Optimal Rehabilitation Times for Concrete Bridge Decks

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The deterioration of reinforced concrete bridge decks—a major problem for highway agencies throughout the United States—is due primarily to corrosion accelerated by deicing salt for snow and ice control. A variety of preventive and corrective treatments have been and are being developed to alleviate corrosion-related distress, ranging from patching compounds and sealers to overlays, chloride inhibitor and removal techniques, and cathodic protection. This array of treatments entails a considerable range of costs, expected lives, and technical issues (e.g., compatibility with existing bridge materials), and the relative costs and benefits of different treatments are not always clear. A life-cycle cost approach to structuring decisions on which treatments are the best to apply, and at what time, for existing bridge decks in a given condition is presented. The analytic approach identifies the minimum cost of each treatment and its optimal time of application, considering agency costs and user costs estimated in three distinct periods: deck service life before treatment, during application, and after treatment. An example problem illustrates the solution in terms of agency cost, user cost, and total cost curves over time, with an explanation and interpretation of trends affecting the optimal time of application and related minimum costs.

Concrete bridge deck deterioration is a major problem for highway agencies throughout the United States. Most bridge decks in the nation were built without a protective system. FHWA has documented that unprotected decks often require major rehabilitation after only 5 to 10 years of service and must be replaced only after 15 years (*1*). Some of the figures reported by different agencies provide an idea of the magnitude of the problem:

- In 1975 FHWA reported the cost of bridge deck rehabilitation in the United States at $200 million/year (*2*).
- In 1977 FHWA reported that 65,705 bridges (about 10 percent of the nation's bridges) had badly deteriorated decks (*2*).
- During the winter of 1977, The Road Information Program (TRIP) indicated that 1,626 bridges were rendered unusable, primarily because of spalling (*2*). The rehabilitation and replacement cost for these bridges was estimated at $1 billion.
- In 1979 the General Accounting Office reported to Congress that the total cost of repair to the nation's bridge decks had grown to $6.3 billion (*3*).

Decks have become, over the past few years, the single bridge element requiring the greatest maintenance effort (*4*). This growing problem is due primarily to the use of deicing salts for snow and ice control. Chloride ions penetrate the concrete cover to the mat of reinforcing steel, causing an electrolytic action that corrodes the steel. The oxide product of this corrosion swells the volume of the steel, causing the concrete cover to crack, delaminate, and ultimately spall. Spalling weakens the deck locally, impairs the riding surface, and exposes the reinforcement to further corrosion. Neglected spalling could eventually lead to local failures in the deck.

A variety of preventive and corrective treatments have been developed to alleviate the distress to reinforced concrete bridge decks caused by corrosion, either through protection, repair, or rehabilitation of the deck. (For purposes of this discussion, routine maintenance activities such as short-lived patching are not considered.) These treatments include several types of patching compounds, sealers, overlays, and cathodic protection. In addition, the Strategic Highway Research Program (SHRP) has investigated new methods of alleviating the corrosion of reinforced concrete bridge members, including chemical and physical techniques of bridge protection and rehabilitation, electrochemical chloride removal, and cathodic protection.

This array of available treatments entails a considerable range of costs, expected lives, and technical issues (e.g., compatibility between the treatment and the existing bridge materials or environment). The difficulty in weighing the relative costs and benefits of multiple treatments while accounting for the many technical details surrounding their correct use complicates the decisions on the following questions:

- What is the best treatment to apply to a bridge deck of given design, construction, environment, and traffic to alleviate the effects of corrosion?
- When is the best time to apply that treatment?

This paper presents an analytic approach to address these questions within the framework of a life-cycle cost analysis. For a given treatment, the approach provides a bridge manager with an optimal application time and the minimum life-cycle costs associated with that decision. For a set of competing treatments, the approach permits a comparison of the least-cost strategies associated with each treatment and identifies the treatment that has the lowest life-cycle cost overall.

The problem of optimal rehabilitation policies for civil facilities has received increased attention in recent years, including the development of new analytic techniques. Fernandez first formulated the problem in terms of optimal control theory and showed how this approach could be used to solve problems in facility investment and maintenance very effi-

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ciently (*5*). Markow and Balta extended this methodology to the optimal overlay frequency and thickness for highway pavements within a fixed planning horizon (*6*), and Tsunokawa solved for the optimal frequency and thickness of highway pavement overlays in an infinite planning horizon (*7*).

The problem of selecting and identifying the optimal time of bridge corrosion treatments as described was also formulated and solved initially using a control theory approach. Not only did this approach yield an efficient closed-form solution, but it also permitted the interpretation of results in terms of their technical and economic implications. In investigating the solution further, we found the control theory approach to have a mathematical limitation that prevented us from expressing the desired dependency between the projected life of a treatment and the deck condition at which the treatment is applied. Consequently, we resorted to a direct numerical evaluation of the relevant mathematical expressions rather than seek the closed-form solution produced by the control theory approach. Although the new solution follows very closely the control theory solution, it is more general and flexible in adapting to variations in the way the problem is formulated, with little loss in the efficiency of computing the result. We have retained, however, the insight into the behavior of the solution that control theory has provided. The resulting formulation and solution, and its application in an example problem, is described later.

The analytic procedure is formulated as a project-level analysis within the context of a life-cycle cost framework. Both agency costs and incremental user costs are included, although bridge managers are free to include only those cost components that they believe are relevant to their particular situations. With respect to the time dimension, the analysis considers three distinct periods in assessing the optimal scheduling of a treatment:

- The time before the treatment, during which the mechanisms of corrosion are developing and distress begins to affect deck condition and motorist response;
- The time during the treatment, which is modeled as a project that incurs both agency costs for work performance and incremental user costs due to congestion or detours around the construction zone; and
- The time after the treatment, during which the projected service life of the treatment is accounted for in calculations that essentially are equivalent to the salvage value of each alternative.

The model development described in the following therefore considers each cost component within each respective period.

DETERIORATION AND COST MODELS

Service Life Before Treatment

Deck Deterioration

The analytic approach is based on a project-level analysis, so bridge deck deterioration is based on the history, condition, and projected performance trend for an individual bridge, rather than a bridge network as is dealt with in bridge management systems. The type of deterioration model that was selected for this application is a deterministic prediction instead of the probabilistic approach that is applied in current bridge management systems such as that by Pontis (*8*). Although the assumption of a deterministic model does not reflect the uncertainties in data and performance prediction that are inherent in life-cycle costs, these problems can be alleviated by a sensitivity analysis that is relatively easy and efficient to include in the model's framework. Moreover, on the basis of the case studies reviewed to date, the model's results appear to be fairly robust over a range of input data, further lessening the impact of uncertain input values. An extensive discussion of the implications of assuming deterministic deterioration predictions is given by Madanat (*9*).

For simplicity and brevity, the following model development focuses on corrective actions: that is, treatments that repair or rehabilitate a bridge deck that has already been subjected to corrosion. A parallel model development also exists for protection treatments that are intended to be used before the onset of corrosion.

The prediction of bridge deck deterioration requires a measure of current bridge condition as a function of time. The measure developed for this project is based on the judgment and experience of the team members in consultation with researchers on concurrent SHRP projects dealing with reinforced concrete bridges. This measure, referred to by the notation $S(t)$, is expressed in terms of the percentage of deck area that is distressed. $S(t)$ is a weighted average of three time-dependent characteristics of the bridge deck: the percentage of deck area that is (a) chloride-contaminated, (b) delaminated but not yet spalled, and (c) spalled.

The deterioration of reinforced concrete bridge decks due to chloride-related corrosion is typically represented in the literature as a piecewise linear function, as shown in Figure 1 (*top*). This curve itself is an approximation of how distress actually progresses in the field. However, the three linear segments (i.e., the regime of zero distress before corrosion initiation at t_0; the intervening period up to the time to deterioration R_t, and the growth of distress thereafter) cannot conveniently be represented by a single equation. The deterioration model assumed in this study is therefore as shown in Figure 1 (*bottom*). This S-shaped, or logistic, curve is a plot of the following model:

$$S_1(t) = \frac{100}{1 + A_1 \exp(-B_1 t)} \qquad S_1(t) < S_{\max} \tag{1}$$

where

$S_1(t)$ = percentage of bridge deck distressed at time t (with subscript 1 denoting period before treatment);
A_1, B_1 = constants controlling rate of deterioration and shape of curve;
t = time since initial construction or last reconstruction; and
$S_{\max}$ = practical maximum level of distress above which bridge would probably be closed.

The logistic curve is a reasonable model of corrosion-induced distress. It tracks the general trend of deck performance ob-

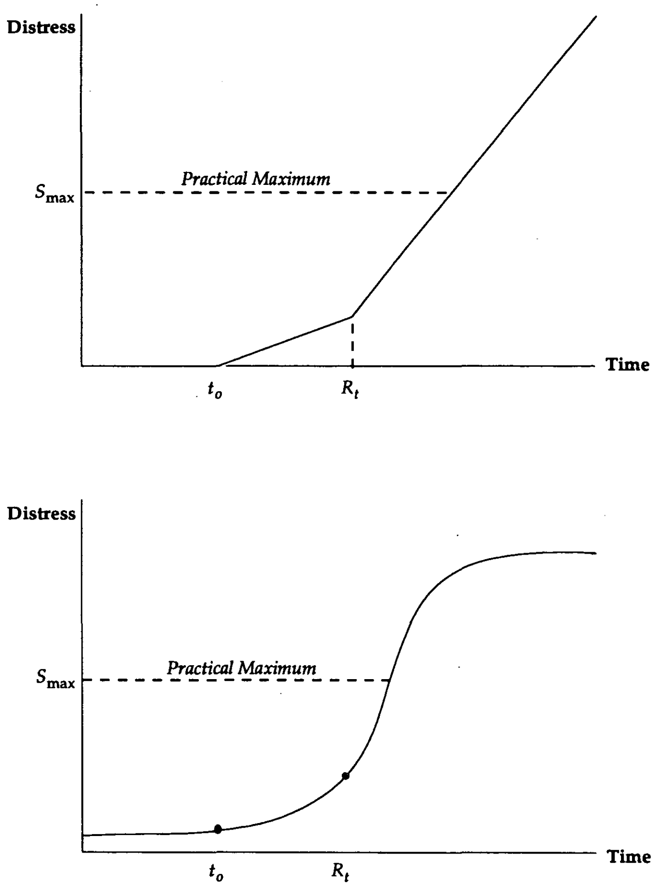

FIGURE 1 Bridge deck deterioration model: *top*, typical in literature; *bottom*, assumed in this study.

served in field studies of corrosion. Furthermore, it is a bounded function, ensuring that the mathematical limit of 100 percent of deck deterioration cannot be exceeded. (More realistic field limits of distress can be imposed as separate mathematical bounds, as shown in Figure 1.)

User Costs

User costs in the period before treatment are a function of the condition of the bridge deck and its effect on traffic flow (hence, travel time costs), vehicle operating costs, and accident costs. In theory, user costs would be expected to increase as bridge condition deteriorates, since a distressed bridge deck impedes speed and may impair vehicle handling somewhat. (Reductions in fuel consumption due to lower speeds may be offset by increases in fuel use during stop-and-go driving and are typically outweighed by the travel time and accident components when total user costs are computed.) Studies of driver behavior and vehicle operating costs on North American highways suggest that (a) for typical road surface conditions, operational characteristics of traffic volume and congestion have a much greater effect on user costs than do surface conditions, and (b) road surface condition does not begin to have a major effect on user costs until the condition becomes very bad (*10*). Furthermore, the lengths of bridges represent a very small segment of most trips.

All of these facts suggest that bridge deck condition does not have a major effect on road user costs unless it is allowed to deteriorate to a very high degree of spalling. A badly spalled deck would impede traffic flow, causing speed reductions, congestion, and increases in travel time costs and vehicle operating costs. (Effects on accident costs are not well researched.) This congestion-related approach suggests an exponential function similar to that used in urban congestion studies, which would predict negligible or very small increases in user costs for decks in good to moderate condition and large increases in user costs for decks in poor condition. The expression for the increase in user costs due to worsening deck condition is as follows:

$$U[S_i(t)] = k_2\left[\frac{S_i(t)}{S_{\max}}\right]^{k3} \tag{2}$$

where

$U[S_i(t)]$ = increase in user costs due to worsening deck condition (\$/vehicle) (subscript i will have a value of 1, denoting period before treatment, or 2, denoting period after treatment);
k_2 = calibrating constant (\$/vehicle); and
k_3 = exponent controlling growth of user costs with decreasing deck condition (assumed to equal 4 in this model).

Costs During Treatment

During the treatment, the deck condition is improved through preventive or corrective actions, or both. This improvement can be modeled as a reduction in the amount of current distress, an adjustment in the subsequent slope of the deterioration curve (a function of the additional life conferred on the bridge deck by the treatment), or a combination of the two. The technical basis for these adjustments is described in the following sections.

Improvement in Deck Condition

At the start of the construction period the deck is at condition $S_1(t^*)$, as illustrated in Figure 2 and where t^* denotes the optimal time of treatment that is to be determined. In most situations the treatment will reduce or eliminate the current distress. The extent of this improvement can be denoted by a proportionality constant I, on a scale of 0 to 1, associated with each treatment. I is defined as the relative degree of improvement: 1 denotes complete improvement, and 0 denotes no improvement. A value between 0 and 1 denotes a partial improvement equal to $IS_1(t^*)$. The residual distress after the repair is therefore $(1 - I)S_1(t^*)$.

The value of I depends on the treatment technology:

- I has a value of 1 for activities such as cathodic protection, deep concrete removal, chloride removal, and inhibitor injection, if these chloride-combatting steps are accompanied by repairs to distress of the riding surface of the deck. $I = 1$ therefore reflects the assumption that these treatments correct

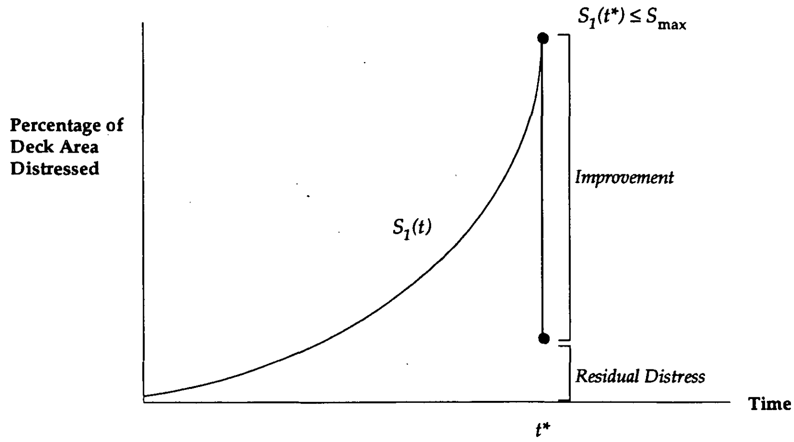

FIGURE 2 Deck condition as result of repair.

all of the spalling and delamination and negate any chloride penetration that has occurred before the time of the project, t^*.

- Treatments such as patching and overlays correct defects in the deck surface but do not remove all the chloride within the concrete. Therefore, the best that these activities can accomplish is to reduce the condition index to a small value (e.g., 12 or less, on the basis of Equation 2). Since the repair must be performed at or before the condition index reaches S_{max}, I in these cases may have a value of 0.6 to 0.75.

Treatment Costs

The cost of protecting, repairing, or rehabilitating the bridge deck is denoted by $C[S_1(t^*)]$, where C denotes the total project cost function and $S_1(t^*)$ is the percentage of deck area that is distressed. The cost function consists of two components:

- A "fixed" component that depends on the area of the entire deck. This component reflects that portion of project costs that is insensitive to the percentage of deck area that is distressed, but depends instead on the deck area as a whole. For example, mobilization costs and the costs of applying a sealer or overlay to the entire deck would be reflected in this term.
- A "variable" component that depends solely on the percentage of deck area that is distressed: that is, for which costs depend on $S_1(t^*)$. For example, the costs of rehabilitation patching [entailing the removal of contaminated concrete and replacement with new portland cement concrete (PCC)] would be represented as a variable cost.

The advantage of this approach is that the two cost terms can be combined when it is appropriate to do so. For example, a bridge manager may wish to estimate the costs associated with a combination of activities: that is, replacement of distressed or chloride-contaminated concrete, followed by a deck treatment such as an overlay. The repair of the distressed areas would be handled in the variable cost component, and the overlay would be included in the fixed cost component.

User Costs and Benefits During Treatment

The major impacts on user costs and benefits during the project period are related to congestion as influenced by the degree of bridge closure and the duration of construction. The basic cost relationship is

$$K[S(t^*)] = k_1 \tau t_c \tag{3}$$

where

$K[S(t^*)]$ = increment in bridge user costs during project period caused by repair work;
k_1 = value of bridge user time while traveling (\$/min/vehicle) averaged over the daily traffic stream;
τ = increment in travel time across bridge, or in detour around bridge, caused by construction project (min); and
t_c = duration of construction project (days).

If the traffic will use the bridge during the project period subject to a partial bridge closure, the impacts of the closure on user costs are estimated as

$$\tau = \tau_0\left[1 + \alpha\left(\frac{q_0}{C'}\right)^{\beta}\right] - \tau_0\left[1 + \alpha\left(\frac{q_0}{C}\right)^{\beta}\right] \tag{4}$$

where

τ_0 = free-flow travel time across bridge (min);
α = constant with typical value of 0.15;
q_0 = average two-way daily traffic volume across bridge (vehicles/day);
C' = two-way capacity of bridge during construction, accounting for typical lane closures throughout project and assuming typical pattern of peak-hour and off-peak demand (vehicles/day);
β = constant with typical value of 4; and
C = two-way capacity of bridge during normal periods (vehicles/day).

Equation 4 may be simplified as follows to yield the incremental travel time due to construction:

$$\tau = \tau_0\alpha\left[\left(\frac{q_0}{C'}\right)^\beta - \left(\frac{q_0}{C}\right)^\beta\right] = \alpha\tau_0\left[\frac{q_0(C - C')}{C'C}\right]^\beta \quad (5)$$

The total duration of the project period is a function of the area of the deck and the anticipated construction productivity (itself a function of local construction technology, practice, and management):

$$t_c = S_1(t^*)D/P \quad (6)$$

where D is the total deck area in square feet and P is the estimated productivity of repair in square feet per day.

Combining Equations 3 through 6 yields the following estimate of incremental user costs per vehicle during the construction period:

$$K[s(t^*)] = k_1\alpha\tau_0\left[\frac{q_0(C - C')}{C'C}\right]^\beta DS(t^*)/P \quad (7)$$

Service Life After Treatment

Basic Economic Assumptions

An economic consideration must also be given to the basis on which to compare the varying lives of the deck resulting from different treatments (and the different timing of these actions that will be recommended by the analysis). For example, one type of treatment may have a life of 8 years, whereas another may have a life of 20 years. The longer-lived repair presumably costs more. The issue is this: over what time period should these alternatives be compared to ensure a fair reading of the relative costs and benefits? If the period is too short (e.g., 8 years), the longer-lived, more costly option will be put at a disadvantage, since only a portion of its benefits (8 years' worth of 20) will be available to justify its higher costs. If the period is too long, the shorter-lived, less costly alternative will be disadvantaged, because a significant portion of its life (Year 9 through Year 20) will be analyzed as though in a failed condition.

This question is usually addressed in one of three ways in engineering economics:

- By comparing alternatives in repetitive cycles through an analysis period that is a common multiple of their lives. In the preceding example, a 40-year period would encompass five cycles of the first alternative and two cycles of the second.
- By specifying an arbitrary analysis period, predicting the costs, benefits, and terminal condition of each alternative, and applying a salvage value to correct for the generally different terminal values of each alternative at the end of the analysis period.
- By employing a capitalized cost analysis that assumes an indefinite analysis period and a continuing, repetitive cycle of each alternative.

None of these methods alone is theoretically any more correct than another; they are simply different ways of establishing an economically equivalent comparison among alternatives. Each of them involves an arbitrary assumption, whether in the number of cycles compared (the first approach), the artificial nature of a bridge deck salvage value (the second approach), or the realism of an infinite number of cycles (the third approach). Inaccuracies due to the approximations inherent in these calculations are more than outweighed by the benefit of analyzing various alternatives on a more comparable basis. The choice of which approach to use is therefore based on the particular problem at hand and on which approach fits the solution in the most convenient, effective way.

Practical considerations of how best to compare alternatives on an equivalent basis led to the following judgments:

- The first approach is often used when the lives of different alternatives are well defined; the analysis period can then be easily specified as a common multiple of these lives. With concrete deck corrosion, however, the life of a repair action depends on the condition of the deck when it is applied. This deck condition is not known beforehand; instead, it must be solved as the result of an iterative series of optimization calculations that will be discussed in a later section. This approach was therefore judged to be untenable for this problem.
- The second approach has been used for capital plant such as factory machinery, in which a market value can be estimated for used items. This approach is less applicable to bridges; however, decks are not removed and reused elsewhere as is factory equipment, and it is difficult to see how the value of recycled deck concrete relates to the remaining service value of a deck in place. The salvage value concept was therefore viewed as artificial and difficult to apply here.
- The third approach was judged to be the most reasonable for this problem. This approach is familiar in pavement management (e.g., in the assumption of a periodic overlay cycle extending infinitely into the future). It avoids the problems inherent in the two other approaches: having to find an analysis period that is a common multiple of a large number of disparate, uncertain repair action lives (as in the first approach) and having to estimate a salvage value (as in the second approach). Admittedly, the assumption of an infinite planning horizon and infinite numbers of repair cycles is unrealistic. However, the practical matter is that when costs and benefits are discounted, those cycles that occur in the future essentially reflect the basic trends of costs established in the first cycle (the one for which the optimal solution is sought). The expressions for the bridge deck service life following repair that will be presented are based on the premise of a capitalized cost comparison.

Deck Performance

The engineering and economic implications of a capitalized cost assumption are illustrated in Figure 3, showing a series of deterioration-and-treatment cycles. The first cycle corresponds to the initial portion of deck life described in the earlier section as the service life before treatment. The time of the treatment is denoted by t^*, and it is this time for which the least-cost solution must be obtained. The assumption governing the service life after treatment is that the deck will undergo a series of deterioration-and-repair cycles, such that each subsequent repair will be accomplished at the same condition as that of the initial treatment. Similarly, the level of

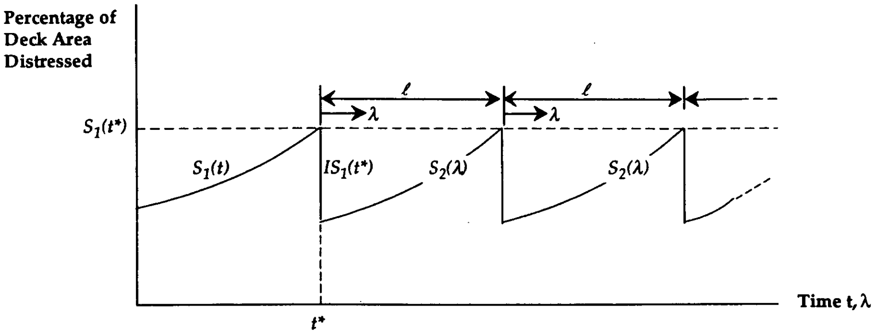

FIGURE 3 Multiple treatment cycles.

improvement obtained in each subsequent treatment is the same as the level of improvement due to the initial treatment.

These assumptions imply the following cyclic behavior of deck performance after treatment:

- Rates of deck deterioration will be the same in each cycle, as denoted by $S_2(t)$ in Figure 3.
- Treatments will be accomplished at the same threshold of distress in each cycle, equal to $S_1(t^*)$.
- The amount of improvement due to the treatment will be the same in each cycle, equal to $IS_1(t^*)$.
- The cycle interval will in each case be equal to a length of time l, with the variable λ measuring time within each cycle. The life of a treated deck, l, is based on an estimated (or nominal) life input by the bridge manager but is adjusted depending on the predicted state of corrosion at the time of initial treatment, t^*.

The cycles in Figure 3 complete the information needed to perform a life-cycle cost analysis of treatments and to solve for t^* for each treatment on the basis of minimum discounted costs. Note that the behavior of the cycles after the treatment at t^* depends on when the treatment at t^* itself is performed. Given the assumptions in Figure 3, it can be shown that the condition or deterioration function for each of the cycles following the treatment at t^* is equal to

$$S_n(\lambda) = \frac{100(1 - I)\exp(B_2\lambda)}{1 + A_1\exp(-B_1t^* - B_3)} \qquad \lambda \in [0, \ell],\ n = 2, 3, 4, \ldots \tag{8}$$

where B_2 and B_3 are calibration constants.

Equation 8 provides a common expression for the periodic deterioration pattern illustrated in Figure 3. Note that the deterioration functions in all periods from 2 on are equal to $S_2(\lambda)$. What remains now is to incorporate this prediction of deterioration within a general economic expression capturing the life-cycle costs in the period following treatment.

Costs and Benefits After Treatment

The general expression for the life-cycle costs and benefits in the period after treatment, as illustrated in Figure 3, is

$$F(t^*) = \lim_{N\to\infty} \sum_{n=2}^{N} \frac{1}{(1 + r)^{(n-2)\ell}} \left\{ \int_O^{\ell} [U(S_n(\lambda)yq_0 + m)]e^{-r\lambda}d\lambda + C[S_n(\ell)]e^{-r\ell} + K[S_n(\ell)]q_0e^{-r\ell} \right\} \tag{9}$$

where

$S_n(0) = S_{n-1}(\ell) \times (1 - I),\ n = 3, 4, \ldots.$

$S_2(t = t^*) = S_1(t^*) \times (1 - I)$

$F(t^*)$ = discounted (present) value at t^* of stream of costs and benefits associated with periodic cycles of deterioration and treatment illustrated in Figure 3;

r = discount rate, expressed as decimal fraction; $1/(1 + r)^{(n-2)\ell}$ is therefore lump-sum or discrete present worth factor to discount a sum at $(n - 2)\ell$ back to t^*;

$U[S_n(\lambda)]$ = bridge user costs as a function of deck condition ($/vehicle) as computed by Equation 2 (note that, as a result of Equation 8, $S_n(\lambda)$ can be replaced by $S_2(\lambda)$;

y = number of days in a year (365);

m = periodic costs of maintaining repair treatment (e.g., cathodic protection maintenance) ($/year);

$e^{-r\lambda}$ = annual present-worth factor to discount costs at λ back to beginning of respective cycle ($\lambda = 0$);

$C[S_n(\ell)]$ = agency cost to perform treatment at end of respective cycle ($\lambda = 1$); and

$e^{-r\ell}$ = lump-sum present-worth factor to discount costs at end of cycle to beginning of that cycle.

Equation 9 thus includes both agency costs for the treatment in each cycle, and the user costs or benefits that result in each cycle in a positive way from treatment, and a negative way from deterioration in deck condition. Note that the user costs or benefits are sensitive to deck condition over time. Similarly, the agency cost for the treatment project is sensitive to the condition of the deck at the time of treatment. Agency costs for maintenance of the treatment are taken as a constant (average) value annually.

Equation 9 can be simplified in the following way, substituting the variable d for the present worth factor $1/(1 + r)$ and the variable M for the integral expression

$$F(t^*) = \lim_{N\to\infty} \sum_{n=2}^{N} d^{(n-2)\ell} \times M = \left[\lim_{N\to\infty} \sum_{n=2}^{N} d^{(n-2)\ell} \right] \times M$$

$$= \left[\lim_{N\to\infty} \sum_{n=0}^{N} (d^{\ell})^n \right] \times M$$

Because

$d^\ell < 1,$

$$\lim_{N\to\infty} \sum_{n=0}^{N} (d^\ell)^n = \frac{1}{1 - d^\ell}$$

hence

$$F(t^*) = \frac{M}{1 - d^\ell}$$

where

$$M = \int_0^\ell \{U[S_n(\lambda)]q_0 + m\}e^{-r\lambda}\, d\lambda + C[S_n(\ell)]e^{-r\ell} + K[S_n(\ell)]q_0e^{-r\ell} \quad (10)$$

FORMULATION AND SOLUTION OF OPTIMIZATION

Optimal Treatment Activity and Timing

Equations 1 through 10 provide the technical and economic basis for solving for the optimal timing of treatment for each activity that the manager defines as being feasible. This optimal timing of treatment will be the value of t^* that yields the lowest discounted life-cycle cost, including both agency costs and user costs, for that activity. An optimal time of treatment and its associated minimum life-cycle cost will be predicted for each activity considered. Managers can then review these results to select the treatment activity to be actually used. In many cases the recommended activity may be the one that has the best cost results (i.e., its optimal life-cycle costs are the lowest among those of all activities considered). In some cases, however, other factors may influence a decision: for example, the local availability of a repair technology or budget constraints that dictate both the level and the timing of anticipated expenditures. As another example, a manager may select an activity that is not strictly optimal mathematically but is nevertheless close enough to rank as a very good solution. In any case, the optimization procedure will give the bridge manager the following information on each activity considered:

- The recommended timing of the activity, on the basis of the minimization of life-cycle costs; and
- The total discounted agency and user costs for that activity if it is performed at the recommended time.

The optimization procedure is formulated in terms of the following objective function:

$$\begin{aligned}\min_{t^*} J = &\int_0^{t^*} U[S_1(t)]yq_0 \exp(-rt)dt + C[S_1(t^*)] \exp(-rt^*) \\ &+ K[S_1(t^*)]q_0 \exp(-rt^*) \\ &+ \frac{\exp(-rt^*)}{1 - d^\ell} \left\{ \int_0^\ell [U(S_n(\lambda))yq_0 + m]e^{-r\lambda}\, d\lambda \right. \\ &\left. + C[S_2(\ell)]e^{-r\ell} + K[S_2(\ell)]q_0e^{-r\ell} \right\}\end{aligned}$$

$$S_1(t) = 100/[1 + A_1 \exp(-B_1t - B_1t_0)]$$

$$t_0 = \text{age at time } t = 0$$

$$S_1(0) = \text{given (input by manager)}$$

$$\begin{aligned}S_2(\lambda) &= 100(1 - I) \exp(B_2\lambda) \\ &\div [1 + A_1 \exp(-B_1t^* - B_1t_0)]\end{aligned} \quad (11)$$

where J is the total discounted life-cycle costs to be minimized for a given repair activity. The total cost J is the sum of the costs in each of the three periods discussed in the preceding sections. The first integral expression (from time = 0 to t^*) represents the discounted user costs before any treatment. (Routine maintenance costs for the deck could also be easily included here, if desired.) The expressions for $C[S_1(t^*)]$ and $K[S_1(t^*)]$ represent the agency costs and the incremental user costs, respectively, during the construction period for the treatment. The expression in the braces represents the cost stream after the treatment.

Mathematical Optimization Procedure

The fact that the cyclic interval l in Figure 3 depends on the optimal solution t^* renders the upper limit of the second integral in Equation 11 a variable, making it difficult to obtain an analytic (or closed-form) solution. However, the existence of well-defined bounds governing this integral and its functions allows an accurate solution using numerical methods. The lower bound of the range of variation of t^* is zero. Its upper bound is a time that we can denote as t_{max}, defined as the time to reach the practical maximum distress S_{max}. Thus, it is easy to tabulate Equation 11 for t^* varying from 0 to t_{max}, and to select the value of t^* for which the objective function J is minimum. The optimization procedure therefore depends simply and directly on the numerical evaluation of Equation 11.

Although the behavior of the objective function 11 is driven by the entire set of parameters that have been defined, the primary sensitivity of total costs is to the unit cost of the treatment, the rates of deterioration before and after the initial treatment (S_1 and S_2), and the discount rate (r). The user cost terms (U and K) do not exert a strong influence over much of the potential range of t^*, but become important only in the case of a badly distressed deck condition (affecting U) or construction project situations involving either extreme congestion or lengthy, costly detours (affecting K). These and other trends are illustrated in the example runs.

EXAMPLE SOLUTIONS OF OPTIMAL DECK TREATMENTS

How the Solution Works

It will assist the interpretation of the following figures to know basically how the numerical solution proceeds. The procedure is simple in concept:

- For a given treatment, the procedure steps through the analysis period year by year.

- In each year it simulates the application of the treatment in that year. For example, if the treatment is a PCC overlay, it begins by simulating the performance of the overlay in Year 1. Next, it simulates the performance of the overlay in Year 2. It continues this procedure for each succeeding year in the analysis period.
- As each treatment is simulated, the procedure tallies all life-cycle costs (essentially the different terms in Equation 11), including both the agency and the user costs before the treatment, during the project to install the treatment, and after the treatment.
- The various cost components are stored in a table organized by the year in which the treatment was simulated to be performed. At the end of the analysis (i.e., when costs have been tallied for all years within the analysis period), the optimal solution may be obtained by inspection as that year having the lowest life-cycle costs associated with treatment performance.
- It is crucial to note that the time dimension is correlated with the condition of the bridge deck. Treatments that occur at the beginning of the analysis will affect the deck in its current condition. Treatments later in the analysis will affect a deck as it has deteriorated from its condition at the start of the analysis.

Example Data

For this example we assumed the following as constant for all the treatments:

- A relatively new, lightly distressed bridge deck with an area of 4,000 ft^2 (four lanes wide totaling 48 ft, with a length of about 83 ft);
- An AADT of 25,000, with a value of time of $10/hr;
- A free-flow crossing time of 0.01894 min (based on the bridge length of 83 ft and a speed of 50 mph);
- A normal two-way capacity of 96,000 vehicles per day and a capacity during the treatment of 57,600 vehicles per day (assuming one lane of the bridge is closed at a time);
- No annual maintenance (either of the bridge deck before the treatment, or of any of the treatments after their installation); and
- A discount rate of 5 percent.

The factors that varied in each of the cases were the characteristics of the treatments. Four basic types of treatments were considered in these examples:

- Rehabilitation patching (i.e., patching with PCC after removal of distressed concrete);
- Application of a sealer after patching of distressed concrete;
- Application of a PCC overlay over the entire deck after patching of distressed concrete; and
- Installation of a cathodic protection system after patching of distressed concrete.

Key values associated with these options are shown in Table 1. Variations in these methods (to reflect different types of patches, sealers, overlays, cathodic protection systems, or other types of treatments) would be reflected by changes in the values of the parameters in Table 1 (or other parameters in the model, such as the extent of bridge closure).

Example of Optimal Timing

An example of the computation of optimal treatment timing is given in Figure 4 for the PCC overlay. Figure 4 shows the life-cycle costs attributable to the simulated performance of the overlay treatment in each of Years 1 through 36 of the analysis period. Three cost curves are shown:

- Discounted agency costs attributable to performing the treatment, plus an amount for the presumed cycle of subsequent repairs (computed in lieu of a salvage value, as explained earlier and shown in Figure 3).
- Discounted incremental user costs attributable (a) to riding on a badly deteriorated deck and (b) to delays due to

TABLE 1 Input Data for Example Treatments

Parameter	Patching	Sealer	PCC Overlay	CP
Service Life, years	6	10	20	30
Cost of patching, $/sf distressed area	35	35	35	35
Project cost (to treat entire deck area, $)	0	1,230	24,000	36,000
Productivity, sf/day for patching	135	135	135	135
Time, days, for project to treat entire deck	0	4	10	20
Improvement factor, I (fraction of distress corrected)	0.765	0.765	0.765	1.0

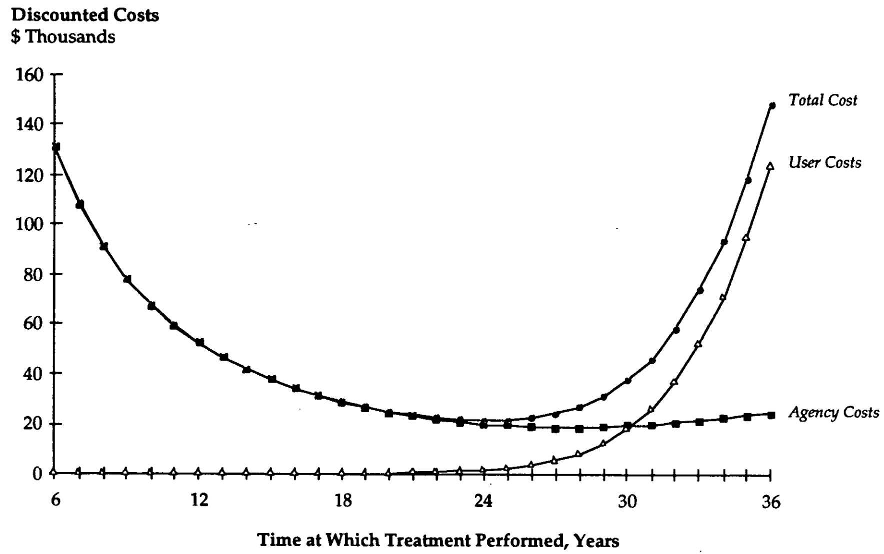

FIGURE 4 Optimal timing of treatment, using PCC overlay as example.

congestion during the treatment project. These costs are tallied not only surrounding the initial treatment (for which the optimal solution is being computed), but also for the series of subsequent repairs computed in place of a salvage value, as mentioned earlier.

- Discounted total costs, representing the sum of the agency costs and the user costs.

The agency cost curve declines over time up to a point, because of the effect of the discount rate. A higher discount rate would result in a steeper decline over time; a lower discount rate, a more gentle decline. At about 27 years, however, agency costs begin to increase, albeit in a very flat region of the curve. The reason for this increase is that the area of the deck requiring patching before overlay is becoming excessive and starting to drive the costs of the treatment higher, at a rate faster than the discount rate can compensate for the increase. Thus, the agency cost curve illustrates a tension in the solution between the rate of discount and the rate of deterioration of the bridge deck and the competitive effects of these two parameters on costs.

The user cost is essentially zero throughout the early part of the analysis, indicating very few incremental costs because the deck is new and has no adverse impacts on speed and travel time. Congestion costs due to the treatment project are negligible in all cases and for all years in this example, because of the relatively short length of the bridge and the modest level of traffic. This situation might change with longer spans in which travel time effects would become more significant, or with detours around the bridge site. Implications of higher traffic volumes are investigated later in the example.

Note that the user costs do not begin to affect the result until the bridge deck condition deteriorates to a condition bad enough to have a noticeable effect on traffic. Furthermore, the increase in user costs may occur at relatively low values of the distress measure $S(t)$ taken over the entire deck area, since distress is likely to be concentrated in the wheelpaths, exhibiting much higher "local" values of $S(t)$. In effect, therefore, the incremental user costs act as an economic justification for bridge deck treatments. Viewed another way, the dramatically higher user costs attributable to worsening deck condition are the penalties that drivers incur for deferred repair of a facility.

The optimal timing of the overlay treatment is obtained from the point of minimum total costs in Figure 4, and in this case is 24 years. This optimum is based on both agency costs and user costs. If user costs were not considered in this problem—that is, if the decision on overlay timing were predicated solely on agency costs—the optimal timing would have been determined to be later, in Year 27. More important, the region of optimality is more clearly defined when user costs are considered. With just agency costs, the least-cost value occurs in a very flat part of the curve, where the costs of deferral may not be significant enough to justify expenditures against competing bridge needs. In other words, considering agency costs alone does not introduce the full scope of the benefits of bridge treatments, or the full scope of penalties if such treatments are deferred or forgone.

Comparison of Treatments

Analyses such as that described for overlays were performed for all four treatments given in Table 1. The results are summarized in Figure 5, showing only the total cost curves for each treatment. Bear in mind that these results pertain to a relatively new (therefore only slightly distressed) bridge deck that was considered in the example. In this light the results are interpreted as follows:

- The optimal time for patching occurs strictly in Year 5, although the total cost curve is so flat that the activity would be justified virtually at any time required in the first 10 or 15 years. The reason that costs are low, of course, is that the deck is relatively undamaged, so the total costs of the treatment are small, even though the unit costs (per square foot of distressed deck area) are relatively high compared with those of the other treatments.

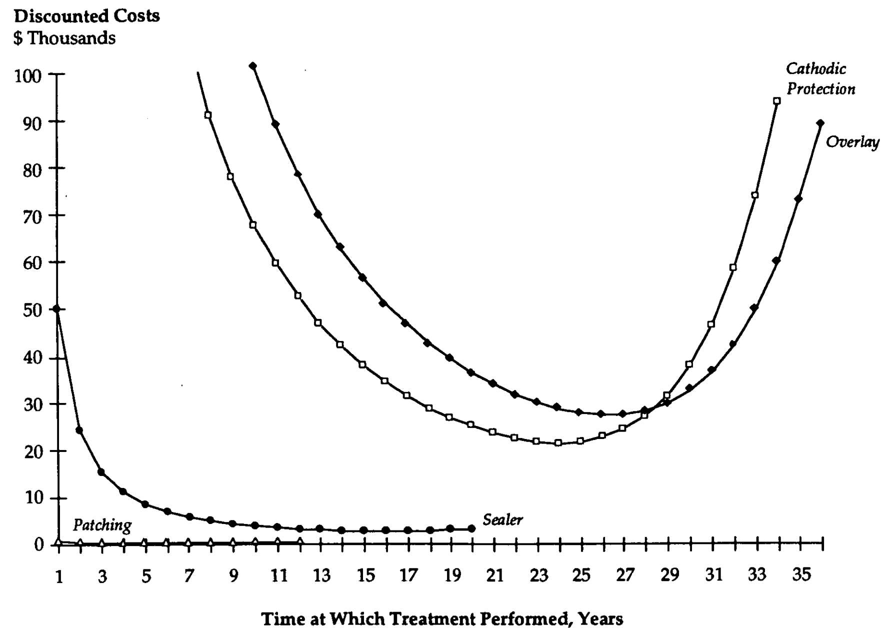

FIGURE 5 Comparison of treatments.

- The optimal time for sealer application occurs strictly in Year 16, although total discounted costs do not vary significantly within 5 years either side of this time. Total discounted costs in this region are somewhat more than for PCC patching, but still relatively low.
- Both PCC overlays and cathodic protection have optimal times much later in a deck's life, and at higher costs than seen for the sealer and patching. For overlays the optimal treatment time is 24 years, as discussed earlier; for cathodic protection it is 26 years. (These treatments may show greater benefits in future versions of the model, as more sophisticated corrosion relationships are included for protection or preventive techniques and better data on service life and degree of improvement are developed through other SHRP projects.)

These results indicate the value of preventive and relatively small-scale corrective activities at this point of the deck's life cycle. Since the current level of distress is small, treatments that can correct this distress or prevent further distress at relatively low cost are preferred.

Effect of Parameter Variation

The parameters identified in Table 1 can be varied to assess their impact on the solution. To give one example, we investigated the effect of an increased traffic volume. The AADT of 25,000 in Table 1 was increased to 55,000 (just below the bridge capacity during the treatment project). Results and comparison with the original case are shown in Figure 6, using the overlay treatment as an example.

The increase in traffic results in an earlier optimal time of treatment (from 24 to 23 years) and a somewhat higher minimum total cost. Both of these effects are expected and consistent with the problem formulation. A higher traffic volume increases the penalties of a deck in bad condition (since more users experience this condition), prompting an earlier treatment. The higher total costs are associated with several effects:

- The earlier performance of the treatment, which means that the agency costs are not discounted as much.
- The greater number of users experiencing a deteriorating deck condition.
- Interactions among users themselves: if the traffic reduces its speed somewhat because of distress in the wheelpaths on the deck, the greater number of users increases the degree of congestion exponentially.

CONCLUSIONS

The example results reported are based on a prototype system intended to demonstrate the basic concepts and the technical and economic relationships embodied in Equations 1 through 11. Additional work is being completed to refine the models and incorporate them within a production version system and a parallel manual that will be available at the conclusion of SHRP. The results of this initial prototype indicate the following:

- The basic economic and technical concepts and relationships in Equations 1 through 11 appear to be reasonable in their behavior and general trends. Detailed enhancements of the models are under way.
- Trial runs with the prototype were conducted on a PC 386. The solution for each treatment was obtained within 1 sec. (with a PC 286 the solution takes about 20 sec.) The execution time appears reasonable for eventual practical application.
- The case study highlights the importance of a consistent and unambiguous understanding of the many concepts and items of information embodied in this analysis, particularly during data collection and reporting. For example:
  - Measures of distress and costs both may be expressed in terms of either overall deck area or distressed area. It must be clear which is meant in each case.

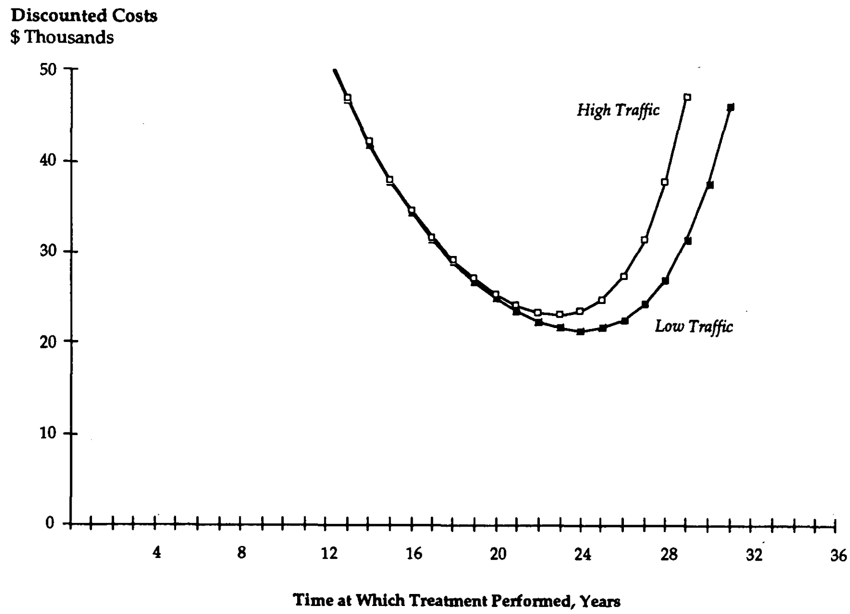

FIGURE 6 Difference of traffic, using overlay as example.

–Similarly, the concept of "service life" needs to be clarified: does it refer to the useful life of the treatment itself, or to the expected life of the deck after the treatment has been applied?

• The required precision of input values may not be a problem:

–The uncertain precision of user costs is typically a concern with engineers. As the results demonstrate, however, the intent of the user cost functions in this model is to reflect the incremental costs of a deck only after it had deteriorated to a condition that affects the motoring public. In other words, user costs define the bounds of acceptable deck condition; before this point, they do not affect the results.

–Although the total cost curves in this example all have a true minimum point that defines the optimal solution, they also exhibit a region in which the total costs do not vary significantly. This region defines a span of, say, 5 to 10 years in which the solution is reasonably close to the optimum (i.e., within 10 percent). Thus, even if some of the input data cannot be specified with exactitude, the solution is robust enough to provide very reasonable results.

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