Representation, Processing, and Interpretation of Fuzzy Information in Civil Engineering

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Some of the fundamental issues in the civil engineering application of fuzzy set theory are addressed, and an overview of various types of solution approaches is presented. Emphasis is placed on the Type II approach, in which the solution model is deterministic and the input is fuzzy. Issues addressed include representation and processing of fuzzy information and interpretation of fuzzy output. The new methods developed and presented in this paper include the interpretation of fuzzy output by the α-level distance model and a new approach to performing multiple linear regression of fuzzy data. An example dealing with fuzzy multiple linear regression is presented to illustrate various aspects of the issues addressed.

There is often a need to elicit numerical input from subjective information in the process of solving many transportation engineering problems. Eliciting numerical input from subjective information naturally induces uncertainty, which is usually of an ambiguous rather than a random nature. In this case, the use of fuzzy set theory rather than probability theory for modeling the ambiguous uncertainty is generally recommended (*1,2*).

Fuzzy set theory was developed in 1965 by Zadeh (*3*), a control engineering professor at the University of California at Berkeley. Since then it has been applied to many disciplines, with the most successful applications in control engineering, decision science, and management. In recent years, "the growth of roughly a billion dollar per year industry in Japanese commercial products (such as air conditioners, washing machines, camcorders, and train controllers) based on various ideas in fuzzy logic" has been reported (*4,* p.83).

Civil engineering applications of fuzzy sets were pioneered by Blockley (*5*), Brown (*6,7*), and Yao (*8*), mainly in the area of structural safety. Numerous applications can now be found in many subdisciplines of civil engineering, including transportation engineering (*9–16*).

The principle of fuzzy sets may be summed up as the transformation of ambiguous and fuzzy information into numerical data in a systematic way so that subjective information such as expert opinions, rules of thumb, and other "nonquantifiable" but significant information can be directly utilized in the solution process.

In this paper practical issues regarding the representation, processing, and interpretation of fuzzy information in depth are discussed from a civil engineer's perspective.

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TYPES OF SOLUTION APPROACHES

There are a number of analytic approaches to solving problems in civil engineering, as shown in Table 1. Of the analytic approaches shown, the Type I approach is most commonly used by the engineer. If the input data are of a quantitative nature (i.e., easy to obtain or measure in crisp, precise numerical terms), they are called nonfuzzy data. If the model is based on well-established, unarguably precise knowledge and the process has no randomness present, the model is referred to herein as deterministic. If both conditions are met, the Type I approach is the most appropriate choice.

To use a Type I approach, the engineer must exercise his or her best judgment in the selection of input data. If a nonrandom uncertainty (*2*) exists in the information from which the data are derived, the engineer will be faced with the burden of eliciting the numerical input from ambiguous or fuzzy information. In this case, considerable engineering judgment is needed to use the Type I approach, which significantly relies on a professional's judgment and is often variable and inconsistent. Hence, the process is subject to scrutiny. A more appropriate way to model this problem is by employing the Type II approach, in where a deterministic model is retained but the fuzzy information is systematically represented by fuzzy sets (fuzzy data).

The Type III approach (the probabilistic approach), in contrast to the Type II approach, assumes that the process is well defined but random. When a random event is considered, the framework for incorporating uncertainty can be precisely defined. However, if the event is not random, as in many transportation problems, the burden of eliciting numerical input to the probabilistic model from ambiguous information must rest on the engineer. Historically, many nonrandom events were modeled with the Type III approach in order to handle the uncertainty involved in eliciting the numerical data from available information because the probability theory was thought to be the only way of handling uncertainty, which is not true (*17*).

If the process to be modeled is random and the input information is fuzzy, the Type IV approach may be used. In this case, a probabilistic treatment of the fuzzy event is deemed necessary. If the process or the cause-effect relationship is fuzzy, a Type V or VI approach may be used, depending on whether the input data are fuzzy.

Many transportation engineering problems that can be modeled by deterministic models often have to deal with fuzzy

TABLE 1 Analytic Approaches to Problem Solving in Civil Engineering

Type of Approach	Type of Input Data	Model	Type of Output
I	non-fuzzy number	deterministic	non-fuzzy number
II	fuzzy number	deterministic	fuzzy number
III	non-fuzzy number	probabilistic	probability distribution
IV	fuzzy number	probabilistic	fuzzy probability
V	non-fuzzy number	fuzzy	fuzzy number
VI	fuzzy number	fuzzy	fuzzy number

input data, and thus are suitable for applying the Type II approach. In this paper the focus is on the Type II approach with emphasis on the representation and processing of fuzzy input data and interpretation of output fuzzy sets.

REPRESENTATION OF FUZZY INFORMATION

Ambiguous or fuzzy input is almost always expressed in linguistic terms, since it is easier to do so. In order to process these linguistic terms, they must be transformed into numerical data. Rather than translating a linguistic term into a certain number (and ignoring the associated uncertainty), a fuzzy number (*18*) may be used.

In a Type II approach, the input data for the engineering parameters (or variables) of a deterministic model are fuzzy numbers, which may be translations of linguistic terms that describe the engineering parameters or direct numerical estimates of these parameters. In either case, these fuzzy numbers can be grouped into four classes, as shown in Figure 1. The Class I fuzzy number is used to represent a fuzzy point estimate (FPE) or a linguistic term of "about m." There are two special cases for the Class I fuzzy number. If the value m is an absolute lower bound or upper bound, the fuzzy number exhibits only one-half of the Class I fuzzy number. In such cases, the fuzzy numbers are labeled Class I-L and I-R, respectively, as shown in Figure 2.

The Class II fuzzy number is used to represent a fuzzy interval estimate (FIE) or a linguistic term such as "about from $m - c$ to $m + c$." FIE may be considered an extension of an FPE. The fuzziness of an FIE, as shown in Figure 1b, exists around lower and upper bounds of the interval. The extent of the fuzziness, represented by the values b and d, may be interpreted in the same way as in the case of an FPE. If the value c approaches 0, an FIE would become an FPE.

The Class III fuzzy number is used to represent the notion of "greater than about m." Since real-world engineering parameters almost always have an absolute upper bound, the Class III fuzzy number may be defined as shown in Figure 1c. The fuzziness in this case exists only around the lower bound. The Class IV fuzzy number, on the other hand, is used to represent the notion of "less than about m." As shown in Figure 1d, the fuzziness exists around the upper bound only, since an absolute lower bound (usually zero) is implied. The Class III and IV fuzzy numbers are needed to complement the Class I fuzzy number.

In summary, fuzzy information commonly encountered in transportation engineering can be represented by one of the four classes of fuzzy numbers shown in Figure 1. Although the concept and use of fuzzy numbers have been discussed in the fuzzy set literature (*18*), the four classes of fuzzy numbers are interpreted and presented in this paper in a way suitable for direct use in civil engineering. The use of fuzzy numbers to represent fuzzy information allows for uncertainty to be systematically evaluated and can aid in making better engineering decisions.

Note that in the four classes of fuzzy numbers defined herein, a triangular shape is assumed. Although the triangular shape is commonly used and deemed appropriate for the application presented in this paper, other shapes may be used.

PROCESSING OF FUZZY INFORMATION

As discussed earlier, the Type II approach is deemed suitable for many transportation engineering problems with fuzzy input data. The solution process of this approach is shown in Figure 3. Three methods are available for processing fuzzy data in a Type II model. One method involves processing fuzzy data by defining new mathematical operations. Since fuzzy set theory may be considered an extension of ordinary set theory, extending ordinary arithmetic to fuzzy arithmetic (*18*) is a natural evolution. Zadeh's "extension principle" (*19*) provides a basis for extending conventional arithmetic into fuzzy arithmetic. It is noted that most implementations of the

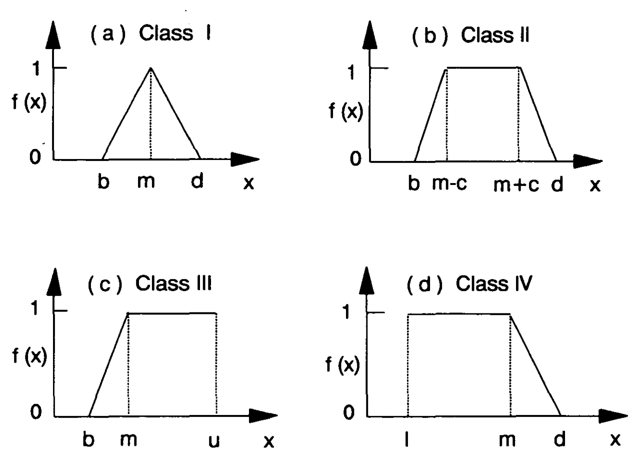

FIGURE 1 Four classes of fuzzy numbers.

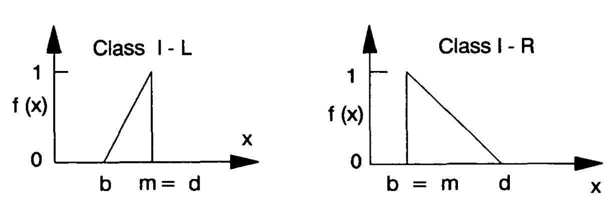

FIGURE 2 Special cases of Class I fuzzy numbers.

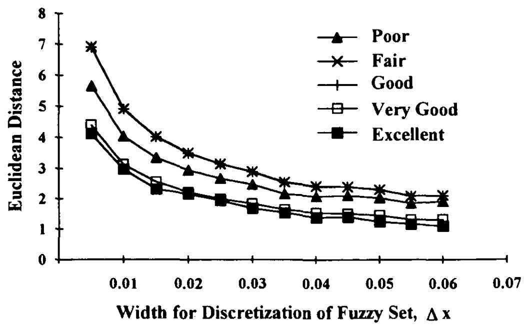

FIGURE 3 Euclidean distances between FCD and standard fuzzy numbers.

extension principle do not ensure uniqueness. However, this may be undesirable in many engineering applications.

Processing fuzzy data by fuzzy arithmetic based on the extension principle is often inefficient (*20–22*). A more efficient, nonfuzzy computational method, called the vertex method (*20*), has been developed.

Although the vertex method has been successfully applied in solving many engineering problems (*10,21,23,24*), there are cases in which a computationally more efficient method is desired. For example, in a recent study on liquefaction susceptibility (*2*), 22 fuzzy variables were involved in a simple deterministic model. Use of the vertex method in this case is very time consuming because of the large amount of interval computations required. In such cases, a technique called the JHE method (*22*) was used for processing fuzzy data. The JHE method is based on the Monte Carlo simulation technique and involves a rigorous treatment of membership functions. Some applications of this method have been reported (*2, 25–28*).

INTERPRETATION OF FUZZY OUTPUT

The output of a Type II approach is a fuzzy set, the output of which presents more information than a single-value output. For example, it gives the lower and upper bounds, the most appropriate value (the mode), and the possibility (membership grade) of each value. In many applications, however, it may be desired to interpret the fuzzy set output. Two common approaches are used: application of a mapping model that measures the fuzzy set (*2*) and translation of the fuzzy set into an appropriate linguistic term.

The translation of the output fuzzy set into an appropriate linguistic term requires three elements: (*a*) a group of standard linguistic terms commonly used to describe the subject matter; (*b*) a group of fuzzy sets, each of which represents one of the standard linguistic terms; and (*c*) a model for determining similarity between the output fuzzy set and each of the standard fuzzy sets. The most appropriate translation is the linguistic term whose fuzzy set is most similar to the output fuzzy set.

The similarity may be measured by the Euclidean distance defined below (*18*):

$$d_j = \sqrt{\sum_x [\mu_A(x) - \mu_j(x)]^2} \tag{1}$$

where

d_j = distance between the output fuzzy set A and the fuzzy set j,
$\mu_A(x)$ = membership grade of x in the fuzzy set A, and
$\mu_j(x)$ = membership grade of x in the fuzzy set j.

Here the Euclidean distance is a measure of similarity between fuzzy sets. Thus, the most appropriate translation is the one with the smallest distance. Although this equation provides a simple measure of the similarity, there are some drawbacks (which will be discussed later). In this paper, a new measure, called α-level (α-cut) distance, is developed. The α-level distance is defined as follows:

$$d_j = \frac{\sum_{\alpha=0}^{1.0} \sqrt{(a_{\alpha,\min} - j_{\alpha,\min})^2 + (a_{\alpha,\max} - j_{\alpha,\max})^2}}{N} \tag{2}$$

where

d_j = α-level distance between the output fuzzy number A and the predefined standard fuzzy number j,
$a_{\alpha,\min}$ = lower bound of the α-cut interval of the fuzzy number A,
$a_{\alpha,\max}$ = upper bound of the α-cut interval of the fuzzy number A,
$j_{\alpha,\min}$ = lower bound of the α-cut interval of the predefined standard fuzzy number j,
$j_{\alpha,\max}$ = upper bound of the α-cut interval of the predefined standard fuzzy number j, and
N = number of α-cut intervals taken.

Note that if the α-cuts are made at an equal spacing of $\Delta\alpha$, then the total number of α-cut intervals will be

$$N = (1/\Delta\alpha) + 1 \tag{3}$$

The α-level distance defined in Equation 2 is a simple average model. Although a weighted average model might be more attractive in theory, Equation 2 is found to be adequate for translating a fuzzy number into the most appropriate linguistic term. Assessment of the above two similarity models is presented in the next section.

HYPOTHETICAL EXAMPLE

In many engineering problems, the basis for deriving a solution is often some rules of thumb provided by experts. For example, the possibility of meeting the Environmental Protection Agency (EPA) requirements for constructing a clay liner to contain hazardous wastes is often assessed with a set of rules of thumb regarding the hydraulic conductivity of the liner. Symbolically, each of these rules of thumb is expressed as follows:

IF X_1 is A_{1j} and X_2 is A_{2j} and X_3 is A_{3j}
THEN Y is B_j.

Here X_1, X_2, and X_3 are linguistic variables representing some factors that are thought to have an important influence on the possibility of meeting the EPA requirements, such as the plasticity index, colloid percentage, and swelling potential of the clay used. The values of the linguistic variables A_{1j}, A_{2j}, A_{3j}, and B_j are descriptions commonly used in the assessment of a clay liner. As a hypothetical example, a rule of thumb may state:

IF the plasticity index is medium, and the colloidal percentage is low, and the swelling potential is high,
THEN the possibility of meeting the EPA liner requirements is very low.

Now, if it is assumed that a group of rules of thumb on this subject is available as listed in Table 2, these rules, as a form of fuzzy information, may be used to establish a predictive equation for assessing the possibility of meeting the EPA liner requirements. To begin with, all possible values of the linguistic variables used in the model need to be translated into fuzzy sets or numbers. Various classes of fuzzy numbers defined earlier can be used to represent the linguistic terms adopted. In this hypothetical example, the actual definitions of the fuzzy numbers used are based on knowledge extracted from the literature. The linguistic terms and the corresponding fuzzy numbers selected are given in Tables 3–6. Using these tables, the fuzzy information (the rules of thumb collected and shown in Table 2) can be translated into some fuzzy numbers, and a set of fuzzy data (Y versus X_1, X_2, and X_3) is thus obtained.

One way to extract knowledge from these fuzzy data is to establish a trend using a regression analysis. In this case, a multiple linear regression can be performed since the dependent variable Y is assumed to be a function of the independent variables X_1, X_2, and X_3. Since all input data are fuzzy numbers and the multiple linear regression is a deterministic process, a Type II approach is appropriate. The fuzzy input data are processed in the framework of a regression analysis and the JHE method (*22*) is readily applicable.

Note that use of the Type II approach and the JHE method for the fuzzy regression analysis is different from reports in the literature. One of the first introductions of fuzzy regression was by Tanaka et al. (*29*). Fuzzy regression analysis, as the name implies, uses the tools of fuzzy set theory to analyze

TABLE 2 Hypothetical Example: Rules of Thumb for Assessing the Possibility of Meeting EPA Clay Liner Requirements

Plasticity index (X_1)	Colloidal Percentage (X_2)	Swelling potential (X_3)	Possibility of Meeting EPA Requirements (Y)
high	high	high	very low
high	high	medium	low
high	high	low	medium
high	medium	high	very low
high	medium	medium	low
high	medium	low	medium
high	low	high	very low
high	low	medium	low
high	low	low	low
medium	high	high	low
medium	high	medium	medium
medium	high	low	very high
medium	medium	high	low
medium	medium	medium	medium
medium	medium	low	very high
medium	low	high	very low
medium	low	medium	low
medium	low	low	medium
low	high	high	low
low	high	medium	medium
low	high	low	high
low	medium	high	low
low	medium	medium	medium
low	medium	low	high
low	low	high	very low
low	low	medium	low
low	low	low	medium

TABLE 3 Linguistic Terms and Their Corresponding Fuzzy Numbers: Plasticity Index

Linguistic Term for Describing Plasticity Index (X_1)	Fuzzy Number Characteristics (see Figure 1)							
	b	m-c	m	m+c	d	l	u	Class
high	25	---[a]	30	---	---	---	50	III
medium	10	15	---	25	30	---	---	II
low	---	---	10	---	15	0	---	IV

[a]Not applicable.

TABLE 4 Linguistic Terms and Their Corresponding Fuzzy Numbers: Colloid Percentage

Linguistic Term for Describing Colloid Percentage (X_2)	Fuzzy Number Characteristics (see Figure 1)							
	b	m-c	m	m+c	d	l	u	Class
high	20	---[a]	25	---	---	---	40	III
medium	5	10	---	20	25	---	---	II
low	---	---	5	---	10	0	---	IV

[a]Not applicable.

TABLE 5 Linguistic Terms and Their Corresponding Fuzzy Numbers: Swelling Potential

Linguistic Term for Describing Swelling Potential (X_3)	Fuzzy Number Characteristics (see Figure 1)							
	b	m-c	m	m+c	d	l	u	Class
high	25	---[a]	30	---	---	---	45	III
medium	5	15	---	25	30	---	---	II
low	---	---	10	---	15	0	---	IV

[a]Not applicable.

fuzzy variables. In contrast to the statistical least-squares criterion, a fuzzy criterion based on a "vagueness" measure for the goodness of the regression was used in the approach of Tanaka et al. Although this approach has been applied to the solution of many engineering problems, some questions remain to be answered. Among them are questions regarding uniqueness of the fitting, selection of the vagueness criteria, and the interpretation of the fuzzy regression. Other fuzzy regression models, including one based on neural networks (*30*), have been reported. The JHE-based approach for fuzzy regression follows conventional regression techniques closely. Comparison of these fuzzy regression methods, however, is beyond the scope of this paper.

Using the data given in Tables 2 through 6, a fuzzy multiple linear regression can be performed using the JHE method (*22,28*). The results of this analysis, including the fuzzy coefficients of the predictive equation (a_0, a_1, a_2, and a_3) and the fuzzy coefficient of determination (FCD), are given in Table 7. The fuzzy number output reflects the uncertainty in the input in this case.

Results of the above fuzzy regression analysis may be interpreted as in conventional multiple linear regression. If the range over which the resulting FCD (a fuzzy number) is defined is very small, the mode (m) of this fuzzy number may be used to represent the FCD. A higher value of the mode, say closer to 1, indicates a better fit. If the FCD is quite fuzzy, an interpreting model is required. One way to interpret the goodness of the fit is to translate the FCD fuzzy number into a linguistic term. A dictionary of linguistic terms for describing the goodness of fit, such as those shown in Table 8, may be defined and used. The translation may be made by measuring the similarity between the resulting FCD fuzzy number and those predefined fuzzy numbers. The concept and formulation defined in Equations 1 and 2 are examined here using the output of this example application shown in Table 7.

Figure 3 shows the Euclidean distances between the resulting FCD fuzzy number (shown in Table 7) and each of the predefined fuzzy numbers shown in Table 8. Since the term "excellent" has the least Euclidean distance, it is the most appropriate translation for the "goodness of fit" represented by the resulting FCD.

Although the Euclidean distance model, such as that in Equation 1, is commonly used in the literature and is able to

TABLE 6 Linguistic Terms and Their Corresponding Fuzzy Numbers: Possibility of Meeting EPA Requirements

Linguistic grade for possibility of meeting EPA requirement (Y)	Fuzzy number characteristics (see Fig. 1)			
	b	m	d	class
very low	0.00	0.00	0.25	I-R
low	0.00	0.25	0.50	I
medium	0.25	0.50	0.75	I
high	0.50	0.75	1.00	I
very high	0.75	1.00	1.00	I-L

TABLE 7 Results of Fuzzy Regression Analysis

Regression Coefficient	Fuzzy Number Characteristics		
	b	m	d
a_0	0.35	0.70	0.91
a_1	-0.0028	-0.0025	-0.0019
a_2	0.01	0.014	0.014
a_3	-0.02	-0.02	-0.016
FCD	0.60	0.90	0.91

The form of the predictive equation is: $Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3$.

TABLE 8 Linguistic Terms and Their Corresponding Fuzzy Numbers: Goodness of Fit

Linguistic term for describing goodness of fit	Fuzzy number characteristics (see Fig. 1)			
	b	m	d	class
poor	0.00	0.00	0.25	I-R
fair	0.00	0.25	0.50	I
good	0.25	0.50	0.75	I
very good	0.50	0.75	1.00	I
excellent	0.75	1.00	1.00	I-L

"pick" the most appropriate translation in this case, a closer look at Figure 3 reveals some drawbacks. First, it may be seen from Figure 3 that the Euclidean distance defined in Equation 1 depends on Δx (a step size or width) selected in the discretization process. The result shows that a smaller Δx yields a larger "calculated distance." Any variation in the calculated distances between the same two fuzzy numbers, caused by use of different Δx, is obviously undesirable.

Second, the distances between the resulting FCD and the fuzzy numbers that represent the terms "good," "fair," and "poor" reveal an inconsistency of the Euclidean distance defined in Equation 1. Here, the distance between the FCD and the fuzzy number representing the term "good" is equal to that between the FCD and the fuzzy number representing the term "fair." In addition, the distance between the FCD and the fuzzy number representing the term "poor" is smaller than that between the FCD and the fuzzy number representing the term "good." Thus, translation models commonly seen in the literature, such as Equation 1, may yield incorrect conclusions.

An improved model for translation of a fuzzy number to a proper linguistic term is presented in this paper (Equation 2). Figure 4 shows the α-level distances between the resulting FCD (Table 7) and each of the predefined fuzzy numbers (Table 8) obtained from this new model (Equation 2). The same conclusion about the most appropriate term for translation is reached from Figure 4. However, it eliminates the two undesirable characteristics observed in Figure 3. As shown in Figure 4, the α-level distance is more or less constant regardless of the $\Delta\alpha$ (step size) used in the discretization. In addition, the distances calculated are consistent with the common intuition.

CONCLUDING REMARKS

An overview of various types of solution approaches for applications of fuzzy set theory in civil engineering is presented. The Type II approach is considered appropriate for solving many transportation engineering problems in which the process (model) is deterministic and the input is fuzzy. Some practical issues in applying the Type II approach, including representation, processing, and interpretation of fuzzy information, have been addressed in depth. The new α-level distance model developed and presented in this paper is shown to be superior to the commonly used Euclidean distance model for interpretation of fuzzy output.

An example dealing with multiple linear regression of fuzzy data is presented to illustrate the concept and method of the Type II approach. This hypothetical example, although abstract in content, has demonstrated the use of the Type II approach.

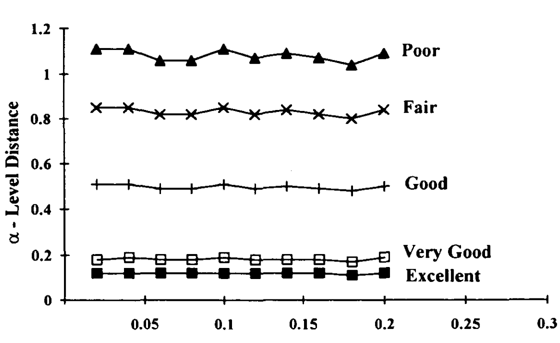

FIGURE 4 α-Level distances between FCD and standard fuzzy numbers.

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