

Unified Autostress Method

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A unified method of inelastic analysis that can be applied in both the American Association of State Highway and Transportation Officials overload and maximum load checks is presented in this paper. Different methods of analysis are presently used in these two checks in the AASHTO alternate load factor design procedures. The new method, called the unified Autostress method, can account for yielding at any number of negative- and positive-bending locations, such as pier and flange-transition locations. It can also be applied in an elastic analysis. The new method is founded on classical indeterminate theory and can be readily adapted for computer programming. In continuous spans, automoments are caused by yielding at various locations and remain after the loading is removed. These automoments are calculated by satisfying a continuity relationship at all pier locations and a rotation relationship at all yield locations. The continuity relationship depends on the stiffness properties along the entire length of the member, and the rotation relationship depends on the properties of the cross sections at the yield locations. An iteration procedure is required for more than two spans; one such procedure, which has been shown to work satisfactorily, is outlined.

The American Association of State Highway and Transportation Officials (AASHTO) now allows alternate load factor design (ALFD) procedures for designing braced steel beams with compact cross sections (1). These ALFD procedures are based on the Autostress method (2) and permit inelastic redistribution of moments in continuous spans. The ALFD procedures, like the load factor design (LFD) procedures (3), require design checks at both overload (1.67 times the specified service load) and maximum load (2.17 times the specified service load). The beam-line method (2) of analysis is normally used in the ALFD overload check and the mechanism method (2) is normally used in the maximum load check.

A unified method of inelastic analysis (4) that can be applied in both the overload and maximum load checks is presented in this paper. The new method, which is called the unified Autostress method (UAM), can account for yielding at any number of positive- or negative-bending locations such as flange-transition and pier (interior support) locations. It can be applied to beams and girders with either compact or noncompact sections if the moment-rotation characteristics of these sections are known. It can also be applied in an elastic analysis.

The new method gives the same results as the beam-line method and more accurate results than the mechanism method. It is easier to understand and apply than the beam-line method, especially for girders with more than two spans or when yielding in positive-bending regions is considered. The new method is founded on classical indeterminate theory and can be readily adapted to computer programming. Iterative procedures are

required for girders with more than two continuous spans. The new method has been used to make trial Autostress designs for noncompact steel girders (4).

The development of automoments caused by yielding at piers and other locations is first explained. If the plastic rotations at all yield locations are known, the automoments can be calculated by classical indeterminate methods and the total moments can be obtained by adding these automoments to the elastic moments caused by the applied dead and live loading. Additional information, however, is required to determine the plastic rotations caused by a given loading.

Next defined are two relationships that can be used to determine the plastic rotations for a given loading: a continuity relationship and a rotation relationship. Both relationships must be satisfied. The continuity relationship interrelates the plastic rotations at all yield locations and the moments at all pier locations; it depends on the stiffness properties of the girder. The rotation relationship interrelates the plastic rotation and moment at each yield location and depends on the properties of the cross sections at those locations.

Finally, calculation procedures used in the new method and application of the method to bridge design are discussed. The discussion of calculation procedures includes a derivation of the three-moment equation used in the UAM and also covers iteration procedures, stiffness properties, sequential loading, and composite sections. The discussion of application of the method to design covers both the AASHTO maximum load and overload checks.

AUTOMOMENTS

Caused by Yielding at Piers

Yielding at a pier location in a continuous span because of a given loading causes a plastic rotation that remains after the loading is removed. This permanent rotation actually occurs over a finite length, but is assumed to occur at a single cross section (over an infinitesimal length) at the pier. Thus, the girder is assumed to be elastic over its entire length and to have all of the plastic rotation concentrated in an angular discontinuity at the pier. This is the usual assumption made in plastic design methods (5).

With this assumption, the plastic rotation is equivalent to an angular discontinuity created by cutting the ends of two beams slightly off square and then welding them together end to end as illustrated in an exaggerated way in Figure 1. When the spliced beam is placed on the abutments and held down against the pier (either by a downward reaction at the pier or by dead weight), moments occur along the beam as illustrated in the figure.

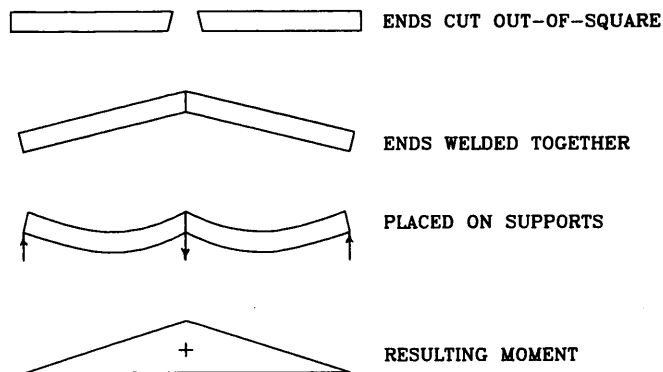


FIGURE 1 Plastic-rotation analogy.

Such moments develop automatically if pier yielding occurs in a continuous girder; hence, they are called automoments. They are proportional to the amount of plastic rotation (or angular discontinuity) at the pier and can be calculated by classical indeterminate theory. If yielding and plastic rotation occur at more than one pier, the automoments caused by the plastic rotation at each pier can be calculated separately and summed to get the total automoments. The automoments are held in equilibrium by reactions at the supports and remain after all loading is removed. The automoments caused by an angular discontinuity (plastic rotation) at Pier 1 of a three-span girder are shown in Figure 2.

The classical three-moment method of indeterminate analysis (6) is ideally suited for calculating the automoments caused by plastic rotation at one or more piers. In this method, the continuous span is treated as a series of simple spans (or hinges are inserted at the piers), and the end moments necessary to restore the continuity are determined. First, end slopes caused by unit end moments are calculated. These end slopes define the stiffness characteristics of the girder. The three-moment equation is then applied at each pier; this gives a sufficient number of simultaneous equations to provide a unique solution.

The three-moment equation defines the plastic rotation at a pier in terms of (a) the end slopes caused in the adjacent spans by the applied load, (b) the stiffnesses of the adjacent spans expressed as end slopes caused by a unit moment at an end, and (c) the moment at the pier and at the two adjacent piers. For an elastic analysis, the plastic rotation, or angular discontinuity, is set to 0 at each pier, and the end slopes caused by the applied loads control the pier moments. In calculating the automoments caused by the plastic rotation, in contrast,

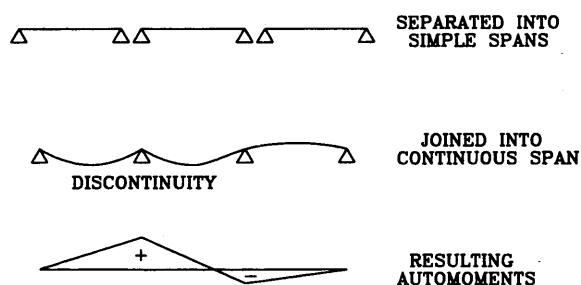


FIGURE 2 Automoments caused by discontinuity at Pier 1.

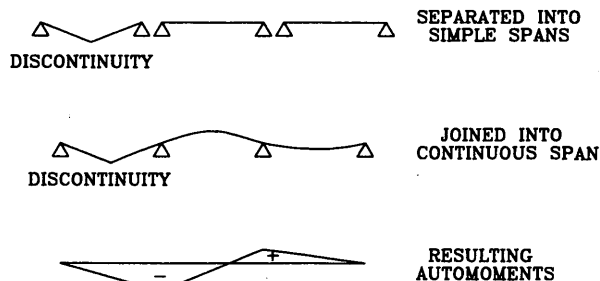


FIGURE 3 Automoments caused by discontinuity in Span 1.

the applied loads and corresponding end slopes are 0 and the pier moments result only from the plastic rotation. A derivation of the three-moment equation, including the effects of plastic rotations at the piers, is given in the section on calculation procedures.

Caused by Yielding at Other Locations

The applied loading may cause yielding at locations other than the piers. For example, yielding might occur at the location of the maximum positive moment; this can occur as a result of residual stresses even if the moment is below the theoretical yield moment. Similar yielding might occur at splice locations where the flange width, thickness, or yield strength is changed. Any such yielding causes automoments similar to those caused by pier yielding.

The yielding at each location can be assumed to occur over an infinitesimal length and be equivalent to an angular discontinuity at that location. The resulting automoments, illustrated in Figure 3, can again be calculated by the three-moment equation. In this case, however, the plastic rotations at the piers are 0 and the automoments result from end slopes caused by the plastic rotation within the span. If an angular discontinuity is inserted into a simple beam, the rest of the beam will remain straight, as illustrated in Figure 3. The resulting end slope is given by

$$S = \frac{aR}{L} \quad (1)$$

where S is the slope at one end, a is the distance from the opposite end to the angular discontinuity, R , and L is the span length.

CONTINUITY RELATIONSHIP

The total moments in a continuous span under a given loading are equal to the algebraic sum of the elastic moments caused by this loading and the automoments caused by plastic rotations at various locations. The plastic rotation at any location may have resulted from either (a) yielding caused by the present loading, or (b) yielding from a previous different

loading. The total moment at Pier 1 can be expressed as

$$M_{1c} = M_{1e} + m_{1p1}R_{p1} + m_{1p2}R_{p2} + m_{1s1}R_{s1} \cdots + m_{1pn}R_{pn} + m_{1sn}R_{sn} \quad (2)$$

In this equation, M_{1c} is the total (continuity) moment at Pier 1, M_{1e} is the elastic moment at Pier 1 caused by the applied loading, R_{p1} is the plastic rotation at Pier 1, m_{1p1} is the automoment at Pier 1 resulting from a unit plastic rotation at Pier 1, m_{1p2} is the automoment at Pier 1 resulting from a unit plastic rotation at Pier 2, R_{s1} is the plastic rotation at a point in Span 1, m_{1s1} is the automoment at Pier 1 resulting from a unit plastic rotation at the point in Span 1, and the other parameters are defined in a comparable way. Automoments caused by yielding at any number of different locations can be included in this equation.

The automoments resulting from unit plastic rotations (m_{1p1} , m_{1s1} , and similar terms) are called automoment coefficients. The actual automoments can be expressed as the product of these automoment coefficients and the corresponding plastic rotations because the automoments are proportional to the plastic rotations. The automoment coefficients are actually stiffness properties of the girder. This automoment-coefficient approach facilitates subsequent calculations, as will be explained later.

Equation 2 provides a continuity relationship that interrelates the pier moment with the plastic rotations at all yield locations. This relationship is based on classical indeterminate theory and depends on the stiffness properties of the girder. A similar continuity equation can be defined at each pier. These continuity relationships are used in conjunction with rotation relationships at the yield locations to determine the correct pier moments for a given loading. The moments at other locations along the span can then be calculated by elastic procedures.

ROTATION RELATIONSHIP

At each yield location, the total moment (elastic moment plus automoments) and the corresponding plastic rotation must fall on the plastic-rotation curve (moment versus plastic rotation) for the cross section at that location. Such a plastic-rotation curve can be obtained from bending-test results by subtracting the elastic rotation from the total rotation as illustrated in Figure 4 (7). The plastic-rotation curve depends primarily on the properties of the cross section, especially the web and flange slenderness ratios. It defines the permanent rotation that remains after a specimen of this cross section has been loaded into the inelastic range and then unloaded.

The permanent rotations result mainly from (a) steel yielding, including the effects of residual stresses, (b) the spread of this yielding along the length as the loading increases, (c) yielding of the rebars in composite sections, and (d) permanent distortions of the cross-sectional shape. Plastic-rotation curves are generally determined in such a way that they give the total plastic rotation over the finite length in which yielding occurs. Thus, the total plastic rotation caused by yielding along the span is accounted for in the UAM method, even though this plastic rotation is assumed to be concentrated in

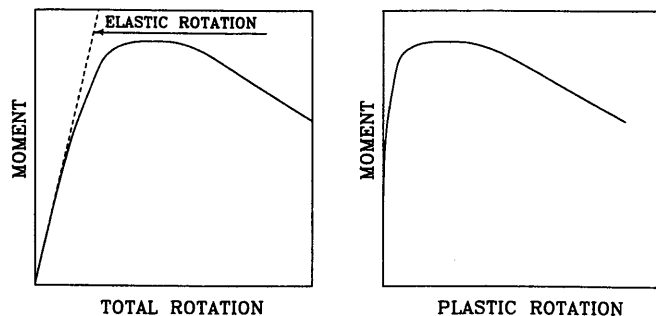


FIGURE 4 Relationship between total and plastic rotation.

an angular discontinuity and the girder is assumed to be elastic over its entire length in the continuity relationship.

At present, plastic-rotation curves can best be determined experimentally, but in the future it may be possible to generate such curves by sophisticated computer modeling (8). Some approximate plastic-rotation curves developed (4) from test results (7,9,10) are given in Figures 5 and 6 for pier sections and positive-bending sections, respectively. These curves, which are plotted in milliradians (radians/1000), were used to make the trial designs mentioned earlier (4). Other appropriate plastic-rotation curves (11,12) could, of course, be used in the unified Autostress method.

The approximate curves in the figures are specifically for composite girders with (a) ultracompact compression flanges in negative-bending regions, (b) a closely spaced stiffener on each side of the pier, and (c) adequate lateral supports. Ultracompact flanges are limited to smaller slenderness ratios than presently allowed by AASHTO (1,13) for compact sections; specifically, the maximum allowable ratios (flange width/2 times flange thickness) are 7.0 and 8.2 for 50-ksi and 36-ksi steels, respectively (5).

Pier Sections

The plastic-rotation curve for pier sections is given in Figure 5. The loading portion of this curve up to M_{max} approximates the plastic-rotation curve given for composite sections in the AASHTO guide specifications for ALFD (1) normalized with

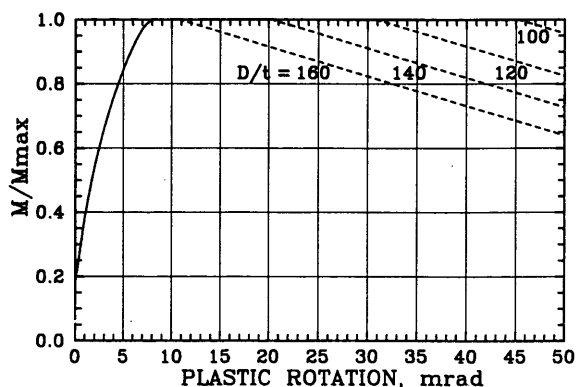


FIGURE 5 Plastic-rotation curve for pier sections.

respect to the maximum-moment capacity, M_{max} . This portion of the curve is controlled primarily by yielding of the steel, including the effects of residual stresses. The curve reaches M_{max} at a plastic rotation, R , of 8.05 mrad. The following empirical equation defines this portion of the curve; R is in milliradians.

$$\begin{aligned} M/M_{max} = & -0.00023R^4 + 0.0046R^3 \\ & - 0.040R^2 + 0.248R + 0.17 \end{aligned} \quad (3)$$

The unloading portion beyond M_{max} is represented by a family of parallel straight lines for different web slenderness ratios, D/t . These straight lines were developed from test results (7,9) and are controlled primarily by buckling and distortion of the cross section. The sloping lines intersect the horizontal line corresponding to $M/M_{max} = 1$ at different values of the limiting plastic rotation, RL , that depend on the web slenderness. The family of downward sloping lines is defined by

$$M/M_{max} = 1.00 - 0.0092(R - RL) \quad (4)$$

in which M is the moment, R is the corresponding plastic rotation in mrad, and RL is limiting plastic rotation in mrad. The plastic-rotation curve is assumed to remain horizontal between 8.05 and RL .

The test results (7,9) for girders with ultracompact flanges showed that M_{max} can be taken as equal to the plastic-moment capacity, M_p , for web slenderness ratios up to 134 and can be obtained from the following equation at ratios between 134 and 170.

$$M_{max}/M_p = 1.41 - 0.00306(D/t) \quad (5)$$

Alternatively, the Q formula that is included in Articles 6.11.5.6 and 6.11.6.6 of the proposed LRFD bridge specifications (3) could be used to define M_{max} .

The following empirical values, derived from test results (4), can be used for RL ; values corresponding to other slenderness ratios can be obtained by interpolation.

D/t	RL , mrad
80	65.1
100	45.2
120	30.8
140	20.2
160	10.7
163	9.3

Positive-Bending Sections

The plastic-rotation curve for positive-bending sections is given in Figure 6; it was obtained from a positive-bending test (10) of a composite girder and is normalized with respect to M_p . The curve reaches M_p at a plastic rotation of 15 mrad and can be assumed to remain constant at M_p thereafter. The unloading portion of the curve is not defined because positive-bending sections are not normally required to sustain plastic

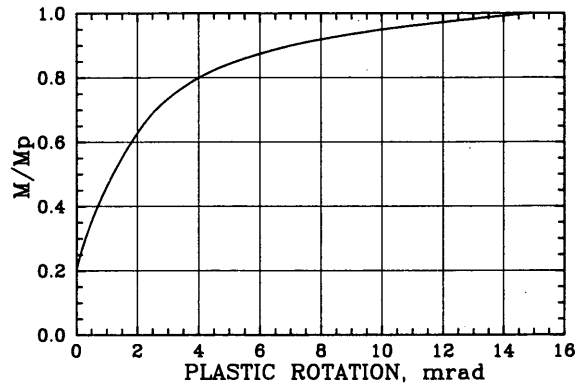


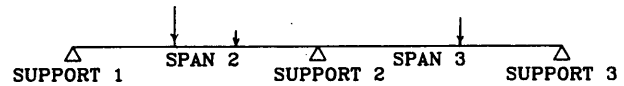
FIGURE 6 Plastic-rotation curve for positive-bending sections.

rotations large enough to cause unloading. The curve is controlled by yielding of the steel and permanent distortions of the concrete at shear studs (10). The shape of this plastic-rotation curve for positive-bending sections has a smaller effect on the behavior of the girder than the shape of the curve for pier sections; therefore, it need not be known with as much accuracy.

CALCULATION PROCEDURES

Derivation of Three-Moment Equation

To derive an equation interrelating the moments in a continuous girder at three consecutive supports (piers and abutments), hinges are placed in the girder at all support locations and the end (support) moments and end slopes for the resulting simple spans are interrelated. A plastic rotation (angular discontinuity) at the center support can be included in the relationship; it is 0 for an elastic analysis. The support moments in a continuous girder with any number of spans can then be calculated for a given set of plastic rotations by writing such a three-moment equation at each interior support (pier) and solving the resulting set of equations simultaneously.



Slope at Right of Span 2:

$$SR_2 = SR_{2a} + sr_{2r}M_2 + sr_{2l}M_1$$

where

- SR_n = slope at right end of Span n ,
- SR_{na} = slope at right end of Span n for applied load,
- sr_{nr} = slope at right end of Span n for unit moment at right end,
- M_n = moment at Support n , and
- sr_{nl} = slope at right end of Span n for unit moment at left end.

Slope at Left of Span 3:

$$SL_3 = SL_{3a} + sl_{3l}M_2 + sl_{3r}M_3$$

where

$$\begin{aligned} SL_n &= \text{slope at left end of Span } n, \\ SL_{na} &= \text{slope at left end of Span } n \text{ for applied load, and} \\ sl_{nl} &= \text{slope at left end of Span } n \text{ for unit moment at left end.} \end{aligned}$$

Equate Slopes at Support 2:

$$R_{p2} = SR_2 - SL_3$$

$$R_{p2} = SR_{2a} + sr_{2r}M_2 + sr_{2l}M_1 - SL_{3a} - sl_{3l}M_2 - sl_{3r}M_3$$

where

$$\begin{aligned} R_{pn} &= \text{plastic rotation (angular discontinuity) at Support } n \\ &\text{and} \\ sl_{nr} &= \text{slope at left end of Span } n \text{ for unit moment at right end.} \end{aligned}$$

Rearrange in Matrix Form: (three-moment equation for Support 2)

$$(sr_{2l})M_1 + (sr_{2r} - sl_{3l})M_2 + (-sl_{3r})M_3 = R_{p2} - SR_{2a} + SL_{3a}$$

$$C_{21}M_1 + C_{22}M_2 + C_{23}M_3 = C_2$$

where

$$C_{nm} = \text{coefficient applied to moment at Support } m \text{ in three-moment equation for Support } n.$$

Define Coefficients: (These coefficients are known for a given loading.)

$$\begin{aligned} C_{21} &= sr_{2l} & C_{22} &= sr_{2r} - sl_{3l} \\ C_{23} &= sl_{3r} & C_2 &= R_{p2} - SR_{2a} + SL_{3a} \end{aligned}$$

Iteration Procedure

The moments and plastic rotations for a given loading can be calculated by inserting hinges at the piers and applying the three-moment equation. At each pier there are two unknowns, the plastic rotation and the moment; there are also two independent equations, one from the continuity relationship and the other from the rotation relationship. At some piers, of course, the plastic rotation may be 0. At each other yield location (those not at piers), there is one unknown, the plastic rotation, and one equation from the rotation relationship. The moments at these locations are defined by the pier moments and the applied loading. Thus there are enough simultaneous equations for a unique solution.

For a two-span girder with yielding only at the pier, the solution can be obtained directly without iteration from the two available simultaneous equations. However, if yielding occurs at other locations, or there are more than two spans, iteration is generally required because the rotation relationship is nonlinear and because the position of the live loading that causes the peak positive moment, and the location of this

peak moment, may vary as the loading increases in the inelastic range. An iteration procedure that has been shown to work satisfactorily (4) is described briefly as follows. Other iteration procedures could be used in the UAM.

The proposed iteration procedure involves iterations of the plastic rotations and is conducted in two stages. Specifically, the plastic rotations at all yield locations are progressively changed until the corresponding moments at these locations satisfy both the continuity and rotation relationships within acceptable limits.

In Stage 1, the correct plastic rotations at all piers (subsequently referred to as pier rotations) are determined for a given set of plastic rotations at positive-bending locations (subsequently referred to as span rotations). During the first application of this stage, the span rotations are assumed to be 0. A trial plastic rotation for Pier 1 is determined by changing this pier rotation until the continuity moment from Equation 2 and rotation moment from Figure 5 are equal at that pier. Other pier rotations, as well as the span rotations, are taken as 0 during this process, which will be referred to as balancing the moments at the pier.

Next, a trial plastic rotation is determined for Pier 2 by balancing the moments at that pier; the trial plastic rotation just determined for Pier 1 is retained and all other pier and span rotations are taken as 0 during this calculation. Next, this process is repeated at all other piers. Then it is again applied to Pier 1 and all other piers. Each time a trial plastic rotation is determined at a pier, all other pier and span rotations are held constant. This process is continued until the moments are balanced within acceptable limits at all piers. In developing the trial designs already mentioned (4), the absolute value of the difference between the moments from the continuity and rotation relationships divided by the elastic moment was not permitted to exceed 0.001.

Next, Stage 2 is applied; it consists of determining the span rotations for the trial pier rotations calculated in Stage 1. These trial pier rotations, together with the applied loadings, define the moments at all positive-bending yield locations. The corresponding span rotation at each location is obtained from the plastic-rotation curve for that location. At some of these locations, the moments may not be high enough to cause yielding so the plastic rotation is taken as 0 at these locations.

Next, Stage 1 is applied again in a manner similar to the first application. In this application, however, the span rotations are held constant at the values determined in Stage 2. The span rotations cause automoments at all piers; these are included in Equation 2 and the similar equations for the other piers. The elastic moments (M_{1e} , M_{2e} , and so on) in these equations are unchanged throughout the iteration procedure. During the second application of Stage 1, the trial plastic rotation at each pier from the first application is retained until a new trial plastic rotation is determined at that pier.

This process of alternate applications of Stages 1 and 2 is continued until differences between the span rotations for successive Stage 2 applications, which progressively decrease, are within acceptable limits. For the trial-design study (4), these differences were not permitted to exceed 0.0001 radians. At this point, the iterations are complete, and all moments and plastic rotations are correct within acceptable iteration limits. In the trial-design study (4), only a few cycles were required for the iteration procedure to converge.

If the applied loading exceeds the maximum strength of the girder, the iteration procedure will not converge. This occurs in the following way. For normal bridge loading patterns, plastic hinges form first at the piers. As the loading is further increased, the moments at these locations decrease in conformance with the unloading portion of the plastic-rotation curve, whereas the positive moments increase. If the maximum positive moment calculated in Stage 2 exceeds M_p in any span, the iteration procedure will not converge. Therefore, the maximum positive moments in all spans in Stage 2 are checked during each iteration cycle; if any one of them exceeds M_p , the iteration process is stopped, and it is concluded that the applied loading exceeds the maximum strength of the girder.

Stiffness Properties

The stiffness properties of the girders must be calculated before the iteration procedure is started. These properties do not change during the iterations. Specifically, it is necessary to calculate the automoments at the piers caused by unit plastic rotations at the piers and other locations (automoment coefficients), and the end slopes caused by unit end moments at the piers (end-slope coefficients). The automoment coefficients are used in the continuity relationship discussed earlier, and the end-slope coefficients appear in the three-moment equations, which are used to calculate both elastic moments and automoments.

The following automoment coefficients and end-slope coefficients need to be calculated for a three-span girder if yielding at the maximum positive moment location in each span is considered. The spans and piers are numbered consecutively from the left.

Automoment Coefficients:

- m_{1p1} = automoment at Pier 1 for unit plastic rotation at Pier 1
- m_{1p2} = automoment at Pier 1 for unit plastic rotation at Pier 2
- m_{1s1} = automoment at Pier 1 for unit plastic rotation in Span 1
- m_{1s2} = automoment at Pier 1 for unit plastic rotation in Span 2
- m_{1s3} = automoment at Pier 1 for unit plastic rotation in Span 3
- m_{2p1} = automoment at Pier 2 for unit plastic rotation at Pier 1
- m_{2p2} = automoment at Pier 2 for unit plastic rotation at Pier 2
- m_{2s1} = automoment at Pier 2 for unit plastic rotation in Span 1
- m_{2s2} = automoment at Pier 2 for unit plastic rotation in Span 2
- m_{2s3} = automoment at Pier 2 for unit plastic rotation in Span 3

End-Slope Coefficients:

- sr_{1r} = slope at right end of Span 1 for unit moment at right end

- sl_{2l} = slope at left end of Span 2 for unit moment at left end
- sl_{2r} = slope at left end of Span 2 for unit moment at right end
- sr_{2l} = slope at right end of Span 2 for unit moment at left end
- sr_{2r} = slope at right end of Span 2 for unit moment at right end
- sl_{3l} = slope at left end of Span 3 for unit moment at left end

To determine the automoment coefficients for plastic rotation at Pier 1 (m_{1p1} and m_{2p1}), a hinge is placed at Pier 1 only, and a unit angular discontinuity is imposed at that point. The resulting moments at Piers 1 and 2 are then calculated by an indeterminate method such as the three-moment method. To determine the automoment coefficients for plastic rotation at a point in Span 2 (m_{1s2} and m_{2s2}), a unit angular discontinuity is imposed at that point, and the resulting moments at Piers 1 and 2 are calculated by an indeterminate method. No hinges are placed in the girder for this calculation. The end-slope coefficients in Span 2 for a unit moment at Pier 1 (sl_{2l} and sr_{2l}) are determined by applying a unit moment to the left end of Span 2 treated as a simple span, and calculating the resulting slopes at the left and right ends.

Sequential Loadings

Automoments are retained in the girder after the loading that caused them is removed. These automoments will not be changed if the same loading is applied again, but may be changed if a different pattern of loading is applied. The final automoments that will result from any sequence of loadings can be determined in the UAM by using the plastic rotations for each loading as the starting values for the analysis of the next loading.

In a bridge girder, the maximum automoments at piers usually result from loading on the two adjacent spans. Such a loading can be caused by two trucks (or strings of trucks simulating lane loading) crossing the bridge at the proper spacing. If there are more than two piers, the automoments caused when the two trucks are straddling the first pier will be changed when the trucks are straddling the second pier. If the same trucks repeatedly cross the bridge at the same spacing, the resulting automoments will eventually stabilize (shakedown) and thereafter remain unchanged. Usually this occurs after only a few cycles. The same thing happens if the piers are alternately loaded (trucks or lane loading in the two adjacent spans) in the UAM.

Composite Girders

In composite girders, a portion of the dead load is usually applied to the steel girder before the slab has hardened. In a continuous span girder, this causes a set of elastic moments, deflections, and end slopes that can be calculated by using the stiffness of the steel section in an indeterminate analysis. When the remaining portion of the loading (dead plus live load) is applied to the composite section, additional elastic

moments, deflections, and end slopes occur and can be calculated by using the stiffness of the composite section. Thus, the total elastic moments, deflections, and end slopes can be calculated by treating the steel and composite sections separately and combining the values from the two separate analyses.

If steel yielding occurs at a pier when loading is applied to the composite section, a plastic rotation develops and causes automoments in the composite section. The total moments are equal to the algebraic sum of the elastic moments in the steel section, the elastic moments in the composite section, and the automoments in the composite section. Thus, M_{1e} in Equation 2 is composed of the elastic moments in the steel and composite sections, each calculated by using the appropriate stiffness in the indeterminate analysis. The automoment coefficients (m_{1p1} , etc.) in this equation are calculated by using the stiffness properties of the composite section.

To be strictly correct, the load applied to the steel section before the slab has hardened should also be accounted for in the rotation relationship. Specifically, a portion of the moment should be applied to the steel section and then additional moment applied to the composite section when generating plastic-rotation curves by either tests or analyses. Plastic-rotation curves generated by applying all moment to the composite section, however, may provide a suitable approximation, especially if it is necessary to use a generic curve (instead of a curve for the particular cross section) anyway.

Generally, for normal bridge loadings, no yielding, or only a very small amount, occurs before the slab has hardened. Consequently, any effect of such yielding can generally be neglected. However, it can be considered in the UAM if desired. To do this, the plastic rotations and automoments for yielding in the steel section are calculated first. These remain unchanged when load is applied to the composite section, but additional plastic rotations and automoments occur in this section. The automoments in the steel section, of course, must be included in Equation 2.

APPLICATION TO DESIGN

Maximum Load Check

In the maximum load check, the specified factored loading (2.17 times the service load) must not exceed the maximum (ultimate) strength of the girder. This requirement is satisfied if the specified loading is applied in the UAM and the iteration procedure converges. Because the maximum load check is based on the worst single loading expected, it is probably not appropriate to consider sequential loadings in this check. Sequential loading is not presently considered in the maximum load check in either LFD (13) or ALFD (1).

Overload Check

At present, both LFD (13) and ALFD (1) limit the maximum stress in positive-bending regions to a fraction of the yield stress to prevent objectionable permanent deflections. This fraction is 0.95 for composite sections and 0.80 for noncom-

posite sections. These specified limits can be checked by the UAM. To do this the specified loading that causes the highest automoment at a pier is applied first; usually, this is truck or lane loading in the two spans adjacent to the pier and will be referred to as negative-bending loading. Then the specified loading that causes the highest stresses in positive-bending regions is applied; usually, this is truck or lane loading in the span containing that region. The algebraic sum of elastic stresses from the second loading and Autostresses from the first loading must not exceed the specified limit.

If the girder has more than two spans, sequential loadings could be considered in the first step by applying the negative-bending loading alternately at each pier until the plastic rotations stabilize. The stabilized plastic rotations after sequential loading are somewhat greater than those for single loading at one pier, but the automoments at that pier are somewhat higher for the single loading (4). Therefore, it may not be necessary to consider sequential loading in most cases.

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REFERENCES

1. *Guide Specifications for Alternate Load Factor Design Procedures for Steel Beam Bridges Using Braced Compact Sections*. American Association of State Highway and Transportation Officials, 1986 (and interim specifications through 1989).
2. G. Haaijer, P. S. Carskaddan, and M. A. Grubb. Suggested Autostress Procedures for Load Factor Design of Steel Beam Bridges. *American Institute of Steel Construction Bulletin* 29, April 1987.
3. *Second Draft LRFD Specifications and Commentary*. National Cooperative Highways Research Program, TRB, National Academy of Sciences, Washington, D.C., June 1991.
4. C. G. Schilling. *A Unified Autostress Method*. Report on Project 51, American Iron and Steel Institute, Washington, D.C., Nov. 1989.
5. *Plastic Design in Steel*. American Society of Civil Engineers, 2nd ed., 1971.
6. L. C. Urquhart. *Civil Engineering Handbook*. McGraw-Hill, New York, 1950.
7. C. G. Schilling and S. S. Morcos. *Moment-Rotation Tests of Steel Girders with Ultracompact Flanges*. Report on Project 188, American Iron and Steel Institute, Washington, D.C., July 1988.
8. D. W. White and A. Dutta. Numerical Studies of Moment-Rotation Behavior in Steel Bridge Girders. In *Proc., Conference of the Structural Stability Research Council*, St. Louis, Mo., April 9–11, 1990.
9. C. G. Schilling. Moment-Rotation Tests of Steel Girders with Ultracompact Flanges. In *Proc., Conference of the Structural Stability Research Council*, St. Louis, Mo., April 9–11, 1990.
10. A. Vasseghi and K. H. Frank. Static Shear and Bending Strength of Composite Plate Girders. *PMFSEL Report 87-4*, Department of Civil Engineering, University of Texas Austin, Tex., June 1987.
11. C. G. Schilling. Moment-Rotation Tests of Steel Bridge Girders. *Journal of Structural Engineering*, American Society of Civil Engineering, Vol. 114, No. 1, Jan. 1988.
12. C. G. Schilling. *Moment-Rotation Tests of Steel Bridge Girders*. Report on Project 188, American Iron and Steel Institute, Washington, D.C., April 15, 1985.
13. *Standard Specifications for Highway Bridges*. 14th ed., American Association of State Highway and Transportation Officials, 1989.