Load Equivalency Concepts: A Mechanistic Reappraisal

ANASTASIOS M. IOANNIDES AND LEV KHAZANOVICH

The history and evolution of load equivalency concepts are traced and discussed, in view of the fact that accommodation of mixed traffic consisting of multiwheel load assemblies is of cardinal importance in any pavement design. It is explained that the equivalent single-axle load concept is considerably different from the equivalent single-wheel load (ESWL) approach, inasmuch as the former is statistical-empirical and is based on the assumption of linear pavement damage accumulation, whereas the latter is soundly mechanistic, if relatively crude, and is merely an innovative means of computing responses under multiwheel gears. A third concept, namely, the equivalent single-axle radius (ESAR), is also mechanistic in nature and dispenses with the arbitrary constant pressure or constant radius assumptions of the ESWL approach. According to the ESAR concept, it is possible to determine with reasonable accuracy a primary structural response, such as the maximum bending stress that occurs in a concrete pavement system, through the use of available closed-form equations for a single-wheel load, into which an equivalent single-wheel radius of a multiple-wheel assembly is substituted. The ESAR concept is currently considered for incorporation in an improved mechanistic "limit state" design procedure for concrete pavements. Because of the far-reaching implications of these proposals, rigorous engineering mechanics derivations are used to verify the applicability of the concept for both the dense liquid and the elastic solid foundation.

Before the AASHO Road Test (1958–1960), the development of pavement design procedures relied heavily on the theoretical investigations of two well-known researchers. With respect to portland cement concrete (PCC) pavement systems, the prominent name is that of Westergaard (*1*), who idealized the PCC slab as a plate resting on a Winkler, or dense liquid, foundation. A couple of decades later, Burmister (*2*) extended Boussinesq's concept of the semiinfinite elastic half-space to develop the layered elastic theory, which has formed the basis for the design of bituminous pavement systems. The solutions derived by Westergaard and Burmister were intended for practical applications, despite the fact that their scope was restricted by a number of limiting assumptions and idealizations. A most important shortcoming common in the derivations of both of these pioneers stemmed from the assumption that the load consisted of a single tire print.

A number of subsequent studies attempted to eliminate the limiting assumptions introduced by Westergaard and Burmister, including the one pertaining to the configuration of the applied loading. Thus, a graphical extension of Westergaard's plate-on-dense-liquid theory to multiple-wheel loads was provided by Pickett and Ray (*3*), who developed the very popular charts, now known under their names. These may be

used to determine a specific response (e.g., deflection or slab bending stress) at a specific location (e.g., interior or edge) for a specific pavement system (i.e., of known radius of relative stiffness, *l*). Note that, in addition to the dense liquid foundation considered by Westergaard, Pickett and Ray derived solutions for the elastic solid idealization. Their charts were patterned after the charts developed some 15 years earlier by Newmark (*4*), which had extended the Boussinesq solution (single wheel on homogeneous foundation) to multiple-wheel loads.

With the advent of the computer, computerized versions of the Pickett and Ray charts were also prepared. Program AIRPORT (earlier name: PDLIB) was coded by Packard (*5*) for the Portland Cement Association (PCA) and may be used for the determination of the maximum dense liquid interior stress under any single- or multiple-wheel load configuration. Similarly, Kreger (*6*) developed the H-51 program for the dense liquid edge loading stress. This program was later expanded by Ioannides (*7*), and the resulting version, H51-ES, may be used to determine the edge stress, assuming either a dense liquid or an elastic solid subgrade. A similar evolution occurred with respect to Burmister's layered elastic theory. The development of the BISAR computer code in the early 1960s enabled engineers to determine the response of any specified multilayered system under a prescribed multiwheel load (*8*).

These computer codes, however, do not address the need for a comprehensive solution applicable to a range of practical problems, which would be incorporated in a design guide in a suitable form, that is, as an equation, chart, or nomogram. In an effort to provide such a solution, Yoder and Witczak (*9*) employed the principles of dimensional analysis in presenting a graphical summary of numerous results obtained using the Pickett and Ray charts. Three dimensionless ratios were introduced for this purpose, namely, (L/l), (x/l), and (d/l), where L is the length of the elliptical tire prints, and x and d are the longitudinal and transverse wheel spacings, respectively. The resulting graphs may be used to determine the edge or interior bending stress under single, dual, or dual tandem wheel loads.

Similarly, a chart for the determination of the dense liquid interior bending stress under dual-wheel loads was prepared by researchers at PCA, using a data base consisting of results from AIRPORT (*10*). Application of the principles of dimensional analysis in the interpretation of the same data base resulted in an even simpler, more concise nomogram (*11*). The dimensionless ratios used in the latter were (a/l) and (S/a), in which a is the radius of each tire print and S is the spacing between the duals. The spacing ratio (S/a) had also

Department of Civil Engineering, University of Illinois at Urbana-Champaign, 205 N. Mathews Ave., #2230C, Urbana, Ill. 61801.

been used in the early 1970s in an Asphalt Institute design procedure (9). In contrast, an approach dating to the 1950s pertaining to the Federal Aviation Agency employed the spacing ratio (S/l) instead (12). Because (S/l) = (S/a) * (a/l), the two spacing ratio forms are interchangeable, although (S/a) is preferable because it is based exclusively on the tire configuration characteristics.

Notwithstanding the contributions made previously, the most prevalent approach in assessing the effect of multiple-wheel loads on pavement systems has been to transform the actual applied multiple-wheel load into some equivalent loading system consisting only of a single wheel, which could then be accommodated in available single-wheel analysis or design procedures. In this paper load equivalency concepts are reviewed, especially as they might apply to the development of an improved mechanistic design procedure. It is demonstrated that despite being relatively crude, the equivalent single-wheel load (ESWL) concept is soundly mechanistic in nature, whereas the very popular equivalent single-axle load (ESAL) concept is entirely statistical-empirical and possibly inappropriate for use in mechanistic design. A less common approach, namely, the equivalent single-axle radius (ESAR) concept, is discussed in detail and a mechanistic justification for its use is developed. This justification is in the form of closed-form equations for a dual wheel load applied on a slab-on-grade concrete pavement system.

ESWL CONCEPT

The earliest documented application of a load equivalency approach in accommodating multiple-wheel loads was devised by the U.S. Army Corps of Engineers in the 1940s. Boyd and Foster (13) developed a method that was aimed at accommodating the B-29 aircraft in the existing single-wheel California-bearing-ratio-based design procedure by reducing the dual-wheel gear of that aircraft to a single tire print. By today's standards, their method was crude, employing as it did Boussinesq's solution for the homogeneous elastic half space, thereby ignoring the layered nature of the pavement system. Yet the Boyd and Foster method was distinguished by the following two characteristics, which remain desirable features of any design procedure to this day:

1. It was mechanistic in nature, being based on a rigorous— if simple—theoretical solution; and
2. It was calibrated and verified using field measurements.

The significance of their method lay in its originality, and, rather unexpectedly for its authors, it begot one of the most pervasive and influential concepts in pavement design history, namely, the concept of the ESWL. As explained in more detail below, this concept should be distinguished from the entirely statistical-empirical ESAL concept, proposed in the 1960s after the AASHO Road Test. Boyd and Foster were, in fact, looking for a method that could "be used to find axle spacings and loads that will produce no greater detrimental effect than is produced by a given load on a single axle." In current conventional terminology, this meant that they sought combinations of total applied load and dual-wheel spacing that would result in a load equivalency factor (LEF) of unity when compared with any "given" single axle. In contrast, the AASHO LEFs developed later were assigned values other than 1 and aimed at converting any arbitrary load to an 18-kip equivalent single axle.

Boyd and Foster were the first to recognize that any load equivalency depended on wheel spacing and repeatedly refer to the "efficiency of variations" in this geometric gear characteristic. Furthermore, they recognized that the equivalency relationship depended on the mechanistic response, for example, stress, strain, or deflection, selected to quantify the "detrimental effect" on the pavement. Also contained in their paper is the earliest acknowledgment that load equivalencies are sensitive to the thickness of the constructed pavement layers and presumably also to their individual moduli. Consequently, these authors appeared to be concerned about the generality of their finding and noted that their comments apply to "this case [alone]." To enhance the generality of the ESWL approach, Boyd and Foster (13) expressed the design thicknesses as dimensionless quantities by "resolving them into ratios of appropriate dimensions of the assembly." They postulated that "the dimension of the assembly that governs the depth at which the dual wheel loads act as independent units is the spacing between the contact areas of the tires," which they designated d. Similarly, "the depth at which the dual wheels act as a single wheel load is governed by the distance between the centers of the wheels," which they designated s. From the geometric configuration of the B-29, design thicknesses were obtained on the basis of ($d/2$) and ($2s$). This is the earliest documented use of the principle's dimensional analysis in the development of a load equivalency concept.

It is significant to note that Boyd and Foster (13) did not consider their method to be a rigorous theoretical solution, but only an approximation whose agreement with "actual observation" was "reasonably close" and "slightly conservative." They never intended the method to be a final solution, but to merely serve as an interim approach until "time and economic considerations permitted the direct development" of design criteria form multiple-wheel assemblies. It appears, however, that the savings in effort that were afforded by the load equivalency approach simply were too attractive, because in the 40 years that have passed since then, no attempt has been made toward such "direct development" of multiple-wheel criteria. Rather, a substantial amount of energy has been devoted to refining an equivalency method that would reduce multiple-wheel gears to an equivalent single-wheel load. In these efforts, however, a number of considerations present in the original Boyd and Foster paper were gradually abandoned or forgotten, so that subsequent "refined" methods were—at least in some respects—less comprehensive and theoretically less rigorous. In addition, each of the later investigators introduced new assumptions, which yet other researchers ignored or forgot as they tried to "re-refine" the already "refined" proposals.

An excellent case in point is offered by the Corps of Engineers study published in 1958 by Foster himself and Ahlvin, whose approach was still mechanistic in nature, and retained Boussinesq's homogeneous foundation assumption (14). Dimensional analysis concepts were adhered to in its derivation: the spacing between the wheels (S) was expressed in terms

of the radius of each wheel (a) as the dimensionless ratio (S/a). Similarly, the geometric coordinates for offset (r, and depth (z), were also expressed in dimensionless forms as (r/a) and (z/a). Yet Boyd and Foster's multiple response considerations were abandoned, and, presumably for simplicity's sake, Foster and Ahlvin (14) retained only the criterion of equivalency of deflections. Thus, the strong dependence of any resulting equivalency relationship on the primary structural response considered was gradually forgotten by many subsequent investigators. A notable exception to this trend is the study performed by the Australian researchers Gerrard and Harrison (15). Interestingly, Foster and Ahlvin (14) recognized that equating the magnitude of the maximum deflection alone would not be adequate, even if one could dispense with all other primary responses (such as stresses and strains), and noted that the shape of the deflection basin should also be considered.

As far as can be ascertained, Foster and Ahlvin (14) were the first to coin the term equivalent single-wheel load. It should also be noted that the ESWL method relied on the use of the principle of superposition, applicable under conditions of elasticity and of similarity of boundary and support conditions. Thus, the extension of their equivalency concept to distresses of a nonrecoverable nature, such as rutting and cracking, is theoretically unjustified. Their main "refinement" consisted of the fact that in obtaining the complete curve of depth (or thickness) versus ESWL, it was no longer necessary to resort to Boyd and Foster's simplistic assumption of an "orderly" (i.e., linear) variation when log-log scales were used. In addition, Foster and Ahlvin (14) abandoned the original constant pressure assumption and equated instead the contact area of the ESWL to that of one wheel of the multiple-wheel assembly. No justification was provided for this choice, but, as Huang (16,17) later showed, the constant radius assumption makes the ESWL calculation easier.

The main contribution of Huang (16) was the introduction of Burmister's layered elastic theory (implemented on an IBM-360 computer) into the computation of ESWLs, an approach adopted also by Gerrard and Harrison (15). Huang presented his results in a "simplified chart such that EDWLs for any combinations of pavement thickness, modulus ratio, wheel spacing, and contact radius could easily be found." This was achieved by using dimensional analysis concepts in the form of dimensionless ratios, for example, (E_1/E_2), (S/a), and (h/a), in which E_1 and E_2 are the moduli of the constructed layer and of the natural subgrade, respectively, whereas h denotes the thickness of the constructed layer. Huang had hoped that "the simplicity of the chart will encourage its use so that more field data will be collected to check the validity of the chart." Following Foster and Ahlvin (14), Huang (16), and later Gerrard and Harrison (15), assumed that the ESWL and each wheel of the dual assembly had the same contact area radius (a). The equivalency relation was based on equality of the maximum vertical deflection at the interface between the two layers in the system, that is, at the top of the subgrade. In addition, because Huang's method is mechanistic, it allocates due importance to a variable ignored in many other pavement investigations, namely, the radius of the applied load (a). In fact, this parameter is so important that it appears in two of Huang's three dimensionless ratios, (h/a) and (S/a).

THE ESAL CONCEPT

In the interpretation of the AASHO Road Test results in the early 1960s, the concept of the equivalent single-axle load (ESAL) was employed. The origins of this concept can also be traced to the early 1940s in a procedure adopted by the California Division of Highways (18,19). However, since its inception the ESAL concept has been drastically different from the ESWL concept: whereas the latter constitutes an attempt to accommodate multiple-wheel loads in the mechanistic determination of pavement responses (i.e., of stresses, strains, or deflections), the original California suggestion was introduced "for the reason that it is necessary to determine some common denominator to which we reduce our axle load determinations" so that distresses due to the "fatigue effect" may be defended against. The historical coincidence between the development of the ESAL concept and the increased interest of highway engineers in the phenomenon of fatigue is extremely significant and must not be overlooked.

The ESAL concept as adopted in the interpretation of the AASHO Road Test data was not entirely faithful to the original California suggestion. Rather than considering solely the "fatigue effect" as Grum (18,19) had suggested, the AASHO statistical regression methodology was based on the equivalency of the empirical and subjective present serviceability index (PSI). This index was later correlated statistically with the combination of several pavement distresses, the primary one being roughness, not fatigue. Nonetheless, AASHO's entirely statistical-empirical ESAL concept is based on the assumption that the destructive effect of a number of applications of a given axle group (defined in terms of load magnitude and configuration) can be expressed in terms of a different number of applications of a standard or base load. Therefore, the ESAL concept presupposes and is based on a concept of linear cumulative damage (quantified in terms of a drop in the PSI), a popular, if arbitrary, extrapolation of Miner's fatigue hypothesis (20).

In the first decade following the AASHO Road Test and the novel application of exclusively statistical concepts that it ushered in, engineers remained skeptical, as witnessed by the discussion comments from several forums at that time. A paper by Deacon published in 1969 (21) appears to mark a turning point in highway engineering history, because it purported to provide much needed intellectual underpinnings for the ESAL concept. This was in the form of a "proof" of the validity of the concept, which also illustrated its intimate connection with the linear cumulative fatigue hypothesis. A closer review of this proof reveals that the validity of the ESAL concept is proven on the basis of the assumption that the concept is valid in the first place, rendering Deacon's exercise an argument in a circle and therefore of little value. Furthermore, the validity of the ESAL concept is based on the assumption of the validity of the linear cumulative fatigue hypothesis, and vice versa. Given the scatter in experimental fatigue data, as well as the specimen size effect involved in any correlation of individual material laboratory test results with the performance of multilayer pavement systems, it is hard to accept that a rigorous theoretical proof of the validity of the ESAL concept can be derived at all. It is worth recalling at this point that the developers of the ESWL concept desisted from providing such proofs for their own, more mechanistic, construct.

An interesting and fortuitous by-product of the statistical manipulations of the AASHO Road Test data has become one of the principles most widely used by practicing pavement engineers. This is the so-called fourth-power law, whose historical origins can be traced to a relatively obscure National Cooperative Highway Research Program (NCHRP) report by Irick and Hudson (22), in which it was attributed to "work that was done at the HRB subsequent to the AASHO Road Test." Its definitive statement, however, did not appear until 1970, when Scala (23) presented it as the "simplest formula" to describe the results of the AASHO Road Test for each given "arrangement of load." He suggested that "accepting that the deflection of a pavement is proportional to the load, the LEF values for a given loading system vary by the fourth power of the ratio of the deflections under the loads." Scala (23) justifies the use of the vertical (elastic) deflection, either measured in the field or calculated from theory, in establishing LEFs by stating that this response "has been proven to be a good indication of the performance of a pavement under various loading systems." The adequacy of deflection as a universal, single indicator of all pavement distresses is certainly debatable. In any case, however, Scala's suggestion would lead to drastically different LEFs than, for example, Deacon's suggestion of using a strain ratio raised to the power 5.5.

The fourth-power approximation represents the simplest "best-fit" equation through data that invariably had a considerable scatter. Scala considered this variation negligible and concluded: "It appears that the LEF values are independent of the road structure [i.e., whether the pavement is rigid or flexible, or what its structural number or slab thickness may be]. Hence, in considering the effect of altering the regulations limiting axle-loads on the life of the highway surface the nature of actual road structure can be ignored." Independent of the validity of this statement vis-à-vis Scala's stated reason for deriving and using the fourth-power approximation (namely, to "advise" with respect to the effect of altering the Australian Motor Vehicle Standards), his comments cannot be interpreted as meaning that the LEFs (and still less the ESWLs) are independent of factors relating to the pavement structure. Certainly earlier investigators have pointed out the influence of such input parameters as layer thicknesses and moduli. Even Scala presents and discusses in detail a long list of such factors and in several instances refers to previous literature, showing that they are indeed important.

The existence of a fourth-power law was recently refuted by theoretical and field investigations conducted by the Organization for Economic Cooperation and Development (OECD) (24). That study "demonstrated that from a strict theoretical viewpoint, it is not possible to prove the existence of a law of equivalence between loads in terms of their damaging effects on pavements." In addition, it was noted that

in view of the statistical nature of [the] supporting equations, [the load equivalence law] is necessarily also a statistical law . . . and the exponent γ has a different value for each type of pavement structure, i.e., flexible, semi-rigid, rigid. For fatigue phenomena and permanent deformation of flexible pavements, it is usual to set γ = 4. For semi-rigid and rigid pavements and in regard to fatigue of hydraulically bound materials, the γ values are between 11 and 33, depending on the material concerned. From the point of view of behavior of these types of structures, it is important to note that such high γ values reflect the pre-

dominant role of heavy loads even if they are infrequent. In fact the actual value of the exponent γ does not play a major role if the standard load is well chosen, this choice being dependent on the actual load distribution. (24)

Data presented from field tests in Italy, France, and Finland show that γ varies between 1.2 and 8 when fatigue cracking and rutting of bituminous pavements are considered. To assess the significance of such differences, a typical flexible pavement design problem was submitted to Belgium, France, the Netherlands, the United Kingdom, and the United States. In accommodating mixed traffic, "all five participating countries used the notion of equivalent traffic." The four European countries used γ = 4, whereas the United States used this only for single axles and a modified equivalence law for multiple axles. Nevertheless, from the responses received "it emerges that there are as many solutions proposed as countries proposing them" (24).

A final comment needs to be made with respect to another practice that has gained considerable popularity in the last 25 years, namely, the use of mechanistic tools such as computerized layered elastic analysis in the determination of LEFs. An early example of this approach is provided by the aforementioned study by Deacon (21), who noted that "empirical determinations [of LEFs] are impractical," since they require "extensive, controlled experiments such as the AASHO Road Test." Therefore, Deacon argued, "theoretical determination of these factors can be of immense significance" provided "suitable analytical techniques" are employed. The approach followed by Deacon, and later by several other investigators, involves the use of Burmister's three-layer theory, implemented in the CHEVRON computer code. Such an exercise, however, constitutes a mechanistic extension introduced into an entirely statistical/empirical framework, a process that "rarely leads to reliable conclusions" (25). As a consequence of the statistical-empirical nature of the ESAL concept and of Miner's fatigue hypothesis, it is impossible to derive reliable and general LEFs using mechanistic tools or measurements of particular pavement structural responses. This is because entirely statistical-empirical concepts cannot be combined with mechanistic procedures in a meaningful way, primarily because of the difference in the understanding of a pavement system inherent in these two approaches. More specifically, exclusively statistical-empirical methods consider the pavement system as the sum of individual, "independent" components and fail to discern the engineering interactions between the geometry of the applied loads, the constructed layers, and the supporting subgrade, which form the basis of a mechanistic understanding of the pavement system (25). In addition, because the distresses considered in the determination of the PSI are permanent and largely irreversible, it is clear that the ESAL concept is in violation of the principles of elasticity and superposition, which are of fundamental significance in mechanistic approaches. The failure of several previous studies to develop LEFs of general applicability, the exclusive use of the AASHTO LEFs in all design procedures that employ the load equivalency approach, as well as the refutation of the fourth-power law by OECD (24), provides ample evidence of the futility of mechanistic LEF development exercises. A good summary of the pertinent literature is provided by Papagiannakis and Haas (26).

THE ESAR CONCEPT

The development of an improved load equivalency concept requires the return to mechanistic principles to which only the ESWL was found to adhere in the preceding review. However, both the equal contact radius and equal contact pressure assumptions made in determining an ESWL are arbitrary and unnecessarily restrictive, imposed only for reasons of expediency. A more general approach would require neither, but would determine instead the radius of an equivalent single wheel that would lead to the same response if loaded by the same total load as the dual-wheel assembly. Such an approach to load equivalency is termed the equivalent single-axle radius (ESAR) concept. The basic idea is already evident in a 1934 paper by Bradbury (27) and was suggested again recently (28) as a means of establishing more reliable mechanistic load equivalency procedures. The importance of load geometry in general and of the load radius (a), in particular may be noted in connection with the governing independent variable (a/l) (29). This dimensionless ratio implies that the sensitivity of the pavement system response to changes in load radius (a) is just as pronounced as the effect of variations in the radius of relative stiffness (l).

As noted earlier, a major step toward an improved load equivalency concept was taken in a published discussion in which application of the principles of dimensional analysis showed that the effect of dual-wheel loads may be quantified by (S/a), where S is the spacing of the two loads (11). Furthermore, during the present investigation it was found that it is possible to determine with reasonable accuracy the maximum bending stress occurring in a slab on grade under interior loading through the use of Westergaard's equation for a single-wheel load, into which the equivalent single-wheel radius (a_{eq}) of a multiple-wheel assembly is substituted. Statistical regression techniques applied to the interior loading maximum bending stress data presented by Tayabji and Halpenny (10) resulted in the following expression of a_{eq}:

$$\frac{a_{eq}}{a} = 1.00 + 0.241683 \left(\frac{S}{a}\right) \tag{1}$$

This formula implies that once the primary structural response is chosen, it is possible to derive with reasonable accuracy an equivalent radius (a_{eq}) for any arbitrary loading gear configuration simply as a function of its geometry (size and spacing of tire prints). The loss of accuracy involved in such a transformation from a multiple- to a single-wheel load has been assessed with reference to data bases compiled at the University of Illinois and elsewhere and has been found to be negligible in many cases when competent slab-subgrade systems are considered.

Investigations into the applicability of the ESAR concept to airport and highway pavement design have been presented in two Ph.D. theses submitted recently to the University of Illinois. In the first, Seiler (30) examined the variation of the equivalent radius (a_{eq}) with the pavement radius of relative stiffness (l) for six multiple-wheel aircraft types, of which three were military and three were commercial. It was shown that for competent pavement sections (a_{eq}) is largely insensitive to l. A similar application of the ESAR concept to highway

pavements was presented in the thesis by Salsilli-Murua (31). Truck configurations consisting of dual, tandem, and tridem axles were examined, and predictive formulas were developed describing the effect of such multiple-wheel loads on the critical tensile bending stress arising in the concrete pavement slab. This approach has recently been used in the evaluation of the detrimental effect of "super-single" or wide-base tires (32).

Unlike the ESAL approach, the ESAR concept addresses all three components of a pavement system, namely, the constructed layers, the supporting natural subgrade, and the geometry of applied loads, primarily the size and spacing of tire prints. Furthermore, in contrast to the ESWL approach, the ESAR concept imposes no a priori assumptions as to the total applied load, contact pressure, or size of tire print, and leads to a reasonably precise estimate of the chosen primary structural response under a multiple-wheel load. This estimate is achieved through the use of available closed-form equations for a single-wheel load into which the equivalent single-wheel radius (a_{eq}) of a multiple-wheel assembly is substituted. Thus, the application of the ESAR concept is akin to Odemark's "method of equivalent thicknesses" (33).

The ESAR concept may be used to address one of the major factors contributing to the complexity of the problem of accommodating the effects of mixed traffic on highway pavements, namely, the very large number of types of trucks that use the national network. Trucks differ from one another not only in terms of size and gross weight, but also in terms of number and spacing of axles, number and spacing of tires per axle, tire type (e.g., radial or bias ply), inflation pressure, and applied contact pressure. Recent collaborative efforts between pavement and truck (mechanical) engineers have indicated that suspension stiffness and damping as well as static and dynamic equalization may also need to be considered (34). The most extensive of such studies was conducted by Gillespie et al. (35) under the sponsorship of NCHRP. These investigators indicated that it is possible to establish a short "baseline matrix" of 13 truck configurations to provide an adequate description of the entire traffic stream. This finding makes much more manageable the problem of mixed traffic. The ESAR concept may now be applied to each of these (and other) truck configurations to provide a mechanistic assessment of their relative damaging effect leading to the development of more efficient and reliable pavement design algorithms. Such algorithms must describe the structural response of the pavement under a number of different loading and support conditions in the context of a new "limit state" design procedure.

Current design methodologies are based on the assumption that pavement distresses accumulate slowly with repeated application of stress or strain cycles at amplitudes significantly smaller than the material's ultimate strength. Consequently, Miner's linear cumulative fatigue hypothesis is often used for distresses such as rutting and erosion, in addition to fatigue cracking. The major highways of the nation, however, which receive not only millions of legally loaded trucks but also relatively fewer heavily overloaded trucks, may not fail in fatigue at all. This possibility becomes even more important when one considers the variation in structural capacity experienced by the pavement system during the annual cycle. Thus, a "limit state" may be expected to occur when heavily

overloaded trucks are applied during periods of considerable loss in pavement strength, such as during spring thaw or partial contact conditions due to a temperature differential through the thickness of the slab. Because of the far-reaching implications of these proposals, a mechanistic justification for the ESAR concept is provided below, where the statistical regression Equation 1 is reproduced from first principles using engineering mechanics, for both the dense liquid and elastic solid foundations.

VERIFICATION OF ESAR CONCEPT

Dense Liquid Foundation

Consider a plate consisting of a linear elastic, homogeneous, and isotropic material resting on a dense liquid foundation. Under a single-wheel load distributed uniformly over a circular area of radius (a), the distribution of deflections $[w(s)]$ may be written as follows (36):

$$w(s) = \frac{Pl_k^2}{\pi D} \left(\frac{1}{a_k}\right)^2 [1 - C_1 \text{ ber } s - C_2 \text{ bei } s]$$

$$\text{for } 0 < s \leq a_k \quad (2)$$

$$w(s) = \frac{Pl_k^2}{\pi D} \left(\frac{1}{a_k}\right)^2 [C_3 \text{ ker } s + C_4 \text{ kei } s]$$

$$\text{for } s \geq a_k \quad (3)$$

in which

- $a_k = (a/l_k)$, which is the dimensionless radius of the applied load;
- $s = (r/l_k)$, which is the normalized radial distance measured from the center of the load;
- $l_k = (D/k)^{1/4}$, which is the radius of relative stiffness of plate-subgrade system for the dense liquid foundation;
- $D = $ flexural rigidity of the plate, which is equal to $Eh^3/12(1 - \mu^2)$;
- $E = $ plate elastic modulus;
- $\mu = $ plate Poisson's ratio;
- $h = $ plate thickness;
- $k = $ modulus of subgrade reaction;
- $P = $ total applied load, which is $p\pi a^2$; and
- $p = $ applied load intensity (pressure).

Note that ber, bei, ker, and kei are Kelvin Bessel functions that may be evaluated using appropriate series expressions available in the literature (37). Constants C_1 through C_4 have been evaluated by Ioannides (38). In the present study, the following simplified expressions have been derived for these constants, valid in the interval $0 < a_k < 0.6$:

$$C_1 = 1 - \frac{\pi}{8} a_k^2 + \left[\frac{5}{4} - \gamma + \ln\left(\frac{a_k}{2}\right)\right] \left(\frac{a_k}{2}\right)^4 \quad (4)$$

$$C_2 = \frac{1}{2} a_k^2 \ln\left(\frac{2}{a_k}\right) + \left(\frac{1}{4} - \frac{\gamma}{2}\right) a_k^2 + \frac{\pi}{64} a_k^4 \quad (5)$$

$$C_3 = -\left(\frac{a_k}{2}\right)^4 \quad (6)$$

$$C_4 = -\frac{a_k^2}{2} \quad (7)$$

in which γ is Euler's constant $= 0.577\ 215\ 664\ 90$.

The distribution of deflections given by Equations 2 and 3 may be used to obtain the corresponding distribution of bending stresses. Graphs presented by Losberg 36 indicate that the tangential stress (σ_ϕ) is more critical than the radial stress (σ_r). Using the mathematical software package MATHEMATICA, the following expression has been derived for σ_ϕ:

$$\sigma_\phi(s) = \frac{6P}{\pi h^2} \left(\frac{1}{a_k}\right)^2 (C_1 J + C_2 K) \quad (8)$$

for $0 < s \leq a_k$, and

$$\sigma_\phi(s) = -\frac{6P}{\pi h^2} \left(\frac{1}{a_k}\right)^2 \left(C_3 L + C_4 M\right) \quad (9)$$

for $a_k < s \leq 1$, in which

$$J = \frac{(1 + 3\mu)s^2}{16} \quad (10)$$

$$K = -\frac{(1 + \mu)}{2} \quad (11)$$

$$L = \frac{\pi}{8}(\mu + 1) + \frac{1}{s^2}(\mu - 1) + \frac{s^2}{16}\left[\gamma - \frac{5}{4} + \mu\right.$$
$$\times \left(3\gamma - \frac{11}{4}\right) + (1 + 3\mu) \ln\left(\frac{s}{2}\right)\right]$$
$$- \frac{\pi s^4}{1,536}(1 + 5\mu) \quad (12)$$

$$M = \frac{1}{4}[1 - 2\gamma - \mu(1 + 2\gamma)] - \frac{1}{2}(1 + \mu) \ln\left(\frac{s}{2}\right)$$
$$+ \frac{\pi s^2}{64}(1 + 3\mu) + \frac{s^4}{384}\left[\frac{-5}{3} + \gamma\right.$$
$$+ \mu\left(5\gamma - \frac{22}{3}\right) - (1 + 5\mu) \ln 2\right] \quad (13)$$

Consider now the same elastic plate resting on a dense liquid subgrade but loaded by two circular loads, each of radius a, spaced at distances S center to center. Once again, it is assumed that the loads are applied at the plate's interior, that is, far from any edges or corners. Under these conditions, the bending stress distribution along a line (O_1-O_2) connecting the centers of the two circular loads may be expected to be a function of the following three dimensionless parameters: $a_k = (a/l)$, the nondimensional radius of the applied loads; $s = (r/l)$, the nondimensional radial distance from the center (O_1) of the first of the two loads; and $\zeta = [(S - r)/l]$, the nondimensional radial distance from the center (O_2)

of the second loaded area. The bending stress from both loads, σ_{II} (a_k, s, ζ) at any point along O_1-O_2 may be obtained by superposition of the contributions of each of the two loads, as given by Equations 8 and 9. Examination of the complete stress distribution during this study revealed that the maximum stress always occurs under each of the two wheel loads. Therefore, for the range $s < a_k$, the combined bending stress is obtained from

$$\sigma_{II}(a_k, s, \zeta) = - \frac{6P/2}{\pi h^2} \left(\frac{1}{a_k}\right)^2 [C_1 J + C_2 K + C_3 L + C_4 M] \tag{14}$$

in which

$$J = \frac{(1 + 3\mu)s^2}{16} \tag{15}$$

$$K = - \frac{(1 + \mu)}{2} \tag{16}$$

$$L = \frac{\pi}{8}(\mu + 1) + \frac{1}{\zeta^2}(\mu - 1) + \frac{\zeta^2}{16}\left[\gamma - \frac{5}{4} + \mu\left(3\gamma - \frac{11}{4}\right) + (1 + 3\mu) \ln\left(\frac{\zeta}{2}\right)\right] - \frac{\pi\zeta^4}{1,536}(1 + 5\mu) \tag{17}$$

$$M = \frac{1}{4}[1 - 2\gamma - \mu(1 + 2\gamma)] - \frac{1}{2}(1 + \mu)$$
$$\times \ln\left(\frac{\zeta}{2}\right) + \frac{\pi\zeta^2}{64}(1 + 3\mu) + \frac{\zeta^4}{384}\left[-\frac{5}{3} + \gamma + \mu\left(5\gamma - \frac{22}{3}\right) - (1 + 5\mu) \ln 2\right] \tag{18}$$

Note that in keeping with the definition of P as the total applied load, the wheel load applied over each area in Equation 14 is $P/2 = p\pi a^2$. The magnitude of the maximum combined stress, σ_{IImax}, may be obtained by differentiating Equation 14 with respect to s and setting the derivative to zero. The following expression has been found to yield a reasonable estimate of the location of the maximum stress (r_{max}):

$$\frac{r_{max}}{l} \approx - \frac{f(0) * (a_k)}{f(a_k) - f(0)} \tag{19}$$

where

$$f(a_k) = \frac{\partial}{\partial s} \sigma_{II}(a_k, s, \zeta)|_{s=a_k} \tag{20}$$

and

$$f(0) = \frac{\partial}{\partial s} \sigma_{II}(a_k, s, \zeta)|_{s=0} \tag{21}$$

Truncated series expressions for $f(a_k)$ and $f(0)$ can be derived by differentiating Equation 15 through 18 and setting $s = a_k$, $\zeta = [(S - a)/l]$, and $s = 0$, $\zeta = (S/l)$, respectively.

The validity of the proposed approximate method for determining σ_{IImax} has been verified by comparison with the exact numerical solution obtained using MATHEMATICA.

A simpler expression may be derived for the bending stress, σ_{II0}, arising under the center of one of the two wheel loads. This stress is expected in most cases to be an adequate approximation of σ_{IImax}. Setting $s = 0$ and $\zeta = S/l = S_k$ in Equation 14, the formula for σ_{IImax} may be simplified by truncation into

$$\sigma_{II0} \approx \frac{P}{h^2} \frac{3(1 + \mu)}{2\pi} \left\{ \frac{\frac{1}{2} - \gamma}{2} + \frac{1}{4} \right.$$
$$\times \left[\frac{1 - 2\gamma - \mu(1 + 2\gamma)}{(1 + \mu)}\right] + \frac{1}{2} \ln\left(\frac{2}{a_k}\right)$$
$$\left. - \frac{1}{2} \ln\left(\frac{S_k}{2}\right) + S_k^2 \frac{\pi}{64} \frac{1 + 3\mu}{1 + \mu} \right\} \tag{22}$$

Having obtained $\sigma_{II0} \approx \sigma_{IImax}$, it is desirable to determine the equivalent radius of a single-wheel load, which would reproduce this maximum stress using available closed-form equations. In this case, the following interior loading equation (39,40) may be used:

$$\sigma_0 = \frac{P}{h^2} \frac{3(1 + \mu)}{2\pi} \left[\ln\left(\frac{2}{a_k}\right) + \frac{1}{2} - \gamma + a_k^2 \left(\frac{\pi}{32}\right)\right] \tag{23}$$

A first estimate (b), say, of the load radius may be obtained from

$$b_k = \exp\left(R - \frac{\sigma_0}{T}\right) \tag{24}$$

in which $b_k = (b/l) \approx a_k$, and

$$T = \frac{P}{h^2} \frac{3(1 + \mu)}{2\pi} \tag{25}$$

$$R = \ln 2 + \frac{1}{2} - \gamma \tag{26}$$

An improved estimate of the radius (a) may then be obtained using the following expression:

$$a_k = \frac{\frac{2}{b_k} - \left(\frac{1}{b_k^2} - \frac{3\pi}{16}\right)^{1/2}}{\frac{\pi}{16} + \frac{1}{b_k^2}} \tag{27}$$

Returning now to Equation 22, a first estimate of the equivalent radius can be written as

$$b_k = R^* (a_k S_k)^{1/2} \left[1 - S_k^2 \frac{\pi}{64} \frac{(1 + 3\mu)}{1 + \mu}\right] \tag{28}$$

in which

$$R^* = \exp\left[R - \frac{\frac{1}{2} - \gamma - \gamma\mu}{(1 + \mu)} - \ln 2 \right] \tag{29}$$

Setting $\mu = 0.15$, Equation 28 results in the following approximate expression for a_{eq}:

$$\frac{a_{eq}}{a} = 1.0674 \left(\frac{S}{a}\right)^{1/2} [1 - 0.0619 S_k^2] \tag{30}$$

Equation 30 shows that a first approximation of the equivalent radius may be obtained merely as a function of the geometry of the applied loads, namely, of the spacing ratio (S/a), provided S_k is not much greater than unity. Equation 30 may also be rewritten in the form of Equation 1 obtained by statistical regression. This is accomplished by considering the minimum (S/a) value of 2 (the two tire prints touch with no clear space between them) and an arbitrary upper limit of (S/a) = $2a + 0.6l$ (corresponding to $S_k \approx 1$). For $S = 2a$, the S_k^2 term in Equation 30 is negligible, resulting in

$$\frac{a_{eq}}{a} \approx 1.50952 \tag{31}$$

Setting $S = 2a + 0.6l$, Equation 30 results in a quasi-linear relationship between (a_{eq}/a) and a_k for values of the latter between 0.05 and 0.3. This relationship may be approximated by

$$\frac{a_{eq}}{a} \approx 1.50952 + \frac{0.1390}{a_k} \tag{32}$$

Therefore, within the range prescribed here,

$$a_{eq} = a_{eq|S=2a} + \frac{a_{eq|S=2a+0.6l} - a_{eq|S=2a}}{0.6l} * (S - 2a) \tag{33}$$

from which

$$\frac{a_{eq}}{a} = 1.046 + 0.2316(S/a) \tag{34}$$

This equation compares favorably with Equation 1, thus verifying mathematically the ESAR concept for the dense liquid foundation.

Elastic Solid Foundation

The corresponding tangential bending stress distribution under a single-wheel load applied on a plate resting on an elastic solid foundation was found in this study to be

$$\sigma_\phi = \frac{6D}{h^2} \frac{P}{\pi a} \frac{2}{C}\left[\frac{1 - \mu}{r} \int_0^\infty \frac{J_1(\alpha r) J_1(\alpha a)}{1 + \alpha^3 l_e^3} d\alpha \right.$$
$$\left. + \mu \int_0^\infty \frac{\alpha J_0(\alpha r) J_1(\alpha a)}{1 + \alpha^3 l_e^3} d\alpha \right] \tag{35}$$

where

C = elastic solid foundation parameter [$E_s/(1 - \mu_s^2)$];
E_s = Young's modulus for subgrade;
μ_s = Poisson's ratio for subgrade;
l_e = radius of relative stiffness of plate-subgrade system for the elastic solid foundation

$$= \sqrt[3]{2\,D/C} \tag{36}$$

J_0, J_1 = Bessel functions of the first kind, order 0 and 1, respectively; and
α = dummy variable.

The combined stress (σ_{II}) due to two circular loads at any distance (r) along the line joining the centers of the two wheel loads (O_1–O_2) is obtained by superposition as

$$\sigma_{II} = \frac{6}{h^2} \frac{P/2}{\pi a_e}\left[\frac{1 - \mu}{s} \int_0^\infty \frac{J_1(\beta s) J_1(\beta\, a_e)}{1 + \beta^3} d\beta \right.$$
$$+ \mu \int_0^\infty \frac{\beta J_0(\beta s) J_1(\beta a_e)}{1 + \beta^3} d\beta$$
$$+ \frac{1 - \mu}{Z} \int_0^\infty \frac{J_1(\beta \zeta) J_1(\beta a_e)}{1 + \beta^3} d\beta$$
$$\left. + \mu \int_0^\infty \frac{\beta J_0(\beta \zeta) J_1(\beta a_e)}{1 + \beta^3} d\beta \right] \tag{37}$$

where

$Z = S - r$,
$\beta = \alpha l$,
$s = r/l_e$,
$a_e = a/l_e$, and
$\zeta = Z/l_e$.

In view of the complexity of Equation 37, a simpler expression may be derived once again for the bending stress (σ_{IIO}) occurring under the center of one of the two circular loads. This is achieved by superposition, using the formula presented by Losberg (*36*) for the stress under the center of one wheel [$\sigma(a_e)$] and the solution for the stress due to a point load, arising at a normalized distance $S_e = S/l_e$ from the load [$\sigma(S_e)$] presented by Hogg (*41*). Both of these important contributions were verified during this study using MATHEMATICA, and the following general expressions were derived:

$$\sigma(a_e) = \frac{P}{h^2} \frac{3(1 + \mu)}{2\pi}\left[\ln\left(\frac{2}{a_e}\right) \right.$$
$$\left. + \frac{1}{2} - \gamma + a_e^2\left(\frac{\pi}{12\sqrt{3}}\right) \right] \tag{38}$$

$$\sigma(S_e) = \frac{6P}{h^2}\left[\frac{1}{8\pi} - \frac{\gamma}{4\pi} - \frac{\mu}{8\pi} - \frac{\gamma\mu}{4\pi} \right.$$
$$\left. + \frac{(1 + 3\mu)S_e^2}{48\sqrt{3}} - \frac{1 + \mu}{4\pi} \ln\left(\frac{S_e}{2}\right) \right] \tag{39}$$

Thus, by superposition:

$$\sigma_{IIO} = \sigma(a_e) + \sigma(S_e) \tag{40}$$

Equation 40 corresponds to Equation 22 derived above for the dense liquid subgrade. Proceeding in the manner outlined above, the following expression may be written for the approximate equivalent radius $b_e = b/l_e$:

$$b_e = R^* (a_e S_e)^{1/2} \left(1 - S_e^2 \frac{\pi}{24\sqrt{3}} \frac{1 + 3\mu}{1 + \mu} \right) \tag{41}$$

in which R^* is as defined by Equations 29 and 26. It is apparent that the only difference between this expression and the corresponding Equation 28 derived earlier for the dense liquid foundation is that $24\sqrt{3}$ replaces 64 in the latter. Thus, Equation 30 becomes

$$\frac{a_{eq}}{l_e} = 1.0674 a_e \left(\frac{S}{a} \right)^{1/2} \left(1 - 0.0953 S_e^2 \right) \tag{42}$$

whence Equation 34 becomes

$$\frac{a_{eq}}{a} = 1.066 + 0.2218(S/a) \tag{43}$$

confirming the validity of the ESAR concept for the elastic solid foundation and by implication for any subgrade type. A similar derivation could verify the validity of the ESAR concept on the basis of deflection considerations.

CONCLUSIONS

Accommodation of mixed traffic consisting of a wide variety of single- and multiple-wheel gear configurations is one of the most critical considerations in pavement design. Closed-form analytical solutions based on the theory of elasticity have hitherto been formulated in terms of a single tire print, whereas graphical and computerized approaches that can accommodate multiple-wheel loads are generally too cumbersome for incorporation in a design guide. For this reason, a number of load equivalency concepts have been promulgated over the last 50 years, with the general aim to transform complex load assemblies into single-wheel loads, which could be accommodated in existing design procedures developed for single-wheel loads.

In this paper, three such load equivalency concepts are reviewed. A critical reexamination of the pertinent literature leads to the conclusion that the ESWL concept is significantly different from the ESAL concept. The ESWL concept constitutes an attempt to provide a general mechanistic solution to the problem posed by multiple-wheel loads on airport and highway pavements using the theory of linear elasticity and the principle superposition, as well as field measurements for verification purposes. In contrast, the ESAL concept is entirely statistical/empirical and violates fundamental precepts of elasticity, as well as the principle of superposition. It seeks to establish "relative damage" effects, quantified as a drop in the PSI, between a standard (base) load level and config-

uration, and any other load level and configuration. In doing so, it fails to recognize the interactions between the characteristics of the three main components of the pavement system, namely, the constructed layers, the supporting natural subgrade, and the geometry of the applied loads. In addition, an intimate connection between the ESAL concept with another statistical-empirical concept, namely, Miner's linear cumulative fatigue hypothesis, has been established. In view of these findings, it is considered inappropriate to continue using ESAL counts as a primary design input. ESAL counts may be retained as indirect inputs to the design process, that is, as indicators of the relative amount of traffic services by any given pavement. As such, they may be used to classify pavements in terms of traffic level and may serve as one of the criteria for selecting the allowable working stress, strain, or deflection level in a pavement system.

A third, less-known load equivalency approach, the ESAR concept, is discussed in this paper, and its advantages over both the ESAL and the ESWL concepts are explained. According to this concept, it is possible to determine with reasonable accuracy a chosen primary structural response occurring in a PCC pavement system through the use of available closed-form equations for a single-wheel load, into which an equivalent single-wheel radius of a multiple-wheel assembly is substituted. Through rigorous engineering mechanics derivations, the concept is verified for both the dense liquid and elastic solid foundation. The ESAR concept can be incorporated into an improved mechanistic "limit-state" design approach and can be instrumental in accommodating the wide variety of load configurations imposed on modern pavements. This may be accomplished through the development of a rating scale based on the level of stress caused by and the probability of occurrence of each of the truck configurations examined. Such a scale will enable pavement designers to select their design-loading configuration as a percentile of the load spectrum, much like a geotechnical engineer selects the flood level for the design of a dam (e.g., the 95th percentile of damaging effect, compared with the 95-year flood). Efforts toward the development of such a design procedure are continuing at this time.

ACKNOWLEDGMENTS

REFERENCES

1. H. M. Westergaard. Stresses in Concrete Pavements Computed by Theoretical Analysis. *Public Roads*, Vol. 7, No. 2, April 1926.
2. D. M. Burmister. The Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways. *HRB Proc.*, Vol. 23, 1943.
3. G. Pickett, and G. K. Ray. Influence Charts for Concrete Pavements. *Transactions, ASCE*, Vol. 116, 1951.
4. N. M. Newmark. *Influence Charts for Computation of Stresses in Elastic Foundations*. Bulletin 338. University of Illinois Engineering Experiment Station, Urbana, 1942, pp. 5–25.
5. R. G. Packard. Computer Program for Airport Pavement Design. PCA, SR029.02P, Skokie, Ill., 1967.

6. W. C. Kreger. *Computerized Aircraft Ground Flotation Analysis-Edge Loaded Rigid Pavement*. Research Report ERR-FW-572. General Dynamics Corporation, Fort Worth, Tex., Jan., 1967.
7. A. M. Ioannides. *Analysis of Slabs-On-Grade for a Variety of Loading and Support Conditions*. Ph.D. thesis. University of Illinois, Urbana, 1984.
8. M. G. F. Peutz, H. P. M. Van Kempen, and A. Jones. Layered Systems Under Normal Surface Loads. In *Highway Research Record* 228, HRB, National Research Council, Washington, D.C., 1968.
9. E. J. Yoder and M. W. Witczak. *Principles of Pavement Design*, 2nd ed. Wiley-Interscience, New York, N.Y., 1975.
10. S. D. Tayabji and D. Halpenny. Thickness Design of Roller-Compacted Concrete Pavements. In *Transportation Research Record 1136*, TRB, National Research Council, Washington, D.C., 1987, pp. 23–31.
11. A. M. Ioannides. Discussion: Thickness Design of Roller-Compacted Concrete Pavements, by S. D. Tayabji and D. Halpenny. In *Transportation Research Record 1136*, TRB, National Research Council, Washington, D.C., 1987, pp. 31–32.
12. E. J. Yoder. *Principles of Pavement Design*. John Wiley and Sons, Inc., New York, N.Y., 1959.
13. W. K. Boyd and C. R. Foster. Design Curves for Very Heavy Multiple Wheel Assemblies. In Development of CBR Flexible Pavement Design Method for Airfields: A Symposium. *Transactions, ASCE*, Vol. 115, Paper 2406, pp. 534–546, 1950.
14. C. R. Foster and R. G. Ahlvin. Development of Multiple-Wheel CBR Design Criteria. *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 84, No. SM2, May 1958, pp. 1647-1–1647-12.
15. C. M. Gerrard and W. J. Harrison. A Theoretical Comparison of the Effects of Dual-Tandem and Dual-Wheel Assemblies on Pavements. *Proc., 5th Conference of the Australian Research Board*, 1970.
16. Y. H. Huang. Chart for Determining Equivalent Single-Wheel Loads. *Journal of the Highway Division, ASCE*, Vol. 94, No. HW2, Nov. 1968, pp. 115–128.
17. Y. H. Huang. Computation of Equivalent Single-Wheel Loads Using Layered Theory. In *Highway Research Record* 291, HRB, National Research Council, Washington, D.C., 1969, pp. 144–155.
18. F. J. Grumm. Designing Foundation Courses for Highway Pavements and Surfaces. *California Highways and Public Works*, Vol. 19, No. 11, Nov. 1941, pp. 4–23.
19. F. J. Grumm. Designing Foundation Courses for Highway Pavements and Surfaces. *California Highways and Public Works*, Vol. 20, No. 3, March 1942, pp. 6–20.
20. M. A. Miner. Cumulative Damage in Fatigue. *Transactions, American Society of Mechanical Engineers*, Vol. 67, 1945, pp. A.159–A.164.
21. J. A. Deacon. Load Equivalency in Flexible Pavements. *Proc., Association of Asphalt Paving Technologists*, Vol. 38, 1969, pp. 465–494.
22. P. E. Irick and W. R. Hudson. *NCHRP Report 2A: Guidelines for Satellite Studies of Pavement Performance*. HRB, National Research Council, Washington, D.C., 1964.
23. A. J. Scala. Comparison of the Response of Pavements to Single and Tandem Axle Loads. *Proc., 5th Conference of the Australian Research Board*, 1970.
24. *Heavy Trucks, Climate and Pavement Damage*. Organization for Economic Cooperation and Development, Paris, 1988.
25. A. M. Ioannides. Theoretical Implications of the AASHTO 1986 Nondestructive Testing Method 2 for Pavement Evaluation. In *Transportation Research Record* 1307, TRB, National Research Council, Washington, D.C., 1991, pp. 211–220.
26. A. T. Papagiannakis and R. C. G. Haas. *Wide-Base Tires: Industry Trends and State of Knowledge of Their Impact on Pavements*. Ministry of Transportation and Communications, Ontario, Canada, Dec. 1986.
27. R. D. Bradbury. Evaluation of Wheel-Load Distribution for the Purpose of Computing Stresses in Concrete Pavements. *HRB Proc.*, Vol. 14, 1934, pp. 225–254.
28. A. M. Ioannides and R. A. Salsilli-Murua. Temperature Curling in Rigid Pavements: An Application of Dimensional Analysis. In *Transportation Research Record 1227*, TRB, National Research Council, Washington, D.C., 1989, pp. 1–11.
29. A. M. Ioannides. Discussion: Responses and Performance of Alternate Launch and Recovery Surfaces (ALRS) Containing Stabilized-Material Layers, by R. R. Costigan and M. R. Thompson. In *Transportation Research Record* 1095, TRB, National Research Council, Washington, D.C., 1987.
30. J. S. Seiler. *A Knowledge-Base for Rehabilitation of Airfield Concrete Pavements*. Ph.D. thesis. University of Illinois, Urbana, 1991.
31. R. A. Salsilli. *Calibrated Mechanistic Design Procedure for Jointed Plain Concrete Pavements*. Ph.D. thesis. University of Illinois, Urbana, 1991.
32. A. M. Ioannides, R. A. Salsilli, I. Vinding, and R. G. Packard. 'Super-Singles': Implications for Design. *Proc., 3rd International Symposium on Heavy Vehicle Weights and Dimensions*, Queens' College Cambridge, United Kingdom, June 28–July 2, 1992.
33. N. Odemark. Investigations as to the Elastic Properties of Soils and Design of Pavements According to the Theory of Elasticity (in Swedish). *Meddelanden fran Statens Vaginstitut* (Sweden) 1949 [English Translation by M. A. Hibbs and J. Silfwerbrand (A. M. Ioannides, ed.), 1990].
34. M. S. Mamlouk. *Vehicle-Pavement Interaction: State of the Art*. Final Report FHWA-AZ90-340. Arizona State University, Tempe, Jan. 1990.
35. T. D. Gillespie, S. M. Karamihas, D. Cebon, M. W. Sayers, M. A. Nasim, W. Hansen, and N. Ehsan. *Effects of Heavy Vehicle Characteristics on Pavement Response and Performance*. Preliminary Draft Final Report, NCHRP Project 1-25(1). University of Michigan Transportation Research Institute, Dec. 1991.
36. A. Losberg. Structurally Reinforced Concrete Pavements, *Doktorsavhandlingar vid Chalmers Tekniska Hoegskola*, 1960.
37. N. W. McLachlan. *Bessel Functions for Engineers*, 2nd ed. Clarendon Press, Oxford, United Kingdom, 1955.
38. A. M. Ioannides. Dimensional Analysis in NDT Rigid Pavement Evaluation. *Journal of Transportation Engineering*, ASCE, Vol. 116, No. 1, Jan./Feb. 1990, pp. 23–36.
39. H. M. Westergaard. New Formulas for Stresses in Concrete Pavements of Airfields. *Transactions ASCE*, Vol. 113, 1948.
40. A. M. Ioannides, M. R. Thompson, and E. J. Barenberg. Westergaard Solutions Reconsidered. In *Transportation Research Record 1043*, TRB, National Research Council, Washington, D.C., 1985.
41. A. H. A. Hogg. Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth. *The London, Edinburgh & Dublin Philosophical Magazine and Journal of Science*, Series 7, Vol. 25, 1938.