Design of Rectangular Rubber Seals on the Basis of Von Mises Stress

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A method for aiding the design of rectangular rubber seals is suggested. Rectangular cross sections are evaluated using plane strain, nonlinear, incompressible, hyperelastic (Mooney-Rivlin) finite element formulation of the software ABAQUS. On the basis of pure shear laboratory tests, a method for determining the limiting stress is suggested. Pure shear homogeneous deformation tests performed on Silicone Dow 888 showed a rearrangement of molecules at 27.5 percent nominal displacement. Even though rubber materials generally do not have a yield point, this point appeared as a monotonic yield point and was used as a design benchmark by comparing it with the equivalent Von Mises stress obtained using the finite element method.

One of the challenges in the study of joint sealing materials is the development of laboratory tests to predict field performance. Traditional tests applied to rubber asphalt sealants include bond, flow, and penetration (hardness) tests. Although these tests have been used for many years, some questions remain unanswered. How well do these tests predict behavior? What do they simulate? Can tests used for other materials be adapted to better predict the behavior of rubber seals?

Bugler stated that it is important to check previous performance of the sealant in question (*1*,p.975). However, if there are no data or previous road performance, then laboratory evaluation should be done. In that case, he suggested more than 20 tests that could be performed, most of which are ASTM standard tests.

Belangie suggested four tests to evaluate sealants for cold temperatures: bond, cold bend, ductility, and force-ductility (*2*). (Cold bend and force-ductility are not standard tests.) He also suggested three tests to evaluate sealants for hot temperatures: flow, penetration, and softening point, all of which are ASTM standard tests.

A number of specifications have been suggested for joint sealants (*1–3*;*4*,p.1009;*5*), but the material properties to consider and the tests used to measure these properties are not always consistent. For example, in the bond-ductility test (*2*), the sealant width is usually kept constant (generally 25.4 mm or 1 in.) for any sealant testing. However, Tons and Roggeveen proposed to change the width dimension in that test to 6.35 mm (¼ in.) in order to more closely simulate field conditions (*6*,*7*). The general belief is that a combination of reasonable tests provides a basis for good specifications that can predict the field performance of a sealant.

None of the methods previously proposed has been based on limiting the stress using the finite element method (FEM) and laboratory tests. This study suggests a new method to design the cross section of a rectangular seal on the basis of the determination of the equivalent Von Mises stress using FEM as compared with the true stress results obtained using a laboratory pure shear homogeneous deformation test.

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LIMITS CONSIDERED IN SEAL DESIGN

In this section is given a background of the important mechanical limits that are considered in seal design. Also, the theoretical aspects involved in the finite element formulation and the Von Mises yield theory are briefly described.

Limits to Horizontal Pavement Movement

Joint expansion, slab warping, crack opening and closing, and pavement deflections were not considered in sealing and resealing operations (*8*). A considerable amount of data at this time, however, are available on concrete pavement joint movements (*9*,*10*). Tons proposed an approach for horizontal joint movements that uses a modified thermal coefficient of expansion (or contraction) and an estimated coefficient of variation (*10–12*). Crack opening and closing are more variable and less predictable. Chong and Phang used 10 mm (0.4 in.) as a value for horizontal crack movement over a yearly cycle (*13*). Belangie estimated crack movement to be between 3.175 and 12.7 mm (⅛ and ½ in.), depending on several factors including location (*2*).

Limits to Bulge and Sag

Extrusion, or bulge, shown in Figure 1 (*middle*), of the sealant material is known to have caused major damage to seals (*9*, *11*). This damage is due to the expansion of concrete, which applies higher compressive displacement on the sealant than expected. If a joint is designed without consideration of this factor, material may bulge out and the passage of traffic over the joint may damage the sealant. With time, complete failure is inevitable

Intrusion, or sag, shown in Figure 1 (*right*), is due to excessive contraction of the concrete, which applies tensile displacement on the sealant. The maximum sag should also be considered in seal design because with high sag, foreign material may intrude into the resulting pocket. When the joint is then under compressive displacements, free movement is restrained because of the existence of foreign material in the

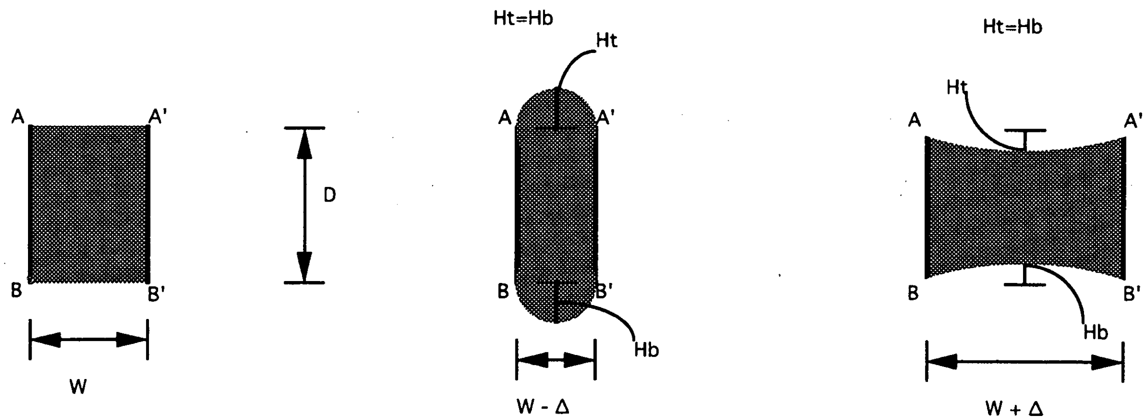

FIGURE 1 Rectangular seal in its undeformed state (*left*), under compressive displacement (*middle*), and under tensile displacement (*right*).

seal structure. Damage to the sealant material may occur, which ultimately tends to cause failure.

It is common, among joint sealant investigators such as Tons (*12*) and Bugler (*1*), to use 5.1 to 6.35 mm (0.20 to 0.25 in.) as a limit for maximum bulge or sag.

Limits to Maximum Strain

One of the problems confronted in joint seal design is to find a limiting value for the strains that determines sealant failure.

Tons, based on experience, suggested the use of a 20 percent maximum strain in the sealant (*11,12*). His design was based on the parabolic behavior of the seal cross section under displacement and on the approximation that strains in the seal along the outer parabolic curve line are uniformly distributed. His suggestions have been widely used and taken into consideration by many sealant investigators. Bugler also suggested that the design of the sealant cross section should be in such a way that the strains stay below 20 percent in the sealant.

Chong and Phang (*13*) and Anderson et al. (*14*) suggested that the maximum allowable sealant tensile strain should be 20 to 30 percent. On the other hand, Lake and Lindley studied the ozone attack and fatigue life of rubber (*15*). They reported that fatigue life is unaffected when the strains are less than 20 percent in natural rubber. Their experiments were based on simple tension tests that apply a homogeneous deformation on a strip of rubber.

Merrett evaluated the sealing of functional joints in structures and suggested a 20 percent limiting strain value (*16*).

Although these suggestions are not based on sound analysis because they do not consider the major factors affecting seal behavior, the fact is that 20 percent nominal strain is the most agreeable number used by sealant investigators.

Limits to Maximum Stress

Very few investigators have studied the maximum stresses generated in a seal structure, specifically at the interface of the sealant and the concrete. This interface is the most complicated and critical area in a seal because most seal failures occur in that region.

Goulden and Thornton made laboratory experiments on bond test to specify silicone sealants (*5*). They tested a thin film of sealant as an adhesive to reduce the problems associated with variations in shape and thickness inherent in laboratory specimens. They determined the bond strength and modulus of elasticity and plotted the modulus of elasticity versus the required bond strength. They concluded that the obtained plot can also be used to evaluate the performance of sealants made from other materials.

This method does not accurately define the performance of the seal because it assumes that the simple tension modulus of elasticity defines the stress-strain behavior of the rubber material in the seal. This may not be sufficient since a simple tension test is applied on a strip of rubber material and may not be applicable to a complicated structure, such as in a rubber seal.

LABORATORY PROCEDURES

The material chosen for experimental analysis was Silicone Dow 888, because its properties do not change significantly with changes in temperature, as reported by Spells and Klosowski (*17*). If another material such as rubber asphalt was chosen, pure shear tests should be done at different temperatures to obtain the elastic properties of the material at various conditions of elasticity.

Pure Shear Specimen Preparation

Silicone Dow 888 was gunned into a wooden mold with dimensions 76.2 × 25.4 × 203.2 mm (3 × 1 × 8 in.). The form was made as if a regular specimen of the these dimensions was to be cast. After the curing process was completed, forms were removed and a band saw was used to cut specimens 76.2 mm (3 in.) long, 25.4 mm (1 in.) wide, and 1.6 mm (1/16 in.) thick. The overall width consisted of 25.4 mm (1.0 in.) of wood from each side and 25.4 mm (1 in.) of silicone in the middle, as shown in Figure 2. Specimens were relaxed for 2

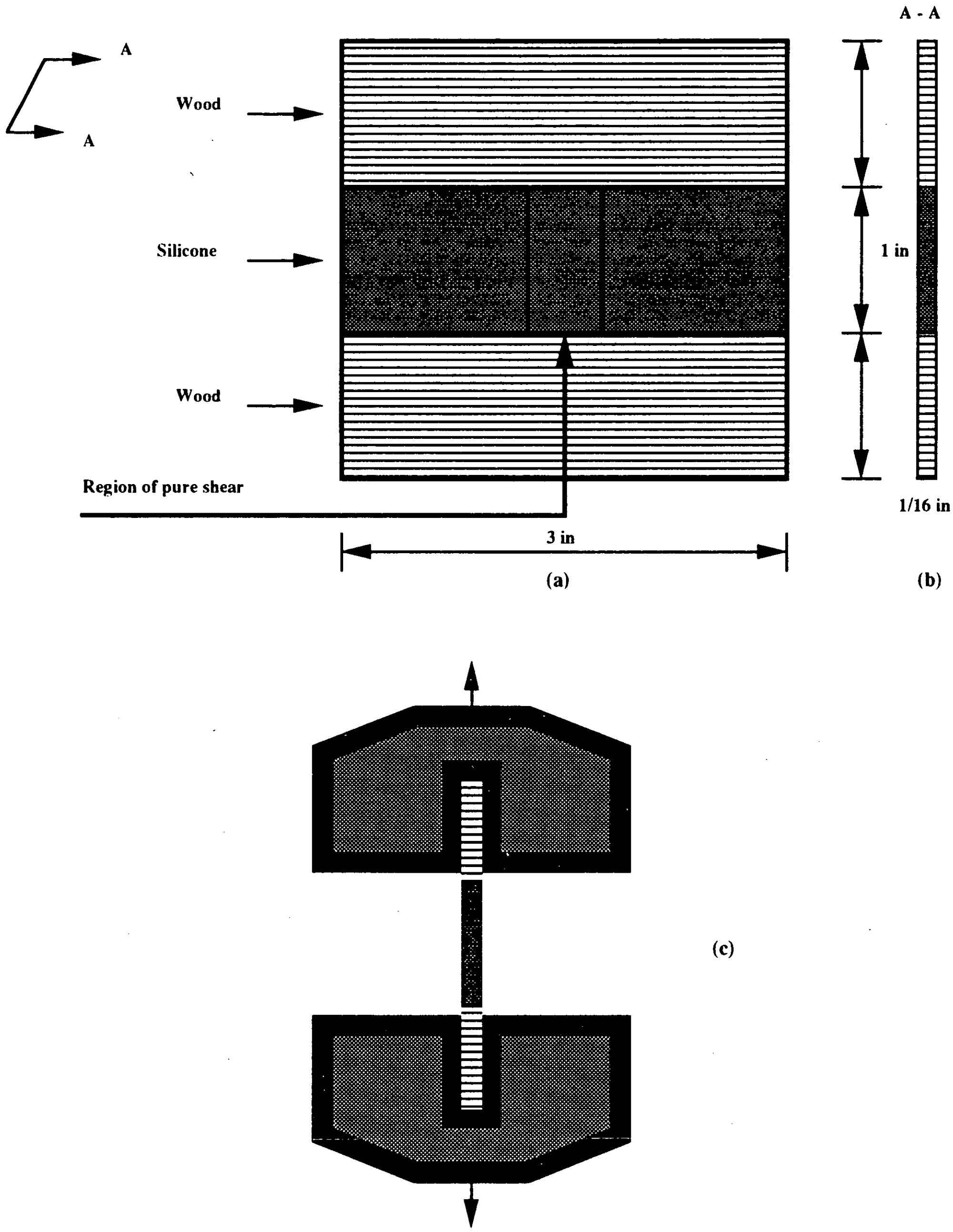

FIGURE 2 Sample specimen used for pure shear tests: (*a*) front view, (*b*) side view, (*c*) INSTRON machine jaws applying pure shear on Silicone Dow 888 specimen.

weeks and then tested in pure shear to determine the elastic properties.

Pure Shear Test and Measurements

A displacement control INSTRON machine (Model 2506) was used for testing. Both sides of the specimens were placed into the INSTRON grips (Figure 2*c*). Pure shear tests were run at 18 in./min in order to minimize viscoelastic effects.

One operator was able to perform the test since it was continuously recorded on an *x-y* plotter. Twelve specimens were tested, and the average performance was determined.

The width of the specimens was only 3 in. Usually the width used in the literature is greater (*18,19*) but only 3-in.-wide specimens were used to accommodate the setup and to symmetrically load the specimens by the INSTRON machine. As explained by some investigators, measurement for the changes in the width can be neglected as compared with the dimension of the width (*18–20*).

The nominal stress is determined by dividing the load over the original undeformed area, and the true stress is determined by dividing the load over the true area. The true area is calculated by assuming an equal original and final volume.

The calculations for true stress and for nominal stress and strain were done in the following manner:

$$A_I * L_I = A_F * L_F$$

but

$$A_F = \text{width}_F * T_F$$

so

$$\text{width}_F * T_F = (A_I * L_I)/L_F$$

$$T_F = (A_I * L_I)/(\text{width}_F * L_F)$$

$$\text{true stress} = \text{load}/(\text{width}_F * T_F)$$

$$\text{nominal stress} = \text{load}/A_I$$

$$\text{nominal strain} = (L_F - L_I)/L_F$$

where

A = area,
L = length,
T = thickness,
Subscript I = initial condition, and
Subscript F = final condition.

A detailed description of the experimental procedures is given elsewhere (*21*).

Design Limits for Silicone Dow 888

As discussed earlier, design limits for rubber seals have been based on fatigue factors and on previous experience observed in the field. The design method presented is based on laboratory evaluation of the material tested in pure shear.

Checking the load-displacement (P-Δ) behavior of the silicone material, it can be observed that there is a point at which the material changes in performance—a point of displacement at which the material has a rearrangement of molecules. This point, for Silicone Dow 888, was found to occur at about 27.5 percent displacement in pure shear, as shown in Figure 3 (*top*). For metals this point is known as yield point, but rubber-like materials, in general, do not yield (i.e., there is no significant permanent deformation in most rubber materials until failure is reached).

The P-Δ values allowed the calculations of the average nominal stresses and the true stress as shown in Figure 3 (*bottom*). It should be noted that during the laboratory tests, end effects were observed—there were displacements at the sides of the specimens, but they were neglected.

Finding a point at which rubber material changes in performance (to be called "rubber monotonic yield point") led to the idea that this point can be compared with the equivalent Von Mises stress obtained from FEM and consequently used as a design limit (*22,23*).

Movements Considered in Design (Tensile/Compressive)

Sealants are usually placed in warm weather, when the joint is at its minimum width. As a result, the sealant tends to remain in tension throughout its life. If the sealant is placed in cold weather when the joint is at its maximum width, the sealant would remain in compression throughout its life. Alternatively, if a joint is sealed in a moderate weather, when the joint width is somewhere between the narrow summer and the wide winter widths, the seal will be subjected to both tensile and compressive movements.

The amount of opening and closing of an in-service joint depends on several factors, such as temperature, moisture changes, spacing between working joints, friction between slab and base, and freeze-up of dowel bars. Some investigators use different models based on changes in temperature and humidity to find the amount of crack movement (*1*). Others make actual hand and electronic measurements (*9*). Tons proposed an approach for the measurements of horizontal joint movements that uses a modified thermal coefficient of expansion (or contraction) and an estimated coefficient of variation (*10,12*). In effect, the prediction of joint movement varies from one investigator to another depending on location, experience, method, and knowledge.

As mentioned, it is desired that the joint movement be kept within 20 to 25 percent (nominal movement), but according to Cook et al. (*9*), movements in some slabs with certain subgrade types may average up to ±5.10 mm (0.2 in.). In this case, the width would have to be at least 25.4 mm (1 in.) in order to accommodate such movements and stay within 20 percent nominal joint displacement. It should be noted that higher than 25.4 mm (1 in.), joint width may be unacceptable and unsuitable for tire noise and ride comfort requirements.

Finite Element Modeling and Analysis

After assuming that the material is fully incompressible (Poisson's ratio = 0.5 as viewed from the compressible field), and assuming that there is a perfect bond between the sealant and the concrete with a free surface at the bottom and top of the sealant, an eight-node plane strain element was used. This element is called CPE8H in ABAQUS (*24*).

The finite element modeling was based on a 10- × 10-element model. Using two-directional symmetry, a 5-x-5 structure is obtained. Figure 4 shows the FEM and the boundary conditions used.

In the analysis of this model, seal cross sections were put in tension and compression. Equivalent Von Mises stresses, shear strains, and maximum displacements (i.e., sag for tension and bulge for compression) were plotted for different computer runs. Charts were made in a normalized form to enable the investigator to pick out the response value for a certain seal shape, given the percentage movement and the material at hand. Response was reported for 10, 20, and 30 percent displacement for three reasons:

- Any desired value in between may be interpolated.
- Values at and higher than 30 percent surpass the fatigue limit of the material.

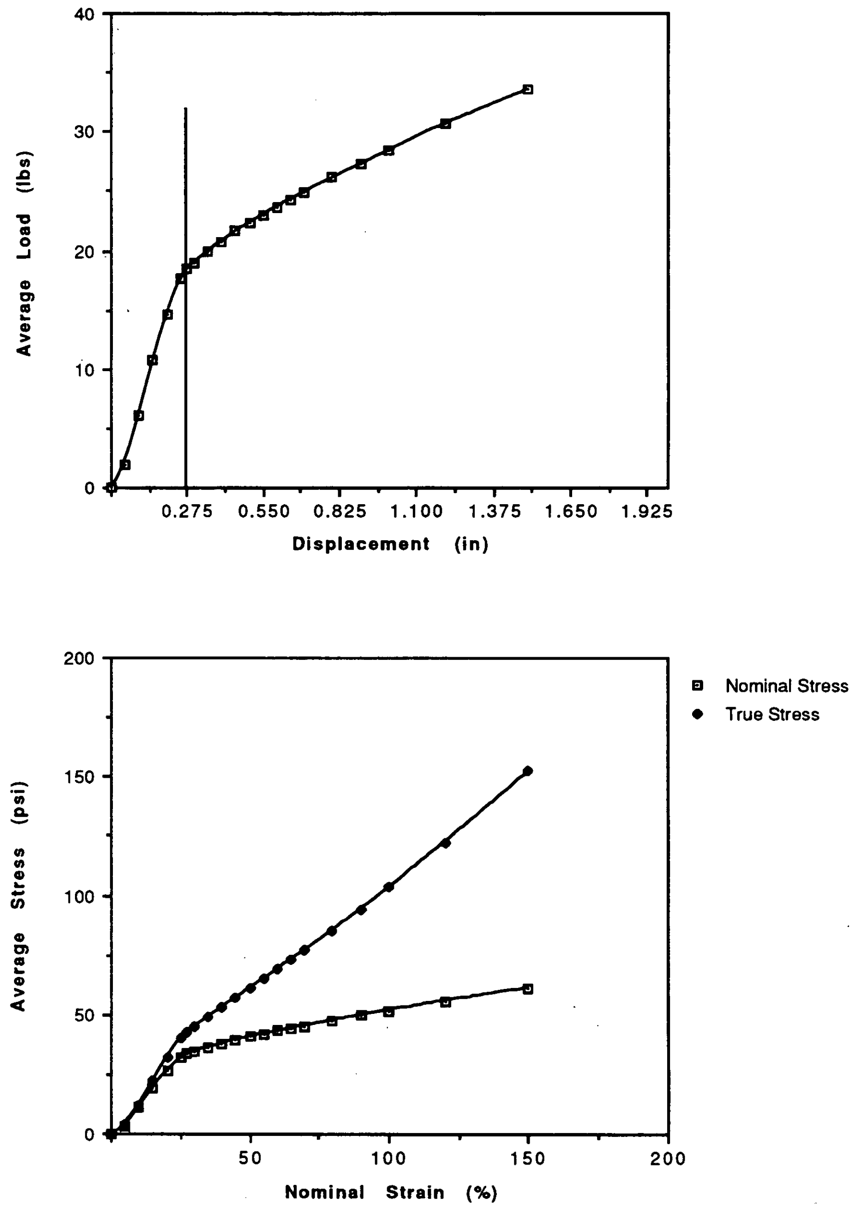

FIGURE 3 *Top*, load-displacement in pure shear; *bottom*, average nominal stress and true stress for pure shear.

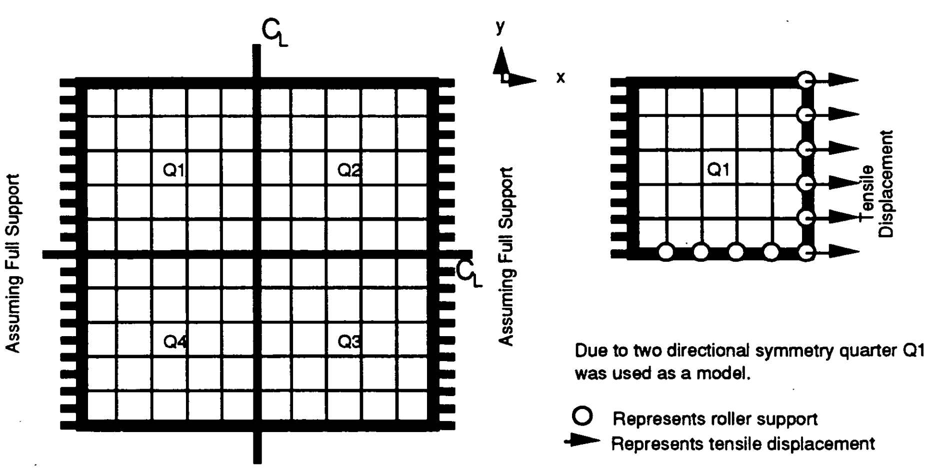

FIGURE 4 Finite element model of rectangular cross section.

- Mooney-Rivlin formulation becomes inaccurate at displacements higher than 30 percent, especially in compression.

From the structural analysis standpoint, it should be noted that two similar cross sections with different dimensions would give the same response. That is to say, two cross sections give similar responses if the ratios of their corresponding dimensions are equal and the displacement (bulge or sag) is a multiple of the ratio of their dimensions. On the basis of this behavior, the dimensions can be normalized in terms of depth/width (D/W).

Using the method provided in ABAQUS, which is based on minimizing the error of the function that fits both simple tension and pure shear tests, the following form of the strain energy function, U, was obtained for Silicone Dow 888:

$$U = C_1(I_1 - 3) + C_2(I_2 - 3)$$

where

C_1 = 0.04922 N/mm² (7.132 psi),
C_2 = 0.05436 N/mm² (7.878 psi),
I_1, I_2 = first and second principal strain invariants of the left Cauchy-Green strain tensor B_{ij}, and
I_3 = third strain invariant (equal to unity since incompressibility condition is assumed) (*18,21,24*).

For a Mooney-Rivlin first-order deformation strain energy function, the stresses τ^{ij} can be obtained using the following:

$$\tau^{ij} = (2C_1g^{ij}) + (2C_2B^{ij}) + (PG^{ij})$$

$$g^{ij} = (g_{ij})^{-1}$$

$$G^{ij} = (G_{ij})^{-1}$$

where

$B^{ij} = (I_1g^{ij} - g^{ir}g^{js}G_{rs})$;
g_{ij}, G_{ij} = metric tensors in undeformed and deformed configurations, respectively;
$B^{ij} = (B_{ij})^{-1}$; and
P = pressure-like variable calculated for every element, just like the displacement.

On the other hand, the strains ε_{ij} are of the form

$$\varepsilon_{ij} = {}^1/_2(B_{ij} - I)$$

where I is the identity matrix.

Detailed descriptions of the mathematical formulation and the choice of the number of elements used are given elsewhere (*21,24,25*).

Determination of Equivalent Von Mises Stress

Von Mises condition asserts that yielding occurs when the second deviator stress invariant reaches a specified value. This value is compared with either simple tension or pure shear laboratory tests and then used to aid the design process.

In terms of principle stresses, the equivalent Von Mises stress (σ_{eq}) may be represented as follows:

$$\sigma_{eq} = [{}^1/_2\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}]^{1/2} \quad (1)$$

The numerical factor "½" in Equation 1 has been chosen so that σ_{eq} equals the yield strength Y. σ_{eq} can be evaluated using FEM, and Y can be determined from the pure shear laboratory test. The reader is referred to Johnson and Mellor (*23*) or Malvern (*22*) for a detailed description of the Von Mises yield theory.

Analysis of Various Cross Sections

Rectangular seal cross sections were analyzed. The widths Wt and Wb were kept constant at 25.4 mm (1 in.), and the depth D was varied with values as follows: 6.35, 12.7, 19.05, 25.4, 31.75, 38.1, 44.45, 50.8, and 76.2 mm (0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, and 3.00 in.). Results were reported at 10, 20, and 30 percent of Wb displacements for both tension and compression.

Under tensile displacement, results for normalized Von Mises stresses (normalized over the initial modulus of elasticity) and shear strains are shown in Figure 5. While under compressive displacements, results for normalized Von Mises stresses and shear strains are shown in Figure 6. Results for maximum sag under tensile displacement and bulge under compressive displacement are shown in Figure 7. Displacement values are normalized over the width of the rectangular seal such that, given any shape, the displacements can be determined through multiplying by W.

DESIGN PROCEDURE

The design procedure can then be done for any material to which simple tension and pure shear tests can be applied, in the following form:

Given

- Slab dimensions and coefficient of expansion of concrete;
- Maximum changes in temperature;
- Climate conditions and season of installation; and
- Material used and initial modulus of elasticity of the sealant material = 6 ($C_1 + C_2$), where C_1 and C_2 are the first and second coefficients of the Mooney-Rivlin strain energy function (*24,25*). Note that if material elastic properties change with temperature, the strain energy function should be determined at various temperature conditions.

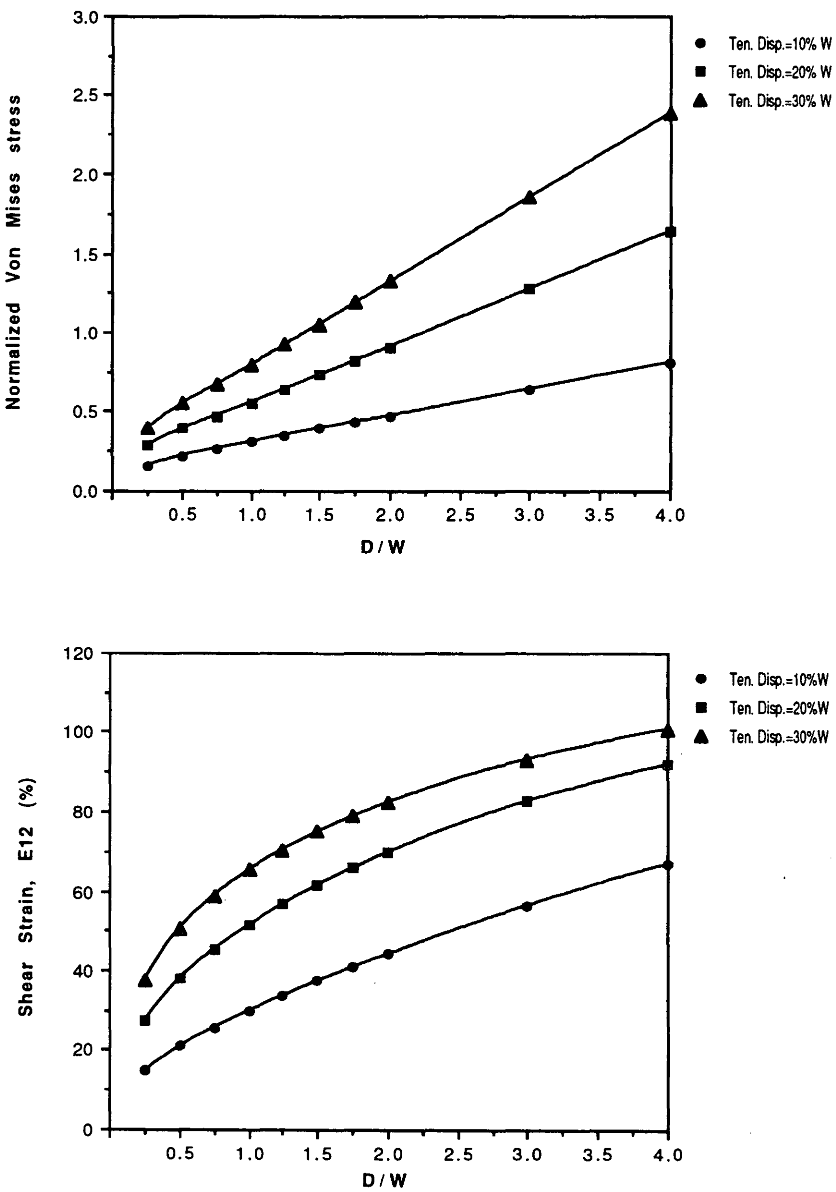

FIGURE 5 ***Top*, normalized equivalent Von Mises stress versus *D/W* in rectangular seal at 10, 20 and 30 percent *W* tensile displacement; *bottom*, shear strain versus *D/W* in rectangular seal at 10, 20 and 30 percent *W* tensile displacement.**

Procedure

- Determine the slab movement;
- Use FEM results to obtain the maximum equivalent Von Mises stresses, strains, and displacements (bulge or sag);
- Perform the pure shear test and then determine the monotonic yield point and use it as a design benchmark;
- Check if bulge or sag are within the acceptable limits;
- Check shear strains or axial strains if design criteria are available; and
- Compare FEM Von Mises stress with the monotonic yield value obtained from the pure shear test. If FEM result is higher, change the design and check again. Otherwise, accept the design.

Design Example

Problem

Assume that the following information is given:

- Slab length: L = 6.096 mm (20 ft.)
- Change in temperature: ΔT = 90°F.

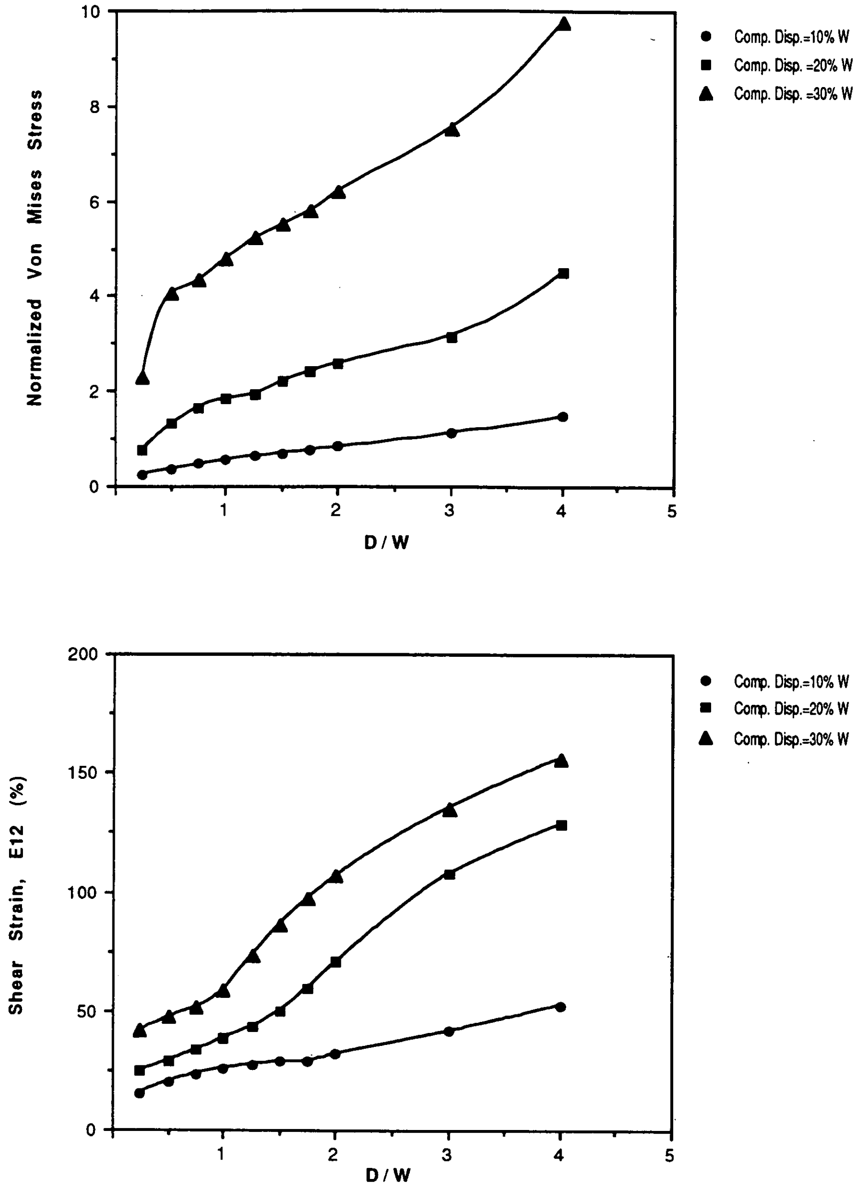

FIGURE 6 ***Top*, normalized equivalent Von Mises stress versus *D/W* in rectangular seal at 10, 20 and 30 percent *W* compressive displacement; *bottom*, shear strain versus *D/W* in rectangular seal at 10, 20 and 30 percent *W* compressive displacement.**

- Depth to which the joint is sealed: D = 6.35 mm (0.75 in.).
- Material used is silicone with initial modulus of elasticity = 0.6211 N/mm² (90 psi). Modulus of elasticity is based on combination of regression analysis curves of simple tension and pure shear laboratory experiments (*25*).
- Maximum allowable bulge or sag = 6.35 mm (0.25 in.).
- Seal is poured in warm weather, which means that the joint tends to stay in tension throughout its life.
- Coefficient of linear expansion of the concrete (α) is 5.5×10^6 in./in./F.
- Width of the joint is to be designed such that it is rectangular, that is, $Wt = Wb$.

Solution

- For the given ΔT, L, and α, the design slab movement is 4.572 mm (0.18 in.) during a yearly cycle. (Value obtained from Figure 8.) Note that different investigators use different movement criteria, and this is used only to illustrate the method.
- Guess $Wt = Wb$ = 19.05 mm (0.75 in.) → movement is going to be:

$$\frac{4.572}{19.05} = 0.24 \text{ or 24 percent of width}$$

$$= \text{percentage displacement.}$$

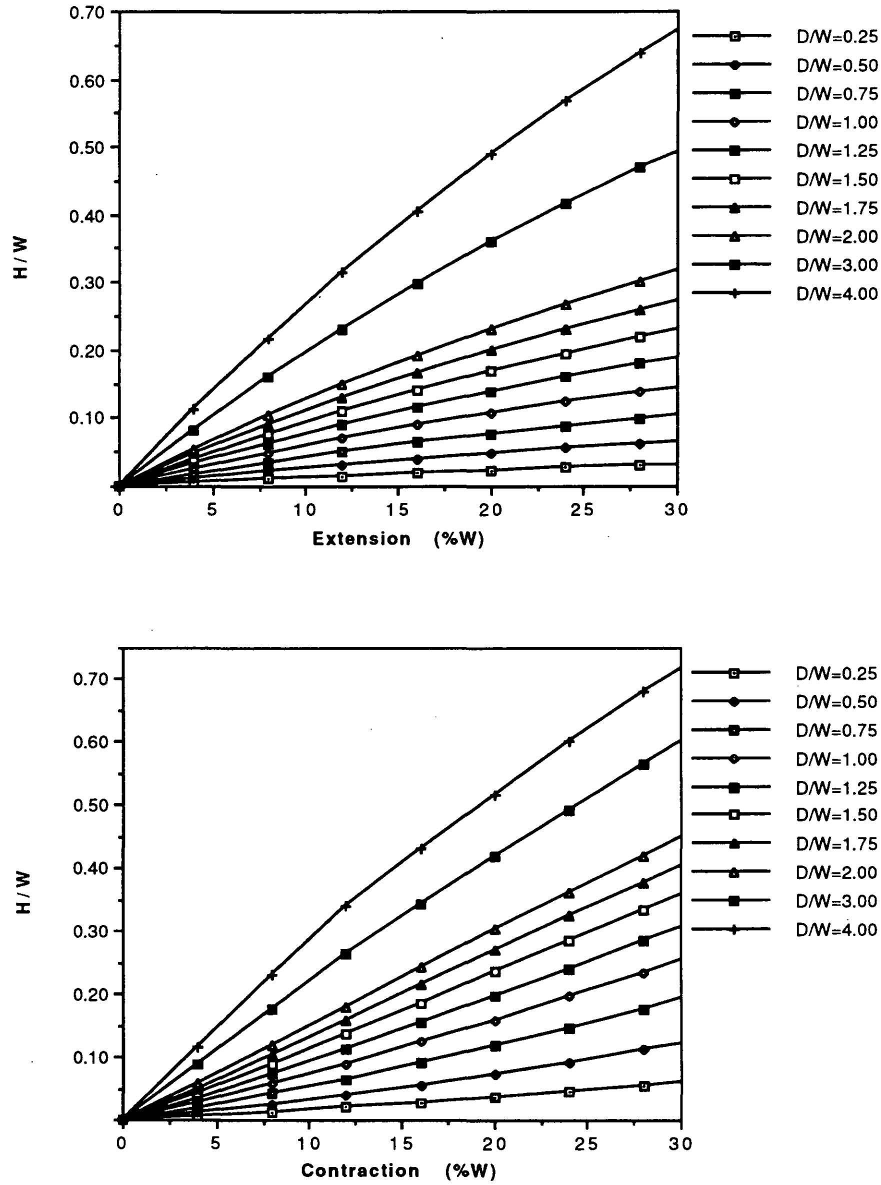

FIGURE 7 ***Top, H/W*** **versus percentage** ***W*** **tensile displacement in rectangular seal at different** ***D/W*** **values;** ***bottom, H/W*** **versus percentage** ***W*** **comprehensive displacement in rectangular seal at different** ***D/W*** **values.**

• For $D/W = 19.05/19.05 = 1$, read from Figure 5 (*top*), the values that correspond to 20 and 30 percent; interpolate to obtain the maximum value of the normalized equivalent Von Mises stress of 0.67 for a displacement of 24 percent.

• For Silicone Dow 888, the Mooney-Rivlin–based initial modulus of elasticity is $E = 6(C_1 + C_2) = 6(0.04922 + 0.05436) = 0.6211$ N/mm² (90.06 psi), but normalized Von Mises $= \sigma_{eq}/E = 0.67$, so $\sigma_{eq} = 0.67 * 0.6211 = 0.4161$ N/mm² (60.3 psi).

• For $D/W = 0.75$, obtain the maximum shear strain ε_{12}; from Figure 5 (*bottom*) this is 54 percent. Note that in this design, it is not important to determine shear strain because design is based on stress criteria.

• From Figure 7 (*top*), read $H/W = 0.125$, so $H = 0.125 * 19.05 = 2.381$ mm (0.094 in.) $<$ 6.35 mm (0.25 in.).

• Compare FEM value of Von Mises stress in Figure 5 (*top*) with the laboratory output of the true stress curve obtained from the pure shear tests in Figure 3 (*bottom*). FEM stress value of 0.4161 N/mm² (60.3 psi) is greater than the laboratory monotonic yield point of 0.3105 N/mm² (45 psi); this means that the design is rejected and another can be tried.

• Guess $W = 25.4$ mm (1 in.) and $D = 19.05$ mm (0.75 in.), then $D/W = 0.75$, and percentage tensile displacement $= 4.572/25.4 = 18$ percent. This means that from Figure 5 (*top*), $\sigma_{eq}/E = 0.4$, therefore $\sigma_{eq} = 0.4 * 0.6211 = 0.2484$ N/mm² (36 psi), and the shear strain $\varepsilon_{12} = 40$ percent from Figure 5 (*bottom*). Also, $H/W = 0.085$, so $H = 0.085 * 25.4 = 2.159$ mm (0.085 in.) from Figure 7 (*top*).

• FEM equivalent Von Mises stress of 0.2484 N/mm² (36 psi) is less than the laboratory monotonic yield point of 0.3105

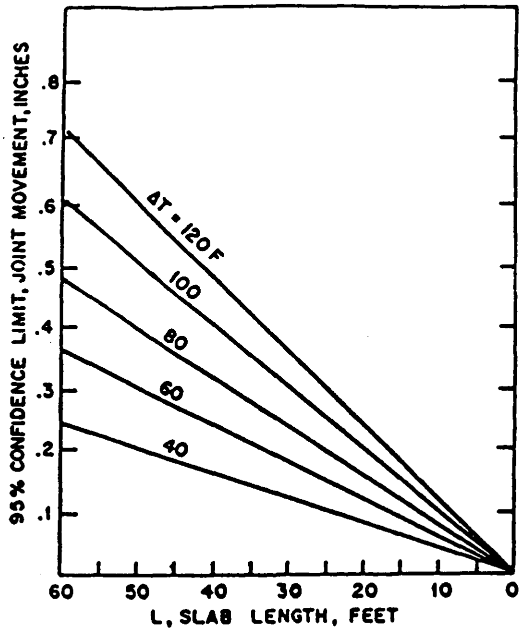

FIGURE 8 Design joint movement for molded in-place seals estimated on basis of thermal coefficient: 30 percent coefficient of variation, 95 percent confidence level.

N/mm^2 (45 psi). Also, the displacements are below the limit of 6.35 mm (0.25 in.). This means that W = 25.4 mm (1 in.) and D = 19.05 mm (0.75 in.) is a good seal cross section for this joint.

It can be observed that decreasing the ratio of D/W from 1.00 to 0.75 may decrease the stress by about 40 percent, the maximum shear strain by about 26 percent, and the maximum displacement by about 9 percent.

Moreover, if W = 38.1 mm (1.5 in.) was chosen, then W displacement = 12 percent, so $\sigma_{eq} = 0.27 * 0.6211 = 0.1677$ N/mm^2 (24.3 psi), ε_{12} = 23 percent, and $H = 0.04 * 38.1 = 1.524$ mm (0.06 in.), which means that as D/W decreases, the maximum values of stress, shear strain, and displacement decrease accordingly: the greater the width W, the lower the response for a constant depth D. It should also be mentioned that a large width is not recommended because it tends to allow the vehicle tire to touch the sealant while passing over the joint, causing the sealant to fail. On the other hand, a wide joint can increase tire noise and riding discomfort.

The design and discussion was based on laboratory stress limits and experience. It was done by determining which seal cross section gives the lowest equivalent Von Mises stress, strain, and displacement and at the same time be practicable.

Generally, the best joint design is obtained when $0.50 < D/W < 0.75$. A D/W lower than 0.5 may cause damage to the sealant, since it will not be able to support a concentrated load applied by a tire pressing on a stone that may be lying on a seal plug. On the other hand, when D/W is greater than 0.75, maximum stress, strain, and displacement may increase beyond the allowable limits, tending to cause failure.

CONCLUSIONS

Rectangular rubber seals were evaluated using a plane strain, nonlinear, incompressible, hyperelastic (Mooney-Rivlin) finite element formulation of the software ABAQUS. Using this evaluation, a method for aiding the design of rectangular seals is suggested. The Von Mises stress results of the finite element method were compared with the true stress results obtained by applying the pure shear laboratory tests. Pure shear tests done on Silicone Dow 888 showed that a rearrangement of molecules was observed at about 27.5 percent displacement, which appeared as a monotonic yield point. This point was used as a design benchmark by comparing it with the equivalent Von Mises stress obtained by FEM.

It is important to note that this method does not consider such factors as viscoelastic effects, temperature effects on elastic properties, adhesion properties (full adhesion was assumed), ozone effects, and long-term expansion and contraction effects. These effects are important and should also be allowed for in the final design.

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