Three-Dimensional Constitutive and Failure Modeling of Polymer Concrete Materials

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A constitutive model based on the theory of plasticity is proposed and used to characterize the stress-deformation behavior of three epoxy concrete materials. It allows for factors such as stress hardening, volume changes, stress paths, temperature, cohesive and tensile strengths, and variation of yield behavior with mean pressure. It is applied to characterize behavior of three epoxy concrete materials. The constants for the model are determined from a series of available laboratory tests. The model is verified with respect to observed laboratory responses. Overall, the proposed model is found to be suitable to characterize the behavior of these three epoxy concrete materials.

Polymer concrete materials have been widely used in the rehabilitation of transportation structures (i.e., bridges and pavement overlays), building structures, and other patching applications. Polymer mortars have been used in construction applications, particularly as structural adhesive for bonding precast units of segmental construction (*1*). It is expected that polymer concrete materials will become more widely used over the next 25 years (*2*,p.413). In transportation engineering, for example, polymer concrete materials have been used in several full-depth bridge deck construction and rehabilitation projects.

Short-term static strengths of polymer mortars are known to be higher than those for portland cement concrete. Severe repeated loading and freeze-thaw exposure are two critical damaging factors for structural materials. The knowledge of degradation resistance of polymer mortars is important for the consideration of the durability of the material.

The literature surveyed regarding polymer concrete, failure criteria, and damage laws leads to some general conclusions (*1,2*). First, polymer mortars are in their early stages of development and characterization as a structural material. Second, when a model is developed for a new material, the initial model is simple with minimal parameters. Third, the material constants must be determined in the laboratory to make use of the model.

STRENGTH, FAILURE, AND CONSTITUTIVE MODELS FOR EPOXY POLYMER CONCRETE

Because polymer concrete materials are used in construction, a fundamental failure and constitutive model is needed for predicting material behavior. The present research is undertaken as a first step toward developing a fundamental failure and constitutive model for epoxy polymer concrete, as well as for providing benchmark data on the strength and failure characteristics of material specimens for future work. The failure model will be developed on the basis of a failure function. This model will predict the changes in constitutive properties and resistance values in aggressive environments.

Since previous work has shown that temperature can affect the strength properties (*3*), temperature was selected as the primary testing variable for investigating ultimate compressive and split tensile strength. A model relating strength to temperature and loading rate had been proposed by Kelsey and Biswas (*4*) and was compared with experimental data.

The proposed model for epoxy concrete materials is developed on the basis of previous models conducted by Salami (*5*). These models have been successfully applied to concrete, soil, and rock materials.

Constitutive Model

Theoretical development of the hierarchical model approach and application to soil, rock, and concrete behavior is given by Salami (*5*), Desai and Faruque (*6*), Desai et al. (*7*), and Desai and Salami (*8,9*). Application of the model for geological materials, including comprehensive modeling and verifications for various geological materials is discussed by Salami (*5*). The hierarchical concept provides a framework for systematically developing models with progressively complex responses: isotropic associative hardening, isotropic nonassociative hardening, anisotropic hardening, and strain softening. As a result, it can be sufficiently simplified in terms of material constants determined from laboratory tests and applications. A brief description of the model is presented in the following.

A compact and specialized form, F, of the general polynomial representation, by Salami (*5*) and Desai and Faruque (*6*) is adopted herein to describe both the continuous yielding and ultimate yield behavior; it is given by

$$F = \{J_{2D} - F_b\, F_s\}_T \tag{1}$$

where

J_{2D} = second invariant of deviatoric stress tensor, S_{ij}, of the total stress tensor σ_{ij};

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F_b = basic function; and
F_s = shape function.

The function F is a continuous function in the stress space with the final curve representing the ultimate behavior. In expanded form, Equation 1 is written as

$$F(J_1, J_{2D}, J_{3D}) = \left\{ J_{2D} - \left(\frac{-\alpha}{\alpha_0^{n-2}} J_1^n + \gamma J_1^2 \right) (1 - \beta S_r)^m \right\}_T = 0 \quad (2)$$

where

$J_1 = \sigma_1 + \sigma_2 + \sigma_3$, the first invariant of σ_{ij};
S_r = stress ratio = $(J_{3D})^{1/3}/(J_{2D})^{1/2}$, which can also be the lode angle;
J_{3D} = third invariant of S_{ij};
α, n, γ, β = response functions;
T = temperature;
α_0 = 1 stress unit; and
m = $-1/2$ response function.

As a simplification, γ and m are assumed to be constants, whereas β is expressed as a function of mean pressure, J_1, to account for the observed yield behavior of geological materials (*5,7–9*). This constitutive model is developed to represent a wide range of materials.

From Equation 2, a new constitutive model is proposed to describe both failure and yielding of the polymer concrete materials. The model agrees with the experimental evidence regarding the shapes of yield surfaces on various planes. Moreover, both ultimate failure and yielding are defined by a single yield surface.

For the ultimate criterion given by Equation 2 to apply to concrete, polymer concrete, and rocks, the cohesion and tensile strength sustained by concrete, polymer concrete, and rocks must be included. This is done by translating the principal stress space along the hydrostatic axis by the addition of a constant stress $R = ap_a$ added to the normal stresses. The modified function is given by Salami (*5*) as

$$\{J_{2D}^* - (-\alpha J_1^{*n} + \gamma J_1^{*2}) (1 - \beta S_r)^{-1/2}\}_T = 0 \quad (3)$$

where

$$J_1^* = \sigma_{11}^* + \sigma_{22}^* + \sigma_{33}^* \quad (4a)$$

$$J_{2D}^* = \tfrac{1}{6}[(\sigma_{11}^* - \sigma_{22}^*)^2 + (\sigma_{22}^* - \sigma_{33}^*)^2 + (\sigma_{11}^* - \sigma_{33}^*)^2] + \sigma_{12}^{*2} + \sigma_{23}^{*2} + \sigma_{13}^{*2} \quad (4b)$$

$$J_3^* = \tfrac{1}{3}\sigma_{ij}^* \sigma_{mn}^* \sigma_{ni}^* = \tfrac{1}{3}(\sigma_{11}^{*3} + \sigma_{22}^{*3} + \sigma_{33}^{*3}) \quad (4c)$$

$$J_{3D}^* = J_3^* - \tfrac{2}{3}J_1^* J_{2D}^* - \tfrac{1}{27}J_3^{*3} \quad (5)$$

The corresponding normal stresses σ_{11}^*, σ_{22}^*, and σ_{33}^* in Equations 4 and 5 at ultimate (failure) state are expressed as

$$\sigma_{11}^* = \sigma_{11} + R \quad (6a)$$

$$\sigma_{22}^* = \sigma_{22} + R \quad (6b)$$

$$\sigma_{33}^* = \sigma_{33} + R \quad (6c)$$

$$R = ap_a \quad (7)$$

where a is a dimensionless number and p_a is the atmospheric pressure. For cohesionless materials such as sand and gravel, $R = 0$, and the function at ultimate in Equation 3 reduces to that in Equation 2.

Growth Function, α

The response function α is the growth or evolution function. In this study, however, α will be made a function of a single parameter, ξ:

$$\xi = \int (d\varepsilon_{ij}^p \, d\varepsilon_{ij}^p)^{1/2} \quad (8)$$

where ξ is the trajectory of plastic strain in a nine-dimensional Euclidean space formed by the components of the incremental plastic strain tensor. With the definition of ξ, we can now define α in the form

$$\alpha = \frac{\alpha_1}{\xi^{\eta_1}} \quad (9)$$

where α_1 and η_1 are the material constants associated with plastic stress hardening.

Elastic-Plastic Constitutive Relations

The principles of continuity and consistency in Drucker's postulate (*10,11*) enable one to decompose an incremental strain tensor into elastic part and plastic part (assuming small strain) as

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (10)$$

where the superscripts e and p refer to elastic and plastic, respectively. The stress-elastic strain relationship can be written in the form

$$d\sigma_{ij} = C_{ijkl} \, d\varepsilon_{kl}^e \quad (11)$$

where C_{ijkl} is the elastic constitutive relation tensor and $d\varepsilon_{kl}^e$ is elastic part of the total incremental strain $d\varepsilon_{kl}$. Substituting for $d\varepsilon_{kl}^e$ in this equation gives

$$d\sigma_{ij} = C_{ijkl} (d\varepsilon_{kl} - d\varepsilon_{kl}^p) \quad (12)$$

In general, the yield function may be written as

$$F = F(\sigma_{ij}, d\varepsilon_{ij}^p) \leq 0 \quad (13)$$

with equality during yielding and negative during unloading. With the assumption of the normality principle and associated flow rule, the increment of plastic strain must be normal to the yield surface; hence,

$$d\varepsilon_{ij}^{p} = \lambda \frac{\delta F}{\delta \sigma_{ij}} \tag{14}$$

where λ is the unknown hardening parameter giving the magnitude of the plastic strain increments, with the direction governed by the normality rule. The stress-elastic strain relationship can be written in final form (*5,10,11*) as

$$d\sigma_{ij} = \left\{ C_{ijkl} - \frac{C_{rskl} \dfrac{\delta F}{\delta \sigma_{kl}} \dfrac{\delta F}{\delta \sigma_{ij}} C_{ijkl}}{\dfrac{\delta F}{\delta \sigma_{ij}} C_{ijkl} \dfrac{\delta F}{\delta \sigma_{kl}} - \dfrac{\delta F}{\delta \varepsilon_{ij}^{p}} \dfrac{\delta F}{\delta \sigma_{ij}}} \right\} d\varepsilon_{kl} \tag{15}$$

or

$$d\sigma_{ij} = \{C_{ijkl}^{e} - C_{ijkl}^{p}\}\, d\varepsilon_{kl} \tag{16}$$

This can be rewritten as

$$d\sigma_{ij} = \{C_{ijkl}^{e-p}\}\, d\varepsilon_{kl} \tag{17}$$

where $\{C_{ijkl}^{e-p}\}$ is known as the elasto-plastic constitutive tensor.

DETERMINATION OF MATERIAL CONSTANTS

General

The proposed model has a number of material constants including the Young's modulus (E); Poisson's ratio (ν); shear modulus (G); and bulk modulus (K). Determination of such constants for any material requires a comprehensive series of laboratory tests with number of loading, unloading, and reloading cycles as shown in Figure 1.

Three polymer concrete materials (epoxy types) were considered to obtain the material constants associated with the proposed constitutive model. These are (a) a low-modulus epoxy, (b) a medium-modulus epoxy, and (c) a high-modulus epoxy. Its application to these materials that were tested by Kelsey (*12*) is presented.

Procedures for Determining Material Constants

Ten material constants are associated with the proposed model as described in Equation 3. These constants can be classified into four categories:

1. Elastic constants: E, ν or G, K
2. Constants for hardening yielding: R, n, β_0, β_1, γ
3. Constants for hardening: a_1, η_1
4. Temperature: T

Elastic Constants

There are two elastic constants for an isotropic material: Young's modulus and Poisson's ratio. Figure 1 shows a typical stress-strain response curve. It may be noted that bulk modulus and shear modulus may also be used. It will be assumed that unloading and reloading is elastic. Thus E and ν can be found from the slope of the unloading-reloading curves. The slope of the unloading-reloading of the mean pressure–versus–volumetric strain curve is shown in Figure 2. [Volumetric strain is given by $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, and mean pressure, by $p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$.] This gives the bulk modulus (K) where K is related to E and ν through the following equation:

$$K = \frac{E}{3(1 - 2\nu)} \tag{18}$$

To obtain E and ν explicitly, a second equation is needed. To determine appropriate values for shear modulus plots of octahedral shear stress versus octahedral shear strain are used. The slope of the unloading-reloading of the τ_{oct}-versus-γ_{oct} curve represents a value equal to twice the shear modulus as

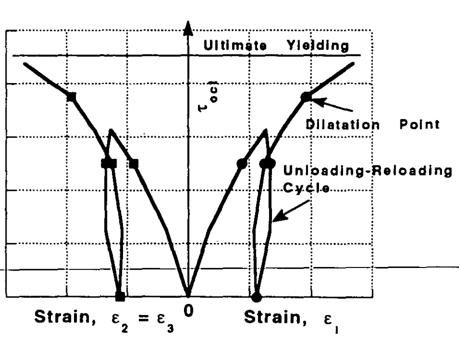

FIGURE 1 Typical stress-strain response curves.

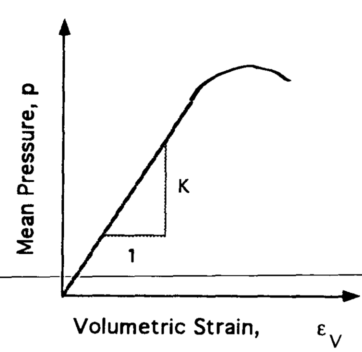

FIGURE 2 Volumetric strain versus mean pressure.

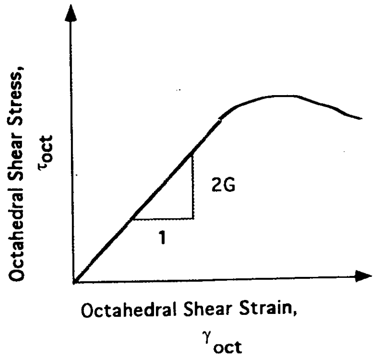

FIGURE 3 Octahedral shear strain versus octahedral shear stress.

shown in Figure 3. [Octahedral shear strain is given by $\gamma_{oct} = \frac{1}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$, and octahedral shear stress, by $\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$]. From these plots a weighted average value of shear modulus (G) was determined. G is related to E and ν through the following equation:

$$G = \frac{E}{2(1 + \nu)} \tag{19}$$

Using Equations 18 and 19, E and ν can be obtained explicitly.

Response Function, n

The value of n can be determined at the state of stress (in the experiment) at which the dilatation occurs—that is, volume change is zero (*5*); n can be determined as:

$$S = \left(1 - \frac{2}{n}\right) \tag{20}$$

where

$$\left[\frac{\left(\frac{\tau_{oct}^2}{J_1^2}\right) \text{dilation}}{\left(\frac{\tau_{oct}^2}{J_1^2}\right) \text{ultimate}}\right] = S \tag{21}$$

The n-value obtained from different stress paths (for different initial confining pressures) will, in general, be different. Thus, an average value of n will be calculated. Here, n is assumed to be 7.

Effect of Tensile Strength, R

If the uniaxial tensile strength, f_t, is not determined experimentally, Salami (*5*) gives an approximate formula relating f_t to the unconfined compression strength, f_c', through the following formula:

$$f_t = \left\{p\, p_a \left(\frac{f_c'}{p_a}\right)^q\right\}_T \tag{22}$$

where

p, q = dimensionless numbers,
T = temperature, and
p_a = atmospheric pressure in same units as those of f_t and f_c'.

Values of p and q have been determined, as described by Salami (*5*), for these three epoxy polymer concrete materials for various temperatures, and their values are given in Table 1. Once f_t is known, the value of R can be computed. R was found to be in the following range as

$$1.003\, f_t \leq R \leq 1.014\, f_t \tag{23}$$

With the estimated value of R, the resulting stresses in Equation 6 are calculated and then substituted into the expression of the stress invariants given in Equations 4 and 5.

Determination of β_0 *and* β_1

Consider the yield condition at ultimate failure at which $\alpha \rightarrow 0$. Then Equation 2 reduces to

$$J_{2D} - (\gamma J_1^2)(1 - \beta S_r)^{-1/2} = 0 \tag{24}$$

β is derived from Equation 24 by Salami (*5*) as:

$$\beta = \left[\frac{(J_{2D}^2)_{TC} - (J_{2D}^2)_{TE}}{(Sr\, J_{2D}^2)_{TC} - (Sr\, J_{2D}^2)_{TE}}\right] \tag{25}$$

It is evident from Equation 25 that β can be determined for a pair of tests (TC and TE) or (CTC and RTE) if the stresses at ultimate failure are known. Also from the following equation

$$\beta = \beta_0\, e^{-\beta_1 J_1} \tag{26}$$

taking natural log on both sides of Equation 26, one obtains

$$\ln(-\beta) = \ln(-\beta_0) - \beta_1 J_1 \tag{27}$$

Equation 27 represents a straight line when plotted in $\ln(-\beta) - J_1$ space as shown in Figure 4. Then β_0 and β_1 are obtained from the intersection and slope of the straight line, respectively.

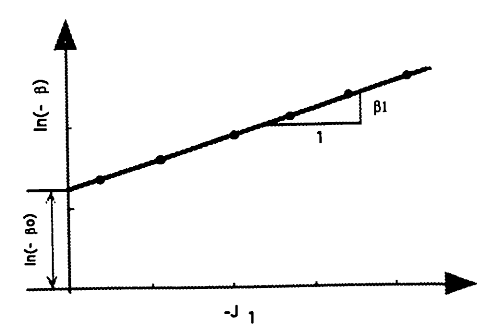

FIGURE 4 Schematic plot to determine material constants β_0 and β_1.

Determination of γ

The value of γ can be determined (*5*) as

$$\gamma = \frac{[J_{2D}\,(1 - \beta S_r)^{1/2}]}{J_1^{\,2}} \qquad (28)$$

Since β is known, γ can be determined for any test if the stresses at ultimate failure are known; γ also can be found graphically from Equation 28 as shown in Figure 5.

Determination of a_1 and η_1

The growth function, Equation 9, after taking natural log of both sides can be written as

$$\eta_1 \ln(\xi) + \ln(\alpha) = \ln(a_1) \qquad (29)$$

Equation 29 can now be used to determine η_1 and a_1. A typical plot is shown in Figure 6.

Material Constants for Three Epoxy Polymer Concrete Materials

Kelsey describes the epoxy polymer concrete materials and presents the test results (*12*). Figures 7 through 9 show stress-strain responses of three epoxy polymer concrete materials for various temperatures. These tests are used to obtain the material constants associated with the proposed model for three epoxy polymer concrete materials.

Values of p and q also were determined for these materials at various temperatures and are given in Table 1. The material constants for these three epoxy polymer concrete materials were obtained by previous procedures.

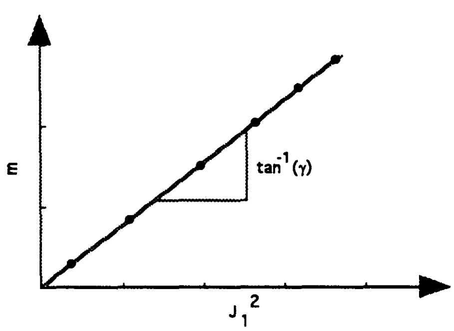

FIGURE 5 Schematic plot to determine material constant γ.

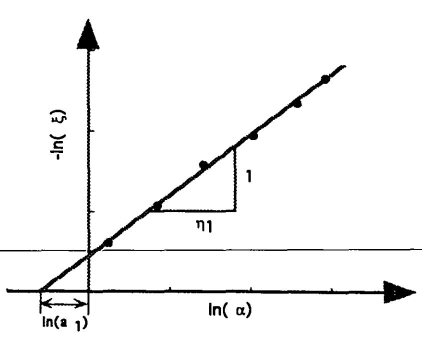

FIGURE 6 Schematic plot to determine material constants a_1 and η_1.

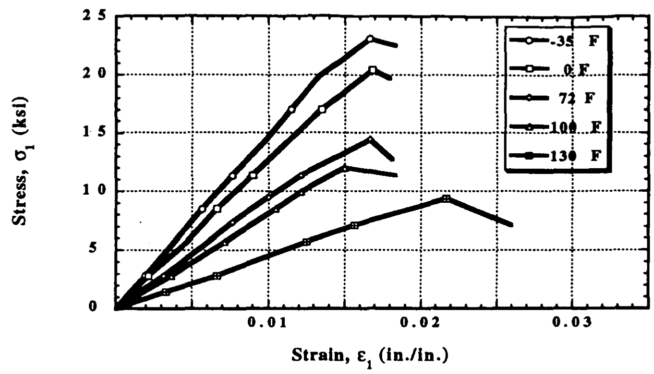

FIGURE 7 Plot of stress versus strain at different temperatures for high-modulus material (AEX1539).

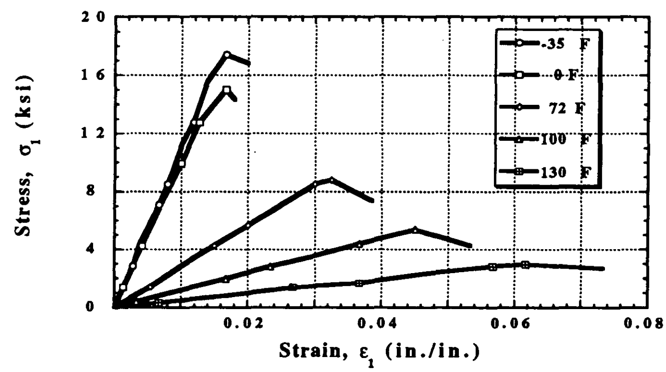

FIGURE 8 Plot of stress versus strain at different temperatures for medium-modulus material (AEX3070).

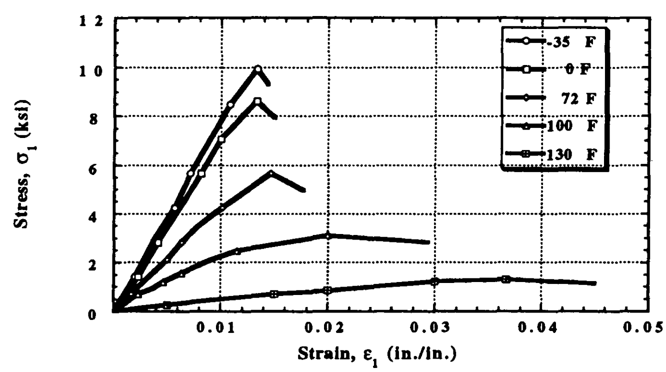

FIGURE 9 Plot of stress versus strain at different temperatures for low-modulus material (FLEXOLITH).

TABLE 1 Values of Parameters *p* and *q* from Developed Model for Various Temperatures for Three Epoxy Polymer Concrete Materials.

Temperature (°F)	*p*	*q*
0	1.6520	0.6509
32	1.5157	0.6720
72	1.5849	0.6745
100	1.4757	0.6813
130	1.9293	0.6593
160	0.3532	0.9400

CONCLUSIONS

A general yet simplified failure and constitutive model is proposed and used to model the behavior of polymer concrete materials as affected by complex factors such as state of stress, stress path, temperature, and volume change. The model allows for continuously yielding and stress hardening, ultimate (failure) yield, and cohesive and tensile strength components. The test results are used to derive material constants. The model is verified with respect to laboratory test data used for finding the constants. The model is found to provide satisfactory predictions for the observed behavior of the epoxy polymer concrete materials. It is also found to provide similar predictions for other concrete and rocks such as westerly granite, Durham dolomite, and sandstone (*5*). The proposed model provides a simple alternative approach for developing constitutive models for epoxy polymer concrete materials.

For the purpose of including reasonable values of tensile strengths in the failure criterion and constitutive model (for frictional materials with effective cohesion), it may be necessary to include the uniaxial tensile strength in the parameter determination. Simple expressions for evaluating the uniaxial tensile strengths for various temperatures on the basis of the uniaxial compressive strength are given. Some sets of uniaxial compressive and tensile tests for various temperatures were performed by Kelsey on three epoxy polymer concrete materials (*12*). The purpose of these tests was to acquire some understanding of the strength behavior of epoxy polymer concrete subjected to compressive and tensile load histories with various temperatures, and the results of these tests were used to calibrate the proposed tensile strength model for predicting the tensile strength of these epoxy polymer concrete materials based on the experimental uniaxial compressive loading.

The proposed tensile strength model has two material constants. Laboratory tests were used to determine these two material constants.

REFERENCES

1. M. Biswas. Precast Bridge Deck Design Systems. *ACI Journal*, Vol. 31, No. 40, 1986.
2. W. O. Nutt and B. Staynes. The Next Twenty-Five Years. *Proc., 5th International Congress of Polymers in Concrete*, Brighton, England, 1987.
3. M. Biswas, O. Ghattas, and H. Vladmirou. Fatigue and Freeze-Thaw Resistance of Epoxy Mortar. In *Transportation Research Record 1041*, TRB, National Research Council, Washington, D.C., 1985.
4. R. G. Kelsey and M. Biswas. Static and Repeated Load Tests of Polymer Mortar Materials. In *Transportation Research Record 1113*, TRB, National Research Council, Washington, D.C., 1988.
5. M. R. Salami. *Constitutive Modelling of Concrete and Rocks Under Multiaxial Compressive Loading*. Doctoral dissertation. Department of Civil Engineering and Engineering Mechanics, University of Arizona, Tucson, 1986.
6. C. S. Desai and M. O. Faruque. A Constitutive Model for Geological Materials. *Journal of the Engineering Mechanics Division*, ASCE, Vol. 110, No. 9, Sept. 1984, pp. 1391–1408.
7. C. S. Desai, G. N. Frantziskonis, and S. Somasundram. Constitutive Modelling for Geological Materials. *Proc., 5th International Conference on Numerical Methods in Geomechanics*, Nagoyo, Japan, April 1985.
8. C. S. Desai and M. R. Salami. Constitutive Model Including Testing for Soft Rock. *International Journal of Rock Mechanics and Mineral Science*, Vol. 24, No. 5, Oct. 1987, pp. 299–307.
9. C. S. Desai and M. R. Salami. A Constitutive Model for Rocks. *International Journal of the Geotechnical Engineering Division*, ASCE, Vol. 113, No. 5, May 1987, pp. 407–423.
10. D. C. Drucker. A More Fundamental Approach to Plastic Stress-Strain Relations. *Proc., 1st U.S. National Congress on Applied Mechanics*, Chicago, Ill., 1951, pp. 487–491.
11. D. C. Drucker and W. Prager. Soil Mechanics and Plastic Analysis of Limit Design. *Quarterly of Applied Mathematics*, Vol. 10, No. 2, 1952.
12. R. G. Kelsey. *Failure Theory for Epoxy Mortar Materials*. Doctoral dissertation. Department of Civil and Environmental Engineering, Duke University, Durham, N.C., 1988.

Publication of this paper sponsored by Committee on Mechanical Properties of Concrete.