (S_g)</td>
<td>40 m</td>
<td>40 m</td>
</tr>
<tr>
<td>Spacing Between Terminal Block and Pier 1 (S_1)</td>
<td>305</td>
<td>170</td>
</tr>
<tr>
<td>Remote Pier Spacing (S)</td>
<td>305 m</td>
<td>450 m</td>
</tr>
<tr>
<td>Terminal Block Width (b_1)</td>
<td>240</td>
<td>250 m</td>
</tr>
<tr>
<td>No. of Gates along The Terminal Block</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Fraction of Total Transfers P</td>
<td>0.65</td>
<td>0.60</td>
</tr>
<tr>
<td>Fraction of Hub Transfers Q</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Fraction of Hub Departs from Their Arrival Pier r</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Walk Cost Ratio +</td>
<td>1.0-5.0</td>
<td>1.0-5.0</td>
</tr>
<tr>
<td>Ride Cost Ratio</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Wait Cost Ratio</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>APM Capital Cost Ratio</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>APM Operating Cost Ratio</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

+ - with respect to time

Note: All cost values have been given with respect to a unit ride cost.
of 450 m between remote piers is used for the calculations. This configuration is similar to the configuration shown in Figure 2c. In the actual New Denver configuration, the gate spacings are not uniform across all piers.

Four groups of configurations, which represent different levels of geometrical constraints as shown in the following, are considered for the analysis. These configurations are compared with respect to different walk/ride cost ratios.

**Configuration**

A  Basic with no additional constraints  
B  Basic for existing number of piers  
C  Basic with first remote pier length specified  
D  No geometrical constraints with gates along the terminal block airside  
E  No constraints and no APM system  
F  Actual (existing) geometry

**Comparison**

First the basic configurations, A, for both terminals are analyzed for different walk/ride cost ratios between 1 and 5. A walk/ride cost ratio with respect to time of 1 assumes walking will not require an additional effort relative to riding. A high value of 5 is selected as the upper limit to study how the optimum number of remote piers increases with the walk/ride cost ratio.

User and operator costs corresponding to the existing geometry and optimum geometries for the other configurations are obtained for walk/ride cost ratios of 1 and 2, respectively. A walk/ride cost ratio of 2 is selected as a reasonable value to account for the disutility of walking. Sensitivity of the optimum geometries to the fraction of hub transfers who transfer from the same pier, $r$, is tested. The results show that the optimum geometries are insensitive to the value of $r$; to the fraction of total transfers, $P$; and to the fraction of hub transfers, $Q$, used for the calculations.

Figure 5 shows how the optimum number of piers for the two terminals increases with the walk/ride cost ratio. However, the rate at which the remote number of piers changes decreases with the walk/ride cost ratio. It is found that the geometries corresponding to walk/ride cost ratios of 1.1 and 1.07 are the closest representations of the actual (existing)
geometries of the Atlanta and New Denver airports, respectively.

Tables 3 and 4 show the optimum number of remote piers, pier lengths, mean walking distance, and total cost of the system for the different configurations considered. Figures 6 and 7 show the variations in walking distance and total cost for the selected configurations.

### Atlanta Airport

Referring to Figures 6 and 7 it can be seen that the no-constraint configuration (D) is the best alternative with respect to total cost irrespective of the disutility of walking considered. As expected, any geometrical constraint tends to increase the total cost. The optimum geometry for a walk/ride cost ratio of 1 is not significantly different from the existing geometry, indicating that the extra disutility of walking has not been considered in the design. The additional tunnel constructed between remote Piers 2 and 3 is a further indication that the existing geometry does not provide low passenger walking distances.

It can be seen that this design can be improved with respect to both the total cost and the passenger walking if the number of remote piers is increased by 2. However, if only passenger walking is considered, the existing configuration (F) becomes the best alternative. This design could also be improved by adding two more piers. If it is necessary to have only three remote piers, the design could be improved by making the pier lengths more uniform.

### New Denver Airport

The no-constraint configuration is the best alternative with respect to the total cost; the existing geometry is the least preferable. This design could also be improved by adding two more piers. If it is necessary to have only three remote piers, the design could be improved by making the pier lengths more uniform.

The best alternatives with respect to walking for walk/ride cost ratios of 1 and 2 are configurations C and A, respectively. The optimum geometry when there is no APM available has two remote piers, and the mean walking distance is 660 m (278 m within piers and 160 m between piers). In Figure 9 the optimum geometries for Configurations A, B, and D are compared with the existing geometry.

### CONCLUSIONS

In general, the optimum geometry consists of a nonuniform set of piers with longer piers toward the terminal block. A configuration with a terminal block pier is the best alternative with respect to total cost of the system. The optimum geometry is sensitive to the disutility of walking; that is, the ratio of the cost of walking to the cost of riding per unit time. Because costly APMs are installed presumably to reduce the disutility of walking, it is essential that this disutility is explicitly considered in the design.

### TABLE 4 Optimum Geometries, New Denver Airport

<table>
<thead>
<tr>
<th>Walk/Ride Cost Ratio</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Walking Distance (m)</strong></td>
<td>241</td>
<td>214</td>
</tr>
<tr>
<td><strong>Total Cost</strong> (+ Cost in $)</td>
<td>2765</td>
<td>2770</td>
</tr>
<tr>
<td><strong>Optimum No. of Piers</strong></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Terminal Block</strong></td>
<td>37</td>
<td>36</td>
</tr>
<tr>
<td><strong>Pier 1</strong></td>
<td>188</td>
<td>198</td>
</tr>
<tr>
<td><strong>Pier 2</strong></td>
<td>2769</td>
<td>3715</td>
</tr>
<tr>
<td><strong>Pier 3</strong></td>
<td>2166</td>
<td>3111</td>
</tr>
<tr>
<td><strong>Pier 4</strong></td>
<td>254</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 5</strong></td>
<td>393</td>
<td>24</td>
</tr>
<tr>
<td><strong>Pier 6</strong></td>
<td>439</td>
<td>17</td>
</tr>
<tr>
<td><strong>Optimum No. of Piers</strong></td>
<td>248</td>
<td>4027</td>
</tr>
<tr>
<td><strong>Terminal Block</strong></td>
<td>2784</td>
<td>3775</td>
</tr>
<tr>
<td><strong>Pier 1</strong></td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 2</strong></td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 3</strong></td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 4</strong></td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 5</strong></td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td><strong>Pier 6</strong></td>
<td>38</td>
<td>23</td>
</tr>
</tbody>
</table>

+ Cost in $ = Cost x Ride cost per unit distance/1000
FIGURE 6  Variation in walking distance: left, Atlanta; right, New Denver.

FIGURE 7  Variation in total cost: left, Atlanta; right, New Denver.
It can be seen that the disutility of walking has not been explicitly taken into account in the designs of Atlanta and New Denver terminals. However, for the Atlanta airport the existing geometry is not significantly different from the constrained optimum geometry if the number of remote piers is fixed a priori at four. The New Denver design can also be improved by increasing the number of piers or adjusting the number of gates in each of the three existing remote piers. However, the nonuniform gate spacing across piers used in the actual design has not been considered in this study.

The proposed method is useful in determining the number of remote piers and their lengths subject to any geometrical constraints. The ability to assess the sensitivity of the selected geometry to the uncertain input parameters is also useful in making a decision.

REFERENCES


APPENDIX A

INPUT PARAMETERS

The input parameters that are required for the PPAPM programs are as follows. The same notations have been used in the following equations.
Terminal Characteristics

- Total number of gates — G
- Spacing between gates — S_g
- Spacing between the terminal block and the first remote pier — S_1
- Spacing between remote piers — S
- Number of remote piers for which the lengths are specified — j
  - Minimum number of remote piers
  - Maximum number of remote piers
  - Optimum number of remote piers — n
- Number of gates along the terminal block (if there are gates) — g_b
- Width of the terminal block (if there are no gates) — b_1
- Number of gates in the terminal block pier (if length is specified) — g_1
- Number of gates in each of the length-specified piers — x_j

Passenger Characteristics

- Fraction of total transfers — P
- Fraction of hub transfers (with respect to total transfers) — Q
- Fraction of hub transfers that are known to depart from their arrival pier — r

Cost Components

- Walking cost per passenger per kilometer — \( \gamma_w \)
- Riding cost per passenger per kilometer — \( \gamma_R \)
- Waiting and access cost per passenger — \( \gamma_A \)
- Capital cost per passenger per remote pier — \( \gamma_c \)
- Operating and maintenance cost per passenger per kilometer — \( \gamma_M \)

TOTAL COST FOR UNCONSTRAINED CONFIGURATION (7)

\[
Z = \frac{\gamma_w}{L} \left( 1 + P - 2PQ \right) \left[ \sum_{j=1}^{k} \frac{x_j^2}{4} + y^2 + by + \frac{b^2}{8} \right] + PQR \left[ \sum_{i=1}^{k} \frac{x_i^2}{3} + \sum_{i,j=1}^{k} F(y) \right] + PQ(1-r) \left[ \sum_{i=1}^{k} \frac{x_i^2}{2} - \sum_{i,j=1}^{k} 6L - \sum_{i,j=1}^{k} x_i^2 \right] + \frac{PQ}{2} \left[ \sum_{i,j=1}^{k} \left( 2y(b_1 + y) + \frac{b^2}{4} \right) \right] + \frac{F_m(y)}{L} \left( 2y + \frac{b}{2} \right) + \gamma_M \left( 1 + P - 2PQ \right) \left[ \sum_{i=1}^{k} x_i \left( \sum_{i,j=1}^{k} (i - k)x_k \right) + \sum_{k,j=1}^{n} (k - i)x_k + \sum_{k,j=1}^{n} (k - i)x_k \right]
\]

\[
+ \frac{\gamma_c}{L} \left[ 1 + P - PQ(1 + r) \right] \gamma A + m \gamma_0
\]

TOTAL COST FOR CONSTRAINED CONFIGURATION

\[
Z_m = \frac{\gamma_w}{L} \left[ 1 + P - 2PQ \right] \left[ \sum_{j=1}^{k} \frac{x_j^2}{4} + y^2 + by + \frac{b^2}{8} \right] + PQR \left[ \sum_{i=1}^{k} \frac{x_i^2}{3} + \sum_{i,j=1}^{k} F(y) \right] + PQ(1-r) \left[ \sum_{i=1}^{k} \frac{x_i^2}{2} - \sum_{i,j=1}^{k} 6L - \sum_{i,j=1}^{k} x_i^2 \right] + \frac{PQ}{2} \left[ \sum_{i,j=1}^{k} \left( 2y(b_1 + y) + \frac{b^2}{4} \right) \right] + \frac{F_m(y)}{L} \left( 2y + \frac{b}{2} \right) + \gamma_M \left[ 1 + P - 2PQ \right] \left[ \sum_{i=1}^{k} x_i \left( \sum_{i,j=1}^{k} (i - k)x_k \right) + \sum_{k,j=1}^{n} (k - i)x_k + \sum_{k,j=1}^{n} (k - i)x_k \right]
\]

\[
+ \frac{\gamma_c}{L} \left[ 1 + P - PQ(1 + r) \right] \gamma A + m \gamma_0
\]

where

\[
y = g_S S / 4
\]

\[
b = g_v S
\]

\[
\gamma_0 = \gamma_c + \gamma_w S
\]

and

\[
F(y) = \left( \frac{8y^3}{3} + 3by^2 + b^2y + \frac{b^3}{12} \right) + \left( 2y + \frac{b}{2} \right)
\]
where

\[
m = \begin{cases} 
    n - 1 & \text{if there are geometrical constraints} \\
    n & \text{otherwise}
\end{cases}
\]  
(8)

\[2L = GS_y = 4y + b + 2\left(\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i\right)\]  
(9)

\[S_o = S - S_1\]  
(10)

and

\[F_m(y) = \left(\frac{8y^3}{3} + 2by^2 + by^2 + b^2y + b^3\right)\]  
\[+ \left(2y + \frac{b}{2}\right)\]  
(11)

The term in the first square bracket in Equations 2 and 7 represents the mean walking distance within piers for arriving, departing, and nonhub transfer passengers. The terms in the second and third square brackets represent the mean walk for \(r\) and \(1 - r\) fractions of hub transfers respectively. The terms in the last two square brackets represent the mean riding distance between piers for arriving, departing, and nonhub transfers and hub transfers, respectively. The last two terms in Equations 2 and 7 represent the mean waiting and access cost per passenger and the operator cost per passenger, respectively.