Current rigid pavement backcalculation procedures use only interior or center slab deflection bowls that are measured far from any crack or joint. Usually two parameters—the rigidity of the slab and the modulus of subgrade reaction—are evaluated. The backcalculated subgrade modulus is recommended for design subject to a correction/calibration factor. However, since the modulus of subgrade reaction depends on the size of the loaded area, various calibration factors may be required for various loading conditions at the time of the testing and for various assumptions in the design procedure concerning the loading condition, namely interior or edge loading conditions. Furthermore, when a stabilized layer is used beneath the concrete slab it is not clear what the backcalculated parameters represent. Does the backcalculated rigidity of the slab include the stabilized layer or is it included in the backcalculated subgrade modulus of reaction? The answers to these questions have an impact on the computation of stresses that are the basis of any rigid pavement design procedure. A procedure for backcalculating the slab rigidity and the modulus of subgrade reaction for either the center slab or the free-edge loading condition is briefly described. The material parameters and the slab rigidity were backcalculated using the plate on Winkler foundation for interior and free-edge loading conditions. The center slab results are compared with those that are backcalculated using linear elastic layer models. The proposed method uses the MODULUS computer program framework in which a data base of theoretical deflection bowls is generated and used thereafter with a pattern search algorithm to find the set of parameters (the characteristic length \( \ell \) and the subgrade modulus of reaction \( k \)) that minimizes the error between measured and computed deflections. Results backcalculated from five Strategic Highway Research Program jointed concrete monitor sites are presented. The slab thicknesses range between 9.3 and 11.4 in., and the base layers are either granular, asphalt, or cement stabilized. Deflection tests were made at three load levels using a falling weight deflectometer. Results include (a) a comparison of \( \ell \) and \( k \) values backcalculated at the center slab and at the edge (it is clearly seen that the \( k \)-values derived from free-edge condition are two to four times larger than those obtained from center slab deflections) and (b) a comparison of results computed using both the Hogg model and the linear elastic layered model. It is clearly seen that the Hogg model produces subgrade modulus similar to those produced using the linear elastic model. Furthermore, with the Hogg model the effect of the stabilized base layer is included in the \( \ell \)-parameter.

The evaluation of the load-deformation characteristics from deflection-based nondestructive deflection testing (NDT) is, in essence, similar for flexible and rigid pavements \( (f) \). However, because of the existence of joints, cracks, and free edges, the boundary conditions prevailing in rigid pavements are more complex than those in flexible pavements. These conditions are best treated with finite element solutions of plates resting on one- or two-layer systems or on a Winkler foundation. When load transfer characteristics are included in the evaluation, the problem becomes too complicated for standard backcalculation procedures.

The approach is presented in Uzan \((I) \) and some is repeated here for completeness. The backcalculation scheme makes use of simple solutions that assume that the cracks or joints are far from the load and do not affect the deflection bowl. These solutions are applicable for plain or simply reinforced concrete pavements in which cracks are widely separated, in contrast to continuously reinforced concrete pavements in which cracks are fairly close. In general, the distance between joints and cracks would be less than eight times the characteristic length of the system \([in which case the size of the plate may be considered as infinite \((2,3)\)]\). The use of the simple solutions constitutes an approximation. The framework presented in the paper is general and finite element programs instead of the simple solutions could be used for deflection computations.

In this paper a backcalculation procedure is described to determine the appropriate layer properties from field deflection measurements. The procedure involves two steps: in the first step, a factorial deflection data base is built using multiple runs of a theoretical model; in the second step, a pattern search and interpolation scheme are used to match field measured and theoretical deflection bowls. The output is the set of layer properties that minimizes the error between measured and computed deflections. In this paper the authors compare the results from three different theoretical pavement models: (a) a multilayer linear elastic model; (b) Hogg model, and (c) Hertz-Westergaard model. Furthermore, calculations with deflections measured near a free edge can be compared with those made with deflections measured at the center of the slab. This analysis enables an examination of the significance of the \( k \)-subgrade modulus of reaction and the procedures of modeling rigid pavements with stabilized base layers.

### Theoretical Models for Computing Slab Deflections

Three types of models are used for evaluating the load-deformation characteristics of pavement materials. These are the multilayer linear elastic model, the Hogg model of a slab supported by an elastic foundation, and the Hertz-Westergaard model of a slab supported by a liquid foundation. These models all use linear load-deformation characteristics, implying con-

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**Backcalculation of Design Parameters for Rigid Pavements**

J. Uzan, R. Briggs, and T. Scullion

The evaluation of the load-deformation characteristics from deflection-based nondestructive deflection testing (NDT) is, in essence, similar for flexible and rigid pavements. However, because of the existence of joints, cracks, and free edges, the boundary conditions prevailing in rigid pavements are more complex than those in flexible pavements. These conditions are best treated with finite element solutions of plates resting on one- or two-layer systems or on a Winkler foundation. When load transfer characteristics are included in the evaluation, the problem becomes too complicated for standard backcalculation procedures.

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stant moduli of elasticity, Poisson's ratios, and modulus of subgrade reaction.

In the multilayer elastic system model, the pavement is represented by linear elastic layers that extend to infinity in the horizontal direction. This model, widely used in flexible pavement evaluation, corresponds to the interior loading case in which both the load and the deflection sensors are far from any joint or crack. The boundary conditions at the interfaces from the layers can be varied from completely rough to smooth (4,5). In the case of a concrete pavement with unbound base material, the interface conditions have negligible effects on deformations and stresses. However, when the base layer is stabilized, the effects of interface conditions on stresses and deformations may be significant.

The second model considered is the Hogg model (6,7). With this model the pavement is represented by an infinite slab resting on an elastic foundation. This model, like the linear elastic model, corresponds to the interior loading case only. The model is in essence a two-layer system with additional assumptions concerning the first layer, which is represented by a slab (8). In this model, the pavement parameters are the subgrade modulus of elasticity, $E_{sg}$, and the radius of relative stiffness of the slab subgrade system $\ell_s$ (which is a function of the elastic parameters of the concrete and of the subgrade and the slab thickness).

The third model used is the Hertz-Westergaard model (9,10). With this model the pavement is represented by an infinite or semi-infinite slab resting on a dense liquid foundation (also known as a Winkler foundation). In this model, two loading cases are considered: (a) the interior loading case (known as the Hertz model) in which the slab is infinite and the load and deflection sensors are far from any joint or crack; and (b) the free-edge case (known as the Westergaard model) in which the slab is semi-infinite and the load and deflection sensors are near a free joint and far from any additional joint or crack. The pavement parameters are the subgrade modulus of reaction, $k$, and the radius of relative stiffness of the slab-subgrade system $\ell_s$ (which is a function of the elastic parameters of the concrete, the subgrade modulus of reaction and the slab thickness).

Closed-form solutions for deflection bowls exist only for the case of a concentrated load (9,11). In the case of a uniform pressure distributed over a circular area, the solution of the concentrated load may be integrated numerically. In the interior loading case, a computer program given by Selvadurai (11) was used to compute the deflections at the surface. In the free-edge loading case, Westergaard equations (12) were integrated over the loading area using the procedure presented by Uzan and Sides (13).

In the three-layer and Hogg models, a rigid base at any depth in the subgrade can be easily included. The Hertz-Westergaard model does not have this capability because the subgrade is represented by a spring or a liquid that is dimensionless.

**BACKCALCULATION PROCEDURE**

The backcalculation of layer properties for rigid pavement follows the MODULUS backcalculation framework already developed for flexible pavements. It is described in detail elsewhere (14,15). For flexible pavements the outputs are the set of layer moduli that minimize the error between measured and theoretically calculated deflection bowls. The existing system is a two-step process. In the first step, a linear elastic program is run for a range of layer moduli, and the resulting deflections are stored in a deflection data base. In the second step, a pattern search and interpolation scheme are used to minimize the error between measured and computed deflection bowls.

The multilayer system model described in this paper is essentially the same as the flexible pavement system. The data base is built by varying the moduli of each pavement layer in turn and storing the computed deflections.

In both the Hogg and the Hertz-Westergaard models, the data base is built by varying the subgrade modulus $E_{sg}$ (or the subgrade modulus of reaction $k$) and the radius of relative stiffness $\ell_s$ (or $\ell_s$). In the analysis discussed for the Hogg model, 10 values of subgrade modulus (range 2,000 to 75,000 lb/in.$^2$) and 10 values of $\ell_s$ (range, 10 to 210 in.) were computed. This resulted in 100 theoretical deflection bowls being generated and stored. For Hertz-Westergaard, a 10 \times 10 factorial was again stored with ranges of $k$ for 25 to 1,500 lb/in.$^2$ and $\ell_s$ from 10 to 140 in. These ranges are intended to represent the limits of possible values.

The program can handle any sensor arrangement. In all cases deflections are computed at the sensor locations used in the field measurements. For the SHRP testing program this is at 0, 8, 12, 18, 24, 36, and 60 in. from the center of the load. Only one data base of computed surface deflections is required for each sensor arrangement, and the first step of the procedure need not be repeated if the data base is already available. The procedure used in this paper can handle any number of sensors or any sensor arrangement. Moreover, the Hogg model includes both the infinite and finite subgrade thickness cases. The procedure presented is unique in that both center and free-edge deflection bowls can be analyzed to backcalculate the modulus of subgrade reaction and the modulus of elasticity of the concrete.

The results of the numerical integration have been checked against other analysis methods, such as the FE results presented in the appendix, and have been found to be comparable. A direct comparison of the results of backcalculation obtained with the procedure described in the paper (named JUSLAB) and with the ILLI-BACK computer program follows. Table 1 presents results of backcalculation for eight rigid pavement sections at three different load levels. The procedures and objective functions of the backcalculation are usually different in JUSLAB and ILLI-BACK programs. For the sake of comparison, the procedure and objective function in JUSLAB were changed to resemble those in ILLI-BACK. In ILLI-BACK, the computation procedure is (a) to obtain the characteristic length from the basin area; (b) to compute a $k$-value for each sensor; and (c) to compute a mean value. It appeared that in several cases analyzed the computed $k$-value from the last sensor deflection was much lower than the other $k$-values obtained from the other six sensors. The procedure of taking the mean is then similar to dropping the last sensor. The results of backcalculation using JUSLAB were obtained by fitting the first six sensors (by means of weighting factors) and minimizing the squared error between measured and computed deflections at these six sensors. The results obtained with JUSLAB and ILLI-BACK compare very well; different
### TABLE 1 Comparison of Backcalculation Results

<table>
<thead>
<tr>
<th>Section</th>
<th>Subgrade Modulus of Reaction in pci, for Load Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JUSLAB</td>
<td>ILLI-BACK</td>
<td>JUSLAB</td>
<td>ILLI-BACK</td>
</tr>
<tr>
<td>I</td>
<td>564</td>
<td>505</td>
<td>555</td>
<td>518</td>
</tr>
<tr>
<td>J</td>
<td>249</td>
<td>252</td>
<td>254</td>
<td>314</td>
</tr>
<tr>
<td>K</td>
<td>162</td>
<td>144</td>
<td>262</td>
<td>323</td>
</tr>
<tr>
<td>L</td>
<td>125</td>
<td>143</td>
<td>226</td>
<td>139</td>
</tr>
<tr>
<td>M</td>
<td>296</td>
<td>143</td>
<td>286</td>
<td>288</td>
</tr>
<tr>
<td>N</td>
<td>591</td>
<td>501</td>
<td>592</td>
<td>618</td>
</tr>
</tbody>
</table>

Results will be obtained if the seventh sensor is not dropped from the analysis and if the objective function to be minimized is changed, for example, from squared error to absolute error.

### CASE STUDY DESCRIPTION

To evaluate the backcalculation procedure, Strategic Highway Research Program (SHRP) falling weight deflectometer (FWD) data were processed on five SHRP general pavement study sites in Texas. Average slab thicknesses and other information are given in Table 2. All the pavements were jointed concrete with asphalt shoulders.

In the SHRP deflection testing procedure, deflections are taken at midslab, edge, corner, and pavement joints. In this analysis the center slab and the edge FWD deflection data were used. SHRP uses three loading levels for rigid pavements (approximately 9, 12, and 16 kips) and collects four replicate drops at each test point. In this analysis the following four replicate drops were normalized to the average load and then averaged so that a single deflection bowl was processed for each load level. This procedure takes care of the random errors, as planned by SHRP. Therefore, the averaging process used includes (a) computing the average load, (b) normalizing each of the four deflection bowls to the average load, and (c) averaging the normalized bowls.

In general, 60 deflection bowls were processed for each site, three load levels at 20 positions. In all cases the load was applied on a 12-in.-diameter FWD load plate. Deflection sensors were located at 0, 8, 12, 18, 24, 36, and 60 in. from the center of the plate.

### PRESENTATION OF RESULTS

The results of this analysis are shown in Figures 1 through 5, details of which follow. The results from all five sites are presented on a single diagram. Within each site the results from various locations and load levels are presented. In each analysis all seven sensors were used in matching measured

### TABLE 2 Layer Characteristics of Study Pavements

<table>
<thead>
<tr>
<th>SHRP ID #</th>
<th>Pavement Characteristics</th>
<th>Joint Spacing</th>
<th>Slab Thickness (Inches)</th>
<th>Base Type</th>
<th>Thickness (Inches)</th>
<th>Subbase Type</th>
<th>Thickness (Inches)</th>
<th>Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>3003</td>
<td>Plain</td>
<td>15 ft</td>
<td>9.3</td>
<td>AC</td>
<td>3.7</td>
<td>LT</td>
<td>7.2</td>
<td>Clay</td>
</tr>
<tr>
<td>3589</td>
<td>Reinforced</td>
<td>15 ft</td>
<td>10.1</td>
<td>GR</td>
<td>6.0</td>
<td>--</td>
<td>--</td>
<td>Sandy/Clay</td>
</tr>
<tr>
<td>3699</td>
<td>Reinforced</td>
<td>60 - 20 ft+</td>
<td>10.1</td>
<td>CT</td>
<td>5.8</td>
<td>LT</td>
<td>6.0</td>
<td>Sandy/Clay</td>
</tr>
<tr>
<td>4142</td>
<td>Reinforced</td>
<td>60 - 20 ft+</td>
<td>9.6</td>
<td>AC</td>
<td>7.9</td>
<td>SS</td>
<td>8.8</td>
<td>Sand</td>
</tr>
<tr>
<td>4152</td>
<td>Reinforced</td>
<td>30 ft++</td>
<td>11.4</td>
<td>CT</td>
<td>6.3</td>
<td>LT</td>
<td>5.6</td>
<td>Sandy/Clay</td>
</tr>
</tbody>
</table>

*Codes: AC, Asphaltic Concrete; GR, Crushed Gravel; CT, Cement Treated; LT, Lime Treated; SS, Select Sand

+ 60 ft 6 in between construction joints with towels 20 ft between warping joints.
++ with transverse cracks in each slab.
and theoretical bowls, and the average error per sensor was computed. Figure 1 shows the percent error per sensor between measured and computed deflection. It is seen that the error is very large, approaching 10 percent in the cases of the Hertz and Westergaard models compared with the Hogg or layered linear elastic model. In the following analysis, all solutions that resulted in more than 6 percent error per sensor (and solutions that resulted in unacceptable results) have been excluded for Figures 2 through 5.

Figure 2 shows the results for the linear elastic layered analysis in terms of the backcalculated modulus of the subgrade and modulus of concrete at center slab. In modeling the pavement, the complete layering shown in Table 2 was used. For example, for section 4152 the pavement was modeled as four layers (11.4 in. of concrete resting on a 6.3-in. cement-treated base over a 5.6-in. lime-treated subbase over a semi-infinite depth subgrade). In all modeling, the subgrade depth was set at semi-infinite. The backcalculated subgrade moduli ranged from 20 to 50 ksi and characteristic length \( \ell_r \) ranged from 25 to 45 in.

Figure 4 shows the backcalculated results obtained using the Hertz model with the center slab deflections. The modulus of subgrade reactions ranged typically from 50 to 200 lb/in.\(^2\), with a few readings reaching 300 lb/in.\(^2\). The characteristic lengths ranged typically from 40 to 70 in.

Figure 5 shows the backcalculated results obtained using the Westergaard edge solution. The modulus of subgrade reactions ranged typically from 200 to 700 lb/in.\(^2\) with characteristic lengths ranging from 35 to 60 in.

**DISCUSSION OF RESULTS**

The following observations are made with regard to the results presented in Figures 1 through 5.

1. Even after screening, the error in the fitting of the deflection bowl seems to be very large in the Hertz and Westergaard models. However, the error is acceptable for the

![Figure 1](https://example.com/fig1.png)

**FIGURE 1** Average error per sensor between computed and measured deflection: (a) linear layered elastic model; (b) Hogg with center deflections; (c) Hertz with center deflections; and (d) Westergaard with edge deflections.
Hogg and layered linear elastic models. In the case of rigid pavements, material behavior is very close to linear and does not induce appreciable systematic error. It may be argued that the error is caused by the boundary conditions that are not fulfilled because of the proximity of the cracks and joints. However, this deficiency exists for both the Hertz and Westergaard models, as well as for the Hogg and layered linear elastic models. It is the authors' opinion that the bad fitting is the result of systematic errors induced by the model representation of the subgrade support, namely, that in the cases of strong subgrade, with elastic moduli of 20 to 45 ksi, the subgrades cannot be simulated as liquid or spring. The above hypothesis should be checked with weaker subgrade materials. If this is found to be the case, the use of the $k$ model should be questioned in rigid pavement design. The backcalculation was made using all sensors, including the seventh sensor, which was found in several cases to be underpredicted. A similar trend is found in ILLI-BACK results in which the error caused by the seventh sensor may be in excess of 30 percent.

2. The modulus of subgrade reaction at the edge is two to four times higher than that obtained from the center slab. A discussion of this finding together with a comparison of earlier results, such as those of Teller and Sutherland (16), is given in the appendix to the paper. The $k$-values at the center of the slab appear very low. As will be discussed later, the modulus of subgrade reaction represents the subgrade layer and does not include the stabilized base material on top of the subgrade. This result suggests that the modulus of subgrade reaction is not an adequate material property for modeling slab response because it is a function of the loading geometry. Using linear elastic theory, one can find that the subgrade modulus of reaction is proportional to the modulus of elasticity and inversely proportional to the size of the loaded area.

3. From Figures 2 and 3 it is observed that the modulus of the subgrade is identical for both the linear layered elastic model and the Hogg model. The Hogg model is a simplified two-layered system, whereas the linear elastic is either a three- or four-layered system. This difference implies that in the Hogg model, the base-subbase layers are included in the plate rigidity to give a composite plate that rests on the subgrade. This result is very important for choosing the computation scheme of the bending stresses. In this case the method of equivalent moment of inertia or thickness should be used in...
computing the bending stresses. Similar behavioral effect is assumed to occur for the Hertz-Westergaard model, namely that the stiffness effect of the base and subbase layers is included in the plate (pavement) rigidity (1).

4. In all cases the $\epsilon$ values backcalculated are relatively large. However, as mentioned earlier, it appears that this value includes a contribution from the base and subbase layers. To evaluate this contribution the equivalent thickness $h_{eq}$ for the slab was calculated using the following formula:

$$h_{eq}^3 = \frac{\epsilon^2 \times 12(1 - v^2)}{E_c}$$

for the Hertz-Westergaard model and

$$h_{eq}^3 = 2C \frac{12(1 - v^2)}{E_c} \epsilon_c$$

$$C = \frac{E_s (1 - v_s)}{(1 + v_s)(3 - 4v_s)}$$

for the Hogg model,

where

$E_c$ = modulus of elasticity of concrete (an average value of 6,500,000 lb/in.$^2$ was used in the calculation),

$E_s$ = modulus of elasticity of subgrade,

$v_c$ = Poisson’s ratio of concrete (a value of 0.2 was used), and

$v_s$ = Poisson’s ratio of subgrade (a value of 0.4 was used).

The equivalent thickness was computed for each location and load level. In Table 3 the average equivalent thicknesses for each section are compared with the actual thicknesses. The equivalent thickness is much larger than the actual concrete thickness (except for the second sections in which a lower $E_e$ should have been used). It approaches the combined thickness of the concrete and the cement-treated base together. This result indicates that there is a good bond between the layers. To derive a relationship between equivalent thickness $h_{eq}$ and pavement variables, it is important to know the exact layer thickness and to collect data on a wider range of pavement sections.

5. The relationship between subgrade elastic modulus $E_{es}$ and the subgrade $k$-value was developed by the Corps of
TABLE 3 Comparison of Actual Slab Thickness with Calculated Equivalent Slab Thickness

<table>
<thead>
<tr>
<th>Section ID</th>
<th>Actual Slab Thickness (inches)</th>
<th>Base Type</th>
<th>Base Thickness (inches)</th>
<th>Figure 3 Hogg (center)</th>
<th>Figure 4 Hertz (center)</th>
<th>Figure 5 Westergaard (edge)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3003</td>
<td>9.3</td>
<td>AC</td>
<td>3.7</td>
<td>12.60</td>
<td>12.62</td>
<td>14.03</td>
</tr>
<tr>
<td>3589</td>
<td>10.1</td>
<td>GR</td>
<td>6.0</td>
<td>9.59</td>
<td>11.63</td>
<td>12.27</td>
</tr>
<tr>
<td>3699</td>
<td>10.1</td>
<td>CT</td>
<td>5.8</td>
<td>16.29</td>
<td>13.20</td>
<td>14.48</td>
</tr>
<tr>
<td>4142</td>
<td>9.6</td>
<td>AC</td>
<td>7.9</td>
<td>14.29</td>
<td>12.15</td>
<td>13.18</td>
</tr>
<tr>
<td>4152</td>
<td>11.4</td>
<td>CT</td>
<td>6.3</td>
<td>14.28</td>
<td>14.56</td>
<td>17.40</td>
</tr>
</tbody>
</table>

CONCLUSIONS AND RECOMMENDATIONS

The use of four different models to backcalculate rigid pavement material parameters has led to the following conclusions:

1. The Winkler foundation model for rigid pavements may not be appropriate for the cases analyzed with strong subgrades. Similar analyses should be conducted on weak subgrades to further evaluate this conclusion.

2. The subgrade modulus of reaction does not seem to be a material property. The k-value for edge conditions is two to four times larger than that for center conditions for the same site. One should then use the values obtained only for the same conditions that prevailed in their derivations. For example, the values obtained for the center conditions can be used only with a procedure (design procedure, for example) that uses the same infinite slab size conditions. This result makes the edge condition backcalculation necessary for use with design procedures that uses the edge conditions.

3. Bending stress computations should be made using the method of equivalent thicknesses. It is recommended that the deflection bowls and the backcalculation method be used with SHRP data to develop and calibrate the method.

ACKNOWLEDGMENTS

This work is sponsored by the Texas Department of Transportation. Jerome F. Daleiden of Brent Rauhut Engineering supplied the deflection data and pavement layer data used in the study.

APPENDIX

The dependence of the backcalculated k on the location of load (center versus free edge) may be questioned, according to earlier results. For example, on the basis of four slabs and one set of basin measurements on each slab (using an average of 12 observations), Teller and Sutherland (16) concluded, "for conditions that are comparable there is rather good agreement between the values of modulus of subgrade reactions, k, as determined by pavement deflection, for the interior and edge loadings but the value for the corner loadings is consistently lower." However, Carlton and Behrmann (18) stated, "from the data obtained in this study, it appears that the effective subgrade modulus is the same for both corner and edge loadings. In previous model tests, however, determinations of the subgrade modulus for the model had shown that for interior loadings, the effective k was 35 pounds per cubic inch. This is approximately half the value of k measured at the edge and at the corner. It is believed that this apparent increase in k near the boundaries of the slab may be explained by additional support derived from the subgrade outside the limits of the slab."

Despite the above contradictory results, it appears that a gradual increase in the subgrade modulus as the load moves from the center to the edge and then to the corner would be acceptable [see work by Ioannides et al. (3)]. The question would then be, How much increase would be acceptable? The results presented in the paper suggested that the ratio of the
backcalculated $k$ values from edge and interior loading can be as large as 2 to 4. As mentioned in the paper, these results are based on field deflections collected by the Long Term Pavement Performance program of SHRP.

The difference between Teller and Sutherland and SHRP deflection results is seen very clearly from the ratios of the maximum deflections per unit load in the center and edge loadings. In the case of Teller and Sutherland, the ratio is about 1 to 4, whereas in the SHRP section case, the ratio is about 1 to 1.3. In the case study reported by Uzan (1), the ratio of the edge to center loading deflections did not exceed 2. It is not clear what the cause is of the discrepancy of the deflection ratios. It may be attributed to any of the following: the dynamic FWD loading as compared with the static loading condition used by Teller and Sutherland, warping of the plates, or the presence of the voids. Readers are given the opportunity to make their own judgments about which value of $k$ to use in the design.

REFERENCES