# Spectral Analysis of Vehicle Speed Characteristics

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Characteristics of individual vehicle speed are important when evaluating the safety of the traveling public, traffic level of service, and driver behavior. Traditional research is based on analysis in the time or space domain, and its scope is sometimes limited because of lack of methodology or limitations of those domains. Results of a study using a maximum entropy spectral analysis approach to evaluate driver behavior related to driving speed under heavy and light traffic conditions, rainy and dry weather conditions, ramp impacts, and different driver operating characteristics are presented. Three test sites were chosen in the Albany area. Speed of a testing vehicle between two fixed points was recorded and transferred to spectral density functions. Impacts of traffic conditions, weather conditions, vehicles entering from ramps, and driver behavior can be identified from these spectral density characteristics. Basic concepts of maximum entropy spectral analysis and results of the field experiments are presented.

In the 1985 *Highway Capacity Manual* (*1*), traffic speed, volume, and density are defined as basic traffic flow measures. Among these variables, characteristics of individual vehicle speed are important factors when evaluating safety of the traveling public, traffic level of service, and driver behavior. In the past, research has evaluated characteristics of vehicle speed and their impacts on environments, as well as impacts of environments on vehicle speed. For example, relationships among traffic flow, speed, and concentration and statistical distributions of traffic characteristics have been addressed and revised since traffic flow theory began developing in the 1930s with applications of probability theory (*2–7*). Other research regarding speed impacts on environments and environmental impacts on speed cover many topics, such as freeway speed profiles and fuel consumption relationships (*8*), measure of level of service (*2,9*), free-flow speed prediction (*10*), enforcement strategy effects on traffic speeds (*11,12*), and freeway weaving impacts (*13,14*). The results have produced good guidelines, references, and specifications for traffic control, development of traffic flow theory, highway construction, design, maintenance, and other highway operations.

However, traditional research is based on analysis in the time or space domain, and its scope is sometimes limited because of lack of methodology or limitations of those domains. The Engineering Research and Development Bureau of the New York State Department of Transportation recently completed a research study to evaluate traffic flow in the frequency domain rather than the time or space domain. Spectral analysis techniques have been used in transportation engineering for many years in such areas as pavement surface roughness (*15*), traffic flow prediction (*16*), and pavement transverse-crack spacing evaluation (*17*). The study reported here evaluated driver behavior related to driving speed as affected by heavy and light traffic, rainy and dry weather, and entering traffic from ramps using a maximum entropy spectral analysis (MESA) approach. The basic idea of MESA is to transfer individual vehicle speed recorded in the time or space domain to spectral density in the frequency domain by the maximum entropy spectral estimate method (*18*). Impacts of traffic conditions, weather conditions, entering vehicles from ramps, and driver behavior can be identified from spectral density characteristics. During this study, field experiments focused on freeway traffic because traffic lights on local roads would stop traffic, which was not desired in this study. I-87, I-90, and I-787 were chosen as test sites.

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## MESA CONCEPT

### Discrete Spectral Transformation

Vehicle speed data sampled at time interval $T$ can be abstracted as a discrete sequence, called the discrete speed sequence or

$$\{V_i\} = \{V_1, V_2, \ldots, V_N\} \tag{1}$$

where $V_i$ = $i$th vehicle speed data sampled at time interval $T$ (sec). Figure 1 shows a typical discrete speed sequence collected from I-87 during a non-rush-hour period. Consider the inverse discrete Fourier transformation of the discrete speed sequence defined by Oppenheim (*19*):

$$V_i = \frac{1}{N}\sum_{k=0}^{N-1} H_k e^{jik2\pi/N} \qquad (i = 1, 2, \ldots, N) \tag{2}$$

where

$N$ = length of the sequence (number of data points in the sequence),
$V_i$ = $i$th vehicle speed data,
$H_k$ = weights ($k = 0, 1, 2, \ldots, N - 1$), and
$j = (-1)^{1/2}$

Equation 2 states that $V_i$ can be considered a weighted summation of sine function $e^{jik2\pi/N}$. If a new variable $\omega_k$ is defined by

$$\omega_k = k2\pi/N \qquad (k = 0, 1, 2, \ldots, N - 1)$$

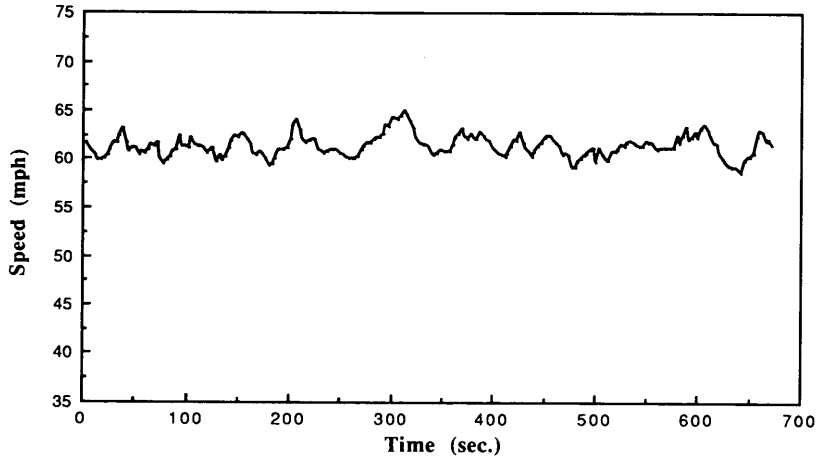


**FIGURE 1 Speed sequence collected from Test Site 1 (light traffic, dry, all lanes).**

then

$$V_i = \frac{1}{N}\sum_{k=0}^{N-1} H_k e^{ji\omega_k} \qquad (i = 1, 2, \ldots, N) \tag{3}$$

and

$$e^{ji\omega_k} = \sin \omega_k i + j \cos \omega_k i$$

Usually, the variable $\omega_k$ is called frequency and is within the range $[0, 2\pi(N - 1)/N]$. From Equation 2, it is known that the larger the $H_k$, the more sine function components with frequency $\omega_k$ the discrete speed sequence $\{V_i\}$ contains. Mathematically, it can be proved that

$$H_k = H(\omega_k) = \sum_{i=-\infty}^{+\infty} V_i e^{-ji\omega_k}$$

$$[\omega_k = 0, 2\pi/N, 4\pi/N, \ldots, 2\pi(N - 1)/N] \tag{4}$$

In other words, $H(\omega_k)$ is the discrete Fourier transformation of $\{V_i\}$ and the function of frequency $\omega_k$. Equation 4 implies that the discrete speed sequence $\{V_i\}$ in the space domain can be transferred into the frequency domain sequence $\{H(\omega_k)\}$, and characteristics of sequence $\{V_i\}$ can be analyzed in the frequency domain—that is, knowing $H(\omega_k)$, one can analyze the characteristics of $\{V_i\}$. Since $H(\omega_k)$ is an imaginary sequence, a real function is defined by

$$S(\omega_k) = |H(\omega_k)|^2 \tag{5}$$

where $S(\omega_k)$ is called the spectral density function of sequence $\{V_i\}$. To calculate $H(\omega_k)$ from Equation 4, the summation should be from $-\infty$ to $+\infty$. In practical engineering cases, sequence length $N$ is finite because one cannot collect infinite sequences of data. The spectral density function $S(\omega_k)$ thus should be estimated from $\{V_i\}$ by some estimation model, instead of using Equation 4.

**Simple Description of Maximum Entropy Spectral Estimation**

In the area of spectral function estimation, several mathematical methods are available, such as fast Fourier transformation (FFT) (*19*), maximum likelihood spectral estimation (*18*), and maximum entropy spectral estimation (MESE) (*18*). The MESE method is one of the best.

The MESE method was introduced by Burg in 1968 (*20*). Like maximum likelihood spectral estimation, MESE is a kind of estimator of parameter estimation. Consider a discrete sequence $\{V_i\}$ with sequence length $N$ and sample interval $T$. If the sequence is a stationary, zero mean, approximately normally distributed, and band-limited stochastic process, then entropy of the sequence is defined as

$$H = \frac{1}{2}\ln(2B) + \frac{1}{4B}\int_{-B}^{B} \ln[S(\omega)]d\omega \tag{6}$$

where $B$ is band width of the sequence and $S(\omega)$ is the spectral density function of the sequence, or

$$S(\omega) = T\sum_{m=-\infty}^{+\infty} R(m)e^{-jmT\omega} \tag{7}$$

In Equation 7, $R(m)$ is defined as the autocorrelation function of sequence $\{V_i\}$

$$R(m) = E\{V_i V_{i+m}\} \tag{8}$$

Combining Equations 6 and 7, entropy is obtained by

$$H = \frac{1}{2}\ln(2B) + \frac{1}{4B}\int_{-B}^{B} \ln\left[T\sum_{m=-\infty}^{+\infty} R(m)e^{-jmT\omega}\right]d\omega \tag{9}$$

Suppose the values of autocorrelation $R(m)$ are given for $m = 0, 1, 2, \ldots, M$. Then the corresponding extension of

the autocorrelation function is defined by the convolution sum

$$R(m) = -\sum_{k=1}^{M} R(m-k)a_k \qquad (m > M) \tag{10}$$

or, equivalently,

$$\sum_{k=0}^{M} R(m-k)\, a_k = 0 \qquad (a_0 = 1, m > M)$$

The method that Burg introduced maximizes entropy $H$ with respect to $R(m)$ ($|m| > M$) with restrained condition Equation 10, so that parameters $a_1, a_2, \ldots, a_M$ can be obtained. Mathematically, this can be expressed as

$$\left.\begin{aligned} &\frac{\partial H}{\partial R(m)} = 0 \qquad (|m| > M) \\ &\sum_{k=0}^{M} R(m-k)\, a_k = 0 \end{aligned}\right\} \tag{11}$$

It can be proved that with the conditions in Equation 11, sequence $\{V_i\}$ can be related by the following autoregression model, called AR(M) model:

$$V_i = -a_1 V_{i-1} - a_2 V_{i-2} - \cdots - a_M V_{i-M} + e_i \tag{12}$$

where $M$ is the order of the AR(M) model and $\{e_i\}$ is an approximately normally distributed disturbance with zero mean value. The estimate of the parameters $(a_1, a_2, \ldots, a_M)$ can be obtained by the Yule-Welker equation

$$\mathbf{R} \cdot \mathbf{A} = \mathbf{P} \tag{13}$$

where $\mathbf{R}$ is the autocorrelation matrix of sequence $\{V_i\}$. $\mathbf{R}$ is called the Toeplitz matrix:

$$\mathbf{R} = \begin{bmatrix} R(0) & R(-1) & \ldots & R(1-M) & R(-M) \\ R(1) & R(0) & \ldots & R(2-M) & R(1-M) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ R(M-1) & R(M-2 & \ldots & R(0) & R(-1) \\ R(M) & R(M-1) & \ldots & R(1) & R(0) \end{bmatrix}$$

and

$$\mathbf{A} = \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{M-1} \\ a_M \end{bmatrix} \qquad \mathbf{P} = \begin{bmatrix} P_M \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix}$$

where $P_M = E\{(e_i)^2\}$.

Finally, with all parameters estimated by the MESE algorithm, the maximum entropy spectral density function can be expressed by

$$S(\omega) = \frac{P_M T}{\left|1 + \sum_{m=1}^{M} a_m e^{-jmT\omega}\right|^2} \tag{14}$$

### MESA of Vehicle Speed Characteristics

From this discussion, it can be understood that the spectral density function $S(\omega_k)$ represents the frequency density distribution characteristics of the discrete speed sequence $\{V_i\}$. The basic idea is that if the discrete speed sequence $\{V_i\}$ changes smoothly, or the driver controls his vehicle in a steady manner, then $S(\omega_k)$ contains relatively numerous low-frequency components. This means that the magnitude of $S(\omega_k)$ in the high-frequency region is fairly low. On the other hand, if the discrete speed sequence changes randomly, or the driver changes his speed abruptly, then $S(\omega_k)$ contains relatively numerous high-frequency components, and the magnitude of $S(\omega_k)$ in the high-frequency region thus is relatively higher.

Conceptually, the spectral density function in the low-frequency region represents contour characteristics or macroscopic characteristics of a speed curve in a long period, but the spectral density function in the high-frequency region represents detail changes or microscopic characteristics of the speed curve in a short period. Since frequency of speed change is limited, spectral density characteristics should be band-limited. Figure 2 shows the spectral density function of the speed curve presented in Figure 1. From this graph, it is known that spectral density function is band-limited, and low-frequency components dominate the whole spectral density function. In fact, the spectral density function shown in Figure 2 is a typical model of speed spectral density characteristics.

## MESA EXPERIMENTS

### Field Test Considerations

Since this study's objective was to analyze driver behavior while moving, stops caused by traffic lights, accidents, or congestion were not considered. Thus only freeways were chosen as field test sites.

The process of sampling field data is relatively simple: the testing vehicle was driven from Site A to Site B and its speed was sampled at 3-sec intervals by an instrument called the Fluke Meter. Then recorded data including speed, traffic condition, test site identification, weather condition, lane change, driver's name, date, and other information were sent to a laboratory for data reduction and analysis.

Basic field test requirements were as follows:

1. The test site should be long enough so the basic dynamic process of changing speed can be recorded. In this study, the length was 10 mi.
2. To find the difference between spectral density characteristics under heavy and light traffic flow conditions, test sites should have heavy flow during rush-hour and light flow during non-rush-hour periods.
3. A few ramps should be included because ramp impacts were to be considered.

By combining these requirements, three test sites in the Albany area were chosen: I-87 between Exits 2 and 9 (southbound), I-90 between Exits 5 and 10 (eastbound), and I-787 between Route 9W and Tibbets Ave. (southbound).

During testing, driver behavior should be as objective as possible—speed control characteristics should change ac-

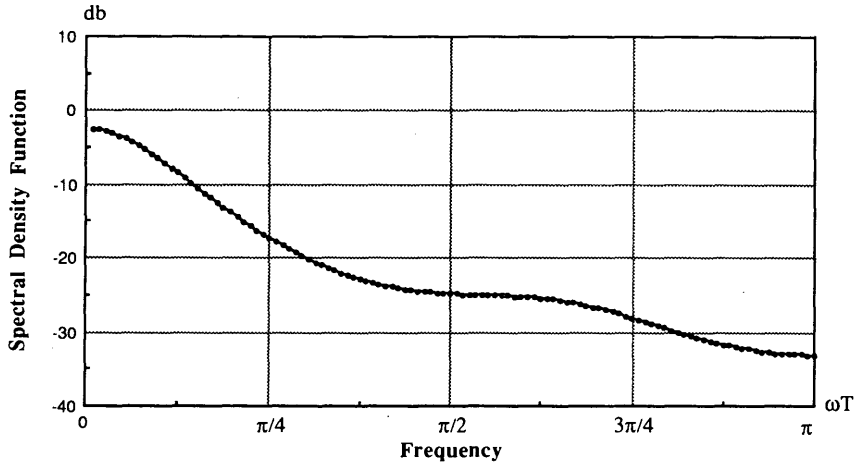


**FIGURE 2 Spectral density function of speed sequence (Test Site 1, light traffic, dry, all lanes).**

cording to traffic conditions, ignoring the fact that the driver is in a test situation. Figure 3 shows the field test factorial. "Right lane" means the testing vehicle always stays in the right lane (to compare ramp impacts), and "all lanes" means the driver can change lanes depending on traffic conditions. In an "all lanes" case, the impact of the ramp is less than that in "right lane."

### Spectral Density Characteristics of Driver Behavior Under Varied Traffic Conditions

Heavy and light traffic conditions are two extremely different cases, in which a driver may control vehicle speed differently. Generally, when traffic is light, vehicle speed is more stable than in heavy traffic conditions. However, this difference may not be easy to identify in the time or space domains. Figures 4, 5, and 6 show speed data collected from Sites 1, 2, and 3, respectively, representing light traffic during non-rush-hour periods and heavy traffic during rush hours. Spectral density characteristics of these speed data are presented in Figures 7, 8, and 9 [vertical scale: $20\log\{S(\omega_k)\}$], from which it can be seen that these characteristics differ significantly under heavy and light traffic volumes, although these differences cannot be easily identified from Figures 4, 5, and 6. Magnitude of the spectral density function under heavy traffic is much greater than under light traffic, which means (as stated earlier) that the driver may change speed abruptly because of heavy traffic ahead of the vehicle. Statistically, the magnitude of the spectral density function resulting from heavy traffic is higher than that from light traffic. Figure 10 shows speed curves collected from Site 1 under very heavy traffic. Figure 11 shows spectral density functions resulting from speed data shown in Figure 1 representing light traffic, from Figure 4 representing heavy traffic, and from Figure 10 representing very heavy traffic. It is known that the heavier the traffic, the higher is the magnitude of the spectral density function.

| | | Rainy | | Dry | |
|---|---|---|---|---|---|
| | | Heavy Traffic | Light Traffic | Heavy Traffic | Light Traffic |
| Driver A | Right Lane | | | | |
| | All Lanes | | | | |
| Driver B | Right Lane | | | | |
| | All Lanes | | | | |

**FIGURE 3 Factorial for field experiments.**

### Weather Condition Impact on Vehicle Speed

In this study, weather condition is described as rainy or dry, and results are based on non-rush-hour traffic flow. Since no heavy rain occurred during testing, heavy rain is not discussed here. A major concern was whether rain had significant impact on individual vehicle speed by spectral analysis. The literature indicates that a wet pavement surface has less skid resistance, which affects driving safety characteristics. But it should be known whether a wet pavement surface significantly affects driver behavior in terms of speed. In this study, a few tests were conducted at the three sites to study rainy weather impact. It would be expected that during rain, a driver might keep cautiously adjusting his speed to find a desired level that he considers safe. He might not accelerate or decelerate quickly, largely because of less skid resistance. Frequent adjustment of vehicle speed could result in a relatively high magnitude of spectral density function in the high-frequency region, but magnitude in the low-frequency region may not change in a non-rain situation because macroscopic characteristics of the speed curve may not show much difference if rain is not heavy. Thus, the shape of spectral density characteristics could be used in analyzing rainy weather impact. A suitable way to identify curve shape is use of normalized spectral density curves. Figures 12, 13, and 14 show normalized spectral density char-

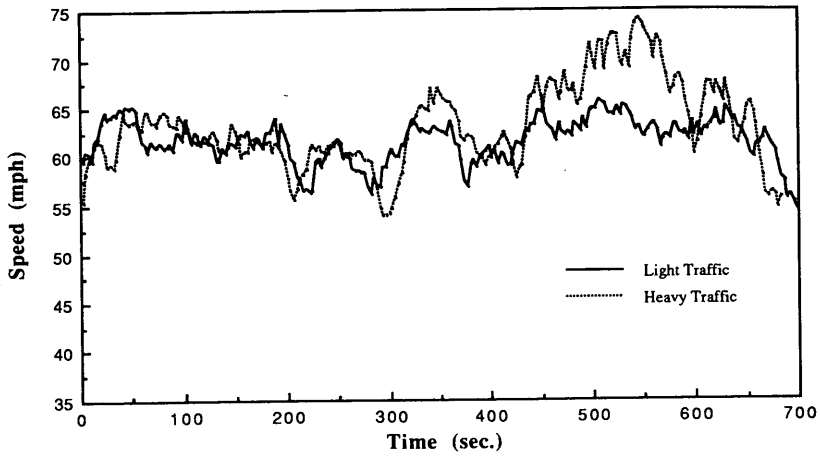


**FIGURE 4 Speed sequence collected from Test Site 1 (dry, all lanes).**

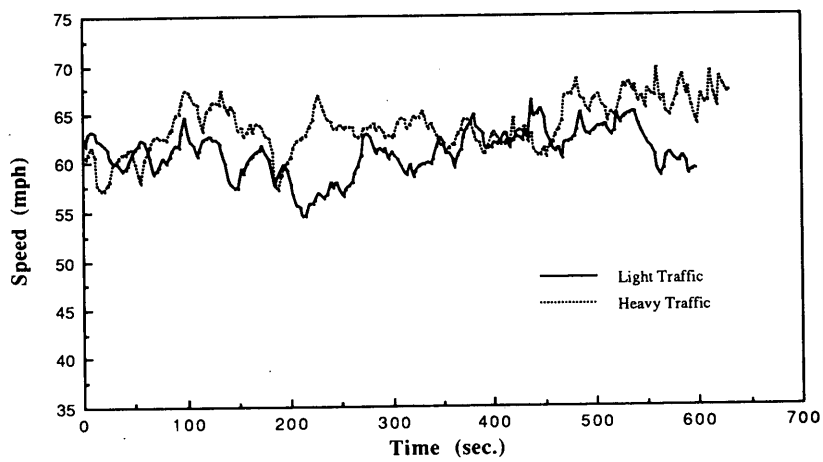


**FIGURE 5 Speed sequence collected from Test Site 2 (dry, all lanes).**

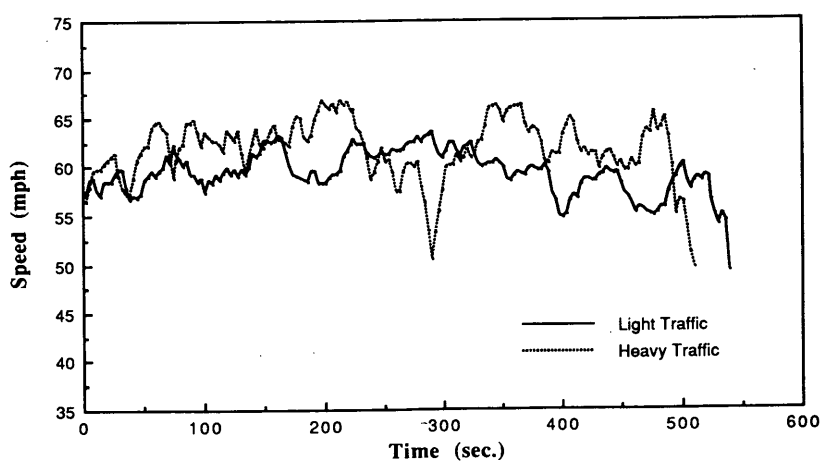


**FIGURE 6 Speed sequence collected from Test Site 3 (dry, all lanes).**

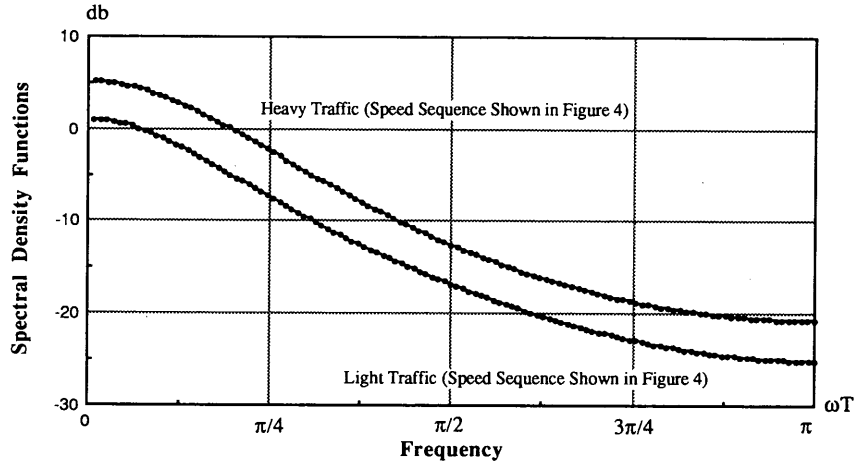


**FIGURE 7 Spectral density functions of speed sequences under light and heavy traffic conditions (Test Site 1, dry, all lanes).**

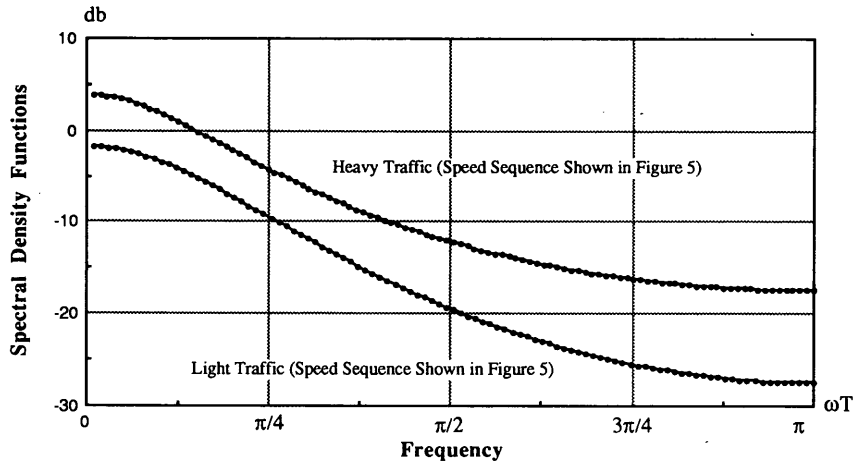


**FIGURE 8 Spectral density functions of speed sequences under light and heavy traffic conditions (Test Site 2, dry, all lanes).**

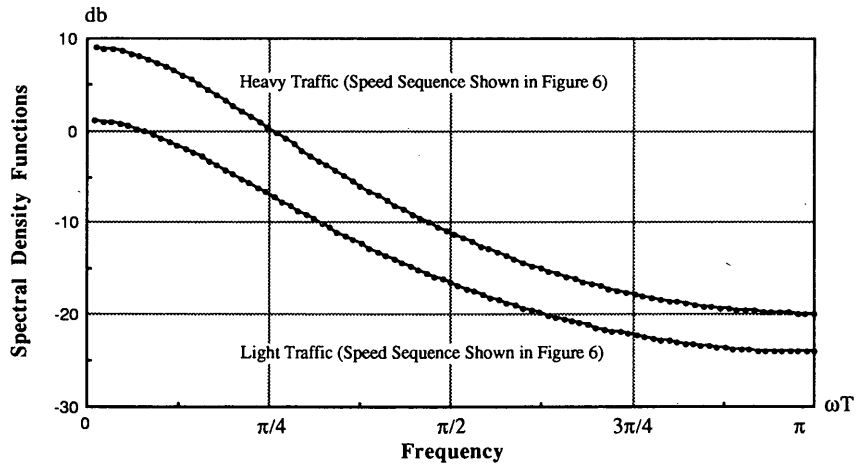


**FIGURE 9 Spectral density functions of speed sequences under light and heavy traffic conditions (Test Site 3, dry, all lanes).**

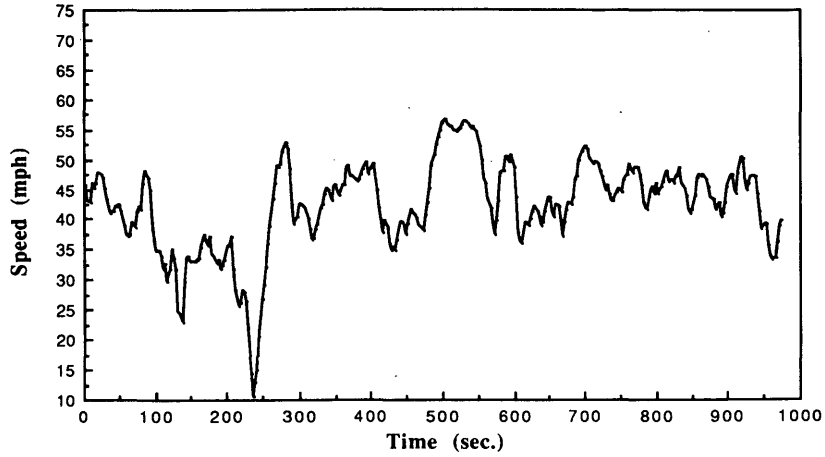


**FIGURE 10 Speed sequence collected from Test Site 1 (very heavy traffic, dry, all lanes).**

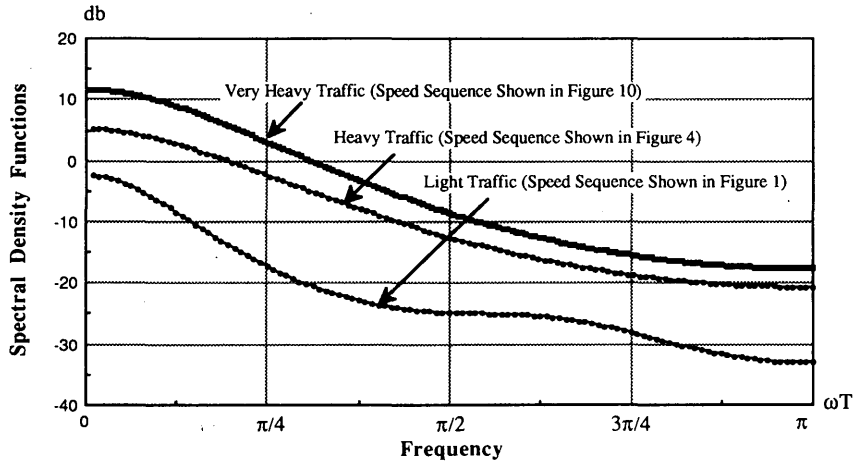


**FIGURE 11 Spectral density functions of speed sequences under light, heavy, and very heavy traffic conditions (Test Site 1, dry, all lanes).**

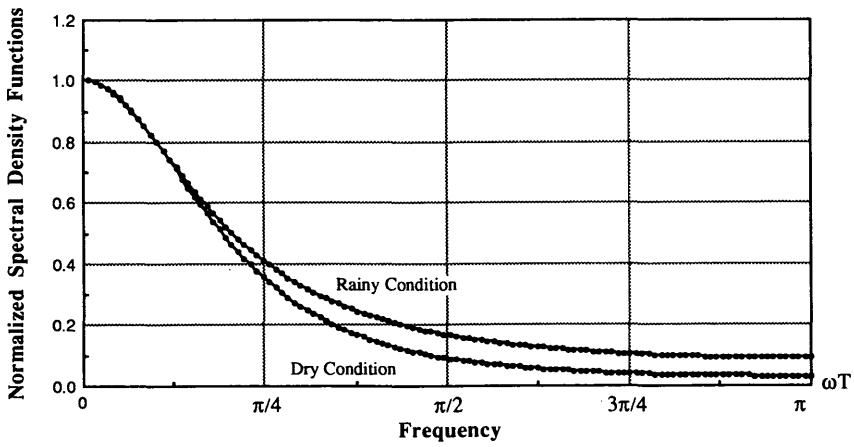


**FIGURE 12 Normalized spectral density functions of speed sequences under rainy and dry conditions (Test Site 1, light traffic, all lanes).**

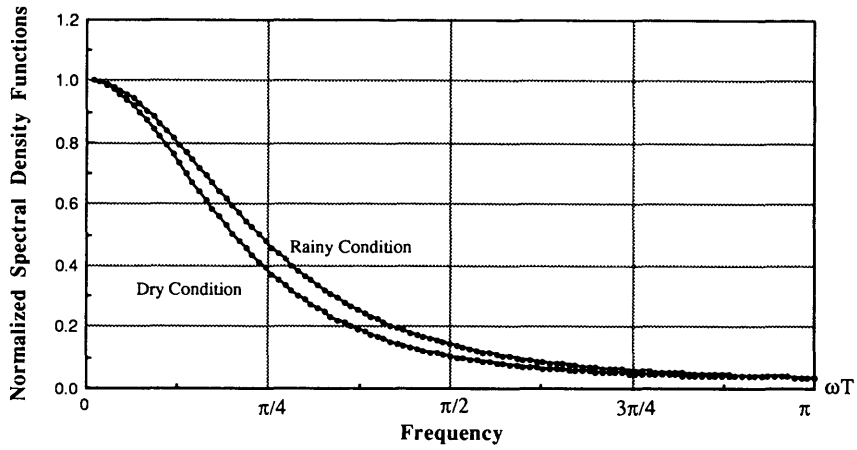


**FIGURE 13 Normalized spectral density functions of speed sequences under rainy and dry conditions (Test Site 2, light traffic, all lanes).**

acteristics under rainy and dry conditions, with speed data collected from Sites 1, 2, and 3. From these graphs, it is apparent that spectral density characteristics of vehicle speed under light rainy and dry weather do not differ significantly (i.e., driver behavior in terms of speed is not significantly affected by wet pavements). However, during field testing, no heavy rain occurred, and the results may not be applicable to such conditions.

### Ramp Impact on Vehicle-Speed Density Characteristics

Vehicles entering from a ramp significantly affect speed characteristics of vehicles already in a freeway. In recent years, research has been done on macroscopic characteristics of ramp impact in the time or space domain. One objective of this study was to evaluate vehicle-speed spectral density characteristics affected by vehicles entering from ramps during a light-traffic condition. It was assumed here that vehicles staying in the right lane were more affected by entering vehicles than vehicles that could change lanes when approaching ramps. Two cases were considered. First, the testing vehicle was allowed to change lanes to avoid ramp impact. In the second, the testing vehicle was directed to stay in the right lane no matter how bad traffic was, and when vehicle speed approached zero the test was considered "fail." Field tests were conducted at Test Sites 1, 2, and 3, and corresponding normalized spectral density functions are shown in Figures 15, 16, and 17, from which differences between "right lane" and "all lanes" can be identified.

### Spectral Analysis of Driver Behavior

The tests discussed so far were based on speed characteristics controlled by a specified driver called Driver A. However, another driver might behave differently—some drivers control vehicles in an aggressive manner and others defensively. In the frequency domain, differences in driving behavior can be spotted. Conceptually, an aggressive driver adjusts his speed more often and more quickly under various traffic conditions

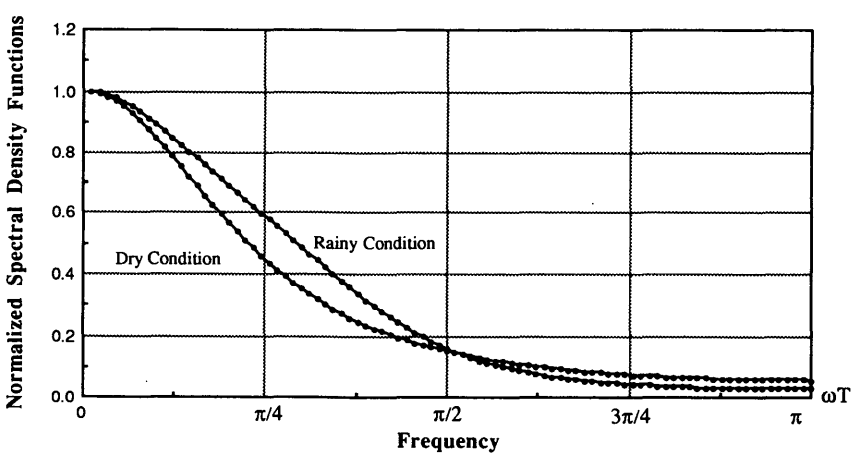


**FIGURE 14 Normalized spectral density functions of speed sequences under rainy and dry conditions (Test Site 3, light traffic, all lanes).**

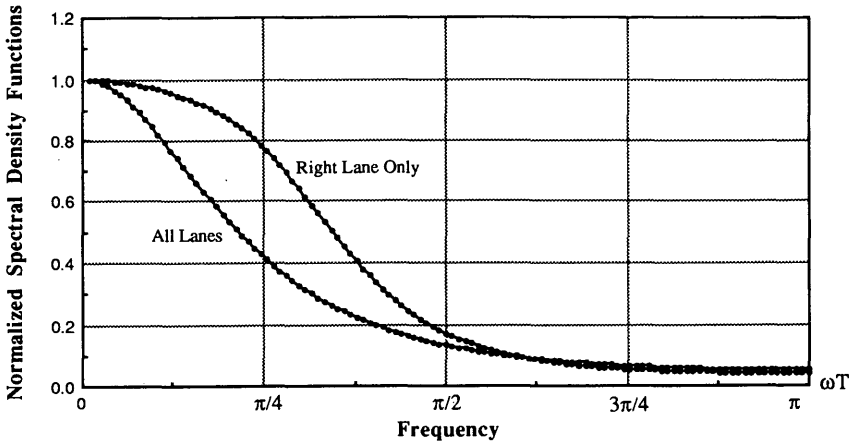


**FIGURE 15 Normalized spectral density functions of speed sequences under right lane and all lane conditions (Test Site 1, light traffic, dry).**

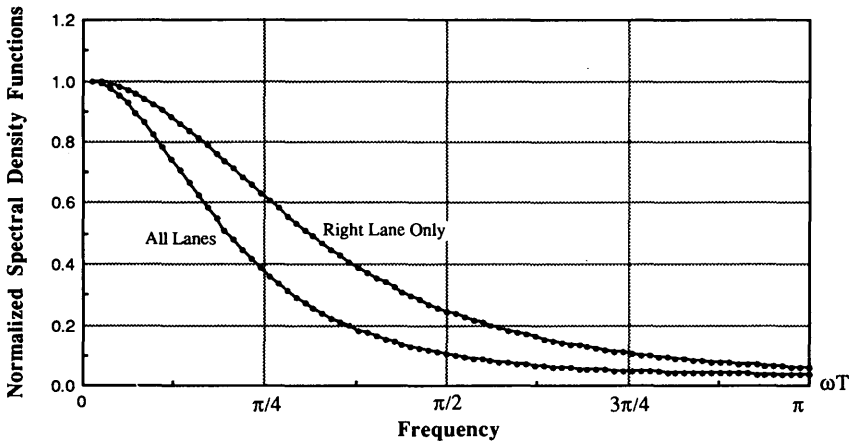


**FIGURE 16 Normalized spectral density functions of speed sequences under right lane and all lane conditions (Test Site 2, light traffic, dry).**

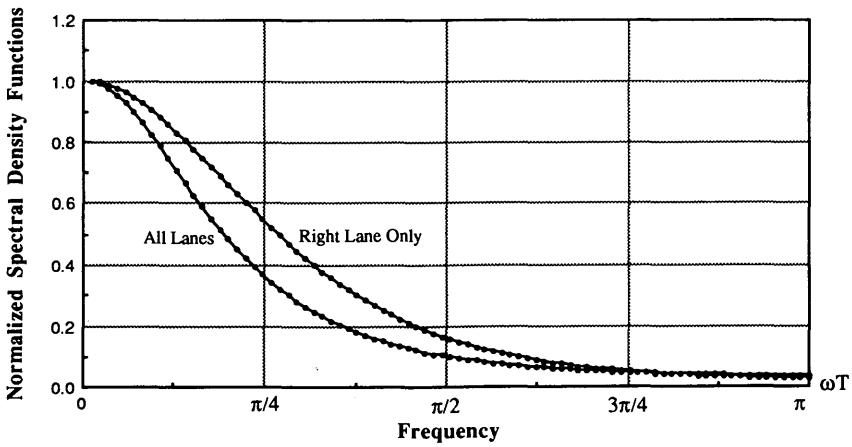


**FIGURE 17 Normalized spectral density functions of speed sequences under right lane and all lane conditions (Test Site 3, light traffic, dry).**

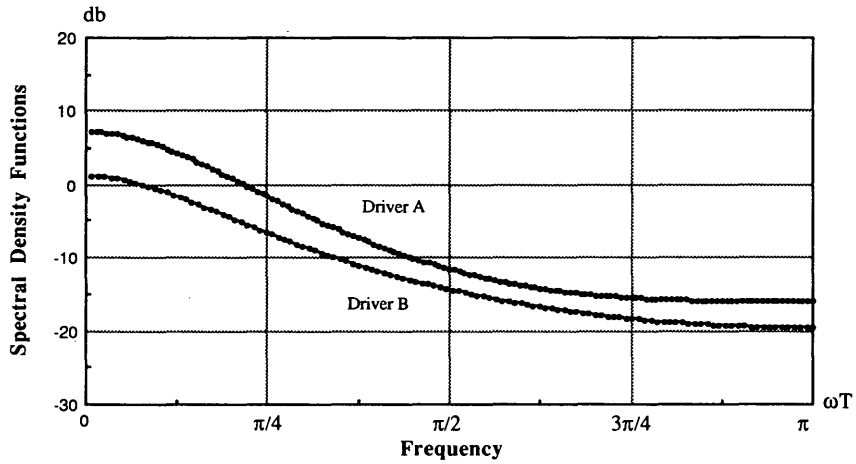


**FIGURE 18 Spectral density functions of speed sequences resulting from Drivers A and B (Test Site 1, light traffic, dry, all lanes).**

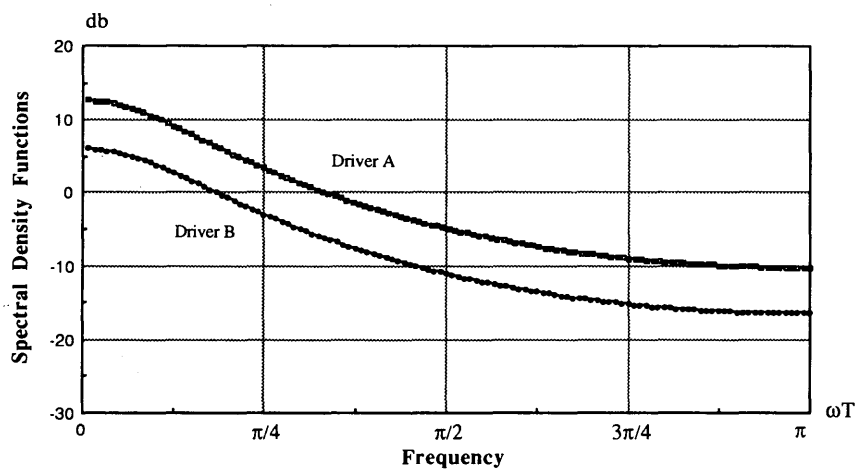


**FIGURE 19 Spectral density functions of speed sequences resulting from Drivers A and B (Test Site 1, heavy traffic, dry, all lanes).**

than a defensive driver, resulting in higher spectral density magnitude in the whole frequency range. Field tests were conducted at Site 1 to examine this assumption. Two drivers were selected from the research staff, and traffic condition (heavy traffic and light traffic) was considered. Figures 18 and 19 show differences between spectral density characteristics of Drivers A and B under heavy and light traffic conditions. From these graphs, Driver A had a higher spectral density magnitude than Driver B, meaning that Driver B controlled his vehicle more defensively than Driver A. However, this difference is smaller when traffic is light compared with when traffic is heavy.

## CONCLUSIONS

1. The spectral analysis technique has been accepted in many engineering areas, but not widely applied or evaluated in transportation engineering. In fact, in addition to the time and space domains, spectral analysis provides another analytical alternative. Some problems that cannot be solved in those domains may be solved easily in the frequency domain.

2. Individual vehicle speed is a stochastic process. If data sampling is limited to a certain time period, this process can be assumed to be symptomatically stationary and approximately normally distributed without obvious constant trends. This assumption makes nonlinear spectral estimation methods applicable.

3. The study reported reflects only initial research results showing how the models work. More effort is needed to evaluate these traffic impacts.

4. Most current vehicle-speed-related research is based on "point detection" or "section detection" (i.e., the mean value of measured speeds is taken at the main variable). In this way more important information is averaged. In fact, such information can be obtained from the detection of the dynamic

speed process, which is called line detection. The technique discussed in this paper belongs to line detection.

5. Data collected from line detection can be analyzed in the time/space domains. Analysis in the frequency domain has been widely used in continuous and discrete control systems and signal evaluation. The spectral analysis technique discussed here can be used to assess highway level of service, traffic congestion, and safety of the traveling public. This technique can also be used in detecting traffic incidents and traffic control, such as in Intelligent Vehicle Highway Systems. However, further research must be conducted to apply spectral analysis techniques to these areas.

6. MESE is one of the spectral estimate methods. For spectral analysis, other estimate methods could be used. Many computer software packages are available.

## ACKNOWLEDGMENTS

William G. Roth performed instrumentation and data reduction work for this study. Field tests were conducted by Robert W. Rider, Andrew A. Dagostino, and Edward W. Bikowitz.

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*Publication of this paper sponsored by Committee on Traffic Law Enforcement.*