Analytical Model for Two-Stage Rotary Snowplows

DAVID J. STEVENS AND KEVIN B. POWERS JR.

An analytical model for the performance of two-stage rotary snowplows is developed to consider both high-velocity operations and the effect of snow properties on machine performance. The analysis determines the power required to cut and disaggregate the snow, the snow removal rate, and the total power required by the system. The purpose of the model is to perform parametric assessments of the rotary snowplow's performance and efficiency. The model results show that the strength of the snow has considerable effect on the power required by the auger to disaggregate the snow and on the total power required for removing the snow. Unfortunately, the available data from field tests of full-scale rotary snowplows are limited and no quantitative verification of the model can be made; however, the model does allow qualitative assessment of the effects of changing the rotary snowplow configuration or the properties of the snow.

In the past, the development of snow removal equipment (both displacement and rotary snowplows) has been performed on a trial-and-error basis. This lack of a rigorous approach was discussed by Minsk (*1*), who pointed out that the "snowplow industry in the United States is composed of small, widely dispersed firms, none with the resources to finance a significant research program. Furthermore, there is no industry association to attempt a cooperative research effort funded by contributions from manufacturers. There is little incentive for manufacturers to spend the necessary amounts required for a comprehensive research program."

In addition, the existing designs and analyses of snow removal equipment have been developed without sufficient consideration of the properties of snow—that is, that "snow is snow." The properties of snow (density and strength) vary greatly from snowfall to snowfall or even hour to hour and, thus, strongly affect the performance of both displacement and rotary snowplows. Proper consideration of the properties of the snow in the design process should aid in improving the performance and efficiency of snow removal equipment.

In this paper, an analytical model for the performance of rotary snowplows is presented. Previous developments (*2,3*) are extended to include high-speed plowing and are improved by considering the effects of snow properties on the power required by the machine.

In the computer implementation of the model, the power requirements of the auger, impeller, and vehicle are evaluated under different operating conditions; the model is applied to a series of forward speeds, auger and impeller rotational speeds, snow depths, snow densities, and snow strengths. Unfortunately, experimental data to provide quantitative verification of the model are not currently available; however, the results appear to be qualitatively reasonable and the model provides a means of assessing the influence of parametric changes.

D. J. Stevens, Southwest Research Institute, 6220 Culebra Road, San Antonio, Tex. 78228. K. B. Powers, New York Department of Transportation, 1220 Washington Avenue, Albany, N.Y. 12226.

BACKGROUND

Unlike the displacement snowplow, which lifts the snow from the surface in front of the vehicle and transfers it to the side, a rotary snowplow cuts or disaggregates the snow and then propels the disaggregated mass away from the machine. A one-stage rotary snowplow uses a single element to perform both cutting and casting. A two-stage rotary snowplow consists of two elements: a rotating auger with three or four helical steel ribbons that disaggregate (or cut) the snow, and an impeller with four or five rotating paddles that discharge the material. Figure 1 shows a schematic of a rotary snowplow, and Figure 2 shows a large rotary snowplow in action. The power requirement of the system includes the power to drive the auger and impeller and the power to propel the vehicle. For large machines, separate engines are commonly used to propel the vehicle and to drive the collecting apparatus; in some commercial machines, a 450-kW (600-horsepower) engine drives the auger and impeller and a 200-kW (260-horsepower) engine propels the vehicle. Typically, these large snowplows are used to clear military and civilian runways, mountain roads, the shoulders of roads after heavy snowfalls, and urban areas where the snow is loaded directly into a truck. High-speed plowing is obviously critical for runway operations.

One of the first studies of rotary snowplows was performed by Croce, who tested and analyzed one-stage machines and primitive two-stage machines (*2*). In a later publication, Croce applied his theories to a single machine (*4*). Croce noted that the mechanical energy needed to cut the snow depends on the hardness, but he did not explicitly consider the effects of snow properties on system performance.

Later, Shalman investigated the energy requirements of the cutting element (auger) and the throwing element (impeller) separately (*3*). Shalman's work was analytical, and no correlation between the analysis and actual machine operations was drawn.

SNOW PROPERTIES

Minsk defined snow removal as a (relatively) simple task of material handling (*1*). The complexity, though, lies in the large variability of the material's properties. Density can range

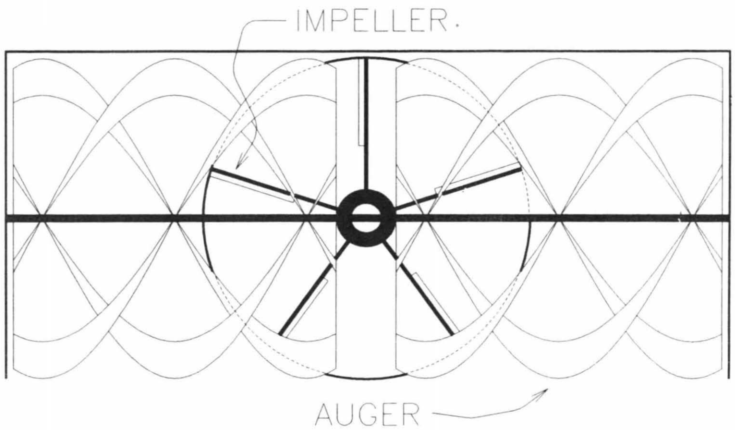

FIGURE 1 Components of a two-stage rotary snowplow: front view.

from 48 kg/m³ (3 lb/ft³) for new snow to 593 kg/m³ (37 lb/ft³) for snow that has been windblown or plowed. In addition, snow can be compressed as much as eight times, or up to 800 kg/m³ (50 lb/ft³). Hardness can range from less than 6.9 kPa (1 lb/in.²) for freshly fallen, low-density snow to 206.9 kPa (30 lb/in.²) for well-bonded, high-density snow.

One of the interesting properties of snow is its ability to increase strength with time after it has been mechanically agitated or processed (*5–7*). For instance, snow that has just been plowed will exhibit an increased density but little immediate strength gain; however, within an hour or two, the windrow can be strong enough to support the weight of a person, even though the density is unchanged. This age strengthening occurs through the process of sintering, in which the number and size of bonds between the snow crystals increase with time (*8*).

Much work has been performed to determine and relate other snow properties, such as density, compressive strength, and shear strength; see work by Powers and Stevens (*9*) for a detailed discussion of these studies. However, for later use, two studies are discussed here.

First, Gold analyzed the dependence of the compressive strength (hardness) of snow on density, temperature, and crystal size (*10*). Gold expressed the relationship between

FIGURE 2 Rotary snowplow in action.

density and hardness as

$$H = 6.67e^{10.24\rho} \qquad \rho \leq 0.15 \text{ g/cm}^3$$

$$H = 35{,}500\rho^{3.92} \qquad 0.15 \text{ g/cm}^3 < \rho < 0.4 \text{ g/cm}^3 \tag{1}$$

where H is the hardness and ρ is the density. It should be noted that, as is typical with most naturally occurring materials, there is significant scatter in the measured data.

Second, Perla et al. used a shear frame with an area of 0.25 m² (38.75 in.²) to measure the shear strength of snow in the field (*11*). They related the shear strength to the density as

$$\tau = \tau_i(\rho/\rho_i)^k \tag{2}$$

where

τ = shear strength,
ρ = density,
ρ_i = characteristic density = 917 kg/m³ (57 lb/ft³),
τ_i = 397.5 kPa (5,760 lb/ft²), and
k = 2.7

ANALYTICAL MODEL

An analytical model of a two-stage rotary snowplow is developed to determine the power required to disaggregate and remove snow. As discussed earlier, a two-stage rotary snowplow contains a rotating auger that cuts the snow and an impeller that discharges the material (Figure 1).

In this analysis, two topics are investigated: the dynamics and kinematics of the auger, impeller, and truck; and the effect of changing snow properties on the system performance. The investigation focuses on the power required to disaggregate the snow by the auger. The analysis of the power required by the impeller for casting is based on previous research (*3*), and the power used in plowing the snow with the back of the auger housing is based on experimental studies (*9*). And, as with previous researchers (*2,3*), a number of simplifying assumptions are made regarding the movement of snow within the auger and impeller because of the complexity of the system.

Auger Mechanics

The investigation of the auger considers the cutting path of the blades, the power required for cutting the snow, and the power required to move the snow to the impeller.

Auger Cutting Path

The cutting path of each auger blade is represented by a set of parametric equations that define the position of the blade:

$$x = \frac{U\theta}{\omega} + R \sin \theta y = R \cos \theta \qquad (3)$$

where

U = vehicle velocity,
ω = auger's angular velocity,
R = auger's radius, and
θ = angular position of blade (Figure 3).

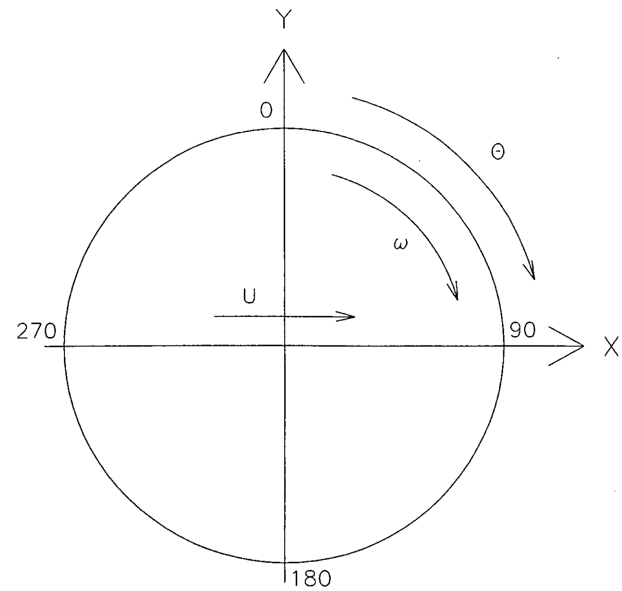

FIGURE 3 Coordinate system of auger.

The blade path can be uniquely determined by the ratio of angular velocity of the auger, ω^*, to forward velocity, U^*.

According to Shalman (*3*), it was assumed that the forward velocity term $U\theta/\omega$ in Equation 3 could be taken to be negligibly small in comparison with the angular velocity; thus, the blade path is a circle, following approximately the same path as the previous blade path and effectively taking only a small "bite" from the snow mass. This assumption only holds for low-speed plowing. For high-speed plowing, the forward velocity term in Equation 3 becomes significant, and the trajectory of the blade is a cycloid with slip, creating loops in the path (Figure 4). Therefore, the length of the cutting arc for the outside and inside of each blade will increase as the overlap of the previous blade path is reduced (Figure 5). The lengths of both the outside and inside cutting arcs reach their maximum when the blade face no longer crosses the previous path. For very high speed plowing, the forward velocity term dominates the equation, and the blade trajectory approaches the path of a true cycloid.

In high-speed plowing, a part of the snow mass is not cut (Figure 6) but is instead plowed by the back of the auger housing or moved directly into the impeller. In this situation, the truck will have to provide more driving power to overcome the plowing force. The proposed model includes an approximation of the plowing effect of the auger housing, as discussed later.

Auger Power

The analysis by Shalman of the cutting force of an auger blade was developed for low-speed plowing and did not consider the forward motion of the machine, which underestimates the cutting force required for high-speed plowing (*3*).

The analysis begins by considering the increment in cutting force as a function of the resistance to cutting:

$$dF = kdB \qquad (4)$$

where

dF = increment in cutting force,
k = coefficient of cutting resistance, and
B = width of the cutting blade.

However, because the cutting resistance is a function of both the shear strength and compressive strength of the snow, it is assumed, in this model, that the cutting resistance can be taken as follows:

$$k = k_c b_\theta + nk_s \qquad (5)$$

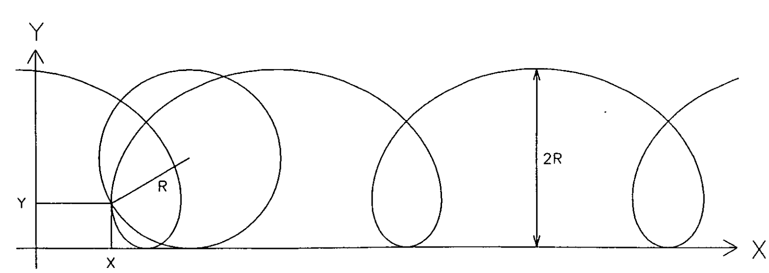

FIGURE 4 Path of a cycloid with "slip."

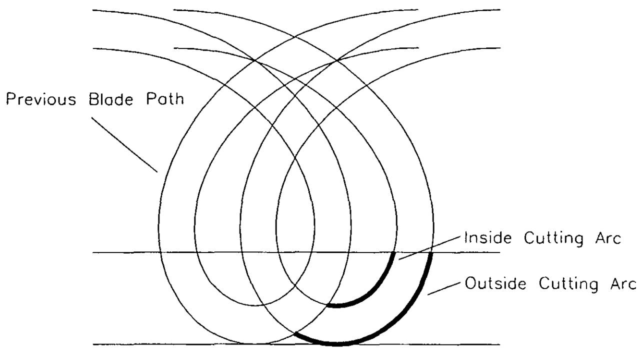

FIGURE 5 Inside and outside cutting arcs of auger.

where

- k_c = compressive strength of snow,
- n = number of shear faces (n = 1 or 2),
- k_s = shear strength of snow (force/unit length), and
- b_θ = thickness of the snow layer being compressed and is a function of the position of the blade in the snow.

The strength value k_c is identical to the hardness H, given in Equation 1, and k_s is the shear strength τ, from Equation 2, multiplied by the length of the shear box. Figure 7 illustrates the shear and compression resistance of the snow. If the snow is deep enough, the auger blade will also cut on the inside edge (i.e., $n = 2$), assuming that the inertia of the snow is sufficient to resist the cutting force.

It is well-known that snow is a rate-dependent material, exhibiting greater strength and stiffness at higher strain rates, but rate effects are not considered in the initial version of this model and will be considered in later development.

The thickness b_θ of the compressed snow layer is a function of the blade position but is limited to the overall width of the blade when the snow is sheared by both edges of the blade ($n = 2$). The determination of b_θ for different plowing conditions when only one edge is cutting ($n = 1$) is discussed in detail by Powers and Stevens (*9*).

The increment of work done to overcome the cutting force is expressed as

$$dW = zdFdl \tag{6}$$

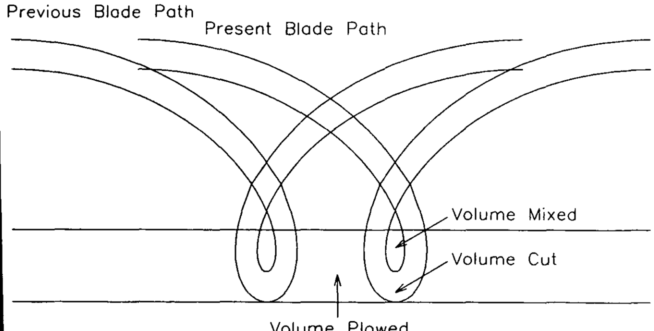

FIGURE 6 Volumes of snow that are mixed, cut, and plowed.

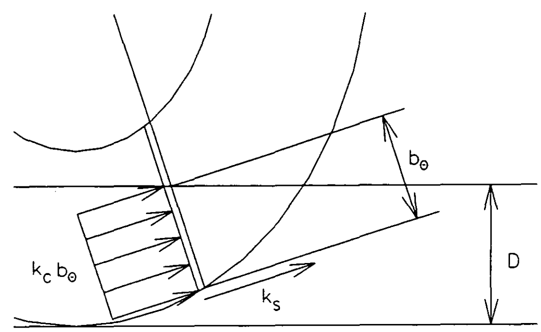

FIGURE 7 Resistance of snow to shear and compression.

where dl is the differential length of the path taken by the blade and z is the number of auger blades. The term dl is derived in terms of the velocity in the parallel and normal direction of travel, and, with a constant angular velocity, it is found to be

$$dl = \frac{\sqrt{U^2 + (R\omega)^2 + 2UR\omega \cos\theta}}{\omega}\, d\theta \tag{7}$$

The work differential is written by substituting Equations 4 and 7 into Equation 6. By considering the full width B, multiplying by the angular velocity, and integrating, the power required for cutting by the auger, P_{a1}, is found as

$$P_{a1} = zB \int_{\theta_{in}}^{\theta_{out}} (k_c b_\theta + nk_n) \times \sqrt{U^2 + (R\omega)^2 + 2\,UR\omega \cos\theta}\; d\theta \tag{8}$$

where θ_{in} and θ_{out} are the angles at which the blade enters and leaves the snow, respectively.

Auger Power Required for Snow Movement

In addition to the power needed to disaggregate the snow, the auger is required to accelerate the snow that is cut (P_{a2}) and move it into the impeller (P_{a3}). Additional power is required for snow that is rehandled in the mixing of the uncut snow (P_{a4}). Due to space limitations, the derivation of expressions for these terms is omitted from this paper; the interested reader is referred to work by Powers and Stevens (*9*), which follows, with some exceptions, the work of Shalman (*3*).

Last, the total power required by the auger is then $P_a = P_{a1} + P_{a2} + P_{a3} + P_{a4}$.

Impeller Power Requirements

Shalman showed that the required impeller power is composed of three components: (a) the power to transfer kinetic energy to the snow moving through the impeller (P_{i1}), (b) the power to overcome the friction of the snow against the impeller housing (P_{i2}), and (c) the power to raise the snow to the height of the discharge chute (P_{i3}). Again, because of space limitations, the derivations of the equations for these terms are omitted and the reader is referred to Powers and Stevens (*9*), which follows Shalman (*3*), with some modifications.

The total impeller power, P_i, required for discharging the snow is given by $P_i = P_{i1} + P_{i2} + P_{i3}$.

Plowing Power

For high-speed removal, part of the snow mass entering the auger is not cut by the auger blades (Figure 6) but is instead plowed by the back of the auger housing. The proposed model includes an approximation of the plowing power (P_p) based on the experimental work by Powers and Stevens (*9*).

Snow Properties

One purpose of this study is to characterize the performance of snow removal equipment with respect to snow properties. Therefore, three general relationships are introduced into the model so that continuity of the material properties is maintained; these relationships are compressive strength versus density (Equation 1), shear strength versus density (Equation 2), and the degree of compaction of the snow during the removal process. All of these relationships are considered as bulk properties and are assumed to be constant for the entire cutting arc; therefore, instantaneous changes are not considered.

The relationships between shear strength and density and between compressive strength and density are used to determine the coefficient of cutting resistance (Equation 5). Thus, given a particular density, the cutting resistance can be determined; however, it should be noted that the model can consider shear strength, compressive strength, and density as uncoupled if so desired.

Last, the amount of compaction (or increase in density) that the snow undergoes during processing by the auger is determined from the experimental observations of Croce (*2*). The relationship between degree of compaction and initial density was digitized and applied in the computer model to the analysis of the mass balance of the snow as it enters and exits the auger.

Computer Model

The computer application of the auger analysis determines the bounds of the integral for P_{a1} in Equation 8, by comparing the entering and exiting positions of two consecutive blade paths. Once the crossing patterns are known, a bisection algorithm is used to determine the angle of intersection. This two-step operation is used to calculate the applicable angles of intersection between the outside edges of the current and previous blade paths and the inside edge of the current blade path and the outside edge of the previous blade path (Figure 5). Numerical integration of Equation 8 is performed by using the trapezoidal method on the subsections of the cutting arc.

The model also evaluates P_{a2}, P_{a3}, and P_{a4}: the amount of auger power required to accelerate the snow, to move it to the impeller, and to mix it, respectively. The power required to accelerate the snow is applied only to the volume of snow directly struck by the blade; however, the power to move the snow into the impeller is applied to the entire volume of snow.

For the impeller, the computer model calculates the thickness of the snow layer (*9*) and compares it with the paddle length. The smaller of the two lengths is used to calculate the absorption capacity. If the mass flow rate exceeds the absorption capacity, the impeller is working to its full capacity and excess snow is plowed in front of the machine; at that point, the operator would have to reduce forward speed or increase the impeller speed. Finally, the three components of impeller power (P_{i1}, P_{i2}, and P_{i3}) are calculated from the amount of snow passing through the impeller.

APPLICATION OF MODEL AND RESULTS

The model is developed so that all significant constants can be changed for parametric evaluations. Thus, the user can evaluate the effect on system performance of changing: snow properties, number and size of auger blades and impeller paddles, gear ratio between impeller and auger, width of the auger or length of the impeller arms, and so forth. In the data presented later, the parameters of a commercially available machine designed to remove snow at a rate of 27 MN/hr (3,000 tons per hour) are used. This machine uses a four-ribbon (or blade) auger, with an outside radius of 0.56 m (22 in.) and an inside radius of 0.43 m (17 in.). Each blade of the auger extends half of the spiral pitch along the auger's axis (Figure 1). The impeller has an overall diameter of 1.14 m (45 in.) and five 0.3-m (12-in.) paddles. The depth along the axis of rotation of the impeller is 0.46 m (18 in.), and the angle of discharge is 60 degrees.

In the following application of the model, three components of the machine's performance were evaluated: the power required by the auger to cut the snow (P_{a1}), the total power required ($P_a + P_i + P_p$), and the percentage of total power required by the auger for cutting $[P_{a1}/(P_a + P_i + P_p)]$.

As reported (*9*), the model has been applied to a range of auger rotational speeds, forward velocities, snow depths, and snow strengths to investigate the effect that operational parameters and snow properties have on the machine operation. Four auger rotational speeds were chosen, from 75 rpm, which is typically used for deep, dense snow, to 150 rpm, which can be used for high-speed removal of "virgin" snow. The depth was chosen in increments of 10.2 cm (4 in.) up to 40.6 cm (16 in.). Finally, the shear strengths included virgin snow, [1.20 kPa (25 lb/ft^2)] and three shear strengths corresponding to processed snow: 4.79, 9.58, and 14.36 kPa (100, 200, and 300 lb/ft^2, respectively). The corresponding densities were back-calculated from Equation 2, and the compressive strength was determined from Equation 1. Again, the material properties did not incorporate rate effects.

In the following, only a few cases are presented; the full set of results is described elsewhere (*9*).

Cutting Power of Auger

The cutting power required by the auger to disaggregate the snow is plotted versus the forward speed in Figures 8 and 9;

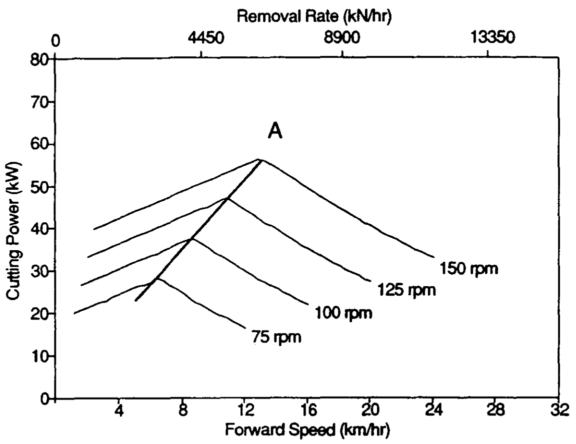

FIGURE 8 Cutting power versus forward speed: snow depth = 10.2 cm (4 in.); shear strength = 4.79 kPa (100 lb/ft²); density = 204.4 kg/m³.

in both figures, the shear strength (and corresponding density and compressive strength) are the same. Two depths of snow are investigated: 10.2 cm (4 in.) in Figure 8 and 30.5 cm (12 in.) in Figure 9.

For a depth of 10.2 cm (Figure 8), the cutting power of the auger is an increasing linear function of forward speed as the cutting arc is increased. The cutting power peaks at Line A when the maximum overall cutting arc is achieved. As the ratio ω^*/U^* continues to decrease past Line A, the relationship between cutting power and forward speed decreases linearly as the overall length of the cutting arc is reduced and less snow is disaggregated by the auger.

For a depth of 30.5 cm (Figure 9), several points of inflection in the relationship between cutting power and forward speed occur because of changes in the cutting paths of the auger blades as the forward speed increases. For low-speed plowing, the cutting power is an increasing linear function of forward speed as the effective cutting arc length is increased. The curve increases sharply at Line A as the inside edge of the auger blade begins to cut the snow. As the inside cutting arc of the auger blade approaches its maximum at Line B, the cutting force increases sharply as the inside cutting arc length increases significantly with the change in forward speed. As the inside cutting arc approaches its maximum length, the cutting force increases sharply, from Point B1 to Point B2 on the 150-rpm curve. The peak power requirement occurs when the maximum outside cutting arc is attained at Line B. For increased forward speeds (lower ω^*/U^* ratio), the relationship becomes a decreasing linear function as the length of the cutting arc is reduced. Finally, at Line C, the relationship decreases quickly because the inside cutting arc is reduced when it overlaps its own path.

In addition, a study of the effects of snow properties was also performed, as reported elsewhere (*9*). For the same depth of snow and auger speed, it was found that, relative to virgin snow with a shear strength of 1.2 kPa (25 lb/ft²), the cutting power increased 255, 705, and 1,175 percent for shear strengths of 4.29, 9.58, and 14.36 kPa, respectively. This range corresponds to density increases of 67, 116, and 151 percent, respectively.

Total Power

The total power [the sum of the auger, impeller, and plowing power ($P_a + P_i + P_p$)] is plotted versus forward speed in Figures 10 and 11 for shear strengths of 1.20 kPa (25 lb/ft²) and 9.58 kPa (200 lb/ft²); the depth is 30.5 cm (12 in.).

For the virgin snow (Figure 10), the total power increases at a constant rate with respect to forward speed. Once the point of maximum cutting power is attained (Line A), the power continues to increase as the amount of snow that is plowed increases sharply.

For processed snow with a shear strength of 9.58 kPa (200 lb/ft²) (Figure 11), the total power required for removal is dominated by the cutting power up to the point of maximum power required, Line A. Once the maximum total power is achieved, the total power remains relatively constant and then decreases. At this point, the rotary snowplow is pushing or plowing the snow instead of processing it through the impeller; the operator would have to reduce the speed.

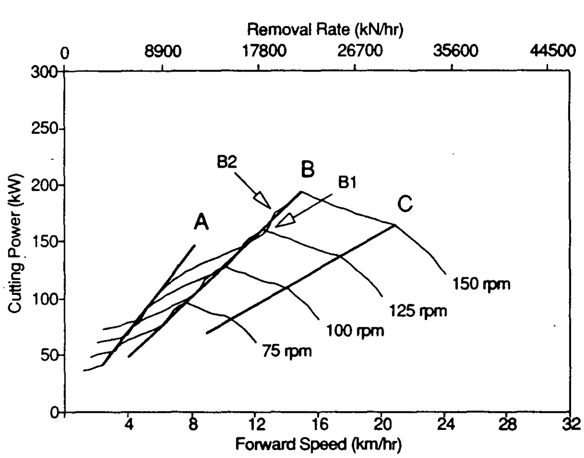

FIGURE 9 Cutting power versus forward speed: snow depth = 30.5 cm (12 in.); shear strength = 4.79 kPa (100 lb/ft²); density = 204.4 kg/m³.

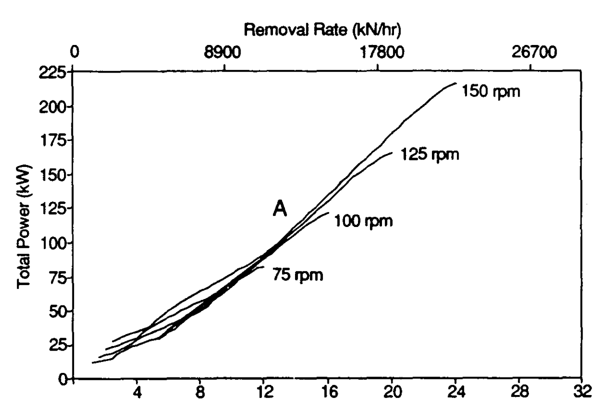

FIGURE 10 Total power versus forward speed: shear strength = 1.20 kPa (25 lb/ft²); depth = 30.5 cm (12 in.); density = 122.2 kg/m³.

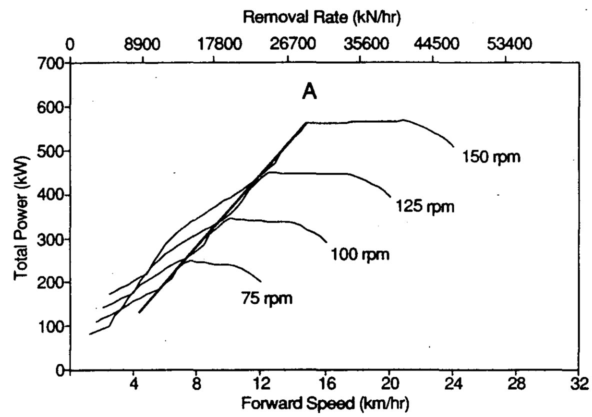

FIGURE 11 Total power versus forward speed: shear strength = 9.58 kPa (200 lb/ft^2); depth = 30.5 cm (12 in.); density = 264.5 kg/m^3.

Percentage of Total Power Used by Auger

Finally, the ratio of cutting power to total power is plotted versus forward speed in Figure 12 for 30.5 cm (12 in.) of snow with a shear strength of 4.79 kPa (100 lb/ft^2).

The percentage of total power required to disaggregate the snow is higher at lower forward speeds since most of the snow is struck by the auger blade and less power is required for mixing and plowing uncut snow. Figure 12 shows that for the high-strength snow, more than half of the total power is required to disaggregate the snow, regardless of the forward speed.

CONCLUSIONS

In this study, an analytical model was developed to study the operation of two-stage rotary snowplows and the effect of the snow properties on the system performance. The model was developed by considering the power requirements in each stage of the rotary snowplow. The model incorporates the dynamics and kinematics of the auger and impeller, the motion of the snow within the system, and the effects of the snow properties on the system. The effect of higher-speed plowing was considered in the derivation.

The results of the cutting power analysis show that the power required to disaggregate snow with the same material properties is a function of the ratio of forward speed to the rotational speed of the auger, indicating that the length of the cutting arc (both inside and outside) significantly affects the required cutting power. The investigation of the effects of material properties showed that an increase in shear strength significantly increased the cutting power requirement. The graphs indicate that, for processed snow and low-speed plowing, the total power required for removal is dominated by the cutting power. And the cutting power, as a percentage of the total power, was highest at lower speeds, since little power is required for mixing and plowing and the impeller is not working to its full capacity. Finally, the percentage of the total power was found to be slightly affected by the rotational speed of the auger.

The casting capabilities of the rotary snowplow were not included in the model because the focus of the model was on the power required for removal. The analysis of casting performance should be considered in future development of the model.

Finally, analytical theories should be validated with full-scale experimental results by instrumenting the engines and drive shafts of a commercial vehicle. Although the present model may not be quantitatively accurate, it appears to be qualitatively correct and provides a means of assessing the influence of changing operational and material parameters.

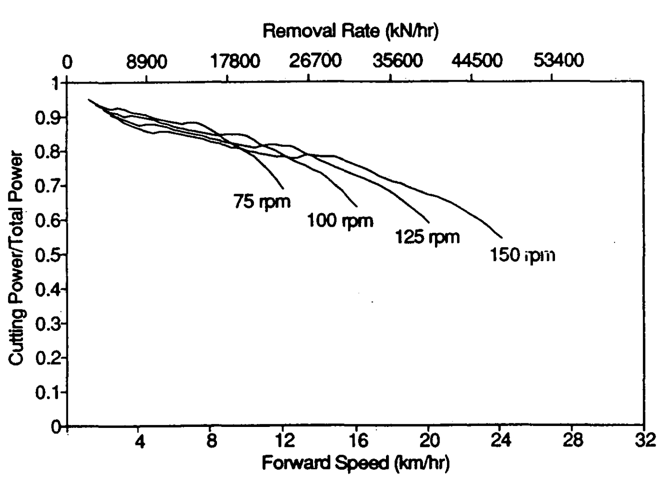

FIGURE 12 Proportion of cutting power to total power: shear strength = 4.79 kPa (100 lb/ft^2); depth = 30.5 cm (12 in.); density = 204.4 kg/m^3.

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