Modified Newton Algorithm for Backcalculation of Pavement Layer Properties

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An efficient algorithm for the backcalculation of pavement layer moduli from measured surface deflections is presented. The algorithm is an iterative one and can use any mechanistic analysis program for forward calculations (presently an extended precision CHEVRON program is used for this purpose). Most mechanistic-based backcalculation methods attempt to find the layer moduli that minimize the weighted sum of the relative or absolute errors between measured and predicted surface deflections. Using a search technique to achieve such a minimization sometimes requires hundreds of calls to a mechanistic analysis program, and some programs try to speed this up by using a previously created data base. The algorithm presented here is different in that it uses a modified Newton method to obtain the least-squares solution of an overdetermined set of equations. This gives the proposed algorithm a robustness that some other approaches appear to lack. For example, the predicted moduli are not too sensitive to the initially assumed seed moduli or the location of the stiff layers (e.g., CRAM section, composite pavements, shallow or deep bedrock, etc.). Further, a set of auxiliary equations that are totally independent of those used in the modified Newton method and that relate surface deflections to the compressions in each pavement layer are used to improve the speed of convergence. The algorithm is also extended to improve incorrectly specified layer thicknesses. The algorithm is being implemented in a new backcalculation program named MICHBACK.

The backcalculation of layer properties from surface deflection measurements is of considerable importance for the accurate evaluation and design of overlays and the management of existing pavements. Most existing methods predict only elastic layer moduli, but often the layer thicknesses are known only approximately and may also need revision.

There are three general classes of backcalculation methods:

1. Iterative methods that repeatedly adjust the layer moduli and call a mechanistic analysis program until a suitable match of the deflection basin is obtained [e.g., CHEVDEF/BISDEF/ELSDEF series (*1*), EVERCALC (*2*)],
2. Methods that match the measured deflection basin with a data base of deflection basins computed in advance for a variety of layer moduli [e.g., MODULUS (*3*)], and
3. Methods that use statistical regression equations [e.g., LOADRATE (*4*)].

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Iterative methods are usually slow since they require numerous calls to a mechanistic analysis program, and sometimes the results are sensitive to the initial seed moduli. Methods that use a data base are fast, but the data base of deflection basins corersponding to the range of expected layer properties must be established before backcalculation is performed, and the results are usually sensitive to the seed moduli. Methods that are statistical regression equations are very fast but usually do not have acceptable accuracy.

Almost all existing iterative methods estimate the layer moduli by minimizing an objective function that is the weighted sum of squares of the differences between calculated and measured surface deflections (*3*), that is,

$$\text{minimize } f = \sum_{j=1}^{m} \alpha_j[w_j - \hat{w}_j]^2 \tag{1}$$

where

w_j = measured deflection at Sensor j,
$\hat{w}_j$ = calculated deflection at Sensor j, and
α_j = weighting factor for Sensor j.

Often, the weighted sum of squares of the relative differences between calculated and measured deflections is minimized by choosing each weight in Equation 1 to be inversely proportional to the measured surface deflections. One of the problems with this approach is that the multidimensional surface represented by the objective function may have many local minima, and as a result the minimum to which a numerical procedure converges may depend on the initial seed moduli supplied by the analyst. Another problem is that convergence can be very slow because numerous calls to a mechanistic analysis program (i.e., forward calculations) are required by most numerical minimization techniques to revise the moduli after each iteration. An efficient and general minimization method (Levenberg-Marquardt algorithm) has been implemented in EVERCALC that makes it converge quickly with only a modest number of calls to the mechanistic analysis program (original CHEVRON). The "____DEF" series of programs also makes only a modest number of calls to a mechanistic analysis program by using empirically determined rules to revise the layer moduli after each iteration, but the results of these programs are sensitive to the initial seed moduli. The EVERCALC program has also been used to successfully estimate the asphalt layer thickness in flexible pavements from theoretical deflection basins (*2*).

IMPROVED INITIAL ESTIMATE OF SUBGRADE MODULUS

It is well known that the deflections measured by the sensors far from the applied load are affected mostly by the deeper pavement layers, and some programs initially estimate the subgrade modulus by using only the furthest sensor. This approach is prone to error, especially if the furthest sensor measurement is inaccurate. Recognizing that the subgrade contributes strongly to the deflection at all sensors, a technique is developed for substantially improving the subgrade modulus using a single call to a mechanistic analysis program.

Consider a pavement with n layers for which m surface deflections are measured ($m \geq n$). Let the vector $\{\hat{w}_j\}$ contain the m deflections computed at the top of the jth layer using current estimates of the layer moduli ($\hat{E}_j$). The vertical compression under the sensors in the jth layer is $\{\hat{w}_j\} - \{\hat{w}_{j+1}\}$. For the last layer we take $\{\hat{w}_{n+1}\} = \{0\}$. The vertical compression in any layer represents the accumulated vertical strain, which is inversely proportional to the layer modulus (i.e., proportional to $\hat{e}_j = 1/\hat{E}_j$). Let the compressions in each layer scaled by the layer modulus be

$$\{a_j\} = \hat{E}_j(\{\hat{w}_j\} - \{\hat{w}_{j+1}\}) \tag{2}$$

and the collection of all such vectors be the $n \times m$ matrix

$$[A] = [\{a_1\}\,\{a_2\}\,\ldots\,\{a_m\}] \tag{3}$$

The sum of the compressions in each layer must sum to the total surface deflection, that is,

$$\sum_{j=1}^{n} (\{\hat{w}_j\} - \{\hat{w}_{j+1}\}) = \{\hat{w}_1\} \tag{4}$$

or, equivalently,

$$[A]\{\hat{e}\} = \{\hat{w}_1\} \tag{5}$$

The following iterative method can be obtained from Equation 5:

$$[A]^i\{\hat{e}\}^{i+1} = \{w\} \tag{6}$$

where $[A]^i$ is computed using the current moduli estimates $\{\hat{E}\}^i$ and $\{w\}$ are the measured surface deflections. The formulation of this iterative process was first suggested by A. R. Raab (unpublished data). The overdetermined system of equations (n equations in m unknowns) is solved using the method of least squares to obtain the revised inverse moduli $\{\hat{e}\}^{i+1}$. It may appear that Equation 6 can be used iteratively to improve the estimates of all the layer moduli, but unfortunately the iteration is very sensitive and rather unstable for estimating all the unknown layer moduli. Often the upper layer moduli become negative. However, since all the surface deflection measurements are strongly influenced by the subgrade modulus, the estimate of the subgrade modulus is greatly improved after a single iteration, irrespective of the initial seed moduli. Equation 6 is therefore used once in the beginning to obtain an accurate initial estimate of the subgrade modulus.

MODIFIED NEWTON METHOD

Consider the Newton method for solving a single nonlinear equation (e.g., estimating a single layer modulus from a single surface deflection measurement). The method is shown in Figure 1. The nonlinear deflection versus modulus curve is approximated by a straight line that is tangent to it at the estimate $\hat{E}^i$. The slope of the straight line, $(dw/dE)|_{E=\hat{E}^i}$, is used to obtain the increment, ΔE^i, which is added to $\hat{E}^i$ to obtain the improved modulus estimate $\hat{E}^{i+1}$. Since the slope is not known analytically, it must be obtained numerically through

$$\left.\frac{dw}{dE}\right|_{E=\hat{E}^i} \approx \frac{w[(1+r)\hat{E}^i] - w(\hat{E}^i)}{r\hat{E}^i} \tag{7}$$

where r is sufficiently small (say 0.05). This requires the additional deflection arising from a modulus of $(1 + r)\hat{E}^i$ to be computed.

For m sensors and n layers, the "slope" is represented by the gradient matrix

$$[G]^i = \left.\left[\frac{\partial\{w\}}{\partial\{E\}}\right]\right|_{\{E\}=\{\hat{E}\}^i} = \left.\begin{bmatrix} \frac{\partial w_1}{\partial E_1} & \frac{\partial w_1}{\partial E_2} & \cdots & \frac{\partial w_1}{\partial E_n} \\ \vdots & & & \vdots \\ \frac{\partial w_m}{\partial E_1} & \frac{\partial w_m}{\partial E_2} & \cdots & \frac{\partial w_m}{\partial E_n} \end{bmatrix}\right|_{\{E\}=\{\hat{E}\}^i} \tag{8}$$

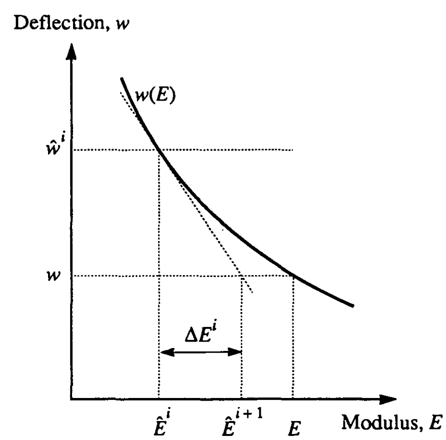

FIGURE 1 Newton's method.

and the element on the jth row and kth column of the matrix is estimated numerically as

$$\left.\frac{\partial w_j}{\partial E_k}\right|_{\{E\}=\{\hat{E}\}^i} \approx \frac{w_j([R]\{\hat{E}\}^i) - w_j(\{\hat{E}\}^i)}{r\hat{E}_k^i}$$

where $[R]$ is a diagonal matrix with the kth diagonal element being $(1 + r)$ and all other diagonal elements being 1 [i.e., the partial derivative is estimated numerically by taking the difference in the jth deflection arising from the use of the moduli $\hat{E}_1^i, \hat{E}_2^i, \ldots, (1 + r)\hat{E}_k^i, \ldots, \hat{E}_n^i$ and the use of the moduli $\hat{E}_1^i, \hat{E}_2^i, \ldots, \hat{E}_k^i, \ldots, \hat{E}_n^i$]. Thus, a separate call to a mechanistic analysis program is required to compute the partial derivatives in each column of the gradient matrix. Increments to the moduli, $\{\Delta E\}^i$, can then be obtained by solving the m equations in n unknowns.

$$\{\hat{w}\}^i + [G]^i\{\Delta E\}^i = \{w\} \quad (9)$$

and the revised moduli are obtained through

$$\{E\}^{i+1} = \{E\}^i + \{\Delta E\}^i \quad (10)$$

One technique for solving the least-squares problem is to solve the $n \times n$ normal equations

$$[G]^T[G]\{\Delta E\}^i = [G]^T\{\{w\} - \{\hat{w}\}^i\} \quad (11)$$

However, the condition number of the matrix $[G]^T[G]$ is the square of the condition number of $[G]$, and hence solving the normal equations can magnify the effect of errors in the elements of $[G]$, errors in $\{w\}$, and round-off errors that accumulate during calculations. The recommended method for solving linear least-squares problems is by using orthogonal factorizations or singular value decomposition (*5*).

The iteration is terminated when the changes in the layer moduli are sufficiently small, that is,

$$\frac{\hat{E}_k^{i+1} - \hat{E}_k^i}{\hat{E}_k^i} < \varepsilon \qquad k = 1, 2, \ldots, n \quad (12)$$

In addition, if the computed and measured deflections match closely, the root-mean-square error defined by

$$\text{RMS error in deflections} = \sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(\frac{\hat{w}_j^i - w_j}{w_j}\right)^2} \quad (13)$$

will also be small. Only for theoretical deflection basins generated by an elastic layer program can the iteration be carried on until the RMS error in the deflections is smaller than a value requested by the analyst. For deflection basins measured in the field, it will usually not be possible to obtain an arbitrarily close match between the computed and measured deflections.

The initial formulation of the Newton method for backcalculating layer moduli was also conceived and first suggested to the research team by Raab (unpublished data). A literature search has revealed that the method was conceived previously and published by Hou (*6*).

In the Newton method, the number of calls made to a mechanistic analysis program is $(n + 1)$ for each iteration. The total number of forward calculations can be reduced by using a modified Newton approach in which several iterations are performed with a gradient matrix before it is revised. The modified Newton method usually converges more slowly than the normal method but saves n forward calculations required to compute the gradient matrix during each iteration. Experience has shown that performing three iterations before revising the gradient matrix yields good convergence with fewer calls to the mechanistic analysis program.

The Newton method is a rapidly convergent algorithm but can sometimes diverge for badly behaved functions if the initial guesses for the solutions are poor. For the pavement backcalculation problem, however, the surface deflections (which are functions of the layer moduli and thicknesses) appear to be well behaved, and for most problems convergence is obtained even for very poor initial guesses (i.e., seed moduli). The modified Newton algorithm is applicable to flexible or rigid pavements as long as an appropriate mechanistic program is used for the forward calculations.

IMPROVING LAYER THICKNESSES

In many situations the thickness of some pavement layers may only be known approximately. Incorrect thickness specifications usually lead to larger errors in the predicted layer moduli. For example, if a layer thickness smaller than the actual one is specified, the modulus backcalculated for that layer will usually be larger than the correct one in order to yield an equivalent layer stiffness. In such situations the analyst may wish to have the backcalculation program improve the incorrect layer thicknesses. Layer thickness improvement has been successfully accomplished for theoretical deflection basins using the EVERCALC program (*2*). The modified Newton method can also be readily extended to include such capability as long as the total number of unknown layer moduli and layer thicknesses does not exceed the number of sensors. For improving l layer thicknesses, Equation 9 is expanded to

$$\{\hat{w}\}^i + [G]^i \begin{Bmatrix} \{\Delta E\}^i \\ \{\Delta t\}^i \end{Bmatrix} = \{w\} \quad (14)$$

where $\{\Delta t\}^i$ is the vector of thickness increments and the augmented gradient matrix is

$$[G]^i = \left.\begin{bmatrix} \dfrac{\partial\{w\}}{\partial\{E\}} & \dfrac{\partial\{w\}}{\partial\{t\}} \end{bmatrix}\right|_{\substack{\{E\}=\{\hat{E}\}^i \\ \{t\}=\{\hat{t}\}^i}}$$

$$= \left.\begin{bmatrix} \dfrac{\partial w_1}{\partial E_1} & \cdots & \dfrac{\partial w_1}{\partial E_n} & \dfrac{\partial w_1}{\partial t_1} & \cdots & \dfrac{\partial w_1}{\partial t_l} \\ \vdots & & \vdots & \vdots & & \vdots \\ \dfrac{\partial w_m}{\partial E_1} & \cdots & \dfrac{\partial w_m}{\partial E_n} & \dfrac{\partial w_m}{\partial t_1} & \cdots & \dfrac{\partial w_m}{\partial t_l} \end{bmatrix}\right|_{\substack{\{E\}=\{\hat{E}\}^i \\ \{t\}=\{\hat{t}\}^i}} \quad (15)$$

A column of the gradient matrix corresponding to a partial derivative with respect to a thickness is estimated numerically by computing the surface deflections due to a slight increase in that thickness. The number of forward calculations during each iteration now increases to $(n + l + 1)$.

It has been found that better overall convergence is achieved if the layer moduli are first estimated with fixed layer thicknesses as discussed in the previous section, and then additional iterations are performed to improve both the layer moduli and thicknesses as outlined in this section.

In principle the technique outlined above can be used to predict any layer property, including Poisson's ratio, as long as the number of unknown quantities does not exceed the number of sensor locations. All that is required is that the partial derivatives of the surface deflections with respect to the unknown quantities be estimated. However, at present the method has only been tested for estimation of layer moduli and thicknesses. Preliminary results indicate that at times the iteration does not converge as the number of unknown quantities is increased.

MICHBACK PROGRAM

The algorithm presented in this paper is being implemented in a new computer program named MICHBACK. The forward calculation program used by MICHBACK is an extended precision version of the CHEVRON program, henceforth called CHEVRONX.

Several elastic layer analysis programs are currently being used in practice for flexible pavement analysis. The CHEVRON program has been widely used, partly because it is in the public domain, and many newer programs are based on it. However, it has been discovered by various researchers that the numerical integration performed in CHEVRON is not sufficiently accurate for stiff pavements, and differences have been observed between results obtained from the BISAR (*7*) and CHEVRON programs, especially in surface deflections close to the applied load. The four-part Legendre-Gauss quadrature used in our version of the original CHEVRON program was extended to 16- and 18-part Legendre-Gauss quadrature over different intervals by L. Irwin of Cornell University to obtain the extended precision program CHEVRONX (which yields results identical to those of the version of the CHEVRON program distributed by Cornell University, CHEVLAY2). Results from CHEVRONX very closely match those produced by the BISAR (*7*) program.

NUMERICAL EXAMPLES AND COMPARISONS

The real test of any backcalculation program should ultimately be based on how well it can predict the properties of real pavements using surface deflections measured in the field. The actual in situ properties of real pavements, however, are seldom known accurately and often sound engineering judgment must be used to ascertain whether the backcalculated properties are reasonable. Whereas backcalculations based on field measurements will be performed in due course using MICHBACK, the initial results presented in this paper are based on "theoretical" deflections basins generated by the elastic layer program CHEVRONX. Note that deflection basins generated by BISAR or CHEVLAY2 would essentially be identical to those generated by CHEVRONX.

Several examples of backcalculation using MICHBACK are given in this section. All the examples are due to a wheel load of 40.034 kN (9,000 lbf) applied to a circular area of radius 150.1 mm (5.91 in.) and seven surface deflections calculated at radial distances of 0, 203.2, 304.8, 457.2, 609.6, 914.4, and 1524 mm (0, 8, 12, 18, 24, 36, and 60 in.) from the center of the loaded area. Surface deflections were rounded to the nearest hundredth of a mil (1 mil = 0.001 in.) before being input to all programs. The moduli backcalculated by MICHBACK are compared with the values obtained from the MODULUS 4.0 and EVERCALC 3.0 programs. Attention was given to making the comparisons as fair and equitable as possible. The forward calculation program used by EVERCALC is the original CHEVRON, whereas that used by MODULUS is WES5 (whose precision is comparable to that of CHEVRONX). To assess whether the results obtained with EVERCALC were sensitive to the difference in precision between the program used to compute the deflection basins (CHEVRONX) and its own forward calculation program (CHEVRON), backcalculations were also performed with EVERCALC using deflection basins generated by the original CHEVRON program. Two sets of backcalculation results are therefore presented for EVERCALC: results tabulated under "EVERCALC" were based on deflection basins generated by CHEVRONX, and the results tabulated under "EVERCALC-Alt" were based on alternative basins generated by CHEVRON.

The MICHBACK and EVERCALC programs require the same types of input parameters. The convergence criterion for the moduli (Equation 12) specified for these programs was $\varepsilon = 0.001$ (0.1 percent), and the seed moduli used in each example are given.

The MODULUS program is somewhat different in its approach, does not allow the anlayst to specify a convergence measure, and requires slightly different input parameters:

1. The most probable value of the subgrade modulus and lower and upper values indicating the range of all other layer moduli are required as input so that a data base of deflection basins can be generated. It was found that the moduli backcalculated by MODULUS were quite sensitive to the initial value of the subgrade modulus and somewhat sensitive to the moduli ranges. To give it a more than fair start, a subgrade seed modulus of 48.26 MPa (7,000 psi) was used for MODULUS, whereas a poorer seed modulus of 20.68 MPa (3,000 psi) was used for the other two programs in all examples. The lower moduli for layers other than the subgrade and concrete slab (in composite pavements) were specified as the seed moduli used for the MICHBACK and EVERCALC programs, whereas for the concrete slab the lower modulus was specified as 13 789.5 MPa (2,000,000 psi). The upper moduli were specifed as 5515.8, 413.69, 275.79, and 41 368.8 MPa (800,000, 60,000, 40,000 and 6,000,000 psi) for AC, base, subbase, and concrete slab moduli, respectively, for the appropriate examples.
2. MODULUS was allowed to automatically select appropriate weights to be applied to the readings of each sensor.
3. Use of a rigid bedrock was suppressed for all examples except for those in which bedrock was present. For the examples in which bedrock was present, MODULUS does not

allow the bedrock modulus to be specified, but assigns it internally (*3*).

4. The "RUN A FULL ANALYSIS" option was used for all examples, so that material types were not required as input.

Three-Layer Pavements

Typical configurations of three-layer pavements with thin, medium, and thick AC layers are given in Table 1. Backcalculation of the layer moduli from the surface deflections were performed using seed moduli of 689.48, 103.42, and 20.68 MPa (100,000, 15,000, and 3,000 psi) for the AC, base, and subgrade layers, respectively. Exact Poisson's ratios and thicknesses were input.

The moduli backcalculated by the MICHBACK, MODULUS, and EVERCALC programs are given in Table 2, together with the maximum percentage error in the backcalculated moduli and the RMS error in the surface deflections described in Equation 13 (multiplied by 100). The MICHBACK program yields excellent results. Whereas the specified tolerance in two consecutive modulus estimates was $\varepsilon = 0.1$ percent, the backcalculated moduli actually have larger errors when compared with the actual moduli. Hence, the specified tolerance, ε, should usually be smaller than the error desired in the backcalculated moduli (perhaps by an order of magnitude). The MODULUS program yields significantly larger errors than MICHBACK, with the error in the base modulus being as much as 4 percent for the "thick" pavement. The EVERCALC program backcalculates progressively poorer results as the pavement becomes stiffer. However, when the alternative deflection basins generated by the original CHEVRON program are used, EVERCALC gives excellent results. This implies that improving the forward calculation program within EVERCALC should enable it to yield excellent results for three-layer pavements.

Four-Layer Pavement

Backcalculated moduli for a four-layer pavement are given in Table 3. The actual properties of the pavement and the seed moduli were thicknesses = 152.4, 254, 152.4, and ∞ mm (6, 10, 6 and ∞ in.); actual moduli = 3447.38, 310.26, 103.42, and 51.71 MPa (500,000, 45,000, 15,000 and 7,500 psi); Poisson's ratios = 0.35, 0.4, 0.4, and 0.45; and seed moduli = 689.48, 103.42, 48.26, and 20.68 MPa (100,000, 15,000, 7,000 and 3,000 psi). The results show that MICHBACK yields more accurate results than the other programs. The error in the subbase modulus backcalculated by MODULUS is very large

TABLE 1 "Typical" Three-Layer Pavements

Layer	Thickness (mm[a])			Poisson's Ratio	Modulus (MPa[b])
	Thin	Medium	Thick		
AC	50.8	127.0	228.6	0.35	3447.38
Base	152.4	203.2	152.4	0.40	310.26
Subgrade	∞	∞	∞	0.45	51.71

[a] 1 mm = 0.03937 in
[b] 1 MPa = 145.038 psi

TABLE 2 Backcalculation: Three-Layer Pavements

Program	Pavement Type	Backcalculated Modulus (MPa[a])			Max. Error in Moduli (%)	RMS Error in Deflections (%)
		AC	Base	Subgrade		
MICHBACK	Thin	3432.95	310.40	51.71	0.42	0.019
	Medium	3446.66	310.24	51.71	0.02	0.032
	Thick	3455.93	307.67	51.74	0.84	0.014
MODULUS	Thin	3346.79	316.47	51.71	2.92	0.370
	Medium	3468.75	307.51	51.71	0.89	0.110
	Thick	3346.71	322.67	51.71	4.00	0.139
EVERCALC	Thin	3472.48	309.54	51.71	0.73	0.020
	Medium	3294.06	317.25	51.71	4.45	0.065
	Thick	3030.65	399.84	51.66	28.87	0.126
EVERCALC-Alt	Thin	3448.97	309.93	51.72	0.11	0.020
	Medium	3467.17	308.71	51.72	0.57	0.024
	Thick	3450.86	315.52	51.72	0.17	0.148

[a] 1 MPa = 145.038 psi

TABLE 3 Backcalculation: Four-Layer Pavement

Program	Backcalculated Modulus (MPa[a])				Max. Error in Moduli (%)	RMS Error in Deflections (%)
	AC	Base	Subbase	Subgrade		
MICHBACK	3448.07	310.93	102.75	51.72	0.65	0.012
MODULUS	3756.95	248.21	153.75	52.34	48.67	0.162
EVERCALC	3282.99	319.28	101.55	51.74	4.77	0.092
EVERCALC-Alt	3413.06	319.04	98.64	51.77	4.62	0.063

[a] 1 MPa = 145.038 psi

(49 percent). Whereas the AC modulus backcalculated by EVERCALC improves significantly when the alternative deflection basin generated by the original CHEVRON program is used, the backcalculated subbase modulus becomes poorer. It has been found that for pavements with more than three layers, MICHBACK generally yields better results than EVERCALC, whereas the MODULUS program yields a poor estimate for at least one layer modulus.

Four-Layer Pavement with Incorrect AC Thickness

The ability of MICHBACK to improve incorrectly specified thicknesses is illustrated by performing a backcalculation of the four-layer pavement analyzed above assuming that the thickness of the AC layer was known only approximately. The AC layer thickness was input as 101.6 mm (4 in.), whereas the actual thickness was 152.4 mm (6 in.), and the MICHBACK program was used to backcalculate the layer moduli as well as the AC thickness. The program backcalculated layer moduli of 3454.83, 312.08, 102,42, and 51.73 MPa (501,081, 45,263, 14,855, and 7,503 psi) for the AC, base, subbase, and subgrade layers, and a thickness of 152.11 mm (5.989 in.) for the AC layer. The maximum error in the backcalculated moduli is 0.97 percent, the error in the thickness is 0.18 percent, and the RMS error in the compound surface deflections is 0.012 percent.

Four-Layer Composite Pavements

Backcalculated moduli for two four-layer composite pavements having a stiff concrete slab as one layer are given in Tables 4 and 5. The actual properties of the pavements and the seed moduli used were as follows:

- Stiff composite pavement (slab above base layer): thicknesses = 152.4, 254.0, 203.2, and ∞ mm (6, 10, 8, and ∞ in.); actual moduli = 3447.38, 31 026.4, 172.37, and 51.71 MPa (500,000, 4,450,000, 25,000, and 7,500 psi); Poisson's ratios = 0.35, 0.25, 0.40, and 0.45; and seed moduli = 689.48, 3447.38, 48.26, and 20.68 MPa (100,000, 500,000, 7,000, and 3,000 psi);
- Flexible composite pavement (slab beneath base layer): thicknesses = 152.4, 203.2, 254.0, and ∞ mm (6, 8, 10, and ∞ in.); actual moduli = 3447.38, 172.37, 31 026.4, and 51.71 MPa (500,000, 25,000, 4,450,000, and 7,500 psi); Poisson's ratios = 0.35, 0.40, 0.25, and 0.45; and seed moduli = 689.48, 48.26, 6894.8, and 20.68 MPa (100,000, 7,000, 1,000,000, and 3,000 psi).

The results indicate that whereas MICHBACK converges reasonably well for both composite pavements, the other programs yield significantly poorer results, especially for the stiff composite pavement. The moduli backcalculated by EVERCALC are extremely poor for the stiff composite pavement when the deflection basin generated by CHEVRONX is used. When the alternative basin generated by CHEVRON is used with EVERCALC, although the error in the backcalculated base modulus is 131 percent for the first composite pavement, the RMS error in the deflections is only 0.06 percent, indicating that the deflections computed by EVERCALC are very close to the input deflections. This implies that EVERCALC converged to a local minimum that does not represent the correct solution. For some other composite pavement sections that were analyzed EVERCALC was able to backcalculate more accurate moduli, but its accuracy was generally poorer than MICHBACK's.

Whereas surface deflections were rounded to the nearest hundredth of a mil in all the examples presented here, improved accuracy was obtained with MICHBACK when the surface deflections were input to greater precision, especially for very stiff composite pavements. The MODULUS and EVERCALC programs do not allow surface deflections to be input to precision greater than a hundredth of a mil. Whereas a precision greater than a hundredth of a mil is usually unrealizable in practice, the observation with MICHBACK indicates the difficulty that can occur when backcalculating moduli of composite pavements owing to sensitivity arising from even rounding of the surface deflections.

TABLE 4 Backcalculation: Stiff Composite Pavement

Program	Backcalculated Modulus (MPa[a])				Max. Error in Moduli (%)	RMS Error in Deflections (%)
	AC	Slab	Base	Subgrade		
MICHBACK	3443.03	31131.9	158.59	51.75	7.99	0.007
MODULUS	3639.74	30827.1	67.57	52.40	60.80	0.068
EVERCALC	10908.93	15838.0	90.95	51.83	216.44	1.526
EVERCALC-Alt	3410.81	29075.9	398.29	51.52	131.07	0.064

[a] 1 MPa = 145.038 psi

TABLE 5 Backcalculation: Flexible Composite Pavement

Program	Backcalculated Modulus (MPa[a])				Max. Error in Moduli (%)	RMS Error in Deflections (%)
	AC	Base	Slab	Subgrade		
MICHBACK	3444.31	172.29	30839.3	51.67	0.60	0.030
MODULUS	3392.90	175.13	30354.1	51.71	2.17	0.078
EVERCALC	4293.42	190.71	27300.9	51.39	24.54	0.753
EVERCALC-Alt	3388.46	176.00	30423.6	51.79	2.10	0.105

[a] 1 MPa = 145.038 psi

TABLE 6 Backcalculation: Three-Layer Pavements over Bedrock

Program	Bedrock Location	Backcalculated Modulus (MPa[a]) AC	Base	Subgrade	Max. Error in Moduli (%)	RMS Error in Deflections (%)
MICHBACK	Deep	3459.07	309.76	51.69	0.34	0.037
	Shallow	3446.14	309.68	51.75	0.19	0.087
MODULUS	Deep	3460.72	308.88	51.71	0.44	0.187
	Shallow	3505.29	299.23	53.09	3.55	0.074
EVERCALC	Deep	5493.76	215.07	51.90	59.36	1.150
	Shallow	4123.02	278.06	52.34	19.60	0.600
EVERCALC-Alt	Deep	3435.39	311.54	51.69	0.41	0.016
	Shallow	3456.94	309.20	51.76	0.34	0.041

[a] 1 MPa = 145.038 psi

TABLE 7 Performance Comparison of MICHBACK and EVERCALC for Four-Layer Flexible Pavement

Program	Number of Times Operation was Performed: CHEVRON Called	Gradients Computed	Iterations
MICHBACK (Newton)	27	5	5
MICHBACK (Modified Newton)	23	3	9
EVERCALC	21	4	4

Three-Layer Pavements Over Bedrock

Backcalculated moduli for two three-layer pavements overlying bedrock are given in Table 6. The only difference in the two pavements was the bedrock depth of 6.096 m (20 ft) for the deep bedrock pavement and 1.219 m (4 ft) for the shallow bedrock pavement. The true location of the bedrock was specified as input to all programs, and the true modulus of the bedrock, 34 473.8 MPa (5,000,000 psi), was input to MICHBACK and EVERCALC (the MODULUS program does not allow the bedrock modulus to be input). All other layer properties and the seed moduli used were identical to the three-layer pavement of "medium" thickness analyzed above (see Table 1). The moduli backcalculated by MICHBACK are somewhat better than those obtained by the other programs. Note that EVERCALC yields excellent results when alternative deflection basins generated by CHEVRON are used.

Performance Comparison

The performances of MICHBACK and EVERCALC are compared in Table 7 for backcalculation of the moduli of the four-layer flexible pavement considered earlier. Both the Newton method (in which the gradient matrix was calculated for each iteration) and the modified Newton method (in which four iterations were performed before updating the gradient matrix) were used in MICHBACK. Four more iterations are required for the modified Newton method than for the Newton method, but the total number of calls to the CHEVRONX program is fewer for the former (i.e., 23 as opposed to 27) since the gradient matrix is computed fewer times. The actual savings varies from problem to problem, but typically the modified Newton method requires about five fewer calls to CHEVRONX. The EVERCALC program calls CHEVRON only 21 times and is therefore marginally more efficient than MICHBACK (EVERCALC also computes a gradient matrix similar to the one computed by MICHBACK). For more complex problems, such as those involving composite pavements, the difference in the number of forward calculations performed by EVERCALC and MICHBACK can be greater, but MICHBACK usually yields more accurate results. In view of MICHBACK's accuracy, its slightly poorer performance appears justifiable. EVERCALC's performance was the same even if a deflection basin generated by CHEVRON was used.

CONCLUSIONS

An algorithm based on the efficient Newton method for the solution of simultaneous nonlinear algebraic equations is presented for the backcalculation of pavement layer properties from measured surface deflections. The method is capable of backcalculating layer thicknesses in addition to the layer moduli and has been implemented in a new backcalculation program named MICHBACK.

Comparisons of moduli backcalculated by the MICHBACK, MODULUS 4.0, and EVERCALC 3.0 programs, using deflection basins generated by an extended precision CHEVRON elastic layer analysis program, are presented for a variety of pavement sections. The results indicate that MICHBACK usually backcalculates more accurate moduli than the other programs. EVERCALC is also able to backcalculate moduli very accurately for three-layer flexible pavements, but it is handicapped by its use of the original CHEVRON program, the accuracy of which is poor for stiff pavements. Even for deflection basins generated by its own forward calculation program (i.e., original CHEVRON), EVERCALC's backcalculation deteriorates for four-layer flexible pavements and stiff composite pavements. MODULUS tends to produce a large error in at least one backcalculated modulus for four-layer flexible and stiff composite pavements.

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