Effects of Location on Congested-Regime Flow-Concentration Relationships for Freeways

PAUL HSU AND JAMES H. BANKS

Two questions related to the effect of location on congested-regime flow-occupancy data and relationships are investigated. A theory accounting for the effect of traffic entering or exiting the freeway between the point of observation and the bottleneck is presented and verified in general terms. According to this theory, maximum flows will decrease in the upstream direction when flow entering exceeds flow exiting and vice versa. The effect of lane drops that occur in the section occupied by the queue is also investigated. It is found that when locations upstream and downstream of the lane drop are compared, the approximate slopes of the congested regime of the flow-occupancy relationship are roughly proportional to the reciprocal of the number of lanes. This implies that average densities (and occupancies) remain approximately unchanged across the lane drop. As this issue was investigated only for the case of drops from five to four lanes, the conclusion must be considered to be tentative.

Recent research has led to a revised understanding of freeway speed-flow and flow-concentration relationships. In a synthesis of this work, Hall et al. (*1*) propose three basic regimes: normal uncongested flow, congested flow, and flow that is accelerating in or just downstream from bottleneck sections (see Figure 1). This interpretation of the data rests on the realization that the entire speed-flow or flow-concentration (flow-density or flow-occupancy) relationship cannot be observed at any one point; instead, the overall relationship must be synthesized from data taken from several different locations, and careful attention must be paid to the way the location affects the interpretation of the data. General explanations of the effect of location on the nature of the data date to Edie and Foote (*2*) and May et al. (*3*), and these efforts have more recently been summarized by May (*4*, p.288).

Two questions related to the effect of location on congested-regime flow-occupancy data and relationships are examined in this paper. Flow-occupancy relationships were investigated primarily because the raw data, which were produced by the San Diego ramp metering system, included flow and occupancy but not speed. The flow-concentration relationship also was chosen because it lends itself readily to theoretical interpretations based on driver behavior. Note that the relationship between density and occupancy is linear except in cases in which vehicle lengths and speeds are correlated; as in the work by Hall et al., the two measures will be considered to be interchangeable. The congested regime was studied because it is perhaps the least understood of the three proposed by Hall and because data interpretation for this regime appeared to involve unresolved (or partially resolved) location-related issues. The questions addressed here are (a) the effect of traffic entering or exiting the freeway between the point of observation and the bottleneck, and (b) the effect of lane drops that occur in the section occupied by the queue.

P. Hsu, California Department of Transportation, 2829 Juan Street, San Diego, Calif. 92186-5406. J. H. Banks, Civil Engineering Department, San Diego State University, San Diego, Calif. 92182-0189.

The first of these questions has been raised in the past, but not clearly. Both Hall et al. (*5*) and Banks (*6*) recognize that apparent discontinuities in the flow-occupancy relationship upstream of bottlenecks may result from the action of the queue in limiting maximum flow at such locations. Also, Banks hypothesized that speed-flow and speed-concentration relations would appear to be truncated in flow upstream of the bottleneck; presumably, this truncation results from traffic entering between the bottleneck and the point of observation, but this is not stated explicitly. More recently, Hall et al. (*1*) propose a three-dimensional diagram of flow and occupancy versus distance in which flows decrease and occupancies increase just upstream of an onramp that adds traffic to a queue; however, they do not elaborate on this point. One objective of this study was to further develop and clarify theory related to the effect of traffic entering or exiting a queue between the bottleneck and the point of observation and to further verify that the truncation does occur.

The second question does not appear to have been raised in the past and was considered here primarily because the study sites happened to contain lane drops in the sections upstream of the bottlenecks. At first glance, there appears to be no reason that the flow-occupancy relationship should be affected by the number of lanes occupied by a queue except that as total flow is limited by the capacity of the bottleneck, flow per lane in the queue upstream should be inversely proportional to the number of lanes. It was found, however, that such lane drops also resulted in shifts in occupancy, at least at the sites studied here.

THEORETICAL BACKGROUND

A basic assumption in the discussion that follows is that there *is* a flow-occupancy relationship in the congested regime—that is, for every level of occupancy there corresponds a particular level of flow. This assumption has not been universally accepted. Congested-flow data tend to display a great deal of scatter, so the relationship can be said to exist only between data averaged over fairly long periods of time. Moreover, not

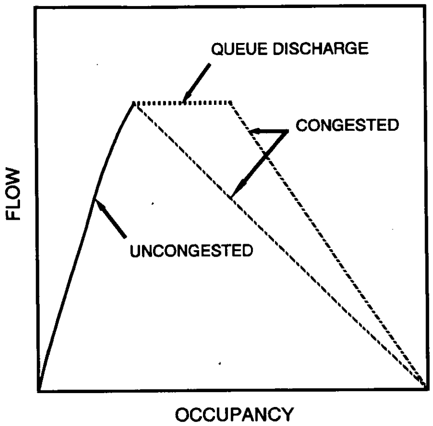

FIGURE 1 Generalized flow-occupancy relationships.

all the variation in the relationship is strictly random, because there appear to be comparatively definite patterns of wave action in congested flow. Congested regime data tend to cluster in certain regions of the flow-occupancy diagram, and some trend can be identified. That being the case, it can at least be said that there should be some sort of distribution of flow for each level of occupancy and vice versa.

There is also no universal agreement on the nature of the flow-concentration trend in congested flow data. Most traditional representations show it as concave to the origin, but Koshi et al. (*7*) show it as convex and Banks (*6*) suggests that it is linear. One difficulty in identifying the shape of the trend line is that data for very high occupancies and very low flows are hard to obtain, since these conditions tend not to occur on the freeways that have been studied recently.

The exact shape of the relationship is not very important to the issues considered here, but the behavioral interpretation of certain features of the relationship is. Banks (*6*) has pointed out that a linear congested-regime flow-concentration relationship, in which the trend line passes through zero flow and 100 percent occupancy (or jam density), implies that the average time gaps between vehicles do not vary with flow. This, in turn, implies that distance gaps vary linearly with speed. This can be extended to state that any straight line on the flow-occupancy diagram passing through zero flow and 100 percent occupancy corresponds to a constant time gap, and that as the line rotates in a clockwise direction, the average time gap decreases. The relationship between the time gap and the slope of any such line is

$$g = -36/b \tag{1}$$

where g is the average time gap in seconds per vehicle per lane and b is the slope of the line in vehicles per hour per lane per percentage occupancy (*8*).

The definition of the time gap discussed elsewhere (*6*) assumes that drivers assign a buffer of fixed length to the vehicle in front of them to provide a margin of safety so that, in behavioral terms, vehicles have effective lengths greater than their physical lengths. Meanwhile, the detectors used to measure occupancy have nonzero length, and the electrical lengths of vehicle are not identical to their physical lengths, so that, for measurement purposes, there is also an effective vehicle length that is greater than the physical length. The assumption that the trend line passes through zero flow and 100 percent occupancy is based on the assumption (*6*) that these two effective lengths are approximately equal. In fact, they are probably different.

If they are different, constant time gaps (in the behavioral relationship) still correspond to a linear congested-regime flow-concentration relationship, but zero flow occurs at some occupancy other than 100 percent.

Let

H_M = measured occupancy;
H_B = "behavioral" occupancy—that is, the fraction of time a point is occupied by vehicles plus any distance buffers assigned to them by drivers of trailing vehicles;
L_M = average effective vehicle length for purposes of measurement;
L_B = average behavioral effective vehicle length—that is, the physical length of the vehicle plus the buffer;
h = average time headway;
q = flow rate = $1/h$; and
u = space mean speed.

Measured occupancy equals the time that the detector is occupied divided by the time headway. If both the time gap and the effective vehicle length are defined in behavioral terms

$$h = L_B/u + g \tag{2}$$

Inverting Equation 2,

$$q = \frac{1}{h} = \frac{u}{L_B + gu} \tag{3}$$

Meanwhile

$$H_M = \frac{L_M/u}{g + L_B/u} = \frac{L_M}{L_B + gu} \tag{4}$$

Solving Equation 3 for u leads to

$$u = \frac{qL_B}{1 - gq} \tag{5}$$

Substituting for u in Equation 4 and simplifying results in

$$H_M = \frac{(1 - gq)L_M}{L_B} \tag{6}$$

Meanwhile, however, the average time gap can be defined as one minus the occupancy (the time not occupied) divided by the flow, or

$$g = \frac{1 - H_B}{q} \tag{7}$$

Solving this for H_B leads to

$$H_B = 1 - gq \tag{8}$$

Substituting for $1 - gq$ in Equation 6 thus leads to

$$H_M = (L_M/L_B)H_B \qquad (9)$$

which implies a linear relationship between the measured and behavioral occupancy. If $L_M < L_B$, zero flow (and thus jam density) corresponds to a measured occupancy less than 100 percent; conversely, if $L_M > L_B$, a measured occupancy of 100 percent is reached at a flow greater than zero. In either case, if the behavioral flow-concentration relationship has a linear trend, so will the measured one. This is of some importance, because it appears from the data considered here that most of the congested-regime flow-occupancy trend lines, if linear, pass through zero flow at measured occupancies of less than 100 percent.

The preceding analysis allows all major features of the congested-regime flow-concentration relationship to be interpreted in average time gaps, although it is important to distinguish among behavioral, measured, and physical gaps. The curvature of the trend line expresses the relationship between the average time gap and flow or speed: concave relationships imply that time gaps increase with increasing speed or flow and convex ones imply that they decrease. The scatter in the data corresponds to the variation in time gaps for different groups of vehicles under similar average flow or occupancy conditions. Any rotation or shifting of the trend line between different locations may be related to increases or decreases in the average time gaps and should occur only where there is a reason for this feature of driver behavior to change.

This analysis leads to the expectation that the underlying flow-occupancy relationship will be relatively stable from one location to another as long as driver behavior is stable. There will still be differences in the appearance of the data, however, because the queueing process will act to limit maximum flows at various points upstream of the bottleneck.

Figure 2 depicts a typical freeway section. For the freeways considered here, data are collected by loop detectors that are normally located just upstream of the on-ramps. In this case, the objective is to determine the relationship between q_k and q_{k-1}, the mainline flows at successive detector stations; let $\Delta q = q_k - q_{k-1}$. Ramp flows are q_{on} and q_{off}; let $q_r = q_{\text{on}} - q_{\text{off}}$. If queues are present and flows are averaged over sufficiently long time intervals, we expect $\Delta q = q_r$, and

$$q_{k-1} = q_k - q_r \qquad (10)$$

Equation 10 establishes the relationship that is to be expected between data taken at adjacent detector locations. At both locations, data are expected to be scattered, but the region occupied by data at the upstream station is expected to be offset from that at the downstream station by a flow equal to the average value of q_r. Note that q_r is positive when traffic entering exceeds traffic exiting and vice versa. The data considered here were taken during the morning peak period, when q_r was positive for almost all sections; consequently, it is to be expected that maximum flows will decrease in the upstream direction. This appears to have also been true for the cases considered in previous literature. Note however, that q_r could be negative and, in that case, maximum flows should increase in the upstream direction. This could happen, for instance, for an afternoon-peak bottleneck created by a lane drop.

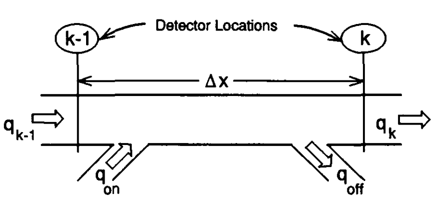

FIGURE 2 Typical freeway section.

DATA

The effects of location on congested-regime flow-occupancy relationships were investigated using morning peak period data from two sections upstream of freeway bottlenecks in San Diego. These sections were Interstate 8 from Jackson Drive to Waring Road and Interstate 805 from Home Avenue to El Cajon Boulevard. Figures 3 and 4 are schematic diagrams of these locations, showing lane configurations and detector locations. Both bottlenecks have been studied extensively in the past *(6,9–11)*. The bottleneck on Interstate 8 is just upstream of the College Avenue on-ramp, so this section actually extends to the second set of detectors downstream of the bottleneck; that on Interstate 805 is on an upgrade between the detectors at University Avenue and those at El Cajon Boulevard. When queues are present just upstream, data at the College Avenue detectors on Interstate 8

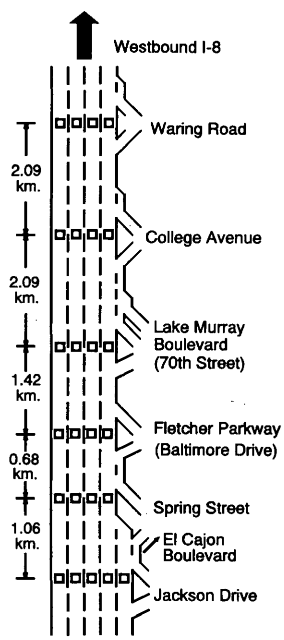

FIGURE 3 Lane configuration and detector location for I-8 (1 mi = 0.62 km).

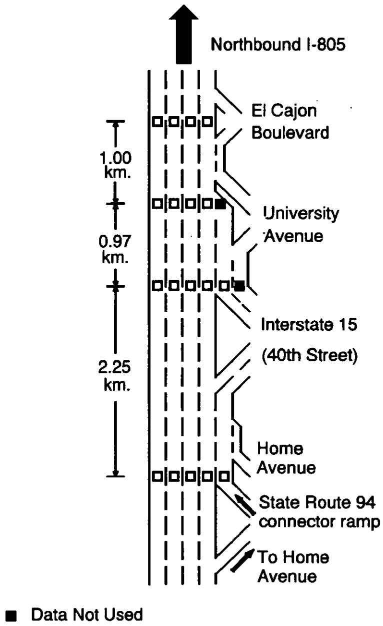

FIGURE 4 Lane configuration and detector location for I-805 (1 mi = 0.62 km).

and the El Cajon Boulevard detectors on Interstate 805 display the characteristics of the bottleneck-acceleration regime identified by Hall (*1*); those further upstream at both sites represent typical congested flow. In addition, some congested flow is encountered at all locations because of incidents downstream.

The data used were produced by the San Diego ramp metering system. Raw data consisted of volume counts and occupancies for each lane, which were recorded every 30 sec. These were aggregated across all lanes and over 6-min intervals, so as to be comparable with those in previous research (*6*). In two cases, there were special problems involved in the aggregation. There are five lanes at the University Avenue detectors on Interstate 805; at the next detector location upstream, just downstream from the branch connector from Interstate 15, there are six. In both cases, the rightmost lane was very lightly used, and data from these lanes were excluded on the grounds that their inclusion would distort volumes and occupancies averaged across all lanes. Unfortunately, use of the rightmost lane at these locations is affected by traffic conditions, with the heaviest use occurring in dense queues. Hence there may be some distortion of the data at these two locations, and they may not be fully comparable with the other locations.

Data were collected for the hours 6:00 to 9:00 a.m. during the months of August 1991 through February 1992. A total of 78 days of usable data were collected for Interstate 805 and 123 days for Interstate 8. Scatter plots of flow versus occupancy were prepared for the data, and data from different detector locations were compared by drawing approximate outer envelopes to the data and by fitting trend lines to the congested-regime data by visual inspection. Outer envelopes were constrained to pass through zero flow and 100 percent occupancy. Two sets of trend lines were prepared: one was constrained to pass through zero flow and 100 percent occupancy and another was unconstrained in its horizontal intercept. A complete description of the data and data analysis process, including scatter plots for all locations, may be found in the literature (*8*).

RESULTS

Figures 5 and 6 show approximate outer data envelopes for each location in the two sections. These tend to verify the theory that maximum flows decrease in the upstream direction in cases in which entering flow exceeds flow exiting. In addition, it can be seen that in some cases the slopes of the data envelopes vary by location.

A more appropriate indicator of the slope of the congested regime is the slope of the trend line. Figures 7 through 10 are sample scatter plots showing both constrained and unconstrained trend lines for four locations upstream of the bottleneck on Interstate 8. As can be seen, the unconstrained trend lines intersect the horizontal axis at occupancies less than 100 percent. Horizontal intercepts for all locations considered (including the ones not covered by Figures 7 through 10) were at occupancies of between 90 and 98 percent, with about 94 percent being most common. This would seem to indicate that

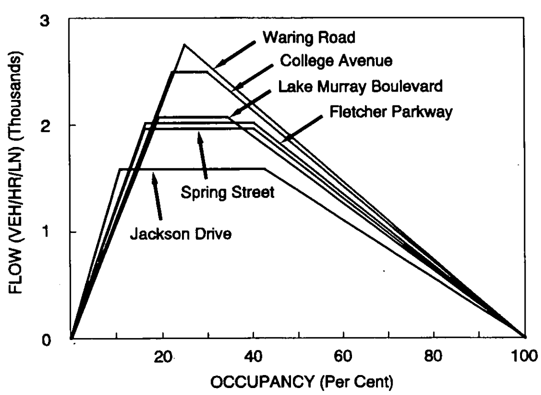

FIGURE 5 Data envelopes for Interstate 8 sites.

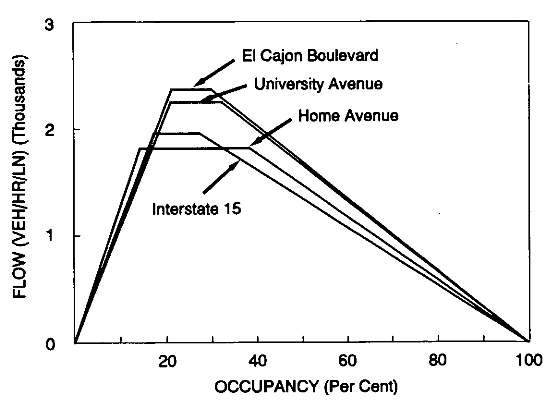

FIGURE 6 Data envelopes for Interstate 805 sites.

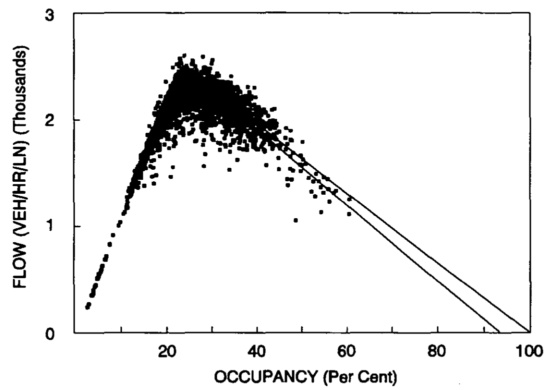

FIGURE 7 Flow-occupancy scatter plot, westbound Interstate 8 at College Avenue.

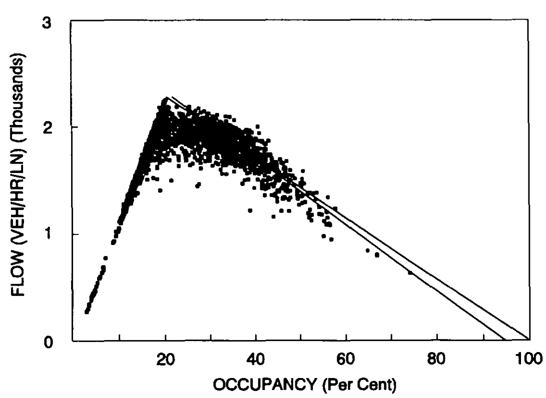

FIGURE 8 Flow-occupancy scatter plot, westbound Interstate 8 at Lake Murray Boulevard.

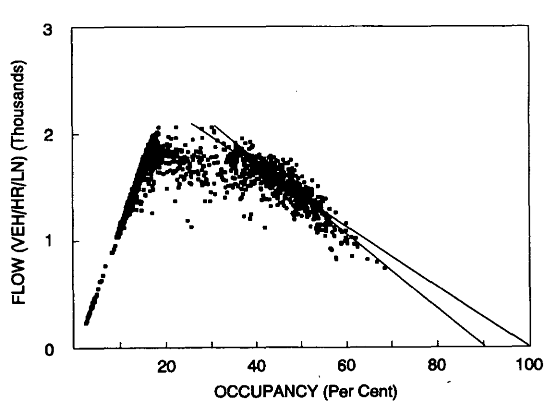

FIGURE 9 Flow-occupancy scatter plot, westbound Interstate 8 at Fletcher Parkway.

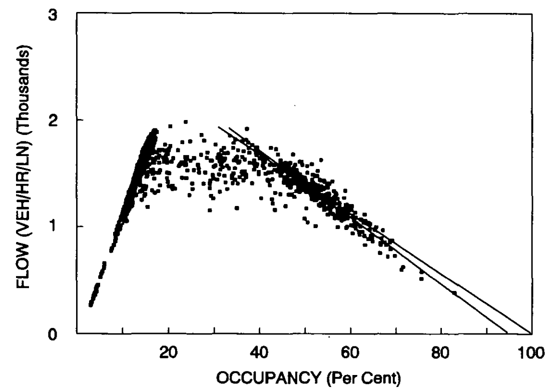

FIGURE 10 Flow-occupancy scatter plot, westbound Interstate 8 at Spring Street.

for the San Diego system, $L_B > L_M$; however, it should be emphasized that this conclusion is based on extrapolations of visually fitted lines, and is at best tentative.

Table 1 gives a summary of the constrained and unconstrained trend-line slopes for the various locations. It can be seen that there are minor variations in slopes among the sites having four lanes in one direction. In particular, constrained slopes at Waring Road and College Avenue on Interstate 8 (which are at or just downstream from the bottleneck) are noticeably steeper than those at 70th Street–Lake Murray Boulevard, Fletcher Parkway, and Spring Street, all of which are definitely upstream of the bottleneck. Constrained slopes for the four-lane sites on Interstate 805 (El Cajon Boulevard and University Avenue)—just upstream and just downstream of the bottleneck, respectively—fall between the two groups on Interstate 8. In all cases, however, slopes for five-lane sites are noticeably flatter than those for four-lane sites.

A rough idea of the relationship between number of lanes and slope of the congested-regime trend line can be gained by comparing the average of the slopes for all four-lane sites in Table 1 with that for all five-lane sites. The average constrained slope for four-lane sites is approximately -29.4, whereas that for five-lane sites is -24.0. For unconstrained slopes, similar averages are -33.5 and -26.7. The ratio of the first two numbers is 0.82 and that of the second, 0.80. In both cases the ratio is approximately 4 to 5—that is, roughly the same as the ratio of the number of lanes downstream to the number of lanes upstream.

This observation provides the basis for an interpretation of the overall effect of the lane drop on flow through the queue. Total flow across all lanes must be unchanged across the lane drop; consequently, as one moves downstream, flow per lane increases in proportion to the number of lanes upstream and downstream.

Let

$$\alpha = N_u/N_d$$

where N_u and N_d are the numbers of lanes upstream and downstream, respectively.

Then

$$q_d = \alpha q_u \qquad (11)$$

TABLE 1 Approximate Slopes for Congested-Regime Flow Concentration Relationships

Freeway	Location	No. of Lanes	Slope: Unconstrained	Slope: Constrained to (0,100%)
I-8	Waring Rd.	4	—	-31.0
	College Ave.	4	-35.5	-31.9
	70th St./Lk. Murray Blvd.	4	-31.7	-27.9
	Fletcher Parkway	4	-35.2	-27.9
	Spring St.	4	-31.7	-27.9
	Jackson Dr.	5	-24.2	-24.0
I-805	El Cajon Blvd.	4	-33.1	-29.0
	University Ave.	4	-33.8	-30.0
	I-15	5	-27.5	-22.9
	Home Ave.	5	-28.4	-25.0

— Insufficient data.

where q_u and q_d are the flows upstream and downstream of the lane drop. Meanwhile, if the trend line is linear and negatively sloped, the equation for q_u is

$$q_u = a - bk_u \tag{12}$$

where k_u is the concentration (density or occupancy) upstream of the lane drop. The rotation of the slope across the lane drop implies that

$$q_d \cong \alpha a - \alpha bk_d \tag{13}$$

Multiplying Equation 12 by α and substituting into Equation 11 yields

$$q_d = \alpha a - \alpha bk_u \tag{14}$$

which implies that $k_u \cong k_d$; in other words, concentrations (either densities or occupancies) are approximately unchanged across the lane drop. This can be verified by superimposing scatter diagrams of data taken just upstream and downstream of the lane drop on one another. Such diagrams are not presented here because the scatter in the data is great enough that data clusters for upstream and downstream locations overlap, making the diagram difficult to interpret.

Note that because $q = uk$, the relationship of speeds upstream and downstream of the lane drop is

$$u_d \cong \alpha u_u \tag{15}$$

Meanwhile, because average time gaps (however defined) are proportional to the reciprocal of the measured trend line, time gaps decrease across the lane drop, so that

$$g_d \cong g_u/\alpha \tag{16}$$

CONCLUSION

Two questions related to the effect of location on congested-regime flow-occupancy data and relationships have been addressed. The first of these was the effect of traffic entering or exiting the freeway between the point of observation and the bottleneck. It has been shown that the clusters of data representing congested flow at adjacent locations should be offset by a flow equal to the average difference between traffic entering and traffic exiting between the two locations. In the case in which flow entering exceeds flow exiting, this means that maximum flows will decrease in the upstream direction; where flow exiting exceeds that entering, the opposite effect is to be expected. This effect, however, is to be expected only over time intervals long enough so the average change in queue density is approximately zero. Data envelopes for several locations on two freeways in San Diego were presented that tend to verify this theory for the case in which flow entering exceeds flow exiting.

The second question concerned the effect of lane drops that occur in the section occupied by the queue. Both freeway sections studied here contained locations upstream of the bottleneck at which the number of lanes decreased from five to four. It was found (somewhat unexpectedly) that the approximate slopes of the congested-regime flow-occupancy relationship were roughly proportional to the reciprocal of the number of lanes at each detector location. This was shown to imply that average densities (and occupancies) are approximately unchanged across the lane drop, that speeds increase in the downstream direction by a factor proportional to the ratio of the number of lanes upstream and downstream, and that time gaps decrease by a factor proportional to the reciprocal of the numbers of lanes.

This result must be considered tentative, as it is based on only three lane drops in two freeway sections. In addition, in the case of Interstate 805, data from little-used lanes were excluded at two key locations, and this exclusion could have distorted the results, although it seems unlikely. If generally true, this finding raises a number of questions.

- First, is the change in the average time gap (and the corresponding rotation of the congested regime slope in the flow-concentration relationship) purely a local phenomenon caused by the lane drop, or are time gaps proportional to the number of lanes, even over extended sections? Two types of evidence could be brought to bear on this question. If the change in time gap is purely local, it should disappear farther upstream. In the case of Interstate 805, one of the sites (Home Avenue) is a considerable distance upstream of the ultimate lane drop, at University Avenue. Note from Figure 3, however, that there is a four-lane section between the Interstate 15 branch connector and Home Avenue, so there are in fact two separate lane drops. Consequently, this type of evidence cannot be brought to bear without extending the study sections. As queues only rarely extend upstream of the locations included in this study, this appears to be impractical. In addition, this question could be addressed by comparing flow-occupancy diagrams for separate freeway sections with different numbers of lanes. Based on the limited amount of comparable data available, it appears unlikely that average time gaps in congested flow are proportional to the number of lanes as a general rule. For instance, the average time gap implied by the data in Figure 2 of Hall and Agyemang-Duah (*12*), for a site with three lanes in one direction, is at least as great as those for the five-lane sites reported here.

• Second, is the change in time gap really proportional to the number of lanes upstream and downstream, or does this apply only to drops from five lanes to four? This question could be addressed by finding lane drops involving different numbers of lanes in sections occupied by queues.

• Third, if the change in the average time gap is purely a result of the lane drop, would the reverse effect occur where lanes are added in a section occupied by a queue?

• Finally, at the microscopic level, what is the behavioral explanation for the change in time gap at the lane drop, and how should it be modeled? Past models of car-following behavior have assumed (or implied) that distance separations are continuous functions of the relative speeds of successive vehicles. Yet here is an example of a situation in which there is apparently a change in the relative speeds of successive vehicles but no change in the distance separation. The only plausible explanation seems to be that drivers increase their distance separations in advance of the lane drop, in anticipation of merging. A microscopic investigation of how and why they do so would be interesting.

ACKNOWLEDGMENTS

This paper is based on data provided by California Department of Transportation. Special thanks are due to Don Day of the Traffic Systems Division of the San Diego District of Caltrans for his assistance in making the data available.

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Publication of this paper sponsored by Committee on Highway Capacity and Quality of Service.