Integrated Pavement and Bridge Management Optimization

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An integrated pavement and bridge management system that allows cost minimization or benefit maximization is presented. It integrates the pavement and bridge systems so that management may allocate scarce resources optimally across the combined systems. Fuzzy set theory is used in these optimizations to better address the desirability or undesirability of the condition states used to categorize the pavement and bridge segments modeled. Both steady-state and multiyear models are presented.

A highway maintenance management system (HMMS) has been developed. This system integrates a pavement management system (PMS), a bridges and structures management system (B&SMS), and a nonpavement management system. A relational data base (Oracle) is used to perform the needed data storage and retrieval functions. This paper focuses on the integration of the PMS and the B&SMS. The full integration with the nonpavement management system may be found elsewhere (*1–3*).

The HMMS is a flexible modular system that can be easily adapted to meet various needs. The particular adaptation presented here is for a given client, but it can be modified easily for other applications. The integrated system allows the optimal allocation of the budget across the various subsystems (e.g., across the PMS and B&SMS in this paper). Thus, it is not necessary to make an arbitrary division of the budget into the subsystem; instead, an optimal division will be determined by the HMMS.

The PMS and B&SMS steady-state and multiyear results may be optimized using either cost minimization or benefit maximization. The PMS is divided into nine strata based on three levels of climate and three functional classes. The condition state variables for the PMS are rutting (three levels), cracking (three levels), delta cracking—1 year change in cracking (three levels), roughness (three levels), and index to first crack (four levels). These variables result in 324 condition states. There are 17 possible maintenance actions with a feasible subset for each condition state. In the cost-minimization models (*2,3*), management specifies desired performance levels and the optimization finds the lowest-cost plan that will meet the performance goals. In the benefit maximization models, benefits based on fuzzy set memberships and importance weights are maximized subject to budgetary controls.

The B&SMS is divided into 43 strata: 36 for bridges, 6 for culverts, and 1 for tunnels. The 36 bridge strata result from 3 climates, 6 major bridge types, and 2 functional classes. Culverts are not subdivided by type in the optimizations and thus have only three climates and two functional classes, resulting in six strata. The condition state variables depend on the stratum. For example, steel bridges have deck (four levels), superstructure (four levels), substructure (four levels), superstructure age (three levels), and substructure age (three levels), for a total of 576 condition states. For this bridge type there are 40 maintenance scopes (e.g., deck repair) with a selected subset feasible for each condition state. Harper et al. describe this in more detail elsewhere (*4,5*).

The PMS and B&SMS are modular systems with prediction, cost, optimization, packaging, and comparator modules. The prediction modules determine the transition probabilities that estimate the degradation rates for the PMS or B&SMS segments. In the PMS a segment is a 1-km single lane of road. In the B&SMS, the definition of a segment depends on the stratum. For steel bridges it is a superstructure span with a substructure pier or abutment. The survey results are converted to condition states as described by Harper et al. (*4*) and are used in Bayesian updating algorithms to adapt the transition probabilities to the actual environmental conditions encountered. The cost module determines the action/scope optimization costs. This paper focuses on the optimization module. The packaging module takes the selected optimal stratum solutions and makes assignments to the actual segments. The optimization selections are made more specific, and detailed cost estimates are created in different formats to satisfy management needs. The comparator module provides feedback on the system performance and implementation.

FUZZY SET THEORY ADAPTATIONS

In classical set theory, either each "object" (e.g., condition state) is a member of a set or it is not. As fractals have stretched the boundaries of many disciplines to consider noninteger dimensions to supplement the integer dimensions found in classical science, fuzzy set theory expands the concept of the membership of an object in a set to be any value on the continuum [0,1.0] with larger values representing a higher or stronger degree of membership in the set. Classical sets are special cases of fuzzy sets in which the membership is restricted to values of 0 (object is not a member of the set) and 1 (object is a member of the set).

Early versions of the cost minimization models (*5*) categorized each condition state into one of the following mutually exclusive categories:

- Desirable,
- Undesirable, or
- Neither desirable nor undesirable.

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Previously the B&SMS categorized as undesirable any condition state that had at least one element (e.g., for bridges, deck, superstructure, or substructure) in critical condition (good, fair, poor, and critical are the possible levels). Though one would surely agree that a bridge segment with deck, superstructure, and substructure all at the critical level is an undesirable condition state, it is not so clear-cut with another segment when the deck is in critical condition and the other two elements are in good condition. It is apparent that the former segment is more undesirable than the latter, which has only one critical element. The previous performance constraints (*5–7*) do not directly account for such distinctions.

Fuzziness is a natural result of the lack of well-defined boundaries. An example would be the set of "rich" people. The transition between nonmembership and membership for this set is gradual and lacks an obvious boundary. Clearly some individuals are rich and would have a membership in this set equal to 1, but for many others it is not so obvious. Zadeh in 1965 published the initial work in this area (*8*). He set the groundwork for a fertile field that is seeing many applications including consumer products.

Confusion about fuzzy set theory often occurs because fuzzy sets are assumed to be related to probabilistic random variables or some form of uncertainty. Instead, fuzziness is a result of the absence of sharply defined criteria of class membership. The fuzziness ensues from the vagueness or imprecision that results from the inability to classify adequately objects using conventional sets. Thus fuzzy sets are essential to address the true situation properly. Zadeh has argued the following: "Indeed, fuzziness is more than a facet of reality; it is one of its most pervasive characteristics—a characteristic rooted in the bounded capacity of the human mind to process and store information" (*9*).

Categorizing a condition state into one of the three categories was a difficult task. These are not black-and-white situations that are readily apparent. Each condition state within one of these three groupings was treated as having equal weight within that category—that is, each condition state had a membership of 1 in the set it was placed and a membership of 0 in the other two sets.

In the optimization models presented here the condition states need not be treated as a member of only one set. Instead, each condition state has a membership in both the desirable and undesirable fuzzy sets. This membership—$\Phi_{id}(s)$ (desirable), $\Phi_{iu}(s)$ (undesirable)—may take any value on the range [0,1.0], that is, $\Phi_{id}(s)$, $\Phi_{iu}(s) \in [0,1.0]$. An extremely desirable B&SMS condition state with all elements good has $\Phi_{id}(s) = 1.0$ and $\Phi_{iu}(s) = 0$. Similarly, an extremely undesirable B&SMS condition state with all elements critical has $\Phi_{id}(s) = 0.0$ and $\Phi_{iu}(s) = 1.0$. A similar situation holds for the PMS. Many condition states will have nonzero memberships in both the desirable and undesirable fuzzy sets. Additional details on the fuzzy set memberships may be found elsewhere (*1*).

STEADY-STATE BENEFIT MAXIMIZATION

The steady-state models in the PMS and B&SMS are solved in order to set 5-year goals for the multiyear planning models. The model is given in the following. The summations over i cover the entire set of condition states for each stratum. Each s representing a stratum is unique. The PMS/B&SMS steady-state model uses the following variables:

Cost minimization and benefit maximization:

$w_{ia}(s)$ = proportion of units in stratum s that are in condition state i and receive action/scope a. These are the decision variables.

$w^*_{ia}(s)$ = optimal output $w_{ia}(s)$.

$P_{iaj}(s)$ = probability of a segment transitioning in 1 year from condition state i to condition state j when action/scope a is applied in stratum s.

$C_{ia}(s)$ = cost of action/scope a for a segment in stratum s in state i.

$C^*(s)$ = optimal steady-state average segment cost for stratum s:

$$C^*(s) = \sum_{i \in I(s)} \sum_{a \in M_i(s)} w^*_{ia}(s) C_{ia}(s)$$

$N(s)$ = number of segments in stratum s.

$I(s)$ = index set of conditions states i for stratum s.

$M_i(s)$ = set of feasible actions/scopes for condition state i in stratum s.

$p_k(s)$ = performance goal upper- or lower-bound for generalized performance constraint k of stratum s.

$\Phi_{ik}(s)$ = generalized performance constraint parameter for condition state i; may be either fuzzy set memberships, $\Phi_{iu}(s)$ or $\Phi_{id}(s)$, or set to other values depending on the form of the generalized constraint k for stratum s.

$\$_k(s)$ = stratum budget limits; they may be used to bound expenditures (upper or lower bound) in stratum s where $\$_k(s)$ is a specified budget limit.

S_P = index set of PMS strata.

S_{BS} = index set of B&SMS strata.

S_{P+BS} = index set of PMS and B&SMS strata.

B_{BS} = total annual budget for bridges and structures.

B_P = total annual budget for pavement.

B_{P+BS} = total annual budget for pavement, bridges, and structures.

Benefit maximization objective function:

α = Lagrange multiplier used to move budget constraint into objective function; this allows separation of the budget integrated optimization into individual stratum problems. The units of α for benefit maximization are (units of benefit)/(units of cost). It is unitless for multiyear cost minimization. This is an output of the optimization process.

$$\alpha \in [0.0,\infty)$$

$N_n(s)$ = normalized number of segments in stratum s; this is the proportion of segments in stratum s relative to the entire subsystem (either PMS or B&SMS).

$W_d(s)$ = importance weight for being in desirable levels in stratum s.

$W_u(s)$ = importance weight for not being in undesirable levels in stratum s.

$\Phi_{id}(s)$ = desirable fuzzy set membership for condition state i in stratum s.

$\Phi_{iu}(s)$ = undesirable fuzzy set membership for condition state i in stratum s.

$\pi_i(s)$ = net worth of condition state i in stratum s that combines the individual desirable/not in undesirable importance weights, $W_d(s)$ and $W_u(s)$, with $\Phi_{id}(s)$ and $\Phi_{iu}(s)$ as follows:

$$\pi_i(s) = W_d(s)\,\Phi_{id}(s) - W_u(s)\,\Phi_{iu}(s)$$

ϕ_{sys} = relative weight of subsystem:

$$\phi_{sys} = \begin{cases} \phi_{B\&SMS} & \text{for a B\&SMS stratum} \\ \phi_{PMS} & \text{for a PMS stratum} \\ \phi_{NPMS} & \text{for an NPMS stratum} \end{cases}$$

The PMS and B&SMS steady-state models are

For the benefit maximization objective function,

maximize

$$N_n(s)\left[\phi_{sys} \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)\pi_i(s)\right] - \alpha\, N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)C_{ia}(s) \qquad (1)$$

For the cost minimization objective function,

minimize

$$N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)C_{ia}(s) \qquad (2)$$

subject to (same constraints for benefit maximization or cost minimization)

$$w_{ia}(s) \geq 0 \qquad \text{for all } i, a, s \qquad (3)$$

$$\sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s) = 1 \qquad \text{for all } s \qquad (4)$$

$$\sum_{a\in M_j(s)} w_{ja}(s) - \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)P_{iaj}(s) = 0 \qquad \text{for all } j, s \qquad (5)$$

$$\sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)\Phi_{ik}(s)\ (\geq \text{ or } \leq)\ p_k(s) \qquad k = 1, \ldots, K(s) \text{ for all } s \qquad (6)$$

$$N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s)C_{ia}(s)\ (\geq \text{ or } \leq)\ \$_k(s) \qquad k = 1, \ldots, K_{BL}(s) \qquad (7)$$

The benefit maximization objective function (Equation 1) maximizes a weighted sum reflecting benefits. The coefficient of $w_{ia}(s)$ is the product of several factors: normalized number of segments $N_n(s)$, $\Phi_{id}(s)$ and $\Phi_{iu}(s)$ that measure the degree of desirable or undesirable membership, importance weights $W_d(s)$ and $W_u(s)$, and the relative subsystem weight ϕ_{sys}. The weights $W_d(s)$ and $W_u(s)$ indicate the relative importance of the difference between proportions of strata in desirable conditions and the proportion not in undesirable conditions, the difference between functional classes, climatic differences, and bridge type (for bridge strata). For steady-state budget integration Equation 1 is summed over all strata, as shown for B&SMS in the following, to incorporate the budget constraint.

$$\sum_{s\in S_{BS}} N_n(s)\left[\phi_{sys} \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)\pi_i(s)\right] - \alpha \sum_{s\in S_{BS}} N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)C_{ia}(s) \qquad (8)$$

The second (Lagrange) term of the benefit maximization objective function enforces Constraint 9, thus ensuring that the budget (B_{P+BS}, B_P, or B_{BS}) is met. Lagrange relaxation is used since it permits the separation of the problem into an equivalent set of individual stratum models without having to actually specify the budget constraint. Each value of α corresponds to a given total budget level. This is a monotonic decreasing function that decrements at discrete levels of α.

$$\sum_{s\in S_{BS}} N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}(s)C_{ia}(s) \leq (B_{BS}, B_P, \text{ or } B_{P+BS}) \qquad (9)$$

(S_{BS} is replaced by S_P or S_{P+BS} as appropriate.)

The cost minimization objective function (Equation 2) minimizes the cost in stratum s. Constraints 3 and 4 ensure that solutions satisfy probability axioms. The variables $w_{ia}(s)$ are elements of a discrete joint probability distribution. Constraint 3 ensures the nonnegativity (implicit in LP) of each individual element in this joint probability distribution, and Constraint 4 forces the sum over the feasible sample space (in a statistical sense) to equal 1. Constraint 5 includes the steady-state equations for a Markov process (force the proportion of the network in condition state i to remain fixed, i.e., at steady state).

Constraint 6 includes generalized performance constraints for each stratum (optional in benefit maximization but almost always necessary in cost minimization). These performance goal constraints allow considerable flexibility and bestow significant management control. Management may make detailed specific goals of relevance to them using these generalized performance constraints. Potential examples of the generalized performance constraints include constraints using fuzzy set goals or the older designations of desirable/undesirable goals. Another option is to set element goals, for example, percentage of decks wanted in at least fair condition (or similar goals on distresses in PMS).

Equation 7 allows the optional inclusion of an upper or lower budget bound for an individual stratum. This is not normally used—usually the Lagrange term is used instead to control the entire network budget.

IMPORTANCE WEIGHTS

This section briefly covers the importance weights that are fully described elsewhere (*1–3*). These are multiplicative weights that are used to derive the $w_d(s)$ and $w_u(s)$ used in the PMS and B&SMS. They are developed within each subsystem (PMS, B&SMS), and then the weights across the subsystems are incorporated as well as weighing desirable versus undesirable.

Within the B&SMS the strata factors depend on whether the stratum is a bridge, culvert, or tunnel stratum. For bridges the stratum factors are bridge type, climate, and functional class. For culverts, only climate and functional class are necessary. Tunnels have only one stratum and thus do not require any further breakdown.

Selected internal B&SMS ranking weights are given below. The ranking weights used by Highway Maintenance Associates (*2,3*) have to be inverted to show importance.

- Functional class
  - –Primary [2]
  - –Secondary [6]
- Climate
  - –Desert [1]
  - –Mountain [1]
  - –Coastal [.75]
- Bridge type
  - –Concrete slab, simple [6]
  - –Concrete slab, continuous [6]
  - –Concrete girders (or R.C. Box) [6]
  - –Steel composite [8]
  - –Prestressed girder [4]
  - –Prestressed box [4]
- Structure type
  - –Bridge [3]
  - –Tunnel [3]
  - –Culvert [8]

Following is an example calculation of how the preceding ranking weights are converted to importance weights used in the optimization. This example deals only with the climatic aspect.

$$\text{desert} = \text{coastal} = 1/[1 + 1 + 1/0.75] = 0.3$$

$$\text{mountain} = (1/0.75)/[1 + 1 + 1/0.75] = 0.4$$

The PMS strata are based on climate (three levels) and functional class (three levels). The same climate weights used for the B&SMS are also used for the PMS. For functional class, the ranking weights established were primary [2], secondary [4], and feeder [8].

Tables 1 and 2 contain selected intermediate importance weights that result from the previous material. They are incorporated with additional weights (e.g., PMS versus B&SMS, and desirable versus undesirable) when the optimizations are run. The results are used in both the steady-state and multiyear optimizations. The following list gives the six bridge types referred to in Table 1:

TABLE 1 Intermediate Importance Weights, Bridges

Func. Class	Climate	Bridge Type	Weight
Primary	Desert	1	1.31
Primary	Desert	4	.99
Primary	Desert	5	1.97
Primary	Mountain	1	1.75
Primary	Mountain	4	1.31
Primary	Mountain	5	2.63
Secondary	Desert	1	.44
Secondary	Desert	4	.33
Secondary	Desert	5	.66

TABLE 2 Intermediate Importance Weights, PMS

Func. Class	Climate	Weight
Primary	Desert	1.54
Primary	Mountain	2.06
Secondary	Desert	.77
Secondary	Mountain	1.03
Feeder	Desert	.39
Feeder	Mountain	.51

1. Reinforced concrete slab bridge, simple span;
2. Reinforced concrete bridges, continuous span;
3. Prestressed girder (I, T, etc.) bridges (or reinforced concrete box girder bridges);
4. Steel composite bridges;
5. Reinforced concrete T-girder bridges; and
6. Prestressed box girder bridges.

MULTIYEAR PMS/B&SMS OPTIMIZATION MODEL

Multiyear budget integration is a complex problem. This section develops a budget allocation such that the first-year budget is met while at the same time providing "smoothing" of the multiyear stratum budgets over the planning horizon leading to the desired steady-state goals. The first-year budget can be achieved if sufficient relaxation of both the performance goals and budget targets is allowed.

The following variables are used (in addition to those defined beforehand under steady-state) in the PMS/B&SMS multiyear optimization:

r = discount rate for computing net present value in cost minimization objective function.

$M_i^1(s)$ = index set of feasible PMS maintenance actions a for pavement in condition state i in stratum s that fix medium raveling, poor friction coefficient, or both.

$M_i^2(s)$ = index set of feasible PMS maintenance actions a for pavement in condition state i in stratum s that fix high raveling.

$\hat{w}_{ia}^1(s)$ = lower bound on the proportion of segments in stratum s that is in condition state i and should receive mandatory maintenance action/scope a in Year 1.

$q_i(s)$ = proportion of segments in stratum s in condition state i at beginning of Year 1.

$q_i^1(s)$ = proportion of pavement that is in PMS stratum s in condition state i at the beginning of Year 1 and that has either medium raveling or poor friction coefficient requiring action in set $M_i^1(s)$.

$q_i^2(s)$ = proportion of pavement that is in PMS stratum s in condition state i at beginning of Year 1 and that has high raveling requiring action in set $M_i^2(s)$.

$w_{ia}^*(s)$ = optimal proportions from the steady-state model for stratum s.
$C^*(s)$ = optimal average segment steady-state cost for stratum s.
$g(s)$ = sixth-year tolerance on steady-state optimal $w_{ia}^*(s)$ for stratum s.
$h(s)$ = sixth-year tolerance on steady-state optimal average segment cost $C^*(s)$ for stratum s.
$n_L^{t,t+1}(s)$ = parameter setting lower bound in budget balancing constraints for stratum s between years t and $t+1$.
$n_U^{t,t+1}(s)$ = parameter setting upper bound in budget balancing constraints for stratum s between years t and $t+1$.
$i_{\text{core}}(s)$ = B&SMS core condition state with same element condition levels as condition state i but does not include element-age parameters.
$I_{\text{core}}(s)$ = set of all core condition states for B&SMS stratum s (maximum of 64 bridges with a separate deck).
$i_{\text{EA}}(s)$ = set of B&SMS full condition states with core condition state $i_{\text{core}}(s)$, with all possible element ages for stratum s; there is a maximum of nine condition states in each set.
$w_{ia}^t(s)$ = proportion of segments in stratum s that is in condition state i and should receive maintenance action/scope a in year t; these are the output decision variables.
$E^t(s)$ = expected expenditures in year t in stratum s; this equals $N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s)\, C_{ia}(s)$.
$K_{\text{GP}}(s)$ = number of user-defined generalized performance constraints in stratum s.
$K_{\text{BF}}(s)$ = number of user-defined budget fluctuation constraints in stratum s.

The PMS/B&SMS optimization model is as follows. The constraints are shown only for an individual stratum s; however, they apply to all strata. This model divides into separable problems using the Lagrange multiplier α. Each problem is an individual stratum linear program. They are tied together externally through the Lagrange multiplier.

Parametric programming on the Lagrange multiplier α allows efficient solution of this problem. It takes only a fraction of the individual stratum solution time to get all solutions over the desired α range with parametric programming once the α = 0 solution is found. This controls the total network budget ensuring that the optimal allocation across all strata meets the desired budget. Using this approach a series of optimal solutions versus budget is created. Management can easily see the advantages of different budget levels.

The PMS/B&SMS multiyear models are as follows:

For the benefit maximization objective function,

maximize

$$\phi_{\text{sys}} \sum_{t=1}^{T} N_n(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s)\pi_i(s)$$

$$- \alpha N(s) \sum_{t=t_L}^{t_u} \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s) C_{ia}(s)/(t_U - t_L + 1) \quad (10)$$

For the cost minimization objective function,

minimize

$$\sum_{t=1}^{T-1} N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} (1+r)^{1-t} w_{ia}^t(s) C_{ia}(s)$$

$$+ \alpha N(s) \sum_{t=t_L}^{t_U} \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s) C_{ia}(s)/(t_U - t_L + 1) \quad (11)$$

subject to (same constraints for cost minimization and benefit maximization)

$$w_{ia}^1(s) \ge \hat{w}_{ia}^1(s) \quad \text{for all } i \text{ in } I(s) \text{ and } a \text{ in } M_i(s), \text{ for Year 1 with mandatory projects in stratum } s \quad (12)$$

$$w_{ia}^t(s) \ge 0 \quad \text{for all } i \text{ in } I(s),\ a \text{ in } M_i(s), \text{ and } 1 \le t \le T \quad (13)$$

{Implicit in LP}

$$\sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s) = 1 \qquad 1 \le t \le T \quad (14)$$

$$\sum_{a\in M_i(s)} w_{ia}^1(s) = q_i(s) \qquad \text{for all } i \text{ in } I(s) \quad (15)$$

$$\sum_{a\in M_i^1(s)} w_{ia}^1(s) \ge q_i^1(s) + q_i^2(s)$$

$$\text{for all } i \text{ in } I(s) \text{ (PMS only)} \quad (16)$$

$$\sum_{a\in M_i^2(s)} w_{ia}^1(s) \ge q_i^2(s) \qquad \text{for all } i \text{ in } I(s) \text{ (PMS only)} \quad (17)$$

$$\sum_{a\in M_j(s)} w_{ja}^t(s) - \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^{t-1}(s) P_{iaj}(s) = 0$$

$$\text{for all } j \text{ in } I(s) \text{ and } 2 \le t \le T \quad (18)$$

$$\sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s)\Phi_{ik}(s)\ (\ge \text{ or } \le)\ p_k(s)$$

$$k = 1, \ldots, K_{\text{GP}}(s) \quad (19)$$

$$E^t(s) - \sum_{t=t_L}^{t_U} [E^t(s)/(t_U - t_L + 1)][1 + \delta_k^t(s)]\ (\le \text{ or } \ge)\ 0$$

$$k = 1, \ldots, K_{\text{BF}}(s) \quad (20)$$

$$N(s) \sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^t(s) C_{ia}(s)\ (\ge \text{ or } \le)\ \$_k(s)$$

$$k = 1, \ldots, K_{\text{BL}}(s) \quad (21)$$

$$\sum_{i\in I(s)} \sum_{a\in M_i(s)} w_{ia}^T(s) C_{ia}(s) \le [1 + h(s)]C^*(s) \quad (22)$$

$$\sum_{a\in M_i(s)} w_{ia}^T(s) \ge \sum_{a\in M_i(s)} [1 - g(s)] w_{ia}^*(s)$$

$$\text{for all } i \text{ in } I(s) \quad (23)$$

$$\sum_{a \in M_i(s)} w_{ia}^T(s) \leq \sum_{a \in M_i(s)} [1 + g(s)] w_{ia}^*(s)$$

$$\text{for all } i \text{ in } I(s) \quad (24)$$

$$\sum_{i \in [i_{EA}(s)]} \sum_{a \in M_i(s)} w_{ia}^T(s) \geq \sum_{i \in [i_{EA}(s)]} \sum_{a \in M_i(s)} [1 - g(s)] w_{ia}^*(s)$$

$$\text{for all } i_{EA}(s) \text{ in } I(s) \quad (25)$$

$$\sum_{i \in [i_{EA}(s)]} \sum_{a \in M_i(s)} w_{ia}^T(s) \leq \sum_{i \in [i_{EA}(s)]} \sum_{a \in M_i(s)} [1 + g(s)] w_{ia}^*(s)$$

$$\text{for all } i_{EA}(s) \text{ in } I(s) \quad (26)$$

The first term of the benefit maximization objective function in Equation 10 maximizes benefits, and the second Lagrange term enforces the first-year budget constraint (though it can be used to control the average budget expenditures also). The first term of the cost minimization objective function (Equation 11) minimizes the average present value cost per segment of maintenance over the time horizon of interest, and the Lagrange second term serves the same purpose as in the benefit maximization objective function.

Constraint 12 handles the mandatory projects. Equation 13 (implicit in linear programming) ensures that the decision variables are nonnegative. Equation 14 forces the sum of the proportions in each year to equal 1, and Equation 15 ensures that the first-year boundary conditions are satisfied. PMS Equations 16 and 17 account for the necessary action upgrades to handle friction and raveling problems. Equation 18 is the probabilistic mass balance (ensures the proper transfer from one year to the next) equation from one year to the next. Equation 19 is the generalized performance constraints that allow considerable flexibility in goal setting by the decision makers. Equation 20 bounds the variability allowed from year to year in the optimal budget. Equation 21 allows the optional inclusion of stratum budget bounds on any given year: these may be upper or lower bounds. Equation 22 enforces the steady-state budget constraint. Equations 23 and 24 are the PMS-only steady-state performance constraints, and Equations 25 and 26 are the same for the B&SMS.

FURTHER DISCUSSION

The HMMS allows both cost minimization and benefit maximization. In cost minimization there is no need for the many parameters introduced that in essence weight some aspect of pavement versus bridges. The coefficients used in the benefit maximization model presented here represent the specific values of one realization of this system: the Kingdom of Saudi Arabia. These values represent the combined interactive efforts of a multinational task force overseen by the World Bank and its consultants. Although such values are not always easy to obtain and agree upon, they do represent rationale tradeoffs for estimating the significance of pavement versus bridges.

In the cost minimization mode, one can minimize cost with or without user cost (*10*). Thus the HMMS allows the minimization of agency cost or user cost in addition to the maximization of benefits as defined in this paper.

Each agency should evaluate its own set of parameters so that the weights are reflective of its values. The sensitivity of the results relative to the parameter values may be readily tested since the key parameters are used in the objective function. As an example, efficient parametric programming may easily determine the impact of changes in ϕ_{sys}.

The benefit maximization run shown in the next section was done for Kingdom of Saudi Arabia. Many additional runs may be found (*1–3*). The run shown in the next section had to meet specified performance goals. Subject to meeting those goals it is clear that the benefit maximization wanted to allocate proportionally more additional funds to the bridge system when more money was available. In this example this is primarily due to the bridges' being weighted more heavily. It can be shown theoretically that the benefit maximization first-year results asymptote as the Lagrange multiplier increases to a cost minimization (with the same performance goals). Thus the higher weighting of bridges versus pavement tends to shift supplemental funding (above the minimum needed to achieve the performance goals) to bridges in this case.

Traffic is introduced into the optimization in two ways. First, the functional class acts as a surrogate for traffic. Second, the condition prediction models in the pavement system (*2*) directly use traffic in distress estimation that results in the transition probabilities.

In the bridge system the secondary functional class includes bridges on secondary and feeder roads. Thus, the secondary bridge functional class weight is between the secondary and feeder functional class weights for pavement.

EXAMPLE RUN

Figure 1 graphs the total PMS and B&SMS network (all strata) budget as a function of the Lagrange multiplier α. The budget is a monotonically decreasing function of the Lagrange multiplier. As the budget is reduced the optimal mix across all bridge and pavement strata is determined. This ensures that the best use is made of the scarce resources available.

In this example the total budget decreases 71 percent over the range of the Lagrange multiplier shown. Most of this comes from a corresponding 76 percent reduction in the B&SMS budget, whereas the PMS budget was reduced only 35 percent. These runs are based on multiyear benefit maximization. In all cases shown the performance goals specified for each stratum were met; however, since this was a benefit maximization run, it attempted to achieve the most benefit possible. For benefit maximization when the Lagrange multiplier $\alpha = 0$, this corresponds to an unconstrained cost situation. So it is not surprising that the budget can be significantly reduced and still meet the performance goals. There is no significant drop in the total budget for values of the Lagrange multiplier larger than shown in Figure 1.

SUMMARY

Optimization models have been presented for steady-state and multiyear pavement and bridge management systems. These optimization models integrate the pavement and bridge management systems so that management can optimally allocate resources across the combined system. The use of importance weights and fuzzy set memberships was discussed.

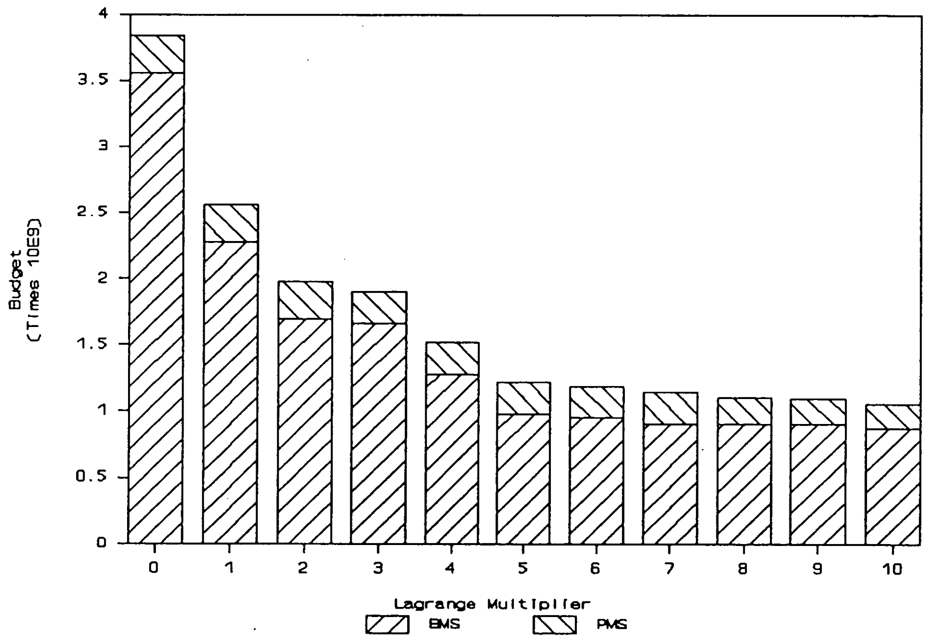

FIGURE 1 Total budget as a function of Lagrange multiplier.

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