Concrete Pavement Backcalculation Results from Field Studies

MICHAEL I. DARTER, KURT D. SMITH, AND KATHLEEN T. HALL

Deflection testing and backcalculation have been conducted on all types of concrete highway and airfield pavements for many years. Backcalculation of the in situ slab elastic modulus and the effective *k*-value or elastic solid subgrade modulus (beneath the slab) from deflection basins was first proposed by Westergaard in 1925 and actually conducted in the 1930s. Over the past decade, backcalculation has greatly improved and is now extremely rapid, highly automated, and reliable. The results from concrete pavement deflection testing and backcalculation have been used for several purposes: to compute the load-carrying capacity of concrete pavement for aircraft and truck loadings, to design structural overlays, and to detect loss of support beneath slab corners. Thus, deflection testing and backcalculation of in situ concrete pavement properties have proven to be very beneficial. The theory and practice of backcalculation for concrete pavements as they have developed over the years are summarized, results from a wide variety of pavements throughout the United States are presented, and implications for evaluation and design of concrete pavements are discussed.

Deflection data have proven to be very useful in the evaluation and structural rehabilitation of concrete pavements. Deflection testing has been conducted on all types of concrete pavements over the past 10 to 15 years on many highways and airfields throughout the United States. The Dynatest falling weight deflectometer (FWD) has been used frequently on such pavements with weights ranging from 5 to 50 kips. The analysis of the deflection basins has greatly improved over the years and is now highly automated and reliable.

The results from deflection testing of concrete pavements and backcalculation have been used for several purposes: to compute the load-carrying capacity of the concrete pavement for aircraft and truck loadings, to design structural overlays, and to detect loss of support beneath slab corners. These results have in turn been used in the selection and design of rehabilitation treatments for many highway and airfield projects.

It is no longer necessary to remove slabs to conduct expensive and time-consuming plate load tests, because the in situ (effective) *k*-value can best be computed from deflections measured at the top of the slab using the FWD. In addition, the in situ elastic modulus of the concrete slab can be directly backcalculated from the same data. If no cores or beams can be taken from the pavement, the flexural strength of the concrete can be roughly estimated as well. These values can be used in the structural evaluation and design of overlays.

The results of many backcalculation studies have led to the conclusion that the in situ *k*-value that actually supports the slab is not necessarily the same as that obtained using the traditional procedures for obtaining the effective *k*-value on top of the base/subbase (where the *k*-value measured on top of the subgrade is increased depending on the thickness and stiffness of the base/subbase layers placed on the subgrade). It is, therefore, not appropriate to describe a *k*-value "on top of a base or subbase layer," only "a *k*-value beneath the slab." This has significant implications for design of new pavements and evaluation of existing pavements.

This paper describes the theory and practice of backcalculation for concrete pavements, presents results from many field studies on diverse pavement and subgrade sections, and discusses implications for evaluation and design of concrete pavements.

BACKCALCULATION THEORY AND PROCEDURES

Concrete pavements have long been characterized using plate theory, in which the slab is characterized by a thickness *h* and a modulus of elasticity *E*, and the foundation is characterized as a dense liquid *k*-value. Westergaard developed stress and deflection equations that have been verified and used to the present day in concrete pavement design (*1*,*2*). In addition, Hogg (*3*) and Holl (*4*) developed the theory to model a slab as a plate on an elastic solid foundation. Equations for deflection basins for a plate on a dense liquid and elastic solid subgrade were presented by Losberg in 1960 (*5*).

Westergaard introduced the term "radius of relative stiffness" to describe the stiffness of a concrete slab relative to that of the subgrade, given by the following equation:

$$\ell_k = \sqrt[4]{\frac{E_{pcc}h_{pcc}^3}{12\,(1 - \mu_{pcc}^2)k}} \tag{1}$$

where

ℓ_k = dense liquid radius of relative stiffness (in.),
E_{pcc} = concrete elastic modulus (lb/in.²),
h_{pcc} = concrete thickness (in.),
μ_{pcc} = concrete Poisson's ratio, and
k = modulus of subgrade reaction (lb/in.²/in.).

Westergaard (*1*) actually proposed in 1925 that the subgrade *k*-value should be backcalculated from deflections at the sur-

ERES Consultants, Inc., 8 Dunlap Court, Savoy, Ill. 61874-9501.

face of the slab rather than measured from load tests on the subgrade:

> It is true that tests of bearing pressures on soils have indicated a modulus, *k*, which varies considerably depending on the area over which the pressure is distributed. Yet, so long as the loads are limited to a particular type, that of wheel loads on top of the pavement, it is reasonable to assume that some constant value of the modulus, *k*, determined empirically, will lead to a sufficiently accurate analysis of the deflections and stresses. . . . The modulus, *k*, enters in the formulas for the deflections of the pavements, and may be determined empirically, accordingly, for a given type of subgrade, by comparing the deflections found by tests of full-sized slabs with the deflections given by the formulas.

To support the statement that bearing tests produce varying *k*-values depending on the loaded area, Westergaard cited the results of field test reported by Bijls in 1923 (*6*), Goldbeck in 1925 (*7*), and Goldbeck and Bussard in 1925 (*8*). However, Westergaard's 1925 paper (*1*) contained only a center-slab deflection equation for a concentrated load. The center deflection equation for a distributed load of radius *a* was presented in Westergaard's 1939 paper (*2*):

$$d_0 = \left(\frac{P}{8k\ell_k^2}\right) \times \left\{1 + \left(\frac{1}{2\pi}\right)\left[\ln\left(\frac{a}{2\ell_k}\right) + \gamma - 1.25\right]\left(\frac{a}{\ell_k}\right)^2\right\} \qquad (2)$$

where

d_0 = maximum deflection at center of load (in.),
P = load,
a = load radius, and
γ = Euler's constant (0.57721566490).

The first known attempts at backcalculation of slab and support properties were at the Arlington, Virginia, experimental tests conducted in the 1930s by the Bureau of Public Roads as part of a major test program to verify the Westergaard equations for stress and deflection. One portion of the program was to develop procedures to determine an appropriate *k*-value. Two different procedures were evaluated: (a) load-deflection testing with a loaded plate directly on the subgrade and (b) load-deflection testing on top of the slab and the use of Westergaard's equations to determine a value for the in situ *k*-value and the in situ slab elastic modulus *E*. The description of the tests and the results of the studies were reported by Teller and Sutherland (*9,10*).

The plate bearing tests conducted directly on the subgrade soil were done using several plate diameters between 2 and 84 in. The test results demonstrated "the important effect of plate area on the pressure intensity required to produce a given plate displacement on the soil in question," particularly for plate diameters less than 26 in. (*10*).

The elastic modulus of the concrete slabs at the Arlington test site was determined by two methods: (a) laboratory testing of cores and beams from the pavement and (b) backcalculation from deflection measurements. Radius of relative stiffness values were determined by matching slab deflection basin measurements to contours developed by Westergaard (*1*) for deflection versus distance from load. This required preparing two-dimensional diagrams of the deflection basins and varying both the horizontal and vertical scales until the measured deflection basins matched the theoretical basins as closely as possible. The subgrade *k*-value could then be backcalculated using Westergaard's deflection equations, and the concrete slab modulus could be backcalculated from the definition of radius of relative stiffness. The backcalculation results are shown in Table 1. Teller and Sutherland (*10*) summarized the results of the backcalculation of *k* and *E* from interior, edge, and corner defections:

> For a given slab thickness, values of the radius of relative stiffness, ℓ, are in good agreement for the three cases of loading (interior, edge, and corner). For conditions that are comparable there is rather good agreement also between the values of modulus of subgrade reaction, *k*, as determined by pavement deflection (on top of the slab), for the interior and edge loadings but the value for the corner loading is consistently lower.
>
> The values of the modulus of elasticity for the concrete, *E*, as determined from the slab deflections are in the same general range as the values that were obtained from the tests of the laboratory specimens (at edge and interior positions).

Although Westergaard proposed backcalculating *k*-values from deflection measurements on full-size slabs in 1925, the fact that he did not present a direct method for doing so, in the words of J. P. Sale (*11*), "has through the years caused Corps' researchers and, we believe, many others considerable concern." The laborious method used to analyze the Arlington test data, in which theoretical and measured deflection basins were matched by hand, was certainly not suited to analysis of large deflection data sets.

It is perhaps because of this lack of a backcalculation method that *k*-values came to be measured by subgrade plate load tests. When bases came into widespread use, it then seemed logical to increase the *k*-value to account for various base types and thicknesses. In time, because of the high costs and long testing times involved, actual plate load testing of subgrades and bases became more uncommon, and typical ranges of *k*-values were identified for various subgrade soils and bases. When very stiff (i.e., stabilized) bases began to come into use, it was naturally expected that the *k*-value would be very high because of the support that this type of base would provide.

Backcalculation methods for multilayer elastic pavement systems first appeared in the 1970s, starting with Scrivner's solution for elastic moduli in a two-layer pavement system in 1973 (*12*). Because of the integral nature of the deflection equations given by elastic layer theory (*13,14*), these backcalculation methods relied on rather complex graphical or computerized solution methods.

A simple two-parameter approach to backcalculation of surface and foundation moduli for a two-layer pavement system was proposed by Hoffman and Thompson in 1981 for flexible pavements (*15*). They proposed the AREA, given by the equation below, to characterize the deflection basin:

$$\text{AREA} = 6 * \left[1 + 2\left(\frac{d_{12}}{d_0}\right) + 2\left(\frac{d_{24}}{d_0}\right) + \left(\frac{d_{36}}{d_0}\right)\right] \qquad (3)$$

TABLE 1 Backcalculation of *k*-Value and Concrete Modulus from Interior and Edge Deflections and Comparison with Subgrade *k*-Value and Laboratory *E* Values, Arlington Test Site, 1930 (*10*)

Load Position	Season	Slab Thick	Effective k-value	Slab E
Interior	Late summer	6 in	195 psi/in	4,140,000 psi
	Winter	7	238	5,750,000
	Summer	7	222	4,670,000
	Winter	8	260	5,500,000
	Late fall	9	203	5,490,000
	Summer	9	220	4,210,000
	Means		223	4,960,000
Edge	Late summer	6	171	4,235,000
	Winter	7	212	5,125,000
	Winter	8	279	5,175,000
	Late fall	9	243	5,220,000
	Means		226	4,339,000

Comparisons: Field static plate bearing tests (using 30-in plate and 0.05-in deflection) directly on subgrade soil:

k-value = 166 (January) to 233 (June) psi / in.

Laboratory static concrete E modulus tests determined from flexural tests on specimens cut from slabs:

Dried for 12 months at normal atmosphere of the laboratory = 4,500,000 psi

Immersed for 10 months in water at laboratory temperature = 6,000,000 psi

where

d_0 = maximum deflection at the center of the load plate (in.) and
d_i = deflections at 12, 24, and 36 in. from plate center (in.).

AREA has units of length, rather than area, since each of the deflections is normalized with respect to d_0 to remove the effect of various load levels and to restrict the range of values obtained. AREA and d_0 are thus independent parameters, from which the surface and foundation elastic moduli may be determined. Hoffman and Thompson developed a nomograph for backcalculation of flexible pavement surface and subgrade moduli from d_0 and AREA.

During the early 1980s, the AREA concept was applied to backcalculation of slab *E* values and subgrade *k*-values for many concrete pavements (*16*). The ILLISLAB finite element program was used to compute a matrix of maximum deflections and AREA solutions by varying the *k*-value and *E* for a given slab thickness. A family of curves was then plotted, as shown in Figure 1, for a given slab thickness. Individual mid-slab deflection basins (AREA and maximum deflection) measured with the falling weight deflectometer could then be plotted on the matrix, and the slab *E* and foundation's *k*-value determined directly. This procedure was used to backcalculate hundreds of concrete pavement deflection basins during the 1980s and produced very reasonable and consistent results for many highway and airfield projects. The only drawback to this approach was its excessive labor intensiveness. An improvement in efficiency came in 1985 when the procedure was computerized by Foxworthy (*17*) using a vectoring scheme. However, the finite element program still had to be run several times to achieve a backcalculated *k*-value and slab *E* modulus (*17*).

The next major advancement was the development of a closed-form solution for backcalculation of the slab *E*, subgrade *k*-value, and subgrade *E* to replace the graphical procedures used previously. This solution was made possible by research by Barenberg and Petros (*18*) and by Ioannides (*19*), which demonstrated that, for a given load radius and sensor arrangement, a unique relationship exists between AREA and the "dense liquid" radius of relative stiffness of the pavement. A different unique relationship was also shown to exist between AREA and the "elastic solid" radius of relative stiffness of the pavement (in which the subgrade is characterized by an elastic modulus and a Poisson's ratio). In 1989, Ioannides et al. (*20*) demonstrated the application of this closed-form approach using the computer program ILLIBACK, which was developed to provide rapid analysis of deflection basins.

Recently, Hall (*21*) developed highly accurate equations for the dense liquid and elastic radii of relative stiffness versus AREA, using the plate-on-a-dense-liquid and plate-on-an-elastic-solid deflection equations presented by Losberg (*5*).

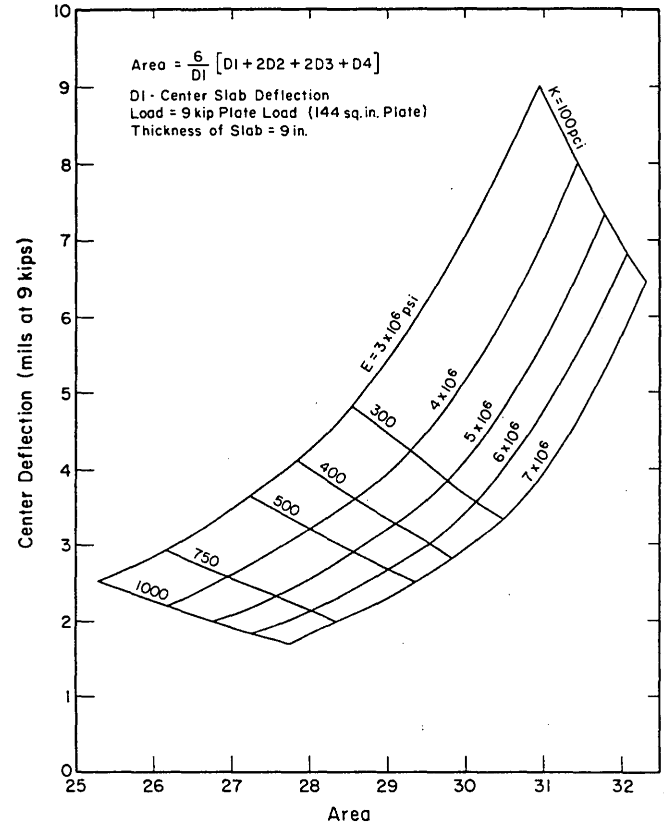

FIGURE 1 Graphical simultaneous solution of E and k for a known slab thickness.

The curve obtained for ℓ_k as a function of AREA is illustrated in Figure 2 and is given by the following equation:

$$\ell_k = \left[\frac{\ln\left(\frac{36 - \text{AREA}}{1812.279}\right)}{-2.55934}\right]^{4.387} \qquad (4)$$

Either the ILLIBACK program or the equations presented in this paper may be used to backcalculate the slab E and subgrade k-value. AREA is calculated from deflection basin measurements (Equation 3) and used to determine ℓ_k (Equation 4). The k-value may then be determined using Westergaard's deflection equation (Equation 2). With ℓ_k and k-value known, the slab Eh^3 value may then be computed from the definition of ℓ_k (Equation 1), and for a known slab thickness h, the concrete modulus E may be determined.

To illustrate this backcalculation procedure, Figure 3 was developed for determination of k-value for a load of P = 9,000 lb and an FWD load plate radius a = 5.9 in. The maximum deflection d_0 measured at loads within about 2,000 lb more or less than 9,000 lb may be scaled to an equivalent 9,000-lb maximum deflection (the AREA is the same regardless of whether the deflections are normalized to 9,000 lb), or the k-value may be determined from Equation 2. Figure 4 was developed for determination of slab E for a load radius of 5.9 in. and a concrete Poisson's ratio of 0.15.

The validity of using center-slab backcalculated k-values and slab E values at the slab edge was studied by Foxworthy and Darter (*17,22*). Results from several pavement sections showed that when the backcalculated k-value and slab E values obtained at the slab center were used at the slab edge, the edge deflections computed by ILLISLAB finite element program (for full contact conditions) agreed with the measured FWD edge deflections, after adjustment for load transfer. The deflections at the slab edge were predicted very well using this procedure, and it was concluded that it was entirely appropriate to backcalculate k and E from the center deflections and to use these values at the edge for stress calculations. This result is consistent with the finding that center and edge k-values at the Arlington test site were very similar.

Very little information is available from side-by-side tests to compare pavement responses backcalculated from FWD deflections measured on concrete pavement slabs to static plate load tests conducted on pavement subgrades. Foxworthy (*17*) compared backcalculated k-values and plate load k-values for seven sites and found that the backcalculated k-values exceeded the plate load k-values by a factor of 2.3 on average. This phenomenon is probably the result of a combination of differences between the two test methods, including testing on the subgrade versus testing on the top of the concrete slab,

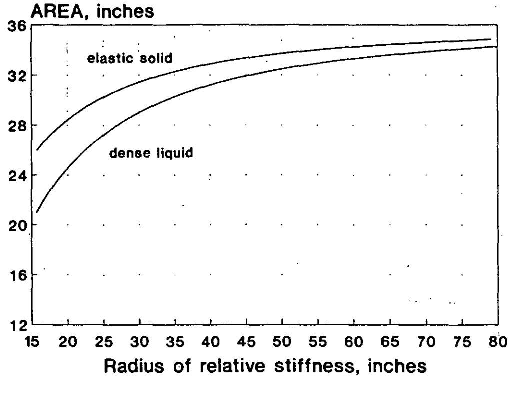

FIGURE 2 AREA versus dense liquid radius of relative stiffness (*21*).

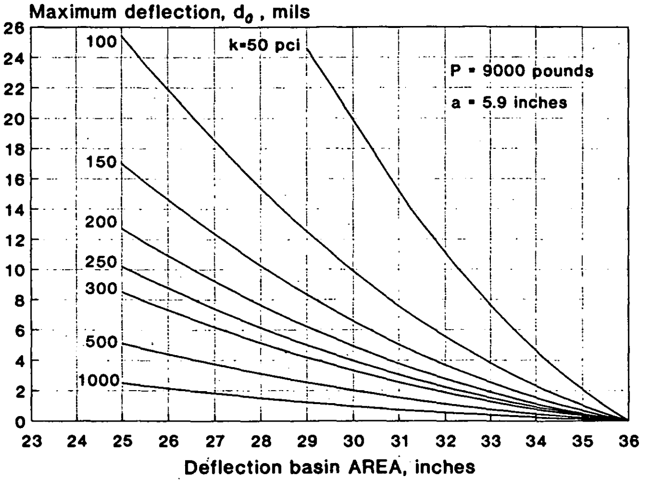

FIGURE 3 Determination of k-value from maximum deflection d_0 and AREA.

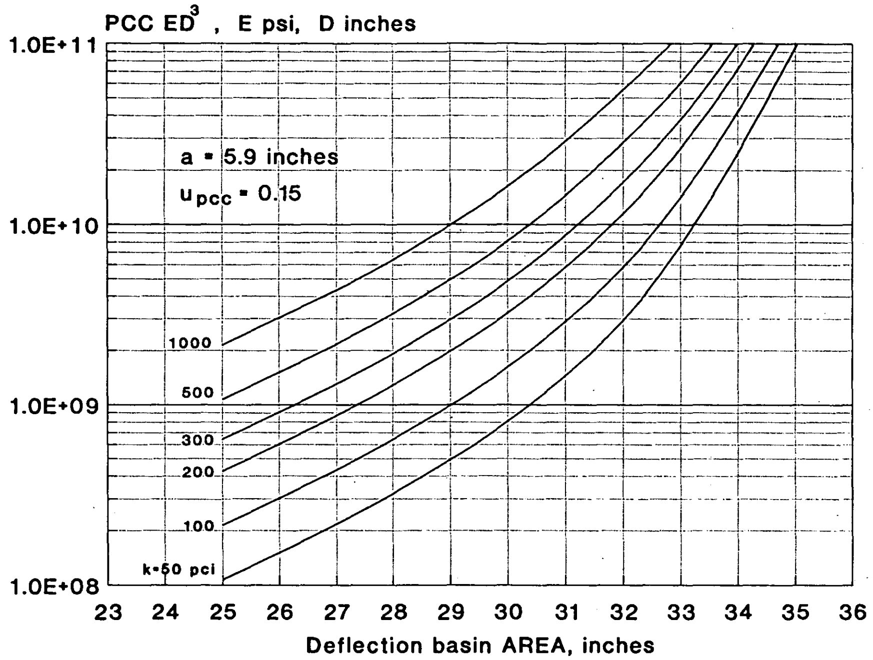

FIGURE 4 Determination of slab *E* from *k*-value, AREA, and slab thickness.

and the dynamic loading of the FWD versus the static loading of the plate load test. The possibility that static versus dynamic loading plays some role is also suggested by plate load tests conducted at the AASHO Road Test (*23*) in which repeated-load *k*-values on the subgrade were found to exceed static *k*-values on the subgrade by a factor of 1.77. Dividing backcalculated dynamic *k*-values by a factor of two to estimate static, plate load *k*-values for use in conventional design procedures (that require the static *k*-value) has yielded very reasonable results in analysis of many highway and airfield pavements over the past 10 years.

FIELD RESULTS FOR SLAB *E* AND SUPPORT *k*-VALUE

Field evaluations were recently conducted on 95 in-service jointed plain concrete pavement (JPCP) and jointed reinforced concrete pavement (JRCP) highway pavements located throughout the United States for FHWA (*24*). These pavements represented a diverse set of designs and climatic conditions. Deflection testing was conducted on these pavements using a Dynatest Model 8000 FWD with a target load of 9,000 lb. The deflection testing was conducted on 10 slabs for each project at slab centers and at joints and corners. The slab center locations were used to backcalculate in situ slab *E* and effective *k*-values using the plate theory concepts described in this paper. The backcalculation results are given in Table 2. This section summarizes the findings from the backcalculation.

In Situ Slab *E* Modulus

A histogram of all in situ slab *E* moduli obtained from backcalculation is shown in Figure 5. The mean of all values was

TABLE 2 Backcalculation Results from Field Studies (*24*)

SECTION	SLAB THICK (IN)	*IN SITU* SLAB E (MPSI)	CONCRETE FLEX STR (PSI)	BASE TYPE	SUBGRADE TYPE	*IN SITU* K-VALUE (PSI/IN)
MN 1-1	9.0	7.09	N/A	AGG	A-6	191
MN 1-2	9.0	7.75	801	AGG	A-6	172
MN 1-3	8.0	6.66	N/A	AGG	A-6	217
MN 1-4	8.0	6.92	552	AGG	A-6	222
MN 1-5	8.0	9.13	N/A	ATB	A-6	304
MN 1-6	8.0	9.36	587	ATB	A-6	314
MN 1-7	9.0	8.30	N/A	ATB	A-6	207
MN 1-8	9.0	7.88	689	ATB	A-6	278
MN 1-9	9.0	6.67	N/A	CTB	A-6	291
MN 1-10	9.0	6.74	735	CTB	A-6	285
MN 1-11	8.0	8.03	N/A	CTB	A-6	245
MN 1-12	8.0	7.79	763	CTB	A-6	239

(*continued on next page*)

TABLE 2 *(continued)*

SECTION	SLAB THICK (IN)	*IN SITU* SLAB E (MPSI)	CONCRETE FLEX STR (PSI)	BASE TYPE	SUBGRADE TYPE	*IN SITU* K-VALUE (PSI/IN)
MN 5	9.0	7.56	N/A	AGG	A-6	156
MN 2-1	9.0	6.78	682	AGG	A-2-7	128
MN 2-2	8.0	8.01	N/A	AGG	A-2-6	127
MN 2-3	9.0	7.32	743	AGG	A-2-6	162
MN 2-4	9.0	6.62	N/A	AGG	A-2-6	178
MN 3	9.0	8.81	N/A	AGG	A-4	256
MN 4	7.5	6.30	832	AGG	A-2-6	222
MN 6	8.0	6.57	616	PATB	A-2-4	199
AZ 1-1	9.0	3.14	687	CTB	A-4	546
AZ 1-2	13.0	3.44	649	NONE	A-6	492
AZ 1-4	13.0	3.49	702	NONE	A-6	344
AZ 1-5	11.0	3.29	761	NONE	A-6	439
AZ 1-6	9.0	3.09	853	LCB	A-6	621
AZ 1-7	9.0	3.69	868	LCB	A-6	584
AZ 2	10.0	5.56	725	LCB	A-6	174
CA 1-1	8.4	6.61	N/A	CTB	A-1-a	232
CA 1-3	8.4	5.24	723	CTB	A-2-4	349
CA 1-5	11.4	5.28	918	CTB	A-1-a	335
CA 1-7	8.4	6.48	781	LCB	A-1-a	433
CA 1-9	8.4	6.95	802	CTB	A-1-a	298
CA 7	10.2	6.26	552	CTB	A-2-4	326
CA 2-2	8.4	7.04	730	PCTB	A-4	1423
CA 2-3	8.4	4.98	644	CTB	A-4	572
CA 8	10.2	6.42	727	ATB	A-7	339
CA 6	9.0	6.71	791	LCB	A-2-4	294
MI 1-1a	9.0	5.45	745	AGG	A-2-4	353
MI 1-1b	9.0	5.79	N/A	AGG	A-2-4	300
MI 1-4a	9.0	5.88	756	PATB	A-2-4	468
MI 1-7a	9.0	6.34	744	AGG	A-2-4	292
MI 1-7b	9.0	6.09	N/A	AGG	A-2-4	269
MI 1-10a	9.0	6.23	N/A	ATB	A-2-4	436
MI 1-10b	9.0	5.28	N/A	ATB	A-2-4	502
MI 3	10.0	4.38	810	PAGG	A-2-4	186
MI 4-1	9.0	4.83	596	AGG	A-4	283
MI 4-2	9.0	4.53	756	AGG	A-4	189
MI 5	10.0	4.49	671	PAGG	A-2-4	233
NY 1-1	9.0	3.81	N/A	ATB	A-2-4	549
NY 1-3	9.0	3.89	809	ATB	A-2-4	503
NY 1-4	9.0	3.89	658	AGG	A-2-4	534
NY 1-6	9.0	4.02	N/A	AGG	A-1-a	619
NY 1-8a	9.0	4.10	684	ATB	N/A	638
NY 1-8b	9.0	3.88	N/A	ATB	A-2-4	548
NY 2-3	9.0	5.10	864	AGG	A-1-a	341
NY 2-9	9.0	5.09	705	AGG	A-1-a	471
NY 2-11	9.0	6.12	812	AGG	A-1-a	296
NY 2-15	9.0	5.52	N/A	AGG	A-1-a	273
OH 1-1	9.0	4.43	686	AGG	A-6	449
OH 1-3	9.0	5.31	N/A	ATB	A-4	440
OH 1-4	9.0	5.23	N/A	ATB	A-4	525
OH 1-6	9.0	4.06	761	AGG	A-4	431
OH 1-7	9.0	4.43	848	AGG	A-4	340
OH 1-9	9.0	3.40	833	AGG	A-6	351
OH 1-10	9.0	4.51	788	AGG	A-6	405
PA 1-1	10.0	4.21	731	CTB	A-2-4	731
PA 1-2	10.0	3.39	720	PATB	A-4	1040
PA 1-3	10.0	3.23	709	PAGG	A-4	538
PA 1-4	10.0	4.53	870	PAGG	A-2-4	747
PA 1-5	10.0	3.62	704	AGG	A-4	540

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TABLE 2 (*continued*)

SECTION	SLAB THICK (IN)	*IN SITU* SLAB E (MPSI)	CONCRETE FLEX STR (PSI)	BASE TYPE	SUBGRADE TYPE	*IN SITU* K-VALUE (PSI/IN)
NJ 2-1	10.0	6.72	700	AGG	A-4	234
NJ 3-1	9.0	5.40	681	PAGG	A-1-a	356
NJ 3-2	9.0	5.33	726	PATB	A-2-4	210
NC 1-1	9.0	4.63	736	AGG	A-2-4	538
NC 1-2	9.0	5.19	674	CTB	A-2-4	347
NC 1-3	9.0	3.97	705	CTB	A-2-6	494
NC 1-4	9.0	4.21	709	AGG	A-2-6	570
NC 1-5	9.0	5.50	674	CTB	A-4	628
NC 1-6	9.0	5.14	559	ATB	A-2-6	672
NC 1-7	8.0	5.09	644	AGG	A-2-4	128
NC 1-8	9.0	4.22	705	AGG	A-2-6	513
NC 2	11.0	5.89	712	LCB	A-4	293
FL 2	13.0	5.55	664	AGG	A-3	378
FL 3	9.0	4.16	599	LCB	A-3	529
CA 3-1	9.0	3.53	N/A	CTB	A-4	286
CA 3-2	9.0	4.17	796	CTB	A-4	312
CA 3-5	9.0	4.38	842	CTB	A-4	397

Note:
AGG = untreated aggregate base
PAGG = permeable aggregate base
ATB = asphalt-treated base
PATB = permeable asphalt-treated base
CTB = cement-treated base
PCTB = permeable cement-treated base
LCC = lean concrete base
SAND = sand base

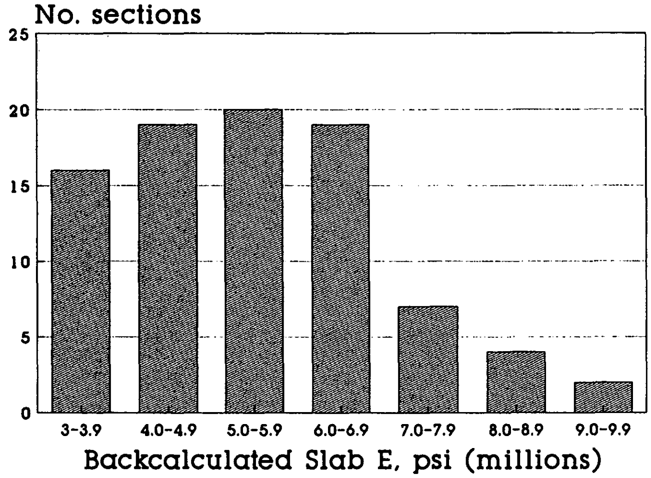

FIGURE 5 Histogram of slab *E* values backcalculated from field studies.

5.4 million lb/in.^{2} with a range of 3 to 9.4 million lb/in.^{2}. These slabs have a mean age of 15 years; therefore, the slab moduli should be expected to be greater than elastic moduli corresponding to typical 28-day-strength values assumed in design.

A comparison of the mean in situ slab elastic moduli for each base type may be made from these data. The results shown below indicate no effect of base stiffness on the backcalculated slab moduli.

Base Type	*No. of Sections*	*In Situ Slab E (lb/in.^{2})*
Aggregate or no base	40	5,400,000
Asphalt-treated	19	5,700,000
Cement-treated	20	5,600,000
Lean concrete	7	5,100,000

A correlation between in situ slab E and the estimated flexural strength of the concrete (from indirect tensile tests on cores) was attempted using the data from Table 2, but no correlation was found. However, this may be partially because only two cores were tested for each section for strength determination. A better correlation was achieved by Foxworthy using data from nine pavement sections, as shown in Figure 6 (*17*).

$$FS = 43.5\left(\frac{E}{10^6}\right) + 488.5 \tag{5}$$

where

FS = flexural strength, estimated from indirect tensile strength (lb/in.^{2}) and
E = in situ modulus of elasticity, backcalculated from FWD data (lb/in.^{2}).

When Foxworthy's prediction model was plotted on the E-versus-FS graph, the curve passed through the center of the scatter of data. This estimate is approximate and should be used with caution when it is not feasible to obtain and test concrete cores.

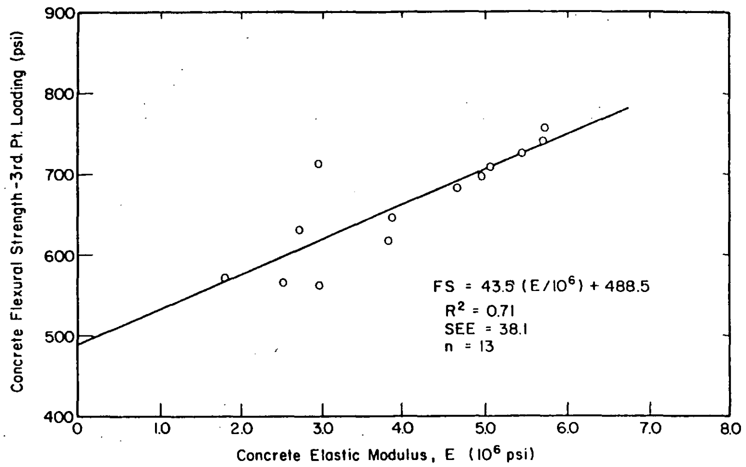

FIGURE 6 Correlation of concrete flexural strength to backcalculated *E*-value from Foxworthy (*17*).

In Situ *k*-Value

A summary of the in situ *k*-values (below the slab) obtained from various sections, sorted by base type and subgrade type, is shown in Table 3. The following observations may be made concerning the results:

1. The in situ *k*-value does not correlate well with subgrade soil classification. The mean *k*-value was computed for fine-grained and coarse-grained soils for each base type. Only the asphalt-treated base course showed a higher *k*-value for coarse-grained subgrade than fine-grained soil. The others were approximately the same.

2. Pavements with asphalt-treated, cement-treated, and lean concrete base layers generally have higher *k*-values than pavements with untreated bases, as shown in the frequency distributions in Figure 7. For example, a high percentage of sections with treated bases had in situ *k*-values greater than 500 lb/in.^{2}/in.

3. The estimated static *k*-values are shown in parentheses. These were obtained by dividing the backcalculated *k*-values by two. For sections with no base or with granular base, the static *k*-values were similar to typical recommended values. For sections with treated bases, the mean values are lower than those that would normally be obtained by conventional methods (e.g., elastic layer program simulation of a plate

TABLE 3 Summary of Backcalculated *k*-Values Obtained from Field Testing (*24*)

Base Type	Backcalculated Mean k-value (psi/in)	Range	Number of Sections
No base	425 (213)*	344-492	3
Aggregate	318 (159)	127-619	35
Permeable-Aggregate	258 (129)	186-356	3
Asphalt-treated	447 (224)	207-672	14
Permeable-Asphalt	534 (267)	199-1040	6
Cement-treated	384 (192)	232-731	19
Permeable-Concrete	1423 (711)	1423	1
Lean concrete	418 (209)	174-621	7

*Estimated static k-value (backcalculated/2)

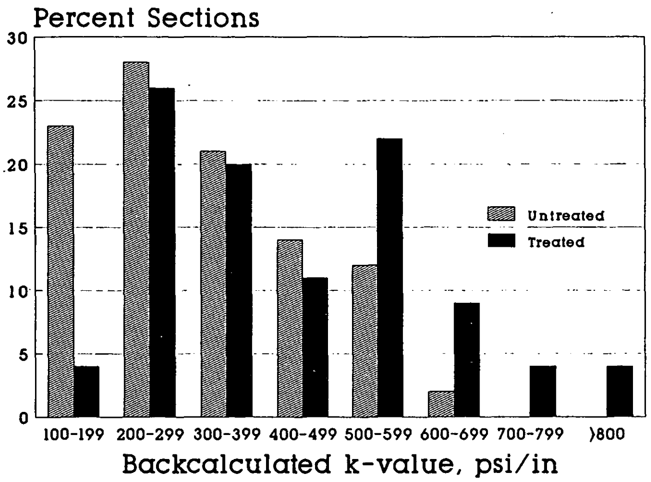

FIGURE 7 Histogram of backcalculated *k*-values from field studies for untreated aggregate and treated bases.

load test) for determining the effective *k*-value on top of a treated base.

IMPLICATIONS FOR PAVEMENT DESIGN AND EVALUATION

For structural evaluation or overlay design, the in situ *k*-value and slab elastic modulus may be easily backcalculated using the techniques presented in this paper or by the ILLIBACK program. The *k*-value and slab *E* backcalculated from FWD deflections should be considered as the moduli that the layers exhibit under moving loads. If the pavement under design is subject to static loads, or if the design procedure requires the input of a static *k*-value (e.g., the FAA and AASHTO procedures), the backcalculated *k*-value should be reduced by approximately 50 percent. The subgrade soil type (fine or coarse-grained) did not appear to significantly affect the in situ *k*-value.

The flexural strength of the slab may be roughly estimated from the slab *E* modulus, but it is preferable to take cores, test them for indirect tensile strength, and estimate the flexural strength. These values, along with the transverse and longitudinal joint load transfer, may then be used to determine the load-carrying capacity of the pavement as well as the required overlay thickness.

For new design, where no slab of similar design exists in the vicinity from which backcalculated slab and support values may be determined, the following *k*-values and the slab moduli shown in Figure 5 may be used as reasonable approximate values.

Base Type	*Backcalculated Mean k-value (lb/in.^{2}/in.)*	*Estimated Static k-value (lb/in.^{2}/in.)*
No base	425	213
Aggregate	318	159
Permeable aggregate	258	129
Asphalt-treated	447	224
Permeable asphalt	534	267
Cement-treated	384	192
Lean concrete	418	209

Existing conventional methods for estimating *k*-values for use in new design may also produce reasonable results, especially for slabs with no base or only untreated aggregate base. However, when a very stiff base is being constructed, the *k*-value used to design the concrete pavement should not be increased to the extent that conventional practice to date suggests.

The *k*-value is not a unique material property but depends on several factors such as slab and loading parameters. The only legitimate way to define a *k*-value is "beneath a slab," and not "on top of a base layer."

CONCLUSIONS

The backcalculation procedures described in this paper provide a rapid and reliable method to determine the in situ slab *E* modulus and the effective *k*-value supporting the slab. These slab and support values should be considered representative of the pavement's response to moving wheel loads. In situ *k*-values backcalculated from FWD deflections must be modified for use in design procedures that are based on static plate load test *k*-values or for design for static loads. Dividing the FWD backcalculated *k*-value by approximately two gives a reasonable estimate of the static *k*-value. It is not necessary to remove slabs and conduct static plate load tests to obtain a reasonable static *k*-value for use in design.

In situ slab *E* moduli were found to vary from 3 to 9 million lb/in.^{2}, with a mean of 5.4 million lb/in.^{2}, in the field tests summarized in this paper. The Arlington tests showed that the backcalculated slab moduli (for static loads) were very close to those obtained from static laboratory testing of samples from slabs.

Further research is needed to develop methods for backcalculation of the *E* moduli of two stiff layers over a dense liquid subgrade (e.g., a concrete slab and a treated base). It is recommended that when a slab and stiff base layer are present they be modeled as two plates over a dense liquid subgrade, instead of as one plate over a dense liquid subgrade.

Further research is also needed to improve methods for estimation of the subgrade *k*-value for pavements with stiff bases. Currently there is no method available to determine the correct *k*-value for use in design of a new concrete pavement when backcalculation of existing similar pavements cannot be done. Conventional methods (plate bearing tests, correlation with soil properties, or estimation from elastic layer analyses) may produce reasonable results for slabs on grade or on untreated aggregate bases. However, increasing the *k*-value to account for a stiff base layer is likely to produce a value higher than the actual in situ value.

One final comment is made regarding the use of elastic layer theory for backcalculating the moduli of concrete slabs and underlying layers. Although this method has been used with reasonable success by the authors and other researchers, the values obtained can be used only with elastic layer theory to compute interior stresses, strains, and deformations. They cannot be used to compute edge or corner stresses because the moduli are not compatible. In addition, no unique relationship exists between the elastic subgrade *k*-value, because the *k*-value also depends on the loading configuration and other factors.

GLOSSARY

The following definitions are given for terms used in this paper.

- Static *k*-value on subgrade: determined from static plate load tests performed on the subgrade using a 30-in.-diameter plate, lb/in.^{2}/in.
- In situ *k*-value beneath slab: backcalculated from FWD deflections on top of the slab at an interior position using Westergaard's center-slab deflection equation, lb/in.^{2}/in.
- Static modulus of elasticity (E) of slab concrete: determined from samples cut from the pavement and tested in flexure in the laboratory under static load.
- In situ slab modulus of elasticity (E): backcalculated from FWD deflections at the center of the slab using backcalculation techniques.

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