Mathematical Model of Pavement Performance Under Moving Wheel Load

PER ULLIDTZ

Vehicle-pavement interaction was studied using the Mathematical Model of Pavement Performance (MMOPP). With MMOPP a length of flexible road is first generated on the computer. Values of layer thicknesses, elastic moduli, plastic parameters, and so forth are generated at points spaced 0.3 m apart in such a way that the pattern of variation is similar to that observed on real pavements. The pavement is then loaded using a quarter car model, and the dynamic load is calculated at each point. The permanent deformation of each layer and the structural deterioration of the asphalt layer resulting from the loads are likewise calculated for each point. The next time increment is then considered, changing the materials characteristics in accordance with climatic changes, applying the loads of that increment, and so on. MMOPP was used on two sections from the AASHO Road Test. The traffic was extended to 20 years, and the effects of varying vehicle characteristics such as spring constant of the tire, damping coefficient of the shock absorber, and use of single or dual tires were studied. It is concluded that the deterioration of pavements is a very complex process, depending on an interaction of pavement parameters, climatic effects, vehicle characteristics, and time. Computer simulation appears to be an efficient and cost-effective means of filling some of the gaps in existing knowledge.

The ride quality of a road pavement is an important indicator of performance. How the ride quality changes over time depends on the longitudinal variation of pavement characteristics as well as on variations in the loading. It is the variation in layer thicknesses, stiffnesses, and plastic and strength parameters in combination with the dynamic effects of the traffic loading that cause a pavement to become rough. Without these longitudinal variations a pavement would remain smooth. It could deteriorate due to rutting and cracking, but it would never get rough.

The Mathematical Model of Pavement Performance (MMOPP) used for this study attempts to model this longitudinal variation on a computer. The first version of the model was developed from 1976 to 1978 (*1,2*), and a later version is described in detail elsewhere (*3*). The model has recently been rewritten for a PC. Only flexible pavements may be simulated by the model.

A brief description of MMOPP is given first, followed by some examples of its application. For this study the model was first used to simulate two of the AASHO Road Test sections. The traffic was then extended over 20 years, for the same sections, and different parameters of the suspension (spring constants, damper coefficients) as well as wheel configuration (dual or single) and traffic mix were used.

Institute of Roads, Transport and Town Planning, Technical University of Denmark, Building 115, DK-2800 Lyngby, Denmark.

MOVING WHEEL LOAD

A road section is first generated on the computer, using a stochastic process, and is then loaded by a moving wheel load, consisting of a mechanical analog of a simple quarter car model (see Figure 1).

The lower system (M2, K2, C2) represents the mass of the axle and the wheel, the spring constant of the tire, and the tire damping. The upper system is the mass of the vehicle and the payload transferred to one wheel, the spring constant, and the damping coefficient of the suspension system. All parameters are considered to be linear, but introducing nonlinear parameters would be easy. The wheel may be either single or dual.

The force exerted at each point of the road surface is calculated using a numerical method. The length of each step in the calculation depends on the resonance frequency and the speed of the vehicle. In most cases a step length of 50 mm could be used.

Parameter values for spring constants and damping coefficients were selected on the basis of information given elsewhere (*4,5*). In the examples given later the following basic parameters were used: spring constant—1242 N/mm for tire and 278 N/mm for suspension; damping coefficients—1 Nsec/mm for tire and 14 Nsec/mm for suspension.

LONGITUDINAL VARIATION OF PARAMETERS

To model the longitudinal variation of the pavement, the road is divided into short lengths, each with a length of only 0.3 m (1 ft). This short length is considered representative of one "point" of the pavement. The layer thicknesses and elastic, plastic, and strength parameters are then varied from point to point. The pattern of variation is very important to the outcome of the simulation.

The variation from point to point is not random. The value at a particular point depends to some extent on the values at preceding points. This may be seen, for example, on the original profile (fall 1958) in Figure 2. This dependency may be described by the autocorrelation function.

An example may illustrate this. Consider a longitudinal road profile where the elevation is measured at points spaced 0.3 m apart and assume that the standard deviation on the elevation is 1 mm. If the values are randomly distributed, this profile would be very rough, with a present serviceability index (PSI) of about 2.5. If, however, the correlation coefficient between consecutive values is 0.9, and 0.8 for points

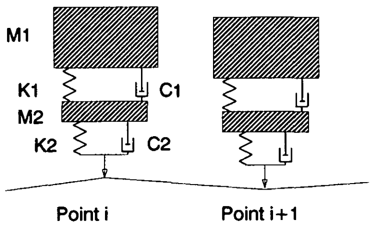

FIGURE 1 Mechanical analog.

0.6 m apart, the profile is very smooth, with a PSI of about 4.0.

A second-order autoregressive process is used to generate the parameters to obtain a distribution with given mean value, standard deviation, and two values of the autocorrelation function.

An example of the variation of layer moduli resulting from the stochastic process is shown in Figure 3. The autocorrelation coefficients for 0.3 and 0.6 m distance were 0.9 and 0.75, respectively, for the asphalt, decreasing to 0.6 and 0.3 for the subgrade.

Very little information is available with respect to actual values of the autocorrelation function for parameters such as elastic modulus, bitumen content, and so forth.

CALCULATION OF PERMANENT DEFORMATION

MMOPP works in increments of time. In the examples below increments of 1 month were used. For each increment the effects of temperature on the asphalt material and seasonal effects, such as spring thaw or wet and dry periods, on the unbound materials (including the subgrade) are first calculated. Then the loads are applied using the mechanical analog described above, and the force (static plus dynamic load) exerted at each point is calculated. The effects of the loads are determined in terms of the permanent deformation of each material and the reduction of the asphalt modulus caused by fatigue.

To calculate the permanent deformations, both the elastic and plastic characteristics of the materials must be known.

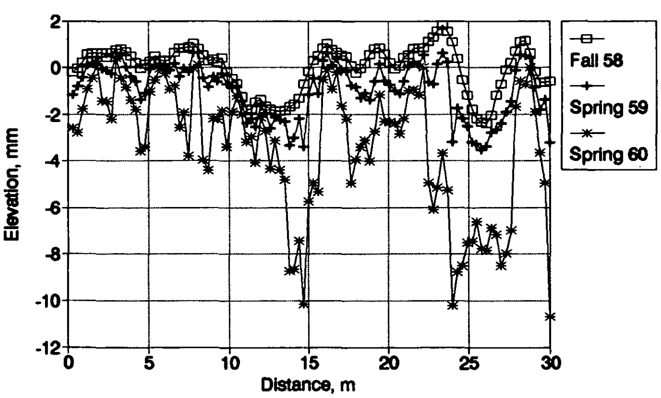

FIGURE 2 Longitudinal profile, simulation of S30, Design 6-9-8.

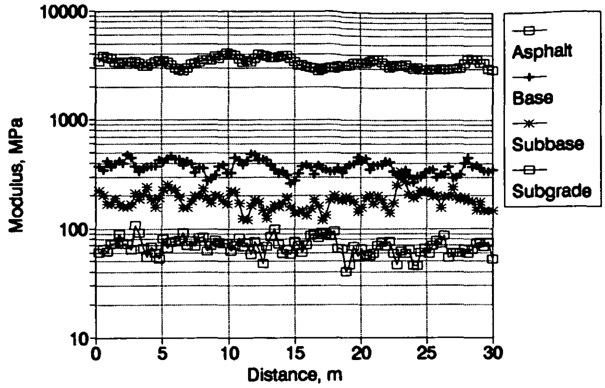

FIGURE 3 Variation of layer moduli, simulation of S30, Design 6-9-8.

The elastic parameters are needed to describe the stress state, and the plastic are needed to get the permanent deformations resulting from the stress state.

The elastic parameters are Young's modulus (E) and Poisson's ratio (ν). For the plastic parameters a decreasing strain rate is assumed as long as the plastic strain is below a critical level. The plastic strain is assumed to be proportional to the number of load repetitions (N) raised to a power (α) (0.1 used as default value) and to the major principal stress (σ_z) raised to another power (β) (1.6 used as default):

$$\varepsilon_p = A \times N^{\alpha} \times \left(\frac{\sigma_z}{\sigma'}\right)^{\beta}$$

A is a constant and σ' is a reference stress.

When the permanent strain reaches the critical level, the strain rate is assumed to be constant, equal to the tangent at the critical level.

The "plastic modulus," E_p, is defined as the ratio of the compressive stress to the plastic strain.

To calculate the permanent deformations Boussinesq's equations are used with Odemark's transformation (*3*). Odemark's equivalent thicknesses, determined from the elastic parameters, define the stress state. The equivalent thickness is calculated from

$$h_{e,n} = f \times \sum_{i=1}^{n-1} \left(h_i \times \sqrt[3]{\frac{E_i}{E_n}}\right)$$

where

h_i = thickness of Layer i,
E_i = modulus of Layer i, and
f = correction factor.

When the equivalent thickness has been calculated, the vertical compressive stress, σ_z, may be found for the centerline of the load from

$$\sigma_z = \sigma_o \times \left\{1 - \frac{1}{\left[\sqrt{1 + \left(\frac{a}{z}\right)^2}\right]^3}\right\}$$

where

σ_o = contact stress under the tire,
a = radius of the loaded area, and
z = depth to the point considered.

For points at a distance R from the center of the loaded area, the vertical compressive stress may be found from the following equation when the distance is larger than the diameter of the loaded area ($R > 2a$):

$$\sigma_z = \frac{3 \times P}{2 \times \pi \times R^2} \times \left(\frac{z}{R}\right)^3$$

where P is the load.

The compressive stress may then be substituted in the equation for the plastic strain, and the permanent, or plastic, deformations can be calculated by integration over the layer thickness. For the center line of the load a closed form solution of the integration exists. The plastic deformation from depth z to infinity may be found from

$$d_p = \frac{(1 + \nu) \times (3 - 2\nu) \times P}{2 \times \pi \times Z \times (2\beta - 1) \times E_p}$$

where ν is Poisson's ratio.

STRUCTURAL DETERIORATION OF ASPHALT

Fatigue of asphalt starts as microcracking in the material. A microcrack reduces the active cross-sectional area of the asphalt layer and thus the modulus of the layer. In the case of uniaxial tension the decrease in modulus is proportional to the loss of area, and the rate of damage may often be expressed by the "kinetic equation" (*6*):

$$\frac{dA}{dN} = K \times \left(\frac{\sigma}{1 - \omega}\right)^n$$

where

dA = decrease in active cross-sectional area caused by dN load repetitions,
σ = mean stress over the area,
ω = damage defined as the lost area divided by the original area, and
K, n = materials constants.

Under a rolling wheel load the stress condition is very complex. To calculate the damage directly would require a sophisticated finite element program. Instead, an empirical relationship with the maximum strain at the bottom of the asphalt layer, ε, is used, having the same format as the kinetic equation:

$$\frac{\Delta E}{E_0} = K \times (\varepsilon)^n \times \Delta N$$

A point is assumed to show a fine crack when the modulus has been reduced to two-thirds the original value and a severe crack at one-third the original value. When the asphalt has reached severe cracking, the modulus is assumed to remain constant. The strain at the bottom of the asphalt is again calculated using Boussinesq's equations with Odemark's transformation.

MMOPP USED ON THE AASHO ROAD TEST

MMOPP produces a graphical output to the screen and stores a number of intermediate values in disk files. At the upper part of the screen the longitudinal road profile is shown as it was generated by the computer and as it changes during loading.

Some examples of longitudinal profiles are shown in Figure 2. The profiles are from a simulation of one of the AASHO Road Test duplicate sections, from Loop 6. The section had 6 in. (152 mm) of asphalt, 9 in. (229 mm) of base, and 8 in. (203 mm) of subbase, and it was loaded by a 30-kip (67 100-N) single-axle load.

The profile from fall 1958 is after 1 month of loading. This profile remains practically unchanged until the spring thaw of 1959, when a considerable permanent deformation occurs. The next major change in the profile is during spring 1960.

In the middle of the screen MMOPP shows the dynamic loading. A horizontal line in the middle indicates the static load (the dead weight), and two other lines indicate a dynamic load of ±20 percent of the static load. The dynamic load is shown for each point of the road.

Static plus dynamic load is shown in Figure 4 at the beginning of a simulation, when the PSI was above 4.0, and toward the end of the experiment, when the PSI was less than 2.0. At the beginning of the test the maximum deviation from the static load level is about 5 percent, and at the end it increases to about 20 percent. The speeds were chosen randomly between 55 and 65 km/hr.

In Figure 5 the asphalt damage is shown for the simulation described above. The values from fall 1958 are after 1 month of loading. The minimum asphalt modulus is assumed to be one-third of the original value. The values shown are at the same reference temperature.

The lower part of the MMOPP screen shows the change in PSI during the duration of the simulation. Figure 6 shows the results of the above simulation of an AASHO Road Test section (Loop 6, design 6-8-9, 30 kips). The AASHO Road Test was started in late 1958 and lasted about 2 years.

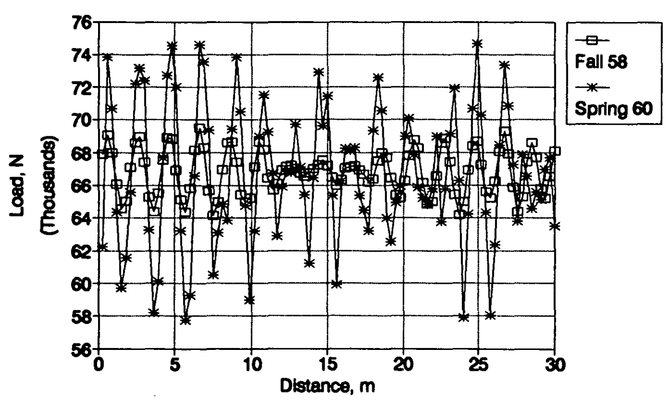

FIGURE 4 Wheel load, simulation of S30, Design 6-9-8.

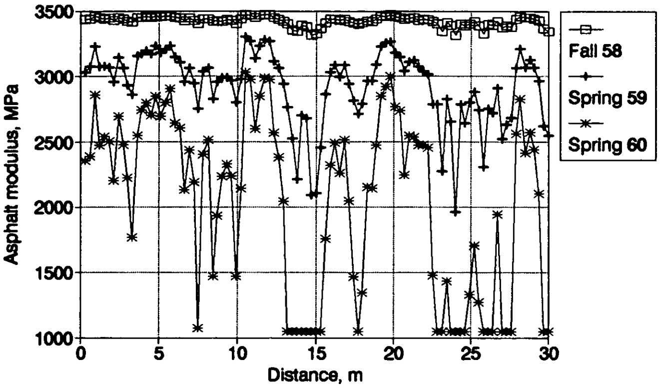

FIGURE 5 Asphalt damage, simulation of S30, Design 6-9-8.

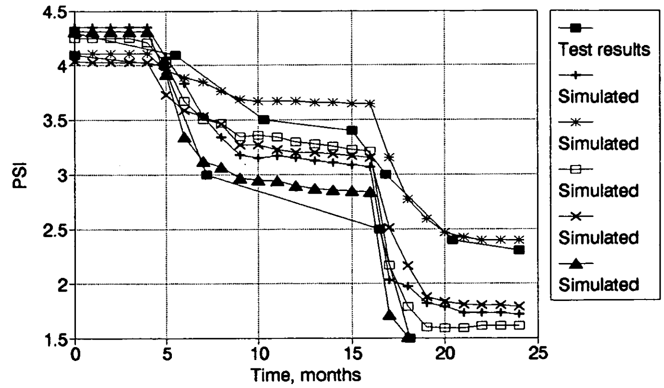

FIGURE 7 Change in PSI, repeated simulations, identical input.

The actual outcome of the road test is also shown for the two identical (duplicate) sections. One section fails in the spring of 1960, whereas the other lasts for the whole experiment. The failure condition was a PSI of 1.5. For both sections most of the damage takes place during spring thaw.

Because the parameters are generated by a stochastic process, the outcome will be different when the program is rerun with the same input parameters, just as was the case for duplicate sections during the actual test. This is shown in Figure 7, where five simulations are compared with the measured performance. Figure 8 shows the corresponding changes in asphalt modulus at a reference temperature. The asphalt modulus is a mean value for the sections. Examples of the variation in modulus along the section are shown in Figure 5.

Another duplicate section, from Loop 2, was also simulated. This section had 2 in. (51 mm) of asphalt, 3 in. (76 mm) of base course gravel, and 4 in. (102 mm) of subbase. The load was a 6-kip (13 360-N) single axle. Figure 9 shows that, again, one of the duplicate sections failed shortly after the spring thaw in 1960, whereas the other lasted to the end of the test.

For both of these simulations the agreement with the performance of the actual test sections is very good. However, a number of input values were unknown and had to be estimated.

EFFECTS OF DIFFERENT LOADING CONDITIONS

The next step in the study was to extend the traffic of the AASHO Road Test over a period of 20 years. During the AASHO Road Test the pavements received approximately 1 million loads, so for a 20-year period this would correspond to 50,000 loads per year.

For the 2-3-4 design from Loop 2 this resulted in a mean life of 3.90 years with a standard deviation 0.81 years, when a PSI of 1.5 was used as the failure criterion. Three simulations are shown in Figure 10. For the 6-9-8 design of Loop 6, the mean life was 3.58 years with a standard deviation of 0.58 years.

Both pavements, in other words, lasted for less than 200,000 loads, although during the AASHO Road Test they had supported more than 600,000 identical loads. This difference is due to the loading rate during the AASHO Road Test, which was much lower during the beginning of the test, including the first spring period, than during the later part of the test. It does, however, emphasize the importance of using a model based on actual distress mechanisms when extrapolating from accelerated tests. A purely statistical extrapolation could lead to very erroneous results.

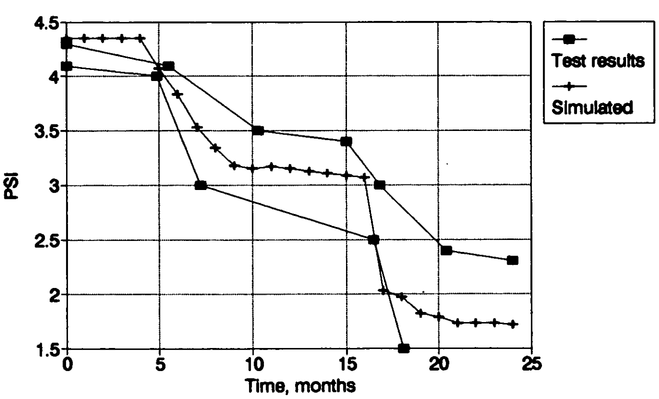

FIGURE 6 Change in PSI, AASHO Road Test results and simulation.

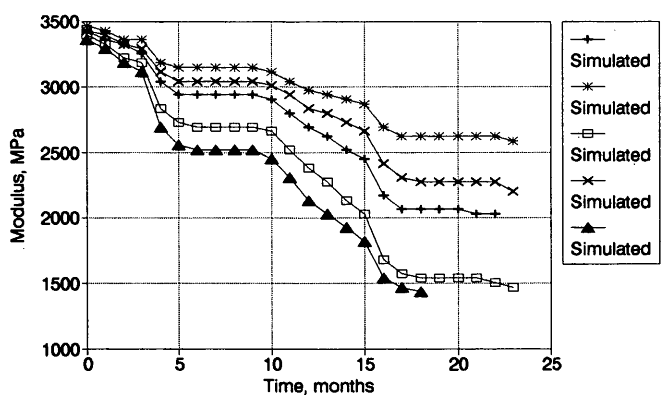

FIGURE 8 Change in asphalt modulus, repeated simulations, identical input.

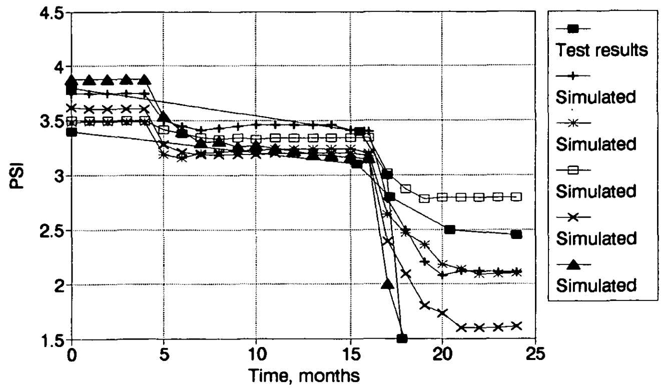

FIGURE 9 PSI, Loop 2, Design 2-3-4, 6 kips.

TABLE 1 Design 2-3-4, 6 kips

	Mean life years	Standard deviation	Light cracking %
AASHO traffic	3.90	0.81	11
½ tire stiffness	5.74	0.99	25
Constant load	4.43	1.08	16
Single wheel	0.62	0.07	0
Mixed traffic	3.12	0.46	26

The amount of light cracking at failure was 11 percent (standard deviation 13 percent) for the 2-3-4 design and 53 percent (standard deviation 17 percent) for the 6-9-8 design.

According to Page (*5*) the tire stiffness is very important to the magnitude of the dynamic load component. Dynamic wheel loads decrease significantly as the tire stiffness is decreased. For the 2-3-4 design a simulation was carried out with the tire stiffness reduced to half the value of the standard wheel. This resulted in a mean life of 5.74 years with a standard deviation of 0.99 years. This corresponds to a 46 percent increase in mean life. The amount of light cracking at the end of the pavement life was 25 percent (standard deviation 10 percent), that is, a larger amount of cracking than observed for the standard wheel, but also after many more load applications. The rate of cracking was 2.2 percent per 100,000 load applications, whereas it was 1.4 percent for the standard load.

Another important parameter is the damping coefficient of the shock absorbers in the suspension system. For a simple damper model it is normally possible to find an optimum value, but actual dampers have very complex characteristics. For the 6-9-8 design a simulation was carried out with a shock absorber having half the damping coefficient of the standard suspension. This resulted in a mean life of 3.13 years (standard deviation 0.70 years), or a decrease of 13 percent. The amount of light cracking was 41 percent at failure, corresponding to a cracking rate of 6.5 percent per 100,000 load applications, whereas it was 7.4 percent for the standard wheel.

Both sections were also simulated using a static wheel load (i.e., without any dynamic effects). This increased the mean life of both sections by about 10 percent and did not change the rate of cracking significantly. It is surprising that the change was not greater. Using the fourth-power rule and a dynamic effect of just 10 percent should result in an increase of about 40 percent.

Changing from a dual to a single wheel on the 2-3-4 design had a very dramatic effect. In all 10 simulations the pavement failed in the first spring period, resulting in a mean life of 0.62 years (−84 percent) (standard deviation 0.07 years). For the 6-9-8 design the change is less dramatic but still important, with a mean life of 2.02 years (−44 percent) (standard deviation 0.37 years).

Finally, the two section were loaded by mixed traffic. For both sections the standard wheel was used as the maximum load and two lighter wheel loads were added. The number of applications of each wheel load were chosen so that the total number of passages would correspond to 50,000 standard wheel loads per year, using a fourth-power relationship.

For both sections mixed traffic reduced the mean life and increased the rate of cracking. For the 2-3-4 design the mean life was reduced by 20 percent to 3.12 years (standard deviation 0.46 years), and for the 6-9-8 by 31 percent to 2.48 years (standard deviation 0.42 years). For the 2-3-4 design the rate

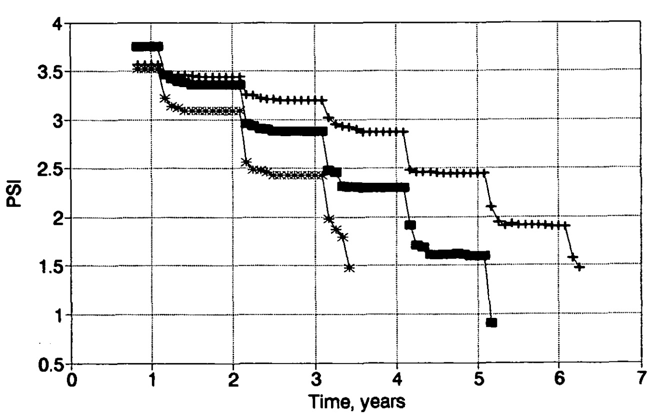

FIGURE 10 Traffic extended over 20 years, Loop 2, Design 2-3-4, 6 kips.

TABLE 2 Design 6-9-8, 30 kips

	Mean life years	Standard deviation	Light cracking %
AASHO traffic	3.58	0.58	53
½ damping	3.13	0.70	41
Constant load	3.83	0.79	44
Single wheel	2.02	0.37	44
Mixed traffic	2.48	0.42	61

of cracking increased to 4.2 percent per 100,000 standard loads (from 1.4 percent for the standard wheel) and for the 6-9-8 design to 12.8 percent (from 7.4 percent).

The results of the simulations are summarized in Tables 1 and 2.

CONCLUSION

The main conclusion is that the deterioration of pavements is a very complex process. It involves an interaction of pavement parameters, climatic effects, vehicle characteristics, and time. Any attempt at isolating the effects of a few parameters will fail if it does not consider the influence of all other parameters.

Sometimes the effects of dynamic loading are considered by adding the dynamic and the static load and calculating the equivalent number of standard axles using a fourth-power law. The results presented indicate that this would be completely erroneous. The effects of dynamic loading may be quite different on different types of distress such as roughness and rutting and cracking, and they will depend on a number of other parameters such as the type of pavement structure.

In the AASHO Road Test some of the important parameters were studied, and the SHRP Long-Term Pavement Performance Program (LTPP) will provide additional information. Even with this information, gaps will still exist. On the basis of the AASHO Road Test and the LTPP data it will not, for example, be possible to evaluate the effects of an improved suspension system or other aspects of vehicle-pavement interaction.

To try to fill some of the gaps in existing knowledge, it is recommended that computer simulation be used, as it is in many other fields of engineering. If a computer model can be developed to fit a number of experimental results, the model may with some confidence be used to extrapolate to other conditions. This may save some costly experiments and may even be used where experiments are not possible.

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