System Identification Method for Backcalculating Pavement Layer Properties

Fuming Wang and Robert L. Lytton

In recent years pavement structural evaluation has relied increasingly on determining material properties by nondestructive deflection testing and backcalculation procedures. The technique used to achieve a convergence of the measured and predicted deflection basins plays an important role in all backcalculation approaches. An iterative method based on the system identification (SID) scheme is developed, and the SID program is used in conjunction with a multilayer elastic model (BISAR program) to backcalculate pavement layer properties. Numerical examples indicate that (a) the moduli backcalculated by the suggested SID method compare well with the results from MODULUS, which is a data base backcalculation program, and other developed iterative backcalculation programs; (b) the SID is a quickly converging procedure, and the influence of seed values, for a relatively wide range, on the derived results is negligible; and (c) it is able to backcalculate pavement layer thicknesses in addition to layer moduli.

Nondestructive testing (NDT) has become an integral part of pavement structural evaluation in recent years. Of many static, vibration, impulse, and vehicular NDT devices, the falling weight deflectometer (FWD) has been most widely used for pavement evaluation (1). By dropping a mass from a predetermined height onto a base plate resting on the pavement surface, the FWD can provide variable and large impulse loadings to the pavement, which to some degree simulate actual truck traffic. Pavement deflection is measured through a series of velocity transducers at various distances from the base plate, and the data can be used to backcalculate the in situ pavement properties, such as layer moduli. This information can in turn be used in pavement structural analysis to determine the bearing capacity, estimate the remaining life, and calculate an overlay requirement over a desired design life.

FWD DATA REDUCTION AND BACKCALCULATION METHODS

The analysis of the FWD test data is an inverse process. Instead of predicting the pavement response, the deflection is measured and the pavement properties are backcalculated.

A variety of different methods and computer programs have been developed for backcalculation of layer moduli from FWD test results. Examples include the MODCOMP program developed by Irwin (2), the "DEF" series of programs described by Bush (3), and the MODULUS program developed by Uzan et al. (4). The MODCOMP program uses the CHEVRON program for deflection calculations and is notable for its extensive controls on the seed moduli and the range of acceptable moduli. The two programs reported by Bush include the CHEVDEF and BISDEF programs, in which the deflection calculations are performed by the CHEVRON (5) and BISAR (6) programs, respectively, and the gradient search technique is used. MODULUS is a data base backcalculation program that departs from the usual microcomputer program pattern. Before the actual backcalculation process, MODULUS computes a series of normalized deflection basins using the BISAR program with layer moduli that cover the range of anticipated values in the field. The deflection basins are stored in a data base for subsequent comparison with measured deflection basins. The pattern search algorithm developed by Hooke and Jeeves (7) and the three-point Lagrange interpolation technique (8) are used to find the layer moduli that minimize the error between measured and computed basins. By replacing the direct computation of deflections with the interpolation scheme, MODULUS is distinctly faster than other iterative backcalculation programs for production cases in which a large number of deflection basins in the same pavement geometry are to be evaluated. When pavement configuration changes, however, the time-consuming task of generating the deflection data base must be repeated.

Most current backcalculation procedures seek only to determine layer moduli and require the thickness of each pavement layer to be specified. The subgrade is assumed to be infinitely thick, or a rigid layer is placed at an arbitrary depth. As reported by other researchers, pavement deflections are sensitive to layer thicknesses. Even modest errors in assumed layer thicknesses can lead to large errors in backcalculated layer moduli (9), and the existence of a rigid layer or bedrock underlying the subgrade has a profound effect on the analysis of deflection data (10). The subgrade modulus may be significantly overpredicted if a semi-infinite subgrade is falsely assumed when actual bedrock exists at a shallow depth, or it may be underpredicted if a shallow rigid layer is arbitrarily introduced when deep bedrock exists.

Pavement thicknesses can be accurately measured through coring, boring, ground-penetrating radar, or seismic tests. However, pavements cover such a large area that it is impractical to use these techniques to determine the layer thick-
nesses at every point tested with deflection devices. Thus advanced backcalculation procedures are clearly needed to determine the layer thickness, especially the subgrade thickness, as well as moduli from the measured deflection information. In this paper an iterative procedure based on the system identification scheme is presented. It may be considered as an alternative approach to the subject.

SYSTEM IDENTIFICATION METHOD

General Procedure

The objective of the system identification process is to estimate the system characteristics using only input and output data from the system to be identified (11). The simplest and intellectually most satisfying method for representing the behavior of a physical process is to model it with a mathematical representation. The model/process is identified when the error between the model and the real process is minimized in some sense; otherwise, the model must be modified until the desired level of agreement is achieved.

There are three general strategies for error minimization in system identification procedures: forward approach, inverse approach, and generalized approach. In the forward approach, the model and the system to be identified are given the same known input, and the output error between the two is minimized. In the inverse approach, the outputs of the model and the system are identical, and their input error is minimized. The generalized approach is a combination of the forward and inverse approaches. In all cases, the minimization of the error between the model and the real process can be conducted with a model parameter adjustment.

The forward approach is not as complicated as the inverse or generalized approaches because, by using a forward model, it is easier to compute the output and to generate the parameter adjustment algorithm. A system identification scheme using the forward approach and parameter adjustment algorithm is shown in Figure 1.

The procedure shown in Figure 1 is exactly analogous to what is being done in backcalculating the moduli of pavements (12,13). However, the system identification procedure can also be applied to determine properties of pavement structures in addition to layer moduli, even including the thickness of the layer as one of the unknown parameters.

Parameter Adjustment Algorithm

The system identification method requires the accurately measured output data of the unknown system, a suitable model to represent the behavior of the system, and an efficient parameter adjustment algorithm that converges accurately and rapidly. If the data and the model are reliable, the success of system identification studies directly relies on the efficiency of the parameter adjustment algorithm.

An algorithm can be developed for adjusting model parameters on the basis of the Taylor series expansion. Let the mathematical model of some process be defined by n parameters:

$$f = f(p_1, p_2, \ldots, p_n; x, t)$$

where $x$ and $t$ are independent spatial and temporal variables. If any function $f_k(p_1, p_2, \ldots, p_n; x_k, t_k)$ is expanded using a Taylor series and only first-order terms are kept, it can be shown that

$$f_k(p + \Delta p) = f_k(p) + \nabla f_k \cdot \Delta p$$

where the parameters have all been collected into a vector

$$p = [p_1, p_2, \ldots, p_n]^T$$

If we equate $f_k(p + \Delta p)$ with the actual output of the system and $f_k(p)$ with the output of the model for the most recent set of parameters $p$, the error between the two outputs becomes

$$e_k = f_k(p + \Delta p) - f_k(p) = \nabla f_k \cdot \Delta p = \frac{\partial f_k}{\partial p_1} \Delta p_1 + \frac{\partial f_k}{\partial p_2} \Delta p_2 + \ldots + \frac{\partial f_k}{\partial p_n} \Delta p_n$$

Note that $e_k$ represents the difference between the actual system output and the model output when the independent variables take on values $x_k$ and $t_k$.

If the error is evaluated at $m$ values ($m \geq n$) of the independent variables, $m$ equations may be written:

$$
\begin{bmatrix}
\frac{\partial f_1}{\partial p_1} \Delta p_1 + \frac{\partial f_1}{\partial p_2} \Delta p_2 + \ldots + \frac{\partial f_1}{\partial p_n} \Delta p_n \\
\frac{\partial f_2}{\partial p_1} \Delta p_1 + \frac{\partial f_2}{\partial p_2} \Delta p_2 + \ldots + \frac{\partial f_2}{\partial p_n} \Delta p_n \\
\vdots \\
\frac{\partial f_m}{\partial p_1} \Delta p_1 + \frac{\partial f_m}{\partial p_2} \Delta p_2 + \ldots + \frac{\partial f_m}{\partial p_n} \Delta p_n 
\end{bmatrix}
$$

Equation 4 can be conveniently nondimensionalized by dividing both sides by $f_k$. Furthermore, if we define matrices $r$,
A sensitivity analysis is performed by repeatedly solving a forward problem with small changes in the parameter values. These parameter changes are then compared with the corresponding changes in the solution. Under some circumstances, these changes are not unique and the problem may be ill-conditioned, which means that the sensitivity values depend on the parameter values. Otherwise the iteration procedure might not converge, or, more often, it may converge very slowly.

In this study the sensitivity matrix is updated when one of the following conditions is encountered:

1. One or more parameters have been increased by more than 100 percent during the past iterations;
2. One or more parameters have been decreased more than 50 percent during the past iterations; or
3. The sensitivity matrix has been used for three iterations, but the 0.5 convergence criterion has not been achieved.

### BACKCALCULATION OF LAYER MODULI

On the basis of the procedure described above, the SID microcomputer program has been developed. In this section the program is evaluated by comparing the backcalculated moduli with the results from MODULUS and other developed programs.

### Comparison with MODULUS

An actual deflection basin is analyzed using the SID backcalculation program, and the results are compared with those from MODULUS. The iteration process is continued until the desired convergence criterion is reached. In this paper the convergence criterion is set to 0.5 percent for α (i.e., the iterative procedure must be repeated until all parameter changes are not more than 0.5 percent).

The sensitivity matrix \( F \) in Equation 6 is generated using a multilayer elastic model (BISAR program). The derivatives \( \frac{\partial f_k}{\partial p_i} \), where \( f_k (k = 1, 2, \ldots, m) \) represent the pavement deflections at the sensor locations of FWD and \( p_i (i = 1, 2, \ldots, n) \) the pavement layer property parameters, are computed as the forward-derived differences. Thus the sensitivity matrix \( F \) can be generated by \( n + 1 \) runs of BISAR.

The sensitivity matrix \( F \) may be used for more than one iteration. If the parameters have been changed “much,” however, it has to be regenerated because it only takes account of the first-order Taylor series and the problem is highly nonlinear, which means that the sensitivity values depend on the parameter values.

As soon as \( \alpha \) is obtained, a new set of parameters is determined as

\[
P^{k+1} = P^k (1 + \alpha)
\]

The iteration process is continued until the desired convergence criterion is reached.
ment surface to bedrock, the moduli backcalculated by MODULUS for AC layer, base, subbase, and subgrade are $E_1 = 140,740$ kg/cm$^2$ (2,000 ksi), $E_2 = 3519$ kg/cm$^2$ (50 ksi), $E_3 = 265,366$ kg/cm$^2$ (3,771 ksi), and $E_4 = 915$ kg/cm$^2$ (13 ksi).

The SID backcalculation program is used to reduce the same data for Section 8. As do other iterative approaches, the SID requires seed moduli values. Three sets of seed moduli are selected to evaluate the effects of seed parameters on derived results.

First, the seed modulus values are assumed to be $E_1 = 105,555$ kg/cm$^2$ (1500 ksi), $E_2 = 4222$ kg/cm$^2$ (60 ksi), $E_3 = 140,740$ kg/cm$^2$ (2,000 ksi), and $E_4 = 704$ kg/cm$^2$ (10 ksi), which are relatively close to the results given by MODULUS. The 0.5 convergence criterion for $\alpha$ is reached after five iterations, and the sensitivity matrix is regenerated after three iterations.

Next, the seed moduli are changed to $E_1 = 70,370$ kg/cm$^2$ (1,000 ksi), $E_2 = 70,370$ kg/cm$^2$ (100 ksi), $E_3 = 70,370$ kg/cm$^2$ (1,000 ksi), and $E_4 = 2111$ kg/cm$^2$ (30 ksi). For this set of seed moduli, only three iterations are needed, but the sensitivity matrix is regenerated after one iteration.

Last, to verify the robustness of the SID approach, the seed moduli are assumed to be significantly different from the previous values: $E_1 = 351,851$ kg/cm$^2$ (5,000 ksi), $E_2 = 35,185$ kg/cm$^2$ (500 ksi), $E_3 = 351,851$ kg/cm$^2$ (5,000 ksi), and $E_4 = 3519$ kg/cm$^2$ (50 ksi).

With these moduli the predicted deflections are approximately four times less than the FWD data, which indicates that very poor seed parameters have been entered. In practice, another set of starting values should be selected in this case. The SID procedure still converges, however. The sensitivity matrix is updated four times, and altogether eight iterations are performed.

The results for the preceding three cases are summarized in Table 1, and the converging process for each case is shown in Figures 2, 3, and 4, respectively. The results backcalculated by the SID program agree very well with those by MODULUS, and the seed values have a negligible influence on the converged results, although they certainly affect the required number of iterations.

### Comparison with Other Iterative Backcalculation Approaches

The SID program is compared with five other iterative backcalculation programs. Pavement data and deflection test data for the comparison are obtained from a real pavement (19). The backcalculated moduli from the various programs are

#### Table 1 Backcalculated Moduli for Different Seed Values

<table>
<thead>
<tr>
<th>MODULI</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
<th>$E_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg/cm$^2$</td>
<td>(140740')</td>
<td>(3519')</td>
<td>(265366')</td>
<td>(915')</td>
</tr>
<tr>
<td>&quot;Seed&quot;</td>
<td>105555</td>
<td>4222</td>
<td>140740</td>
<td>704</td>
</tr>
<tr>
<td>Backcalculated</td>
<td>150451</td>
<td>3504</td>
<td>262128</td>
<td>906</td>
</tr>
<tr>
<td>&quot;Seed&quot;</td>
<td>70370</td>
<td>7037</td>
<td>70370</td>
<td>2111</td>
</tr>
<tr>
<td>Backcalculated</td>
<td>150451</td>
<td>3502</td>
<td>262269</td>
<td>906</td>
</tr>
<tr>
<td>&quot;Seed&quot;</td>
<td>351851</td>
<td>35185</td>
<td>351851</td>
<td>3519</td>
</tr>
<tr>
<td>Backcalculated</td>
<td>151437</td>
<td>3478</td>
<td>265507</td>
<td>906</td>
</tr>
</tbody>
</table>

1 kg/cm$^2$ = 14.21 psi
* moduli backcalculated by MODULUS (18).
The SID program is evaluated by comparing the backcalculated layer moduli and thicknesses with hypothesized theoretical values. The comparison is done by assuming three pavement sections with different thicknesses and moduli. Surface deflections of the assumed pavement section are calculated using the BISAR program and are used to backcalculate the layer thicknesses as well as the layer moduli.

The theoretical values and the backcalculated results for the three pavement sections are presented in Table 3. The SID program always converges toward the correct modulus and thickness for all layers.

### Backcalculation of Hypothetical Pavement Layer Moduli and Thicknesses

The SID program is evaluated by comparing the backcalculated layer moduli and thicknesses with hypothesized theoretical values. The comparison is done by assuming three pavement sections with different thicknesses and moduli. Surface deflections of the assumed pavement section are calculated using the BISAR program and are used to backcalculate the layer thicknesses as well as the layer moduli.

The theoretical values and the backcalculated results for the three pavement sections are presented in Table 3. The SID program always converges toward the correct modulus and thickness for all layers.

### Application Using Actual Deflection Data

The SID program is applied to determine the subgrade thickness of Section 8 from the FWD data given in Table 1. Since increasing the number of unknown parameters requires more data points to ensure the system overdeterminism, and because of the likelihood of large measurement errors in real data, backcalculating more than four parameters is not recommended without performing the dynamic analysis of FWD data. Therefore the process is divided into two steps:

1. The four layer moduli are backcalculated by assuming the subgrade to be of infinite thickness. The results are compared with those backcalculated previously by introducing a 635-cm (250-in.) depth from surface to bedrock. The subbase and subgrade moduli are much more sensitive to the subgrade thickness than the AC and base moduli. Thus the backcalculated results are close to those from the other programs.

### TABLE 2 Summary of Backcalculated Moduli (kg/cm²)

<table>
<thead>
<tr>
<th>Test Site</th>
<th>Program</th>
<th>AC Surface</th>
<th>Aggregate Base</th>
<th>Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BISDEF</td>
<td>13652</td>
<td>1776</td>
<td>809</td>
</tr>
<tr>
<td>2</td>
<td>BOUSDEF</td>
<td>11470</td>
<td>1809</td>
<td>788</td>
</tr>
<tr>
<td>3</td>
<td>CHEVDEF</td>
<td>12371</td>
<td>1738</td>
<td>851</td>
</tr>
<tr>
<td>4</td>
<td>ELSEF</td>
<td>14074</td>
<td>1661</td>
<td>823</td>
</tr>
<tr>
<td>5</td>
<td>MODCOMP2</td>
<td>11456</td>
<td>2350</td>
<td>739</td>
</tr>
<tr>
<td>6</td>
<td>SID(BISAR)</td>
<td>15474</td>
<td>1527</td>
<td>809</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Site</th>
<th>Program</th>
<th>AC Surface</th>
<th>Aggregate Base</th>
<th>Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BISDEF</td>
<td>12223</td>
<td>1084</td>
<td>739</td>
</tr>
<tr>
<td>2</td>
<td>BOUSDEF</td>
<td>11097</td>
<td>1070</td>
<td>697</td>
</tr>
<tr>
<td>3</td>
<td>CHEVDEF</td>
<td>10605</td>
<td>1168</td>
<td>739</td>
</tr>
<tr>
<td>4</td>
<td>ELSEF</td>
<td>12244</td>
<td>1070</td>
<td>732</td>
</tr>
<tr>
<td>5</td>
<td>MODCOMP2</td>
<td>9254</td>
<td>1907</td>
<td>654</td>
</tr>
<tr>
<td>6</td>
<td>SID(BISAR)</td>
<td>11498</td>
<td>1217</td>
<td>704</td>
</tr>
</tbody>
</table>

The theoretical values and the backcalculated results for the three pavement sections are presented in Table 3. The SID program always converges toward the correct modulus and thickness for all layers.

### TABLE 3 Summary of Backcalculated Layer Moduli and Layer Thicknesses

<table>
<thead>
<tr>
<th>MODULI (kg/cm²)</th>
<th>THICKNESSES (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>h₁</td>
</tr>
<tr>
<td>E₂</td>
<td>h₂</td>
</tr>
<tr>
<td>E₃</td>
<td>h₃</td>
</tr>
</tbody>
</table>

| Seed            | 35185 2111 1407  | 25.4 25.4 635.0 |
| Backcalculated  | 42222 2815 1759  | 30.5 30.5 762.0 |
| Theoretical     | 42222 2815 1759  | 30.5 30.5 762.0 |

| Seed            | 35185 2111 1407  | 25.4 25.4 635.0 |
| Backcalculated  | 70370 4222 2815  | 38.1 30.5 889.0 |
| Theoretical     | 70370 4222 2815  | 38.1 30.5 889.0 |

1 cm = 0.3937 in.
1 kg/cm² = 14.21 psi

**BACKCALCULATION OF LAYER MODULI AND LAYER THICKNESSES**

By considering the layer thicknesses as unknown parameters, the SID program can be used to backcalculate the layer thicknesses as well as layer moduli. This ability is illustrated by using hypothetical three-layer pavement structures and the real FWD data for Section 8.
This paper describes a backcalculation method based on the system identification scheme. The SID program is used with the multilayer elastic model (BISAR program) to backcalculate pavement layer properties. Backcalculated moduli are compared with those from other developed programs, and good agreement is observed. The ability to backcalculate pavement layer thicknesses is illustrated by using hypothetical pavement sections and real FWD data.

The backcalculated results for Section 8 indicate clearly that the subgrade thickness should be carefully determined for the pavement under analysis. The backcalculated subgrade modulus assuming infinite thickness may be twice that obtained from an analysis in which the depth to bedrock is arbitrarily selected, such as 610 cm (20 ft). The SID program promises to give more reliable results by considering the subgrade thickness as one of the unknown parameters to be identified.

The SID method is a very powerful and versatile analysis tool and can be applied to a variety of backcalculations. As has been successfully accomplished at Texas A&M University, the parameters of the creep compliance of the AC layer and the dynamic multilayer viscoelastic model UTFWIBM (20) or SCALPOT (21), and the fracture properties of asphalt concrete materials can also be backcalculated from fatigue test data using the SID program and the microcrack model MICROCR (22).

ACKNOWLEDGMENTS

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REFERENCES

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Table 4: Summary of Backcalculated Results for Section 8

<table>
<thead>
<tr>
<th>Subgrade Thickness (cm)</th>
<th>Program</th>
<th>Backcalculated Moduli (kg/cm²)</th>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
<th>E₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite (Assumed)</td>
<td>MODULIS</td>
<td>140740 4081 91833 1970</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SID(BISAR)</td>
<td>145736 3941 101051 1900</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>561 (Assumed)</td>
<td>MODULIS</td>
<td>140740 3519 265365 915</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SID(BISAR)</td>
<td>150451 3519 262128 915</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1082 (Backcalculated)</td>
<td>SID(BISAR)</td>
<td>145736 3941 151858 1407</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 cm = 0.3937 in
1 kg/cm² = 14.21 psi


