System Identification Method for Backcalculating Pavement Layer Properties

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In recent years pavement structural evaluation has relied increasingly on determining material properties by nondestructive deflection testing and backcalculation procedures. The technique used to achieve a convergence of the measured and predicted deflection basins plays an important role in all backcalculation approaches. An iterative method based on the system identification (SID) scheme is developed, and the SID program is used in conjunction with a multilayer elastic model (BISAR program) to backcalculate pavement layer properties. Numerical examples indicate that (*a*) the moduli backcalculated by the suggested SID method compare well with the results from MODULUS, which is a data base backcalculation program, and other developed iterative backcalculation programs; (*b*) the SID is a quickly converging procedure, and the influence of seed values, for a relatively wide range, on the derived results is negligible; and (*c*) it is able to backcalculate pavement layer thicknesses in addition to layer moduli.

Nondestructive testing (NDT) has become an integral part of pavement structural evaluation in recent years. Of many static, vibration, impulse, and vehicular NDT devices, the falling weight deflectometer (FWD) has been most widely used for pavement evaluation (*1*). By dropping a mass from a predetermined height onto a base plate resting on the pavement surface, the FWD can provide variable and large impulse loadings to the pavement, which to some degree simulate actual truck traffic. Pavement deflection is measured through a series of velocity transducers at various distances from the base plate, and the data can be used to backcalculate the in situ pavement properties, such as layer moduli. This information can in turn be used in pavement structural analysis to determine the bearing capacity, estimate the remaining life, and calculate an overlay requirement over a desired design life.

FWD DATA REDUCTION AND BACKCALCULATION METHODS

The analysis of the FWD test data is an inverse process. Instead of predicting the pavement response, the deflection is measured and the pavement properties are backcalculated.

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A variety of different methods and computer programs have been developed for backcalculation of layer moduli from FWD test results. Examples include the MODCOMP program developed by Irwin (*2*), the "___ DEF" series of programs described by Bush (*3*), and the MODULUS program developed by Uzan et al. (*4*). The MODCOMP program uses the CHEVRON program for deflection calculations and is notable for its extensive controls on the seed moduli and the range of acceptable moduli. The two programs reported by Bush include the CHEVDEF and BISDEF programs, in which the deflection calculations are performed by the CHEVRON (*5*) and BISAR (*6*) programs, respectively, and the gradient search technique is used. MODULUS is a data base backcalculation program that departs from the usual microcomputer program pattern. Before the actual backcalculation process, MODULUS computes a series of normalized deflection basins using the BISAR program with layer moduli that cover the range of anticipated values in the field. The deflection basins are stored in a data base for subsequent comparison with measured deflection basins. The pattern search algorithm developed by Hooke and Jeeves (*7*) and the three-point Lagrange interpolation technique (*8*) are used to find the layer moduli that minimize the error between measured and computed basins. By replacing the direct computation of deflections with the interpolation scheme, MODULUS is distinctly faster than other iterative backcalculation programs for production cases in which a large number of deflection basins in the same pavement geometry are to be evaluated. When pavement configuration changes, however, the time-consuming task of generating the deflection data base must be repeated.

Most current backcalculation procedures seek only to determine layer moduli and require the thickness of each pavement layer to be specified. The subgrade is assumed to be infinitely thick, or a rigid layer is placed at an arbitrary depth. As reported by other researchers, pavement deflections are sensitive to layer thicknesses. Even modest errors in assumed layer thicknesses can lead to large errors in backcalculated layer moduli (*9*), and the existence of a rigid layer or bedrock underlying the subgrade has a profound effect on the analysis of deflection data (*10*). The subgrade modulus may be significantly overpredicted if a semi-infinite subgrade is falsely assumed when actual bedrock exists at a shallow depth, or it may be underpredicted if a shallow rigid layer is arbitrarily introduced when deep bedrock exists.

Pavement thicknesses can be accurately measured through coring, boring, ground-penetrating radar, or seismic tests. However, pavements cover such a large area that it is impractical to use these techniques to determine the layer thicknesses at every point tested with deflection devices. Thus advanced backcalculation procedures are clearly needed to determine the layer thickness, especially the subgrade thickness, as well as moduli from the measured deflection information. In this paper an iterative procedure based on the system identification scheme is presented. It may be considered as an alternative approach to the subject.

SYSTEM IDENTIFICATION METHOD

General Procedure

The objective of the system identification process is to estimate the system characteristics using only input and output data from the system to be identified (*11*). The simplest and intellectually most satisfying method for representing the behavior of a physical process is to model it with a mathematical representation. The model/process is identified when the error between the model and the real process is minimized in some sense; otherwise, the model must be modified until the desired level of agreement is achieved.

There are three general strategies for error minimization in system identification procedures: forward approach, inverse approach, and generalized approach. In the forward approach, the model and the system to be identified are given the same known input, and the output error between the two is minimized. In the inverse approach, the outputs of the model and the system are identical, and their input error is minimized. The generalized approach is a combination of the forward and inverse approaches. In all cases, the minimization of the error between the model and the real process can be conducted with a model parameter adjustment.

The forward approach is not as complicated as the inverse or generalized approaches because, by using a forward model, it is easier to compute the output and to generate the parameter adjustment algorithm. A system identification scheme using the forward approach and parameter adjustment algorithm is shown in Figure 1.

The procedure shown in Figure 1 is exactly analogous to what is being done in backcalculating the moduli of pavements (*12,13*). However, the system identification procedure can also be applied to determine properties of pavement structures in addition to layer moduli, even including the thickness of the layer as one of the unknown parameters.

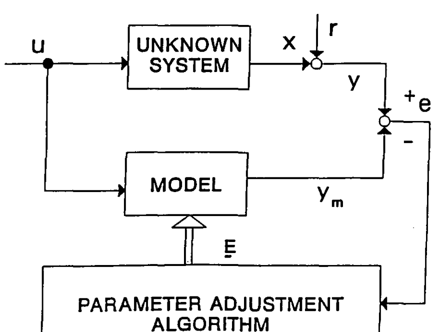

FIGURE 1 System identification (forward approach).

Parameter Adjustment Algorithm

The system identification method requires the accurately measured output data of the unknown system, a suitable model to represent the behavior of the system, and an efficient parameter adjustment algorithm that converges accurately and rapidly. If the data and the model are reliable, the success of system identification studies directly relies on the efficiency of the parameter adjustment algorithm.

An algorithm can be developed for adjusting model parameters on the basis of the Taylor series expansion. Let the mathematical model of some process be defined by n parameters:

$$f = f(p_1, p_2, \ldots, p_n; x, t) \tag{1}$$

where x and t are independent spatial and temporal variables. If any function $f_k(p_1, p_2, \ldots, p_n; x_k, t_k)$ is expanded using a Taylor series and only first-order terms are kept, it can be shown that

$$f_k(p + \Delta p) = f_k(p) + \nabla f_k \cdot \Delta p \tag{2}$$

where the parameters have all been collected into a vector

$$p = [p_1, p_2, \ldots, p_n]^T$$

If we equate $f_k(p + \Delta p)$ with the actual output of the system and $f_k(p)$ with the output of the model for the most recent set of parameters p, the error between the two outputs becomes

$$\begin{aligned} e_k &= f_k(p + \Delta p) - f_k(p) \\ &= \nabla f_k \cdot \Delta p \\ &= \frac{\partial f_k}{\partial p_1} \Delta p_1 + \frac{\partial f_k}{\partial p_2} \Delta p_2 + \cdots + \frac{\partial f_k}{\partial p_n} \Delta p_n \end{aligned} \tag{3}$$

Note that e_k represents the difference between the actual system output and the model output when the independent variables take on values x_k and t_k.

If the error is evaluated at m values ($m \geq n$) of the independent variables, m equations may be written:

$$\left.\begin{aligned} e_1 &= \frac{\partial f_1}{\partial p_1} \Delta p_1 + \frac{\partial f_1}{\partial p_2} \Delta p_2 + \cdots + \frac{\partial f_1}{\partial p_n} \Delta p_n \\ e_2 &= \frac{\partial f_2}{\partial p_1} \Delta p_1 + \frac{\partial f_2}{\partial p_2} \Delta p_2 + \cdots + \frac{\partial f_2}{\partial p_n} \Delta p_n \\ &\vdots \\ e_m &= \frac{\partial f_m}{\partial p_1} \Delta p_1 + \frac{\partial f_m}{\partial p_2} \Delta p_2 + \cdots + \frac{\partial f_m}{\partial p_n} \Delta p_n \end{aligned}\right\} \tag{4}$$

Equation 4 can be conveniently nondimensionalized by dividing both sides by f_k. Furthermore, if we define matrices r,

F, and α as

$$r = [r_1\ r_2\ .\ .\ .\ r_m]^T$$

$$r_k = \frac{e_k}{f_k} \qquad k = 1, 2, \ldots, m$$

$$F = [F_{ki}]$$

$$F_{ki} = \frac{\partial f_k}{\partial p_i} \cdot \frac{p_i}{f_k} \qquad k = 1, 2, \ldots, m \qquad i = 1, 2, \ldots, n$$

$$\alpha = [\alpha_1\ \alpha_2\ .\ .\ .\ \alpha_n]^T$$

$$\alpha_i = \frac{\Delta p_i}{p_i} \qquad i = 1, 2, \ldots, n$$

respectively, Equation 4 may be rewritten as

$$r = F\alpha \tag{5}$$

or

$$F^T r = F^T F\alpha \tag{6}$$

The vector r is completely determined from the outputs of the model and the real system. The matrix F is usually called the sensitivity matrix, because its element F_{ki} reflects the sensitivity of the output f_k to the parameter p_i, and it can be generated numerically if the analytical solution is not available. The technique used for generating the sensitivity matrix F and when it should be updated will be discussed later in this section.

The unknown vector α reflects the relative changes of the parameters. If the sensitivity matrix F or the system of equations is well behaved, it can be obtained by using a generalized inverse procedure to solve Equation 5 (*14,15*). However, there might be column degeneracies in the sensitivity matrix F. This condition may be encountered when two or more parameters have similar effects, or any parameter has a negligible effect, on the behavior of the model f. In these cases Equation 5 may be ill conditioned from the mathematical point of view, and the singular value decomposition (SVD) technique (*16*) is one of the alternative approaches to give a stable solution. SVD diagnoses the sensitivity matrix by calculating its condition number, which is defined as the ratio of the largest of the singular values to the smallest of the singular values. F is singular if its condition number is infinite, but the more common situation is that some of the singular values are very small but nonzero, thus F is ill conditioned. Then SVD gives a solution by zeroing the small singular values, which corresponds to deleting some linear combinations of the set of equations. The SVD solution is very often better (in the sense of the residual $|F\alpha - r|$ being smaller) than LU decomposition solution or Gaussian elimination solution. However, the SVD user has to exercise some discretion in deciding at what threshold to zero the small singular values. In this study the iteration method developed by Han (*17*) is used to solve Equation 6. Han's method not only gives the exact solution if Equation 6 is well posed but also gives a stable solution if Equation 6 is ill posed without deleting any equations.

As soon as α is obtained, a new set of parameters is determined as

$$P^{k+1} = P^k(1 + \alpha) \tag{7}$$

The iteration process is continued until the desired convergence is reached. In this paper the convergence criterion is set to 0.5 percent for α (i.e., the iterative procedure must be repeated until all parameter changes are not more than 0.5 percent).

The sensitivity matrix F in Equation 6 is generated using a multilayer elastic model (BISAR program). The derivatives $\frac{\partial f_k}{\partial p_i}$, where f_k ($k = 1, 2, \ldots, m$) represent the pavement deflections at the sensor locations of FWD and p_i ($i = 1, 2, \ldots, n$) the pavement layer property parameters, are computed as the forward-derived differences. Thus the sensitivity matrix F can be generated by $n + 1$ runs of BISAR.

The sensitivity matrix may be used for more than one iteration. If the parameters have been changed "much," however, it has to be regenerated because it only takes account of the first-order Taylor series and the problem is highly nonlinear, which means that the sensitivity values depend on the parameter values. Otherwise the iteration procedure might not converge, or, more often, it may converge very slowly. In this study the sensitivity matrix is updated when one of the following conditions is encountered:

1. One or more parameters have been increased by more than 100 percent during the past iterations;
2. One or more parameters have been decreased more than 50 percent during the past iterations; or
3. The sensitivity matrix has been used for three iterations, but the 0.5 convergence criterion has not been achieved.

BACKCALCULATION OF LAYER MODULI

On the basis of the procedure described above, the SID microcomputer program has been developed. In this section the program is evaluated by comparing the backcalculated moduli with the results from MODULUS and other developed programs.

Comparison with MODULUS

An actual deflection basin is analyzed using the SID backcalculation program, and the results are compared with those from MODULUS. Deflection data were obtained using the FWD (Dynatest Model 8000) on Section 8 at the Texas A&M Research Annex (*18*). Section 8 consisted of 12.7-cm (5-in.) AC, 30.48-cm (12-in.) crushed limestone base, and 30.48-cm (12-in.) cement-stabilized subbase (very stiff layer) over clay subgrade. The FWD geophones were located at 0, 30.48, 60.96, 91.44, 121.92, 152.4, and 182.88 cm (0, 12, 24, 36, 48, 60, and 72 in.) from the center of the load plate, which had a radius of 15 cm (5.91 in.).

By using the BISAR program to generate the deflection data base and assuming a 635-cm (250-in.) depth from pavement surface to bedrock, the moduli backcalculated by MODULUS for AC layer, base, subbase, and subgrade are E_1 = 140 740 kg/cm² (2,000 ksi), E_2 = 3519 kg/cm² (50 ksi), E_3 = 265 366 kg/cm² (3,771 ksi), and E_4 = 915 kg/cm² (13 ksi).

The SID backcalculation program is used to reduce the same data for Section 8. As do other iterative approaches, the SID requires seed moduli values. Three sets of seed moduli are selected to evaluate the effects of seed parameters on derived results.

First, the seed modulus values are assumed to be E_1 = 105 555 kg/cm² (1500 ksi), E_2 = 4222 kg/cm² (60 ksi), E_3 = 140 740 kg/cm² (2,000 ksi), and E_4 = 704 kg/cm² (10 ksi), which are relatively close to the results given by MODULUS. The 0.5 convergence criterion for α is reached after five iterations, and the sensitivity matrix is regenerated after three iterations.

Next, the seed moduli are changed to E_1 = 70 370 kg/cm² (1,000 ksi), E_2 = 7037 kg/cm² (100 ksi), E_3 = 70 370 kg/cm² (1,000 ksi), and E_4 = 2111 kg/cm² (30 ksi). For this set of seed moduli, only three iterations are needed, but the sensitivity matrix is regenerated after one iteration.

Last, to verify the robustness of the SID approach, the seed moduli are assumed to be significantly different from the previous values: E_1 = 351 851 kg/cm² (5,000 ksi), E_2 = 35 185 kg/cm² (500 ksi), E_3 = 351 851 kg/cm² (5,000 ksi), and E_4 = 3519 kg/cm² (50 ksi).

With these moduli the predicted deflections are approximately four times less than the FWD data, which indicates that very poor seed parameters have been entered. In practice, another set of starting values should be selected in this case. The SID procedure still converges, however. The sensitivity matrix is updated four times, and altogether eight iterations are performed.

The results for the preceding three cases are summarized in Table 1, and the converging process for each case is shown in Figures 2, 3, and 4, respectively. The results backcalculated by the SID program agree very well with those by MODULUS,

TABLE 1 Backcalculated Moduli for Different Seed Values

MODULI (kg/cm²)	E_1 (140740*)	E_2 (3519*)	E_3 (265366*)	E_4 (915*)
"Seed"	105555	4222	140740	704
Backcalculated	150451	3504	262128	906
"Seed"	70370	7037	70370	2111
Backcalculated	150451	3502	262269	906
"Seed"	351851	35185	351851	3519
Backcalculated	151437	3478	265507	906

1 kg/cm² = 14.21 psi

* moduli backcalculated by MODULUS (18).

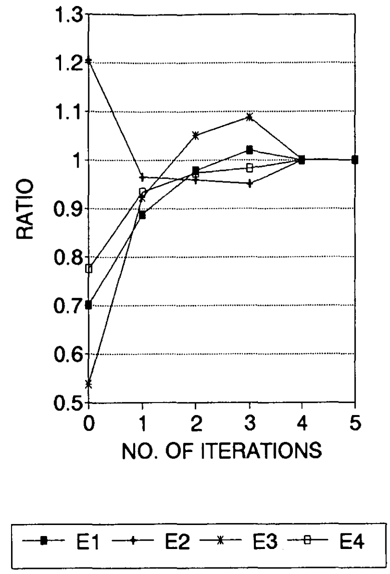

FIGURE 2 Converging process (first set of seed moduli).

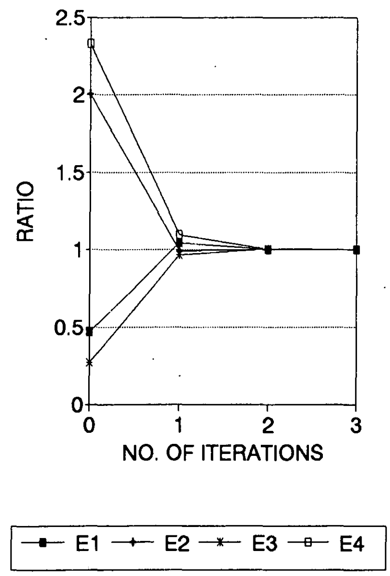

FIGURE 3 Converging process (second set of seed moduli).

and the seed values have a negligible influence on the converged results, although they certainly affect the required number of iterations.

Comparison with Other Iterative Backcalculation Approaches

The SID program is compared with five other iterative backcalculation programs. Pavement data and deflection test data for the comparison are obtained from a real pavement (*19*). The backcalculated moduli from the various programs are

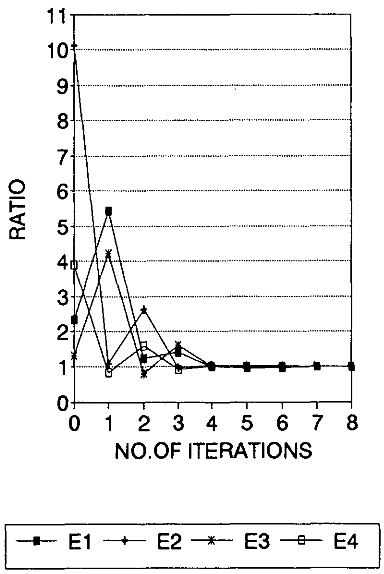

FIGURE 4 Converging process (third set of seed moduli).

TABLE 2 Summary of Backcalculated Moduli (kg/cm²)

Test Site	Program	AC Surface	Aggregate Base	Subgrade
	BISDEF	13652	1776	809
	BOUSDEF	11470	1809	788
1	CHEVDEF	12371	1738	851
	ELSDEF	14074	1661	823
	MODCOMP2	11456	2350	739
	SID(BISAR)	15474	1527	809
	BISDEF	12223	1084	739
	BOUSDEF	11097	1070	697
2	CHEVDEF	10605	1168	739
	ELSDEF	12244	1070	732
	MODCOMP2	9254	1907	654
	SID(BISAR)	11498	1217	704

1 kg/cm² = 14.21 psi

given in Table 2. The results from SID are close to those from the other programs.

BACKCALCULATION OF LAYER MODULI AND LAYER THICKNESSES

By considering the layer thicknesses as unknown parameters, the SID program can be used to backcalculate the layer thicknesses as well as layer moduli. This ability is illustrated by using hypothetical three-layer pavement structures and the real FWD data for Section 8.

Backcalculation of Hypothetical Pavement Layer Moduli and Thicknesses

The SID program is evaluated by comparing the backcalculated layer moduli and thicknesses with hypothesized theoretical values. The comparison is done by assuming three pavement sections with different thicknesses and moduli. Surface deflections of the assumed pavement section are calculated using the BISAR program and are used to backcalculate the layer thicknesses as well as the layer moduli.

The theoretical values and the backcalculated results for the three pavement sections are presented in Table 3. The SID program always converges toward the correct modulus and thickness for all layers.

Application Using Actual Deflection Data

The SID program is applied to determine the subgrade thickness of Section 8 from the FWD data given in Table 1. Since increasing the number of unknown parameters requires more data points to ensure the system overdeterminism, and because of the likelihood of large measurement errors in real data, backcalculating more than four parameters is not recommended without performing the dynamic analysis of FWD data. Therefore the process is divided into two steps:

1. The four layer moduli are backcalculated by assuming the subgrade to be of infinite thickness. The results are compared with those backcalculated previously by introducing a 635-cm (250-in.) depth from surface to bedrock. The subbase and subgrade moduli are much more sensitive to the subgrade thickness than the AC and base moduli. Thus the backcal-

TABLE 3 Summary of Backcalculated Layer Moduli and Layer Thicknesses

	MODULI (kg/cm²)			THICKNESSES (cm)		
	E_1	E_2	E_3	h_1	h_2	h_3
"Seed"	35185	2111	1407	25.4	25.4	635.0
Backcalculated	42222	2815	1759	30.5	30.5	762.0
Theoretical	42222	2815	1759	30.5	30.5	762.0
"Seed"	35185	2111	1407	25.4	25.4	635.0
Backcalculated	70370	4221	2815	38.1	30.5	889.0
Theoretical	70370	4222	2815	38.1	30.5	889.0
"Seed"	35185	2111	1407	25.4	25.4	635.0
Backcalculated	70370	1055	704	15.2	38.1	457.2
Theoretical	70370	1056	704	15.2	38.1	457.2

1 cm = 0.3937 in.

1 kg/cm² = 14.21 psi

TABLE 4 Summary of Backcalculated Results for Section 8

Subgrade Thickness (cm)	Program	Backcalculated Moduli (kg/cm^2)			
		E_1	E_2	E_3	E_4
Infinite (Assumed)	MODULUS	140740	4081	91833	1970
	SID(BISAR)	145736	3941	101051	1900
561 (Assumed)	MODULUS	140740	3519	265365	915
	SID(BISAR)	150451	3519	262128	915
1082 (Backcalculated)	SID(BISAR)	145736	3941	151858	1407

1 cm =0.3937 in

1 kg/cm^2 =14.21 psi

culated AC and base moduli are fixed as known parameters, and the derived subbase and subgrade moduli are taken as the seed values for the next backcalculation step.

2. The subbase and subgrade moduli together with the subgrade thickness are backcalculated. The seed subgrade thickness is selected as 762 cm (300 in.), and a 1082-cm (426-in.) thickness is derived.

The backcalculated results for Section 8, based on three different subgrade thicknesses, are summarized in Table 4. The subgrade thickness significantly affects the backcalculated subbase and subgrade moduli. The subgrade modulus assuming an infinite thickness is approximately twice the value backcalculated assuming a subgrade thickness of 561 cm (221 in.). The backcalculated subgrade modulus with the backcalculated subgrade thickness of 1082 cm (426 in.) from the SID program is in between.

This example clearly illustrates one substantial problem in most current backcalculation procedures. If the subgrade is assumed to be infinitely thick, or a depth to bedrock is arbitrarily selected, the backcalculated subgrade modulus from these two assumptions may be quite different. Since the stiffness of the supporting subgrade is a basic parameter in pavement structural analysis, over- or underprediction of subgrade modulus may lead to under- or overconservative results in pavement evaluation and overlay design.

The SID procedure can be considered as an alternative approach for backcalculating pavement layer moduli and at the same time estimating the subgrade thickness from the FWD data. By using the relatively simple multilayer elastic model to represent the complex behavior of pavement structures, this estimation gives a more consistent prediction in the sense of "equivalent subgrade thickness." The derived layer moduli based on such an equivalent subgrade thickness should be more reliable than that from an analysis based on the assumption of infinite subgrade thickness or the selection of an arbitrary subgrade thickness.

SUMMARY

This paper describes a backcalculation method based on the system identification scheme. The SID program is used with the multilayer elastic model (BISAR program) to backcalculate pavement layer properties. Backcalculated moduli are compared with those from other developed programs, and good agreement is observed. The ability to backcalculate pavement layer thicknesses is illustrated by using hypothetical pavement sections and real FWD data.

The backcalculated results for Section 8 indicate clearly that the subgrade thickness should be carefully determined for the pavement under analysis. The backcalculated subgrade modulus assuming infinite thickness may be twice that obtained from an analysis in which the depth to bedrock is arbitrarily selected, such as 610 cm (20 ft). The SID program promises to give more reliable results by considering the subgrade thickness as one of the unknown parameters to be identified.

The SID method is a very powerful and versatile analysis tool and can be applied to a variety of backcalculations. As has been successfully accomplished at Texas A&M University, the parameters of the creep compliance of the AC layer can be backcalculated from FWD data using the SID program and the dynamic multilayer viscoelastic model UTFWIBM (*20*) or SCALPOT (*21*), and the fracture properties of asphalt concrete materials can also be backcalculated from fatigue test data using the SID program and the microcrack model MICROCR (*22*).

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