Development of New Criteria for Control of Hot-Mix Asphalt Construction

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Statistically based quality control/quality assurance procedures are designed to control accuracy in achieving target values and variability. Properties of construction materials, such as hot-mix asphalt, are known to be normally distributed, and this is used in the establishment of limiting criteria. However, the use of nuclear asphalt content and nuclear density gauges has increased the potential for process manipulation to achieve average values approximating target values. Owner agencies often resort to the use of the mean of absolute deviations from target values instead of the mean of arithmetic deviations to control process manipulation. Distributions of absolute deviations are not normally distributed (for small sample sizes) and, therefore, properties of normal distributions cannot be used directly to establish criteria limits. Distributions of absolute values from target values were examined and statistics of the distributions computed. Procedures for using the statistics of distribution of absolute deviations to produce consistent mathematically correct limiting criteria are demonstrated. These procedures are simple and control both central tendency and variability, thus reducing possibilities for process manipulation.

The move toward statistically based quality control/quality assurance (QC/QA) construction specifications is motivated by the desire to control the quality of the finished product while maintaining reasonable costs. Quality is judged by accuracy and precision of selected properties of the finished product. Accuracy is measured in terms of the proximity of average measured values to target values. Precision is measured in terms of variability of measured values.

An important part of QC/QA specifications is the limiting criteria for controlling central tendency and variability. Statistical concepts applied to historical construction data are used to set specification limits, and the methodology developed must control both central tendency and variability.

As applied by some agencies, "mean deviation" or "variability known" procedures do not control variability and may lead to process manipulation. The proposed methodology uses absolute deviations from target values and will control variability and prevent process manipulation. The quality level analysis as proposed by FHWA (*1*) and adopted by the Western Association of State Highway and Transportation Officials (WASHTO) (*2*) controls both central tendency and variability. However, some agencies have been reluctant to adopt these procedures. Reasons given include the complexity of required computations and a lack of understanding of the consequences of application of the procedures. Contractors should reasonably expect to know the standards by which they are to be judged (acceptable or achievable accuracy and precision) and be able to understand the consequences of performance above or below the accepted norm.

This paper is focused on the construction of hot-mix asphalt pavements, but the principles are applicable to the production and placement of any construction material. The concepts of statistical QC/QA procedures and the process of developing limiting criteria will be discussed. Examples of procedures used to set limiting criteria for hot-mix asphalt construction will be examined. A simple but statistically correct method that maintains consistent levels of control for both central tendency and variability of absolute deviations from the job mix formula (JMF) will be presented.

STATISTICAL QC/QA CONCEPTS

The objective of the use of construction control procedures is to ensure that quality products are produced. A critical aspect of these procedures is the selection of control properties that are important in determining product performance. All properties that influence product performance cannot be measured during construction, but if advantage is taken of the interrelationships among properties, a practical, manageable subset may be selected. For example, asphalt content, gradation, voids, and voids filled with asphalt are important properties of hot-mix asphalt. However, because of their interrelationship, it is not necessary to control all of these properties.

Historical data and experience provide the basis for determining (a) properties that are important, (b) realistically achievable quality (central tendency and variability), and (c) at least qualitatively, how quality level influences product performance. These three factors may be used to determine which properties are controlled and their limiting criteria.

Limiting criteria should be designed to achieve target values (central tendency) and to control variability. These concepts are shown in Figure 1. Figure 1 (top) shows two distributions with means equal to the target value, but with different standard deviations. The second distribution, with a smaller variability, represents a higher level of control. Figure 1 (bottom) shows two distributions, each with different means and standard deviations. The first distribution, with mean equal to the target value, has better central tendency control, but the second distribution, with the smaller standard deviation, has better variability control.

Criteria with limits set about target values are designed to control the mean as well as variability. Two-sided limits around either side of the target value are shown in Figure 2. However,

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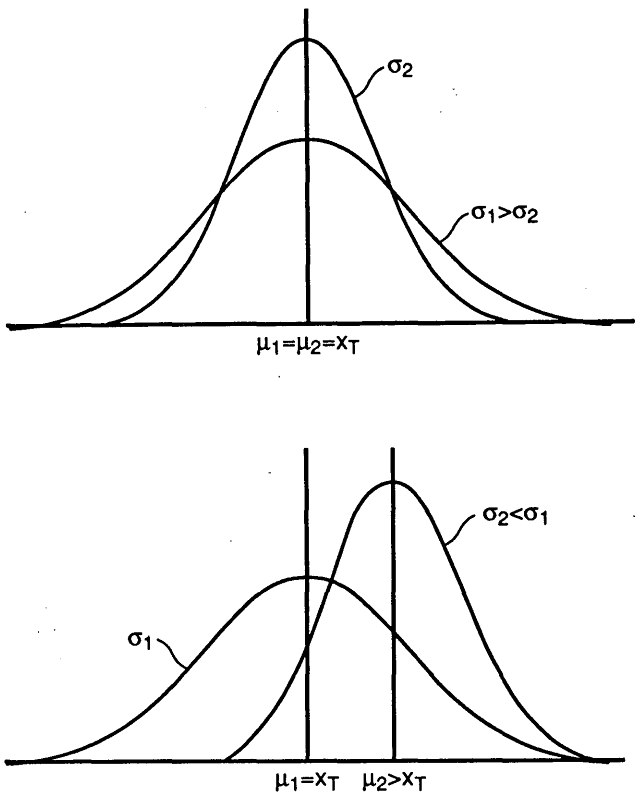

FIGURE 1 Central tendency and variability control: *top*, distributions with same mean and different variabilities; *bottom*, distributions with different means and variabilities.

one-sided limits may be applicable in instances where values higher or lower than the target are undesirable. Symmetrical criteria about the target are shown in Figure 2, but unsymmetrical criteria may be applicable if the underlying distribution is skewed or if there is reason to believe that high or low values affect product performance differently.

Allowable deviation about the target value is set at $C\sigma$, where C is a constant and σ is the standard deviation of the measured property. The standard deviation, based on historical data, provides a basis for the variability that can be realistically achieved. The value of C selected is a rather subjective management decision, but should be supported by available historical data and knowledge of statistical procedures. Intuitively, the decrease in product performance should be related to the deviation from the target value, larger than historical variability, or both. Quantification of this decrease has not been established, and selection of C is often based on tolerable probabilities for pay reduction. For example, limits for 100 percent pay are often set from $\pm 2\sigma$ to $\pm 3\sigma$. If the average of all test data is equal to the target value X_T, and if variability is consistent with historical data, limits of $\pm 2\sigma$ to $\pm 3\sigma$ will mean probabilities for pay reductions of 4.55 to 0.27 percent, respectively. The hatched areas in the tails of the distribution in Figure 2 represent these probabilities. The hatched areas also represent the seller's risk (α), which is the probability that a satisfactory product will be rejected. If the average of all test data is not equal to the target value or actual variability is greater than historical variability or both, the probabilities for pay reductions will be greater. Likewise, if actual variability is less than historical variability, probabilities for pay reductions may be smaller or larger depending on the magnitude of differences between mean and target values.

To decrease the buyer's risk (β) and to break production into manageable size portions (LOTs) for application of pay adjustments, limiting criteria are included for the mean of multiple samples. Buyer's risk is the probability that an unsatisfactory product will be accepted. Consistent criteria for multiple samples are based on the concept that the variability of distributions of mean values can be calculated from the variability of the distribution of individual values using the following equation:

$$\sigma_n = \sigma / n^{1/2} \tag{1}$$

where

σ_n = standard deviation of mean values,
σ = standard deviation of individual values, and
n = sample size.

The reduced variability of means is shown in Figure 3.

Limits for consistent seller's risk (α) may be set for the mean of multiple samples by using Equation 1. For example, if limits for 100 percent pay are set at $\pm 2\sigma$ for individual values, consistent criteria for the mean of multiple samples will be $\pm 2\sigma_n$. These criteria will give the same probability for a pay reduction (4.55 percent) and represent the same area in the tails of the distribution means.

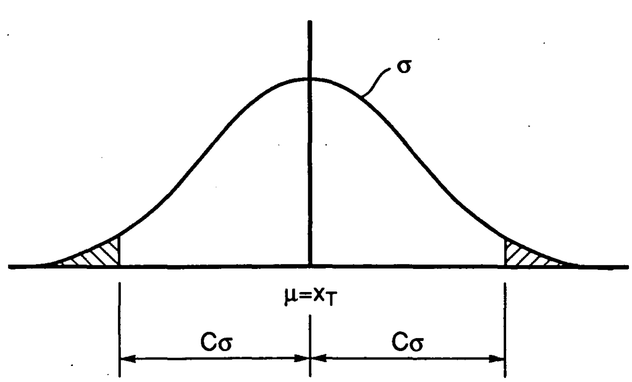

FIGURE 2 Criteria limits.

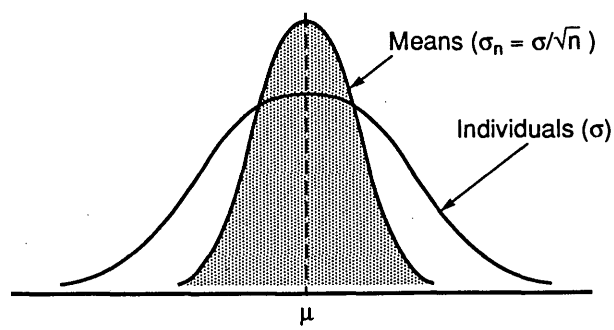

FIGURE 3 Distribution of individual values and means.

The use of multiple samples is desirable to reduce buyer's risk. Limits set at $\pm 2\sigma$ or $\pm 2\sigma_n$ provide the same level of seller's risk (i.e., $\alpha = 4.55$ percent) for any number of samples. However, buyer's risk is reduced from $\beta = 50$ percent for $n = 1$ to $\beta = 2.3$ percent for $n = 4$.

Selection of consistent limits for application of pay reductions or bonus payments is similar to selection of acceptance criteria limits. The decision process is somewhat subjective but should be based on assessments of the influence of material quality on product performance. This applies to pay reductions for quality less than design or bonuses for quality better than design. Bonus payments are not made as often as pay reductions because of the perception that it is more difficult to assess the influence of quality better than design on final product performance than the influence of quality poorer than design.

The usual approach is to set limits, which are intuitively correct and consistent with concepts of causes of failures in final products (i.e., the probability of failure increases as deviation from the target increases). Table 1 presents statistics for setting criteria limits. It follows that pay reductions should increase as deviation from the target increases. The system may be extended to means for any sample size. It also provides the producer with expected probabilities for achieving pay adjustments. For example, if the job mean equals the target and job variability equals historical variability, there would be a 3.32 percent probability of obtaining a pay factor (PF) of 95 percent.

DEVELOPMENT OF LIMITING CONTROL CRITERIA

To illustrate the development of limiting control criteria incorporating the statistical concepts just discussed, an example case will be considered. The example involves the asphalt content of hot-mix asphalt. In the total study from which the data were extracted, asphalt content and air voids of laboratory compacted specimens were selected as the two properties for controlling quality of produced hot-mix asphalt. Level of mat compaction was selected as the property for controlling placement quality.

Historical Data

Historical data are required to establish realistic expectations for variability and for achieving target values. Asphalt content data were collected by the Alabama Highway Department on 11 resurfacing projects during the summer of 1991. A total of 517 measurements was taken using nuclear asphalt content gauges. The variable analyzed was the difference between measured asphalt content and JMF asphalt content as defined by the following equation:

$$\Delta = X - JMF \qquad (2)$$

where

Δ = deviation of individual measured asphalt content from JMF asphalt content,
X = individual measured asphalt content, and
JMF = JMF asphalt content.

The mean deviation for the data set was $\Delta = -0.01$ percent, and the standard deviation was $\sigma = 0.218$ percent. These values indicate an ability to achieve target asphalt content and variability that is similar to that reported by FHWA (*3*) and provide a basis for establishing limiting control criteria that can be reasonably achieved.

TABLE 1 Statistics for Setting Criteria Limits

Pay Factor (PF)	Criteria Limits	Probability of PF
100%	$\pm 2.0\sigma$	95.44%
95%	$\pm 2.5\sigma$	3.32%
90%	$\pm 3.0\sigma$	0.98%
80%	$>3.0\sigma$	0.26%

Setting Limits for Control Criteria

Following the procedure outlined in the previous section, historical data may be used to set limits for control criteria. Since the mean of the data set was near zero, symmetrical limits about a mean of 0 will be set using $\sigma = 0.22$ percent (0.218 percent rounded). This is shown in Figure 4 for sample size $n = 1$, with limits of ± 2, 2.5, and 3σ defining PFs of 100, 95, and 90 percent, respectively. A PF of 80 percent applies if Δ lies outside the 3σ limits.

The percentages for the various areas under the curve represent the probability that a PF will be obtained on a project that has expected variability and a mean asphalt content equal to the JMF asphalt content. For example, there would be a 3.32 percent probability of obtaining a PF of 95 percent. If the JMF was 5 percent, samples with measured asphalt con-

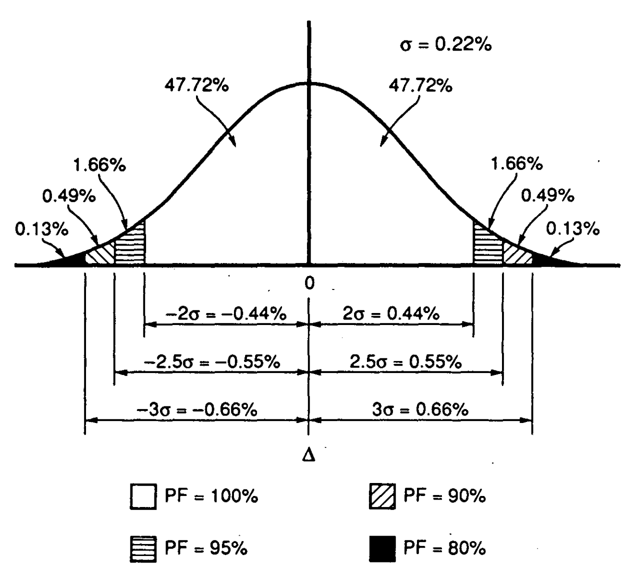

FIGURE 4 Example arithmetic deviation criteria limits for $n = 1$.

tents of 5.45 to 5.55 percent or 4.55 to 4.45 percent would give this PF.

The use of individual values for control is discouraged and the use of means of multiple samples is encouraged to decrease the buyer's risk (β). Consistent criteria limits for $n = 4$ are shown in Figure 5. These criteria use Equation 1 to reduce the standard deviation and, as a result, criteria limits. Again, there would be a probability of 3.32 percent of obtaining a PF of 95 percent. For a target JMF of 5 percent, four samples with mean measured asphalt contents of 5.23 to 5.28 percent or 4.77 to 4.72 percent would give this PF.

Process Manipulation

Criteria limits, as shown in Figures 4 and 5, control central tendency, but provide no control of variability caused by process manipulation. For hot-mix asphalt, the ability to manipulate the construction process is accentuated by the use of nuclear gauges for asphalt content and mat density. These gauges provide almost instant results, which allow for process manipulation during subsequent sampling to ensure that mean values approximating target values are achieved.

For example, assume the JMF asphalt content is 4 percent and that four samples are to be taken from a LOT. Samples of 2 percent, 2 percent, 6 percent, and 6 percent will result in a mean deviation of 0 percent and a 100 percent PF. This will occur despite all individual measurements being well outside the 100 percent PF limits.

Because more than one property is often used to control quality and because of the interaction between properties, process manipulation may be restricted. Nevertheless, it is a concern, particularly when only one property, such as mat density, is used. In response, some agencies modify control procedures to control process modification.

The simplest method, and one often used, is to use the mean of absolute deviations from target values instead of the mean of arithmetic deviations. In the example of the four asphalt contents considered previously, this would have given a mean absolute deviation of 2 percent, which is more representative of the quality of the hot-mix asphalt produced.

Application of criteria that specify absolute instead of arithmetic deviations from target values often does not take into consideration that absolute values are not distributed normally. The use of statistics based on normal distributions will then result in inconsistent criteria for sample sizes greater than one (i.e., the use of Table 1 and Equation 1 is no longer valid). To address this problem, procedures that permit use of absolute deviations from target values were developed.

Proposed Procedure for Setting Limiting Criteria

If the absolute value of Δ (ABSΔ) represents the random variable for the absolute difference between measured asphalt content and JMF asphalt content, and if Δ is normally distributed with mean 0 and standard deviation 1, then the distribution of ABSΔ is defined as:

$$f(\text{ABS}\Delta) = [2/(2\pi)^{1/2}]e^{-(\Delta^2/2)} \qquad 0 \le \text{ABS}\Delta \le \infty$$

This distribution is plotted in Figure 6 with the standard normal distribution. It is apparent that the ABSΔ distribution has smaller variance. The mean or first moment of the distribution is found by integrating (ABSΔ)·f(ABSΔ) over its range, 0 to ∞. The second moment is found by integrating (ABSΔ)^2·f(ABSΔ) over the same range. The variance is then found by subtracting the (mean)^2 from the second moment.

Following these procedures the distribution of ABSΔ will have a mean equal to

$$\mu' = (2/\pi)^{1/2} \qquad (3)$$

and with a second moment equal to one, the standard devia-

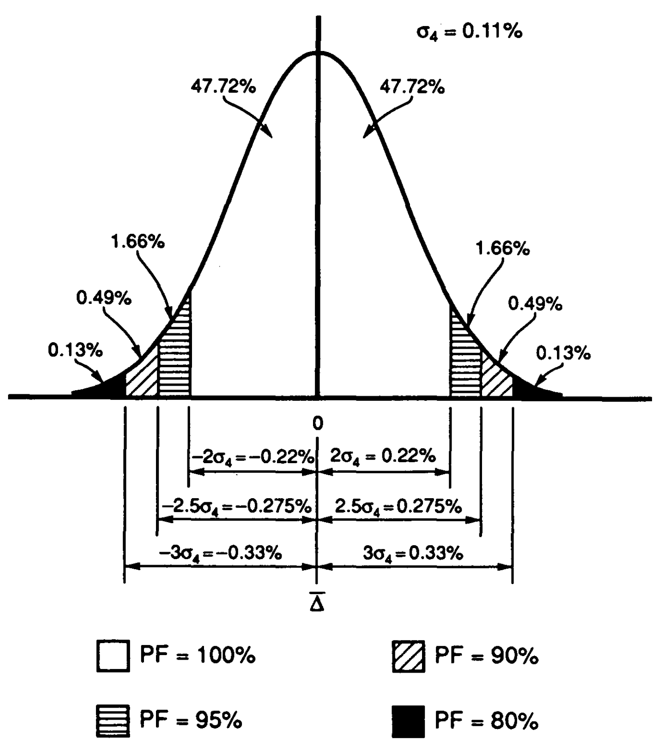

FIGURE 5 Example arithmetic deviation criteria limits for $n = 4$.

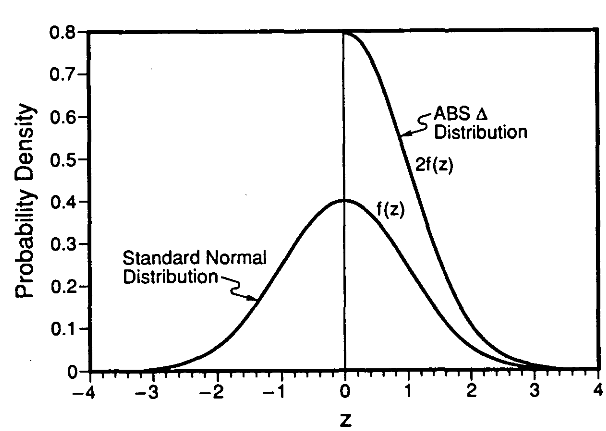

FIGURE 6 Distribution of ABS in comparison with standard normal distribution.

tion is equal to

$$\sigma' = (1 - 2/\pi)^{1/2} \tag{4}$$

Normalized histograms for the average absolute deviations from the arithmetic mean computed numerically from normal distributions of arithmetic deviations are shown in Figure 7 for $n = 1$ to 6. For $n = 1$, the histogram shows the probability that Z, the standard normal deviate, lies between 0 and 0.20 is 0.16. This histogram was generated by manipulating values from the standard normal distribution table.

The other five histograms were developed by numerically estimating the probabilities within given ranges using the normalized histogram for individual values in Figure 7. For example, the probability that the average of two samples ($n = 2$) is between 0 and 0.2 is approximately $(0.16)^2 + 1/2[(0.16)(0.15) + (0.15)(0.16)] = 0.0496$. The values 0.16 and 0.15 are probabilities for individual ($n = 1$) samples.

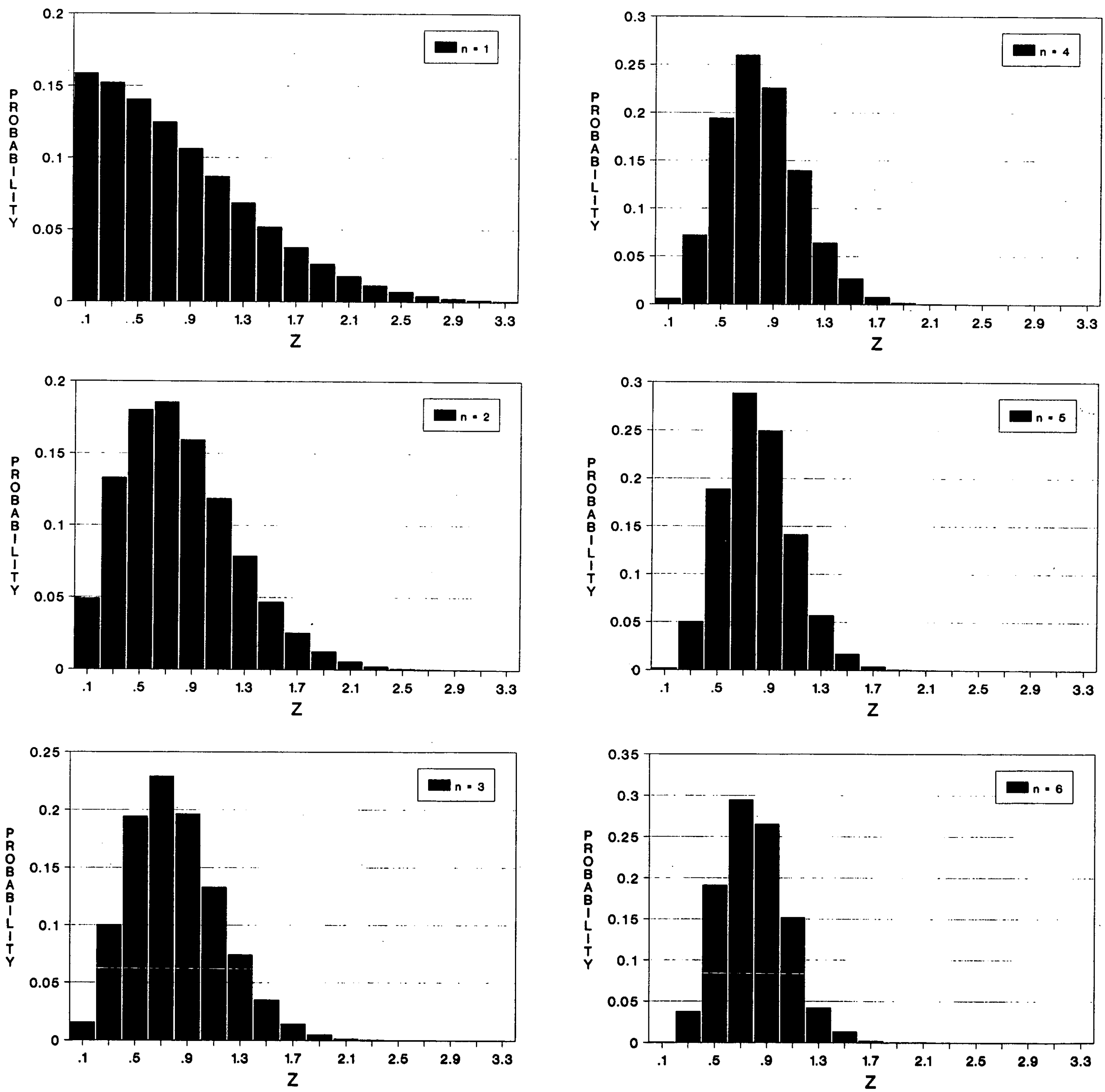

FIGURE 7 Normalized histograms for absolute deviations from arithmetic mean.

As shown in Figure 7, the discrete distribution for $n = 1$ follows that of the continuous ABSΔ distribution in Figure 6. However, the shapes of the other distributions are a function of n and, as n increases, they approach normal distributions according to the Central Limit Theorem.

Standard deviations for $n > 1$ are computed as follows:

$$\sigma'_n = \sigma'/n^{1/2} \quad (5)$$

To check the shape of the distributions and the equations for computing mean and standard deviations, the data set of 517 asphalt content measurements was analyzed. Histograms of arithmetic and absolute deviations for n = 1, 2, and 4 are shown in Figure 8. Comparing these shapes with the normalized histograms in Figure 7 for the same n values reveals good agreement. Means and standard deviations for the distributions are compared with values computed using Equations 1 and 3–5 in Table 2. Excellent agreement is indicated.

Properties of the distributions of absolute values can be used to develop consistent criteria limits when absolute deviations from target values are used to control process manipulation. The normalized histograms in Figure 7 are numerically integrated to determine offsets from the arithmetic mean (Z values) that give areas in the tail of the distributions

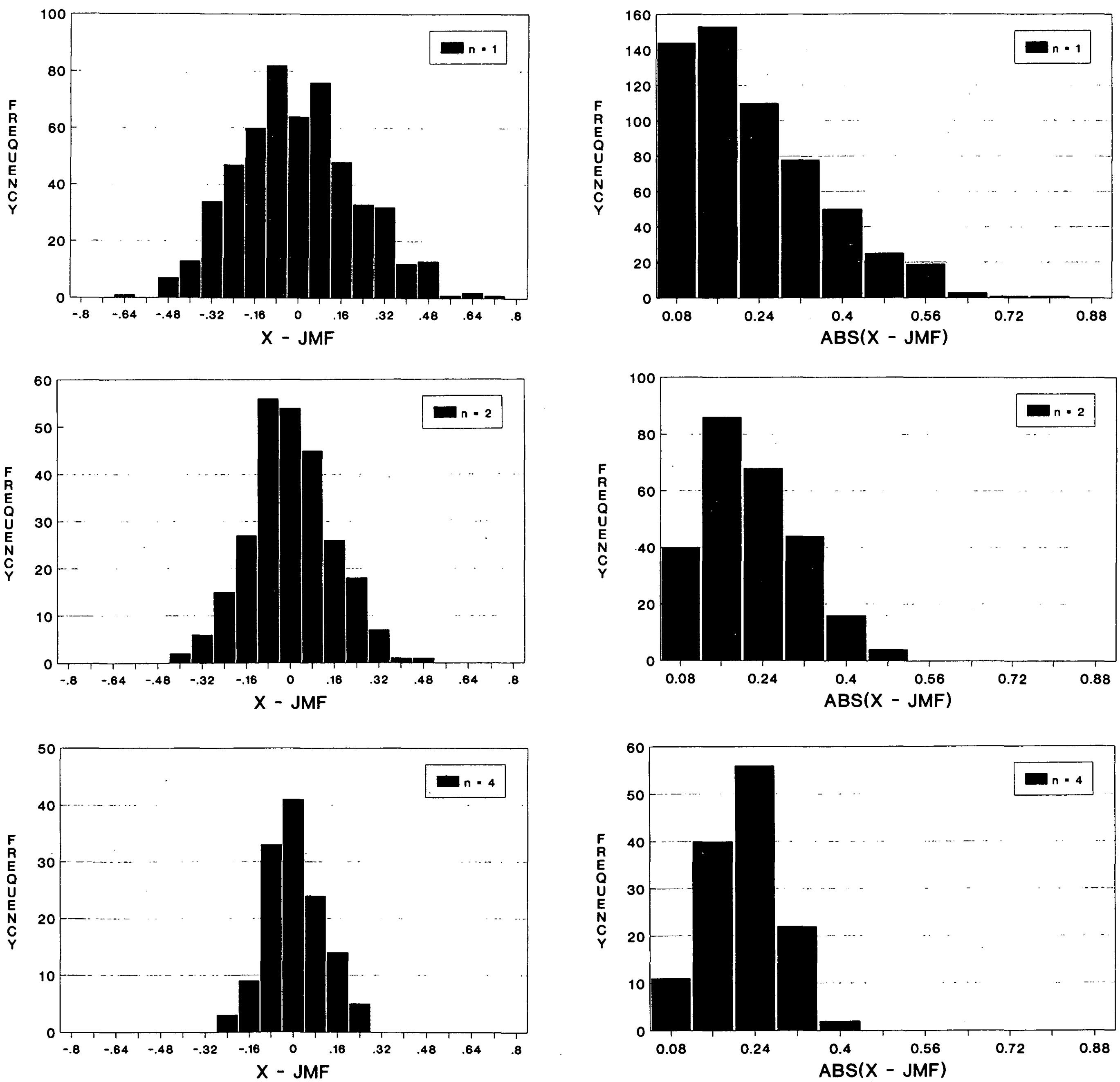

FIGURE 8 Histograms for arithmetic and absolute deviations from target.

TABLE 2 Comparison of Means and Standard Deviations

	$\overline{\Delta}$	σ		$\overline{\|\Delta\|}$		σ'	
n	Distribution	Distribution	Equations	Distribution	Equations	Distribution	Equations
1	-0.010	0.218	--	0.176	0.174	0.129	0.132
2	-0.001	0.155	0.154	0.176	0.174	0.094	0.091
3	-0.001	0.122	0.126	0.176	0.174	0.075	0.074
4	-0.001	0.103	0.109	0.176	0.174	0.067	0.065

TABLE 3 Offsets, Z, from Arithmetic Means

n	α = 4.55%	α = 1.24%	α = 0.60%	α = 0.26%	α = 0.05%
1	2.000	2.500	2.750	3.000	3.500
2	1.625	1.944	2.127	2.298	2.615
3	1.463	1.727	1.851	1.982	2.322
4	1.375	1.585	1.713	1.795	2.134
5	1.318	1.514	1.590	1.720	1.989
6	1.214	1.381	1.480	1.576	1.938

equivalent to the area in both tails of the normal distribution with Z values of ± 2, 2.5, 2.75, 3, and 3.5. These Z values are presented in Table 3 for $n = 1-6$ and correspond to areas in the tails of the distribution of 4.55, 1.24, 0.60, 0.26, and 0.05 percent.

It should be noted that the Z values for $n = 1$ in Table 3 are the same as for a normal distribution. This is because the distribution of the absolute deviations from the mean for $n = 1$, shown in Figure 6, has ordinate values twice the normal distribution. Therefore, the area under the absolute deviation distribution curve at a particular offset from the arithmetic mean will be twice the area corresponding to a normal distribution.

Application of the Proposed Procedure

To demonstrate application of Table 3, limits for $n = 4$ were developed with the asphalt content data used previously for the example in Figure 5. To develop criteria consistent with those shown in Figure 5, Z values of 1.375, 1.585, and 1.795 were selected from Table 3. These values were multiplied by the standard deviation of the historical data, $\sigma = 0.22$ percent, to give offsets from the arithmetic mean of 0.30, 0.35, and 0.39 percent. These offsets, corresponding PFs and areas representing probabilities are shown in Figure 9. These limits are statistically correct when the absolute deviation from target asphalt content is used as the control. The limits are also less restrictive than two-sided limits developed for arithmetic deviations that are often applied to absolute deviations. For example, LOTs with average absolute deviations less than or equal to 0.30 percent would have PFs of 100 percent based on the limits in Figure 9, whereas, the limit from Figure 5 for normal distributions of arithmetic deviations would be 0.22 percent.

Table 4 presents a set ($n = 1, 2,$ and 4) of consistent criteria limits for both arithmetic and absolute deviations from the target JMF asphalt content. Limits for absolute deviation are shown in parentheses and are numerically larger than limits for arithmetic deviations.

Producers are interested in the consequences of noncompliance and knowledge of the possibilities of noncompliance. Using Figures 4, 5, or 9, simple explanations are readily avail-

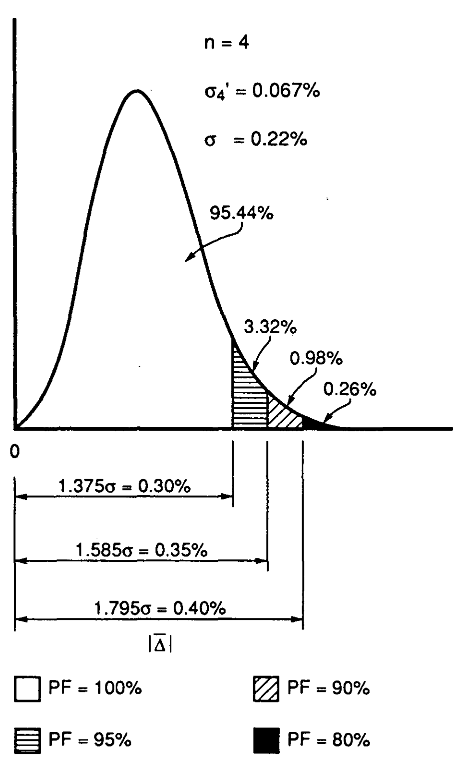

FIGURE 9 Example absolute deviation criteria limits for $n = 4$.

TABLE 4 Example Criteria Limits for Arithmetic and Absolute Deviations

Pay Factor	Sample Size 1	Sample Size 2	Sample Size 4
100	0 to ± 0.44 (0 to 0.44)	0 to ± 0.31 (0 to 0.35)	0 to ± 0.22 (0 to 0.30)
95	± 0.45 to ± 0.55 (0.45 to 0.55)	± 0.32 to ± 0.38 (0.36 to 0.42)	± 0.22 to ± 0.28 (0.31 to 0.35)
90	± 0.56 to ± 0.66 (0.56 to 0.66)	±0.39 to ± 0.46 (0.43 to 0.50)	± 0.29 to ± 0.33 (0.36 to 0.39)
80	< -0.66 or > +0.66 (> 0.66)	< -0.46 or > +0.46 (> 0.50)	< -0.33 or > +0.33 (> 0.39)

Limits for absolute deviations in parentheses.

able. A producer with product quality comparable to average historic quality can expect a pay reduction only 4.55 percent of the time. If a LOT is equal to 1 day's production, this translates into about 1 production day in 20. Furthermore, the producer has some idea of the probability of application of a particular PF. For example, a PF of 80 percent can be expected 0.26 percent of the time or translated into a day's production 1 day in 400.

CONCLUSION

A simple, statistically correct procedure for using absolute deviations from target values to control hot-mix asphalt construction was developed. Methods currently in use either do not use absolute deviations, which can lead to process manipulation in order to control central tendency, or incorrect statistics are used with absolute deviations. Statistics in Table 3 can be used with historical data to develop statistically sound specifications that use absolute deviations from target values and control both central tendency and variability.

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