Simplified Direct Calculation of Subgrade Modulus from Nondestructive Pavement Deflection Testing

ANDREW M. JOHNSON AND RONALD L. BAUS

The 1986 *AASHTO Guide for Design of Pavement Structures* proposed that subgrade moduli underlying existing pavement can be determined in a direct, closed form by using peak deflections measured at a distance from the applied load during nondestructive deflection testing. The presence of the pavement layers above the subgrade could lead to significant error in the calculated subgrade modulus when this technique is applied. Subgrade correction factors are developed by calculation of error on the basis of analysis of linear elastic simulations. Least-squares regression analysis is then used to develop an equation for predicting the error. Pavement properties calculated from field data using corrected subgrade modulus are shown to vary less with time when compared with the same properties calculated using uncorrected subgrade moduli. Using field data, the corrected, directly calculated subgrade moduli are shown to compare well to the results obtained from multilayer iterative backcalculation procedures.

The 1986 *AASHTO Guide for Design of Pavement Structures* proposed that subgrade moduli underlying existing pavement can be determined in a direct, closed form by using peak deflections measured at a distance from the applied load during nondestructive deflection testing. If the subgrade modulus is known before backcalculation of pavement modulus, calculating the pavement properties through equivalent modulus, closed-form solutions, or iterative basin-matching techniques is simplified. The modulus calculation technique, which is shown in the 1986 *AASHTO Guide for Design of Pavement Structures* (*1*), is based on the Boussinesq solution of a point load acting on the surface of a linear elastic half-space.

Using a two-layer linear elastic half-space (combined pavement layer over infinite subgrade) with a circular distributed load to simulate deflection testing, theoretical linear elastic deflections may be calculated using a computer program, such as ELSYM5 (*2*). When the calculated theoretical deflections are then used to solve for subgrade modulus using the direct method, significant errors will occur because of the presence of stiffer pavement layers above the subgrade. If the subgrade modulus is in error, then the corresponding backcalculated pavement properties will also be in error. Therefore, a modified direct procedure for calculation of subgrade modulus is proposed.

A. M. Johnson, Research and Materials Laboratory, South Carolina Department of Highways and Public Transportation, P.O. Box 191, Columbia, S.C. 29202. R. L. Baus, Department of Civil Engineering, University of South Carolina, Columbia, S.C. 29208.

CALCULATION OF SUBGRADE MODULUS ON THE BASIS OF POINT LOADING

The Boussinesq equation for off-axis vertical surface deflection resulting from a point load acting on a linear elastic half-space is as follows (*3*):

$$d_z = \frac{P(1 - \mu^2)}{\pi E r} \tag{1}$$

where

d_z = vertical surface deflection at distance r from the applied load,
P = load,
E = half-space modulus of elasticity,
r = distance from load to point of deflection measurement, and
μ = Poisson's ratio.

To solve for E, Equation 1 may be rewritten as follows:

$$E = \frac{P(1 - \mu^2)}{\pi d_z r} \tag{2}$$

A modified form of Equation 2 is given on page III-86 and in Figure III-5.5 of the AASHTO Guide (*1*).

Ullidtz (*4*) refers to the results of Equation 2 as the "surface modulus." The surface modulus is purported to represent the approximate weighted mean modulus of the layered half-space at a given distance away from the test load. For a pavement overlying a linear elastic subgrade, the surface modulus should theoretically reach an asymptotic value representative of the subgrade modulus at the distance from the load at which vertical surface deflections are due entirely to strain in the subgrade layer. In the AASHTO Guide (*1*), the distance at which the surface modulus becomes asymptotic is referred to as the effective radius of subgrade stress (a_e). This relationship is shown in Figure 1(*a*) (*5*). Most nonlinear subgrades have increasing moduli with decreasing levels of vertical stress. Therefore the surface modulus is expected to increase with increasing distance from the load (*4*). This relationship is shown in Figure 1(*b*).

Estimating a_e requires making a number of assumptions about the pavement properties. Because the subgrade modulus should theoretically be represented by the minimum calculated surface modulus, it is proposed to calculate subgrade

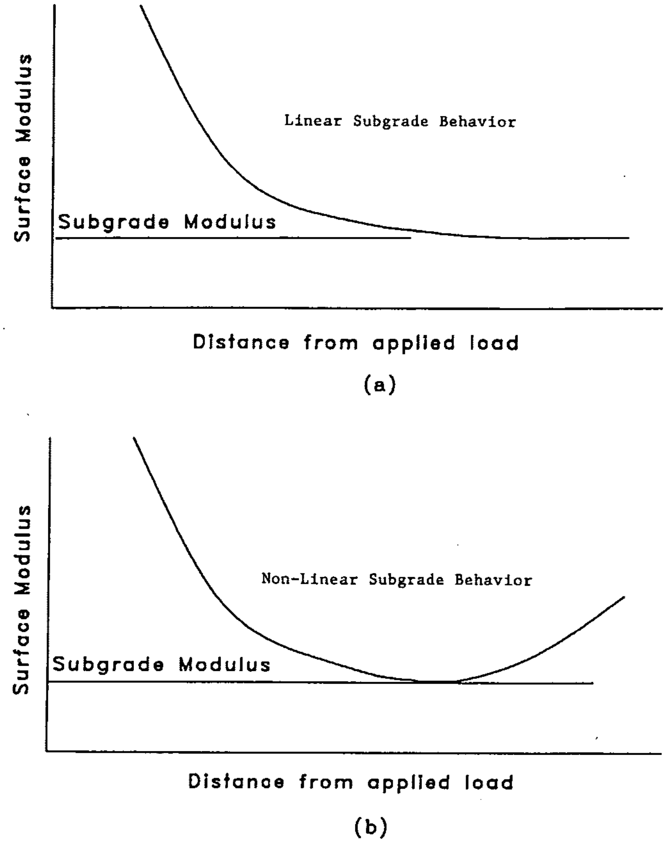

FIGURE 1 Typical expected variation in surface modulus with distance from applied load for pavements with (*a*) linear elastic subgrade and (*b*) nonlinear elastic subgrade.

moduli by calculating surface moduli at all deflection measurement locations. The subgrade modulus is then assumed equal to the minimum measured surface modulus, thus eliminating the need to make assumptions about the pavement and subgrade moduli in order to estimate a_e. However, as will be shown, the subgrade modulus (E_{sg}) may not be represented by the minimum surface modulus, and further correction is necessary.

POINT LOAD APPROXIMATION EFFECTS

The load applied by a falling weight deflectometer (FWD) or another pavement deflection measuring device is typically applied through a circular plate that is in contact with the pavement surface. However, at some distance from the applied load, the difference between the point and the distributed load cases becomes negligible for a homogeneous elastic half-space (*4*). Because it is proposed to choose the subgrade modulus based on the minimum surface modulus, in some cases the point of calculation for the surface modulus may be too close to the applied circular load to use the point load approximation.

No closed-form solution for the analysis of off-axis deflections from a circular distributed load on a linear elastic half-space is available. Ahlvin and Ulery (*6*) present a tabular solution for the calculation of off-axis ($r \neq 0$) vertical deflections for circular loading (*3*). The solution for surface vertical deflection from a circular loaded area is

$$d_z = \frac{pRH(1 - \mu^2)}{E} \tag{3}$$

where

p = pressure,
R = radius of loaded area, and
H = $f(r/R)$ (see Table 1).

To correct for the error induced by the point load approximation, the value of r used in Equation 2 may be transformed to an adjusted radius (r_{adj}). To find r_{adj} for a given load radius (R) and true deflection measurement distance (r), Equation 1 is set equal to Equation 3. The equation for r_{adj} then becomes

$$r_{adj} = \frac{R}{H} \tag{4}$$

where R and H are as defined in Equation 3. Using a load radius of 5.9 in., the actual and adjusted radii used for this study are presented in Table 2. Deflection measurement locations were adjusted to a distance of 4 load radii (23.6 in.) from the center of loading.

Figure 2 is a typical plot of surface modulus versus distance obtained using South Carolina FWD field data. The plot shows the changes in backcalculated moduli when adjusted radii are used and typical nonlinear soil behavior.

THEORETICAL ERROR IN SUBGRADE MODULUS CALCULATION

To test the accuracy of the direct subgrade modulus calculation method using adjusted radii, the pavement/subgrade

TABLE 1 Off-Axis Surface Deflection Factors (*3*)

Load Radius/Distance (r/R)	Deflection Factor (H)
1.0	1.27319
1.2	0.93676
1.5	0.71185
2.0	0.51671
3.0	0.33815
4.0	0.25200
5.0	0.20045
6.0	0.16626
7.0	0.14315
8.0	0.12576
10.0	0.09918
12.0	0.08346
14.0	0.07023

TABLE 2 Actual and Adjusted Geophone Radii Used for Testing Program

Geophone Number	Actual Distance from Load Center (inches)	Adjusted Distance from Load Center for Analysis (inches)
2	7.9	7.095
3	11.8	11.414
4	23.6	23.410
5	35.4	Not Adjusted
6	53.1	Not Adjusted
7	70.9	Not Adjusted

1 inch = 25.4 mm

system was modeled as a two-layer linear elastic half-space. The pavement was assumed to have an average structural layer coefficient from 0.10 to 0.40 structural number (SN)/in. and a structural number from 2 to 9. The subgrade modulus was assumed to range from 5 to 60 ksi. The thickness of the theoretical pavement layer was calculated using the relation

$$h_t = \frac{SN}{a_{avg}} \tag{5}$$

where

h_t = total pavement thickness (inches),
SN = pavement structural number, and
a_{avg} = average structural coefficient (SN/in.).

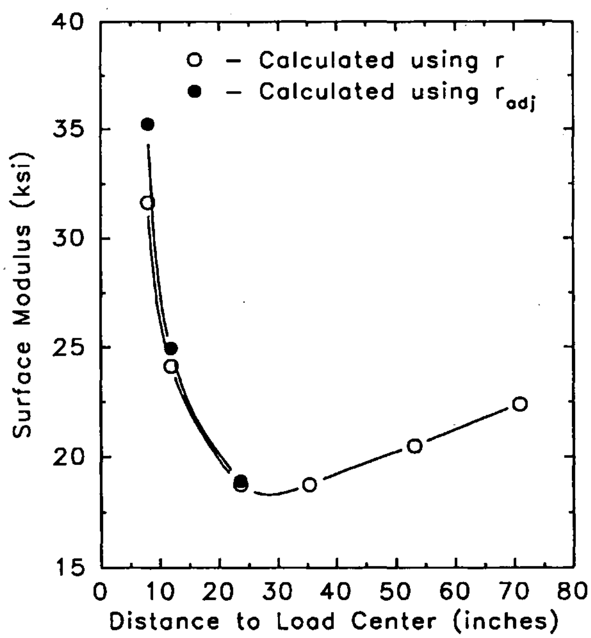

FIGURE 2 Example of variation in measured surface modulus with distance from load center (1 in. = 25.4 mm, 1 ksi = 6.89 MPa).

The value of a_{avg} was then converted to equivalent elastic modulus using the equation

$$E_e = \left(\frac{a_{avg}}{0.0043}\right)^3 (1 - \mu_e^2) \tag{6}$$

where E_e is composite pavement modulus (psi) and μ_e is pavement Poisson's ratio (*1*, Appendix PP).

Both subgrade and pavement Poisson's ratio are assumed to be 0.35. Cases for which h_t was greater than 30 in. were discarded as unrealistically thick.

Using ELSYM5, deflection basins were predicted for the theoretical pavements under a 9,000-lb loading applied to a loading plate with a 5.91-in. radius. The direct method (using minimum surface modulus computed for Equation 2 with the adjusted geophone radii given in Table 2) was then used with the theoretical deflection basins to predict subgrade modulus E_{sg}. The error in calculated E_{sg} is defined as the assumed E_{sg} minus the backcalculated E_{sg}.

Examples of the errors in predicted subgrade modulus for the ELSYM5 deflection basins are shown in Figure 3(*a*). For the E_{sg} = 20 ksi case shown in Figure 3(*b*), the backcalculated subgrade modulus was always lower than the correct value.

The effect of the errors in subgrade modulus on calculated SN are shown in Figure 4(*a*). Values of composite pavement modulus (E_e) were determined by using erroneous calculated values of E_{sg} to match correct values of under plate deflections. These values of E_e were converted to SN using the following equation given in the AASHTO Guide (*1*):

$$SN = 0.0043\left[\frac{E_e}{(1 - \mu^2)}\right]^{1/3} \cdot h_t \tag{7}$$

The error in the calculated structural number is defined as the true (assumed) SN minus the backcalculated SN.

Figures 5(*a*) and 6(*a*) show the error in calculated E_{sg} and SN for all pavement thickness and subgrade modulus combinations. Based on a two-layer elastic analysis, using the direct method of subgrade modulus calculation can result in errors in the predicted structural number of up to ±0.5.

EMPIRICAL CORRECTION OF CALCULATED SUBGRADE MODULI

A careful examination of the error data presented in Figure 5(*a*) indicated that a second-order linear regression provided an excellent fit for error versus pavement thickness, pavement stiffness, and subgrade modulus computed using minimum surface modulus and adjusted radii. Therefore, a quadratic least-squares regression was performed using h_t, the initially calculated structural number, and the initially calculated subgrade modulus to predict the error in the initially calculated subgrade modulus. The following correction equation was developed:

$$\begin{aligned} E_{sgerr} = {} & 313.964 - 482.307SN_1 + 62.40h_t - 0.06219E_{sg1} \\ & + 64.812SN_1^2 - 1.841(h_t \cdot SN_1) - 1.544h_t^2 \\ & - 0.01823(E_{sg1} \cdot SN_1) + 0.003959(E_{sg1} \cdot h_t) \\ & + 0.000000671E_{sg1}^2 \end{aligned} \tag{8}$$

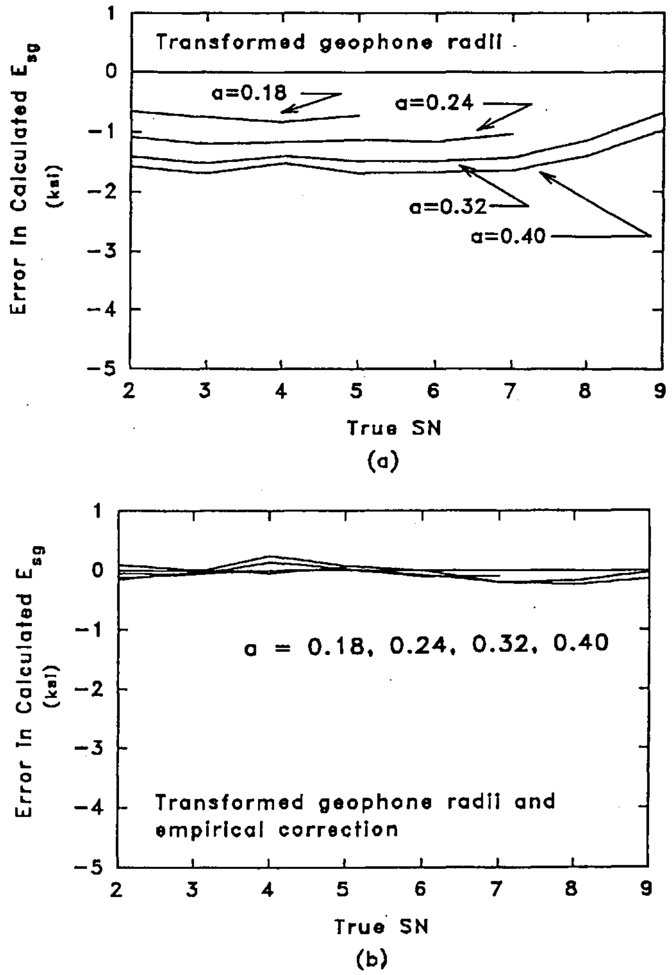

FIGURE 3 Variation of error in calculated subgrade modulus with SN for various average structural layer coefficients, (*a*) without and (*b*) with empirical correction, E_{sg} = 20 ksi (1 ksi = 6.89 MPa).

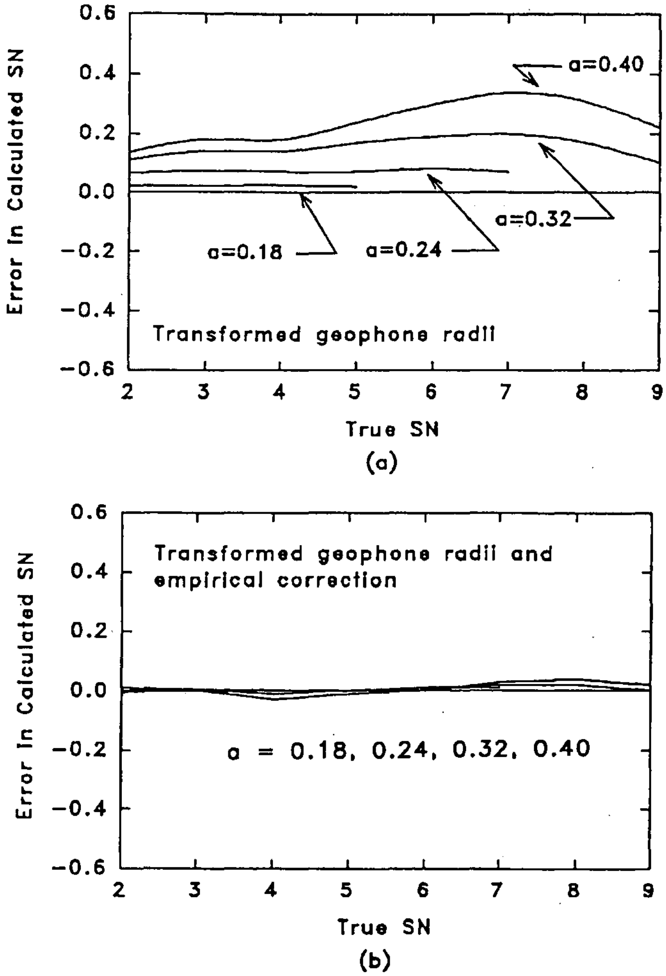

FIGURE 4 Variation in error of calculated structural number with correct structural number for various average structural layer coefficients, (*a*) without and (*b*) with empirical subgrade modulus correction, E_{sg} = 20 ksi (1 ksi = 6.89 MPa).

where

E_{sg1} = initially calculated subgrade modulus (based on minimum surface modulus and adjusted radii) (psi),
SN_1 = structural number calculated using E_{sg1},
h_t = total pavement thickness (inches), and
E_{sgerr} = error in initially calculated subgrade modulus, which is equal to E_{sg1} − correct E_{sg}.

This equation was incorporated into the University of South Carolina's backcalculation program SCSN (*7*), which was then used to recalculate E_{sg} and SN for the theoretical, ELSYM5-derived deflection basins. SCSN uses the minimum surface modulus based on adjusted geophone radii and Equation 8 for subgrade modulus error correction. The recalculated results are shown in Figures 3(*b*), 4(*b*), 5(*b*), and 6(*b*). Within the range of reasonable E_e, E_{sg}, and h_t, the corrected direct calculation technique provides an excellent estimate of lower layer modulus for a two-layer linear elastic system. Transformed geophone radii do not appear to make a large contribution to improving estimates of lower layer modulus because minimum surface modulus is usually computed using deflection measured at a distance greater than 23.6 in. (Geophone 4 location) from the center of the load plate.

COMPARISON OF CORRECTED AND UNCORRECTED DIRECT METHOD SUBGRADE MODULUS ON FIELD TEST DATA

Field deflection measurements were taken bimonthly from January 1989 to June 1990 at 14,500-ft-long test sites throughout South Carolina using a Dynatest 8000 FWD. Two additional sites were tested bimonthly from October 1989 to May 1990. The FWD deflection sensors were positioned as shown in Table 2. The FWD drop height was adjusted to provide nominal loads of 6,000, 9,000, 12,000, and 16,000 lb. Test locations were located at 50-ft stations within the test sections. Details of the pavement structure at each site are shown in Table 3. Interstate 26 in Orangeburg County was rehabilitated throughout 1989, resulting in the eventual relocation of Site 1. Site 12 was overlaid in May 1990. Further details on the deflection testing and FWD test sites are given in work by Baus and Johnson (*7*).

To investigate the improvement gained by performing the suggested subgrade correction method, the temperature corrected SNs were computed based on the nominal 9,000-lb applied load for all stations and dates at each site both with and without subgrade correction. Temperature corrections to

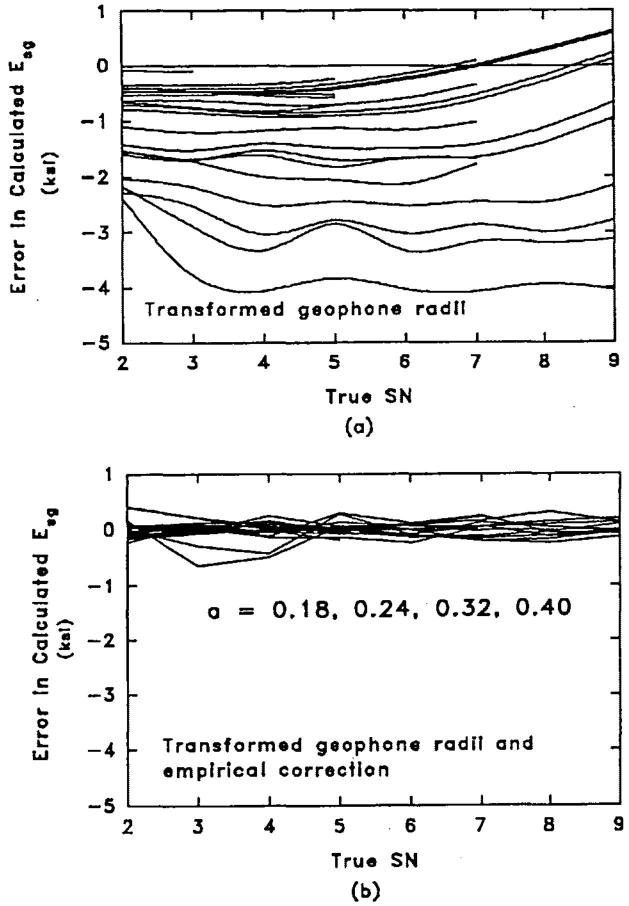

FIGURE 5 Variation of error in calculated subgrade modulus with correct structural number for E_{sg} from 5 to 60 ksi and all a_i, (*a*) without and (*b*) with empirical subgrade modulus correction (1 ksi = 6.89 MPa).

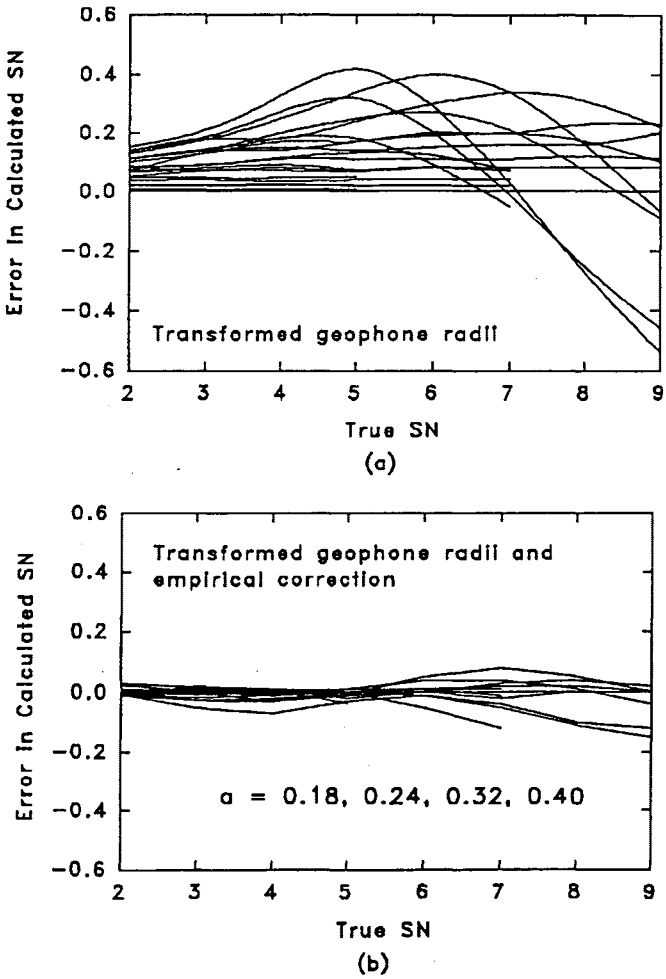

FIGURE 6 Error in calculated structural number versus correct structural number for E_{sg} from 5 to 60 ksi and all a_i, (*a*) without and (*b*) with empirical subgrade modulus correction (1 ksi = 6.89 MPa).

SN were made using the procedure described in work by Johnson and Baus (*8*). On the basis of the results at all stations, the site average temperature corrected SN was calculated for each date and site both with and without subgrade modulus correction. The standard deviations of the site average SN values with and without subgrade modulus correction are plotted against each other in Figure 7. Site 1 was omitted from Figure 7 because of its relocation during testing. Results for Sites 15 and 16 were omitted because of the relatively short time period testing was performed at these sites.

Figure 7 clearly shows that using the proposed subgrade modulus correction technique results in more uniform pavement properties over time. Because the tests represented in Figure 7 were performed over a wide range of temperatures, it is probable that the backcalculated subgrade moduli obtained without correction were affected by the variation of the overlying pavement stiffness with temperature.

COMPARISON OF DIRECTLY CALCULATED SUBGRADE MODULUS WITH BASIN-MATCHED ITERATIVE SUBGRADE MODULUS

To provide comparisons with iterative, basin-matching procedures, the EVERCALC (*9*), MODULUS (*10*), and BOUSDEF (*11*) programs were used to analyze the collected field data to determine the subgrade modulus. EVERCALC analyzed several load levels in its calculations, then normalized the subgrade modulus to a 9,000-lb applied load. For MODULUS and BOUSDEF, as well as SCSN, a single drop at a nominal 9,000-lb load level was used. The results of the comparisons are shown in Figures 8–10. Because of the time involved in performing the iterative, basin-matching procedures, only the deflections from the first station at each site were used for each date.

Generally good agreement between basin-matching and directly calculated subgrade moduli is shown, except for Sites 8–10. Data for these sites are highlighted in Figures 8–10. Pavement structures at Sites 8 and 9 are thick. The pavement structure at Site 10 is thin. In these boundary cases of thickness, the multilayer, basin-matching programs tend to assign a low value of modulus to the unbound base course and high values of modulus to both the asphalt concrete–bound toplayer and subgrade. When Site 8–10 results are taken out of consideration, the corrected subgrade moduli have a correlation coefficient (r^2) of 0.80 to 0.82 with the other backcalculation programs. When compared to each other, the basin-matching programs have r^2 values from 0.90 to 0.95. Where

TABLE 3 FWD Test Site Locations

Site No.	Road and County	Pavement Structures
1	I-26 Calhoun County	9.0 inches AC Bound* 16.0 inches Earth Type Base *Resurfaced during testing period giving 10.1 inches AC Bound
2	I-26 Orangeburg County	11.3 inches AC Bound 14.0 inches Earth Type Base
3	SC-31 Charleston County	3.2 inches AC Bound 11.5 inches Fossiliferous Limestone Base
4	US-17 Charleston County	3.5 inches AC Bound 6.2 inches Fossiliferous Limestone Base
5	US-17 Charleston County	4.9 inches AC Bound 7.4 inches Fossiliferous Limestone Base
6	US-321 Fairfield County	6.2 inches AC Bound 3.5 inches Unbound Granular Base 12.0 inches Cement Stabilized Earth Base
7	SC-9 Chester County	10.8 inches AC Bound 6.0 inches Earth Type Base
8	I-26 Newberry County	9.0 inches AC Bound 16.0 inches Macadam Base
9	I-77 Richland County	18.1 inches AC Bound 6.0 inches Cement Modified Earth Subbase
10	S-1623 Lexington County	1.3 inches AC bound 6.0 inches Macadam Base
11	I-20 Lexington County	12.4 inches AC Bound
12	US-76/378 Sumter County	6.6 inches AC Bound* 12.0 inches Earth Type Base *Resurfaced during testing period giving 8.1 inches AC Bound
13	US-76 Marion County	10.2 inches AC Bound
14	US-76/301 Florence County	7.0 inches AC Bound 4.5 inches Stabilized Earth Base 8.0 inches Earth Type Subbase
15	I-385 Greenville County	6.3 inches AC Bound 8.0 inches Macadam 4.0 inches Cement Modified Subgrade
16	US-176 Union County	4.6 inches AC Bound 8.0 inches Macadam 6.0 inches Cement Modified Subbase

1 inch = 25.4 mm

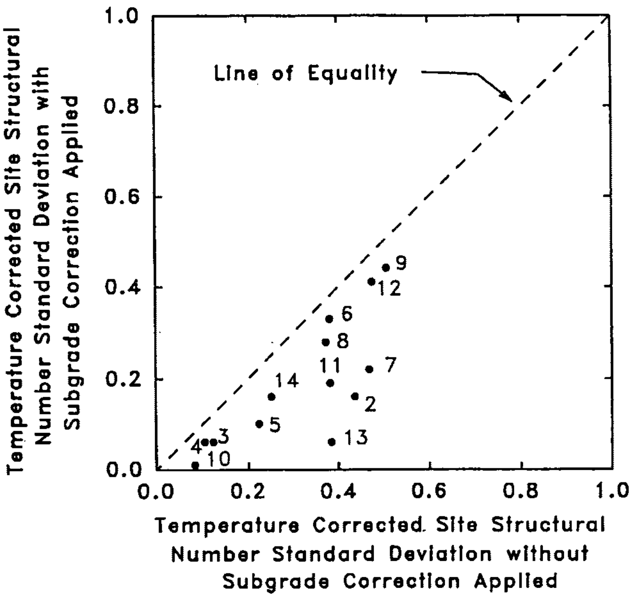

FIGURE 7 Comparison of test site average structural numbers calculated with and without empirical subgrade correction.

disagreement is substantial between SCSN and the basin-matching programs, SCSN almost always predicts a lower subgrade modulus. Extensive comparisons of SCSN and basin-matching program results are presented by Baus and Johnson (*7*).

CONCLUSIONS

Consistent analysis of subgrade stiffness is important in the backcalculation of pavement properties. A small variation in calculated subgrade modulus may lead to substantial variation in backcalculated pavement stiffness. Based on two-layer, elastic layer theory, an empirical correction is described that significantly improves direct (noniterative) computation of subgrade modulus from FWD surface deflections. The direct calculation method with empirical correction can accurately calculate the lower layer modulus of a two-layer linear elastic system. When field data are used to compare subgrade moduli calculated with iterative, multilayer backcalculation programs to corrected directly calculated subgrade moduli, good agreement is shown in the majority of cases. When substantial disagreement is found between the results of field data anal-

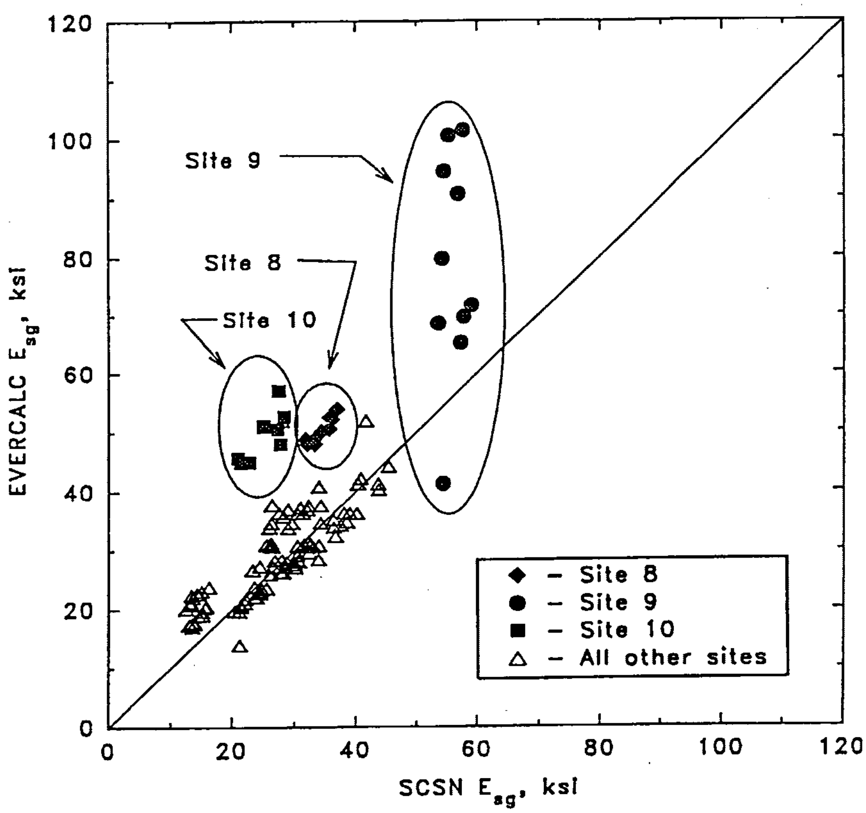

FIGURE 8 Comparison of subgrade moduli calculated directly with empirical correction and using program EVERCALC (1 ksi = 6.89 MPa).

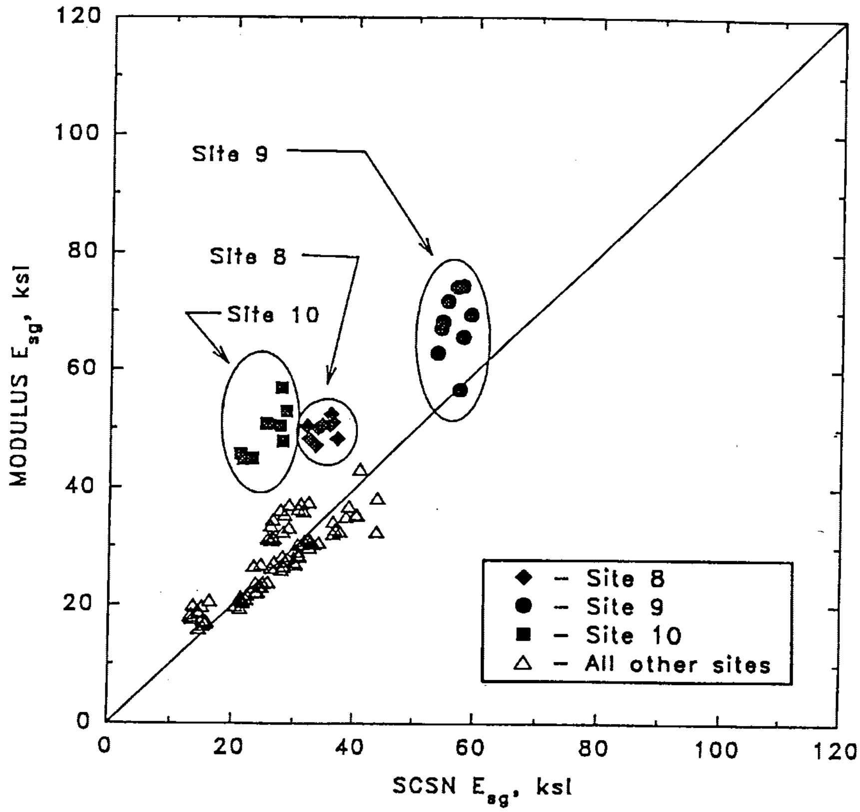

FIGURE 9 Comparison of subgrade moduli calculated directly with empirical correction and using program MODULUS (1 ksi = 6.89 MPa).

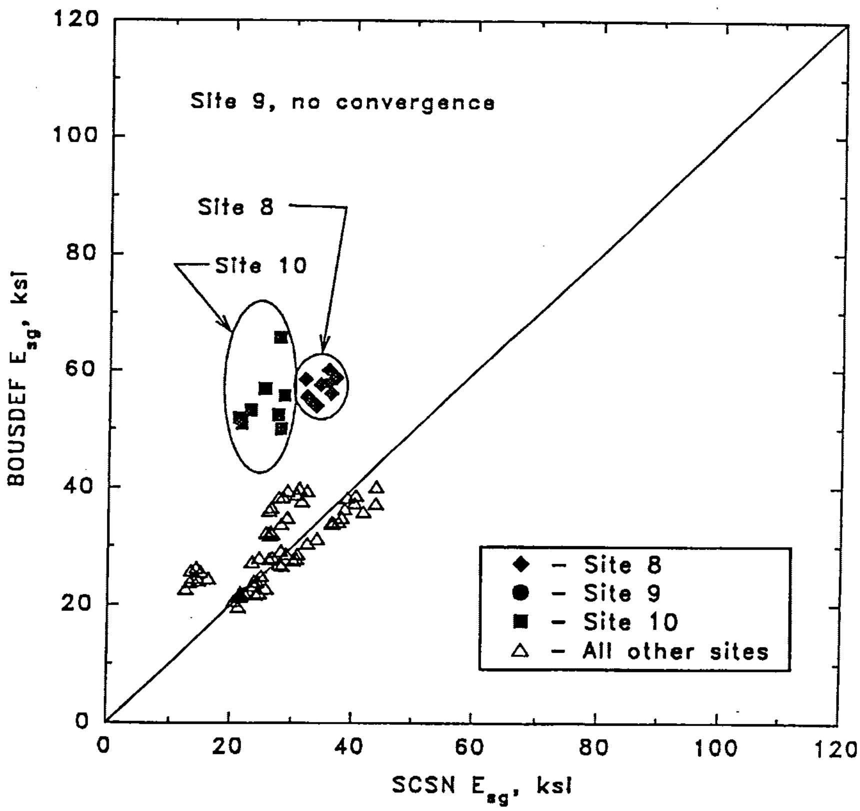

FIGURE 10 Comparison of subgrade moduli calculated directly with empirical correction and using program BOUSDEF (1 ksi = 6.89 MPa).

ysis using iterative and direct methods, the direct method almost always provides a lower value of subgrade modulus, which in some cases appears to be more reasonable. When the proposed subgrade correction technique is applied to field data, the stability of pavement properties over time improves compared with analyses without subgrade correction.

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