Cut-and-Cover Tunnel Subgrade Modeling

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Mathematical models that can be used to construct reinforced-concrete "box" tunnels using the cut-and-cover method are reviewed and critiqued. Newer models consistently provide better agreement between calculated and observed behavior than traditional ones do (the "rigid method" and Winkler's hypothesis with a constant coefficient of subgrade reaction). In addition, determining the appropriate model parameters is conceptually straightforward with the newer models. With current computer analysis capabilities, there is no reason to continue using traditional models that were reasonable alternatives when only manual calculations could be performed. Detailed recommendations for modeling subgrades in practice are presented with consideration given to the capabilities of commercially available structural analysis computer software. Other factors that influence the structural analysis of this type of tunnel are also discussed.

Reinforced-concrete "box" tunnels constructed using the cut-and-cover method have been used for decades for transportation applications and are an important part of several major projects currently under design. This latter fact has resulted in renewed interest in the way to model the soil subgrade underlying the base slabs of box tunnels, especially in consideration of the dramatic advances in the computational capability of microcomputers within the past few years. Mathematical models, also called subgrade models, used for constructing reinforced-concrete box tunnels are reviewed and critiqued. Models that can realistically be used in routine engineering practice are emphasized, although other models are noted for the sake of completeness.

CURRENT PRACTICE: REVIEW AND DISCUSSION

Parameters of Interest

A subgrade model is not necessarily a general soil model. It is intended to approximate only the important aspects of a particular soil-structure interaction problem. Therefore, the accuracy of subgrade models should be judged on consideration of the parameters that are important to a particular type of structure. Bending moments in the base slab and slab settlements, both total and differential, are of primary importance in box-tunnel design. The slab-subgrade contact stress, which will be referred to as the "subgrade reaction stress" or simply "subgrade reaction" as suggested by Liao (*1*), is generally of secondary interest, as is shear within the base slab.

A key premise of this paper is that an acceptable subgrade model should produce accurate estimates of both moments and settlements from a single analysis. This is consistent with other problems in modern structural analysis in which accurate estimates of loads and deformations of structural members are expected from the same analysis. However, this philosophy has not, in general, been used in past analyses of tunnels.

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Problem Components

A base slab and the subgrade that supports it are integral parts of the overall tunnel structure (see Figure 1). The reason is that the load-deformation behavior of any one component (roof slab and walls, base slab, or subgrade) depends on the behavior of the other two. Therefore, these three components should be analyzed as a single composite unit. This renders the problem highly indeterminate. With modern computer software, the structural components can be accurately analyzed. However, the way in which the subgrade effects are modeled has generally not improved by using computer applications. Therefore, this paper focuses on the issue of improving the subgrade modeling.

It is useful to define the coefficient of subgrade reaction:

$$k(x) = \frac{p(x)}{w(x)} \tag{1}$$

This parameter is completely general and independent of a particular subgrade model. An important point is that $k(x)$ is an observed result and not an assumption. However, this fact has not been emphasized in the past and will be used subsequently as one way of evaluating the accuracy of subgrade models.

Traditional Methods of Analysis

Rigid Method

Although numerous subgrade models have been developed over the years (*2*), only two have been used extensively in U.S. practice. The simpler one, which assumes that the base slab is perfectly rigid and has a straight-line distribution of subgrade reaction, is the Conventional Method of Static Equilibrium (CMSE). It is often referred to as the "rigid method." The CMSE is not a true subgrade model because it does not produce an estimate of total settlement of the base slab, although it does imply that there is no differential settlement. The base slab is treated as a footing and settlements are estimated separately using any one of several methods developed for footings. This is a classic example of the traditional approach in which separate analyses are used to estimate moments and settlements.

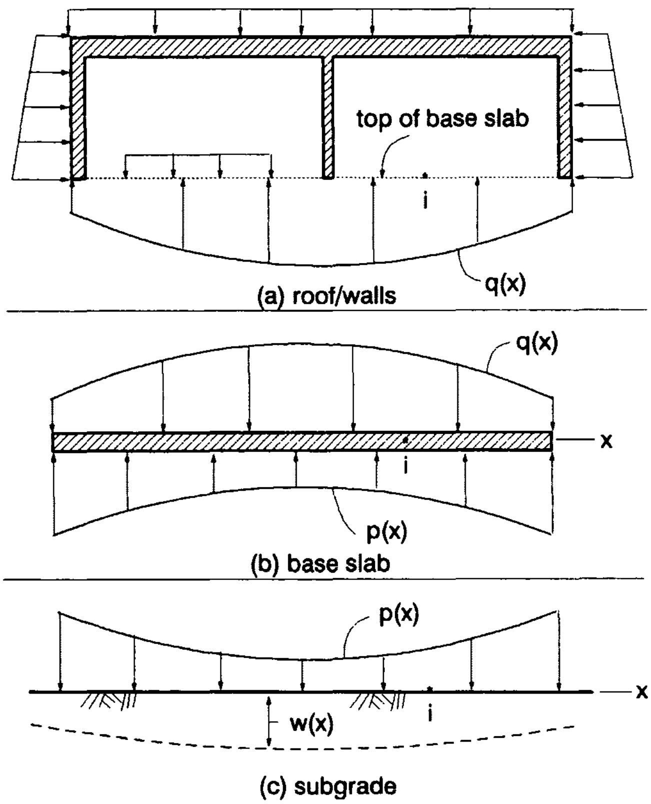

FIGURE 1 Components of typical box tunnel.

The concept that a base slab is essentially a large footing has unfortunately created a persistent perception that design requires only some "allowable bearing pressure." This is incorrect because footings and base slabs are significantly different in this regard. For example, for a footing supporting a building wall, the width is varied to match some "allowable bearing pressure" that is based on consideration of allowable settlement and the safety factor against a bearing failure under service load. On the other hand, the width dimension of a base slab is predetermined by the width of the tunnel. Thus the designer has relatively little control over the magnitude of the total settlement and bearing stresses that will result and must decide whether these parameters are of acceptable magnitude. If not, either a deep foundation alternative is used or the design of the structure is modified.

Winkler's Hypothesis

The other traditional subgrade model is Winkler's hypothesis. In its basic form, it is simply an assumption that the settlement (w_i) at some point i on the subgrade surface is caused only by the applied vertical normal stress (subgrade reaction) (p_i) at that point [see Figure 1(*c*)]. Mathematically, this is expressed as

$$p_i = k_{w_i} w_i \quad (2)$$

where k_{w_i} is the Winkler coefficient of subgrade reaction at Point i. The parameter k_{w_i} is sometimes referred to as the "soil spring constant" or by a similar term because the most common physical interpretation of the abstract behavior defined by Equation 2 is that of an independent spring oriented vertically. For an arbitrary number of contiguous points along the x-axis, the general form of Winkler's hypothesis is

$$p(x) = k_w(x)w(x) \quad (3)$$

where $k_w(x)$ is Winkler's coefficient of subgrade reaction for the general case. An important point is that Winkler's hypothesis forces the coefficient of subgrade reaction to become a known problem input rather than a calculated problem result. This was not emphasized in the past.

There is nothing inherent in Winkler's hypothesis that requires the Winkler coefficient of subgrade reaction to be constant in the x-direction, nor is there any reason why the value of $k_w(x)$ at some point cannot vary with the magnitude of applied load at that point. However, a constant value of $k_w(x)$, independent of load magnitude, is assumed traditionally. This evolved from pre-computer days when the only chart or tabular solutions available for a beam on a Winkler subgrade ("beam on elastic foundation") were based on a constant value of Winkler's coefficient of subgrade reaction. Assuming a constant value, Equation 3 becomes

$$p(x) = k_{w_0} w(x) \quad (4)$$

where k_{w_0} is a constant.

A contentious issue that has interested both practicing engineers and researchers for decades is the appropriate value for k_{w_0} in a specific problem. Methods for estimating k_{w_0} fall into two broad categories: (*a*) tabular or chart values and (*b*) those with some link to the theory of elasticity. The most widely referenced table or chart is from Terzaghi (*3*). Elasticity-based methods are generally based on collocation (matching) of Equation 2 with a p_i and w_i from some closed-form solution for an elastic continuum. A detailed discussion and comparison of several of these methods are provided by Horvath (*4*). However, any discussion of the accuracy of different methods for estimating k_{w_0} is relative because Winkler's hypothesis with a constant value of the Winkler coefficient of subgrade reaction is a poor model of the behavior of an actual soil subgrade, a fact recognized for more than 50 years. This is illustrated using the two limiting cases of a flexible and rigid foundation as shown in Figure 2. For an actual subgrade, the resulting coefficient of subgrade reaction, $k(x)$, is not uniform in either case. This is because of the mechanism of "load spreading," which is primarily the result of vertical shearing within the soil. Using the spring analogy, the springs in actual soil are not independent (as Winkler's hypothesis implies); they are coupled or linked together so that an applied load at one point produces settlement at many points. Conversely, the settlement at some point is influenced by applied loads at other points.

There are many reasons why Winkler's hypothesis with a constant coefficient of subgrade reaction is still used extensively in routine design practice despite its poor representation of actual subgrade behavior. Probably the most significant, practical reason (even in the current computer age) is that it allows the subgrade reaction, $p(x)$, to be eliminated as a variable in the problem solving. For example, the behavior of the

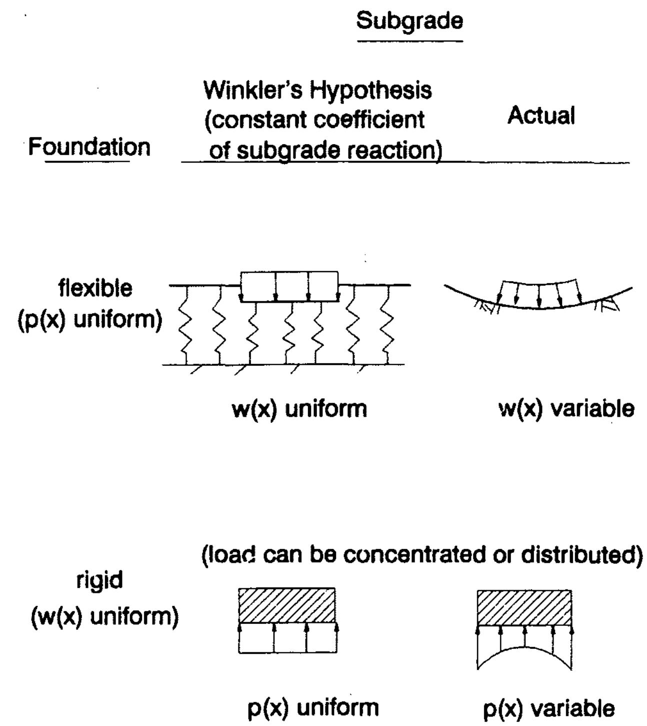

FIGURE 2 Comparison of subgrade response.

base slab shown in Figure 1(*b*) is given by

$$D\frac{d^4w(x)}{dx^4} + p(x) = q(x) \tag{5}$$

where D is the flexural stiffness of the slab (assumed constant). Combining Equations 4 and 5 yields the behavior of the combined base slab and subgrade:

$$D\frac{d^4w(x)}{dx^4} + k_{w_0}w(x) = q(x) \tag{6}$$

Thus the subgrade effects are easily accounted for in the stiffness matrix of the base slab as the external, independent springs transverse to the slab (*5*). Commercially available structural analysis software that is used in practice can easily accommodate such a model.

Although Winkler's hypothesis does produce an estimate of total and differential settlements, traditional practice is to "believe" only the moments calculated using this subgrade model. The calculated settlements are often ignored, which means that a separate analysis must be performed to estimate the expected settlements. This is based on recommendations by Terzaghi (*3*).

New Methods of Analysis

Pseudo-Coupled Concept

In an effort to retain the accustomed mathematical and modeling simplicity of Winkler's hypothesis with a constant coefficient of subgrade reaction, yet provide results closer to reality, "pseudo-coupled" subgrade models have been developed relatively recently (*5*). Examination reveals that this is not a new type of subgrade model at all but simply a return to the general form of Winkler's hypothesis (Equation 3). The assumption of a variable Winkler coefficient of subgrade reaction mimics the effects of coupling of the soil springs without explicitly doing so mathematically. Therefore, existing software can be used because the soil springs are still independent; they just vary in magnitude.

Any number of variations in $k_w(x)$ might be assumed when the pseudo-coupled subgrade concept is used. As a result, several versions of this concept have been proposed to date. They can be grouped into two categories:

- Generic variations that can be applied to any problem and
- Problem-specific variations.

The generic variations now in use assume an increase in $k_w(x)$ near the edges of the base slab. This comes from Figure 2 where, for relatively simple loading, the coefficient of subgrade reaction will always increase at the edges of the foundation, regardless of the relative foundation-subgrade stiffness. The simplest suggestion is to double the traditional constant value of Winkler's coefficient of subgrade reaction, k_{w_0}, along the edges of the base slab (*5*). The basic value of k_{w_0} would have to be determined using the methods discussed by Horvath (*4*). A somewhat more sophisticated version of this, which is discussed by Bowles (*5*), is to use a variation in the Winkler coefficient of subgrade reaction based on the theory of elasticity (usually the solution for a uniformly loaded flexible area). Typically, this produces a Winkler coefficient of subgrade reaction that is also about twice as large along the edges of the base slab than in the center, but with a gradual change in between. Again, the basic value at the center of the base slab to which this empirical variation would be applied would have to be determined from the methods discussed by Horvath (*4*).

Examples of the problem-specific category are provided by Liao (*1*) (a summary of key aspects are also provided by Liao [*6*]) and the Discrete Area Method (*7*). In these methods, both the magnitude and variation of $k_w(x)$ are predicted. Liao developed a solution for a variable Winkler coefficient of subgrade reaction that is based on a uniformly loaded slab of different relative stiffnesses and widths under plane-strain conditions and is supported on an isotropic, homogeneous linear-elastic continuum of finite thickness. He developed his solution specifically for analyzing tunnel base slabs.

The Discrete Area Method is completely general in the type of foundation to which it can be applied, although it has been used primarily for building mat foundations. This method requires separate but parallel structural and geotechnical analyses in which the foundation element (base slab, mat, etc.) and subgrade are divided into the same checkerboard of areas (discrete areas), using an arbitrary number and shape of areas. In the structural analysis, each arbitrary area of the foundation is supported on an independent spring of potentially different stiffness from the others. In the geotechnical analysis, an elastic continuum is subjected to perfectly flexible loaded areas on its surface, with each load uniform but potentially different in magnitude. Spring stiffnesses (in the structural analysis) and applied surface loads (in the geotechnical analysis) of each discrete area are varied in an iterative, trial-and-error process until the deformation patterns from the two analyses match within some acceptable difference. It is assumed that this match provides the unique solution to the problem. Thus the major difficulty inherent in the pseudo-coupled concept is overcome because the variable Winkler coefficient of subgrade reaction used in the structural analysis (*a*) is determined specifically and uniquely for a given problem by matching the patterns of one analysis to those of a separate, rigorous analysis of the subgrade and (*b*) does not depend on some generic solution. Although the Discrete Area Method is perhaps the ultimate pseudo-coupled method and appears to give good results consistently for building mats (*7*), its inherent iterative, trial-and-error nature has discouraged its widespread use in practice.

Although the pseudo-coupled concept appears to be the long-sought improvement to the traditional use of Winkler's hypothesis with a constant coefficient of subgrade reaction, the actual improvement in a given problem is subject to significant variability that can be difficult to assess. The reason for the difficulty is that the degree of improvement depends on how closely the actual problem being analyzed matches the assumed problem that was used to develop the variation in the values of $k_w(x)$. Thus, the fundamental deficiency of the pseudo-coupled concept is that the correct answer must be known beforehand to be able to choose values of $k_w(x)$ that will result in calculating the correct answer. The reason for this circular logic is that the pseudo-coupled concept, which is just Winkler's hypothesis, does not incorporate soil-spring coupling inherently so it is incumbent on the engineer to input the coupling effects. Thus, the accuracy of results depends on how accurately the actual coupling effects can be estimated beforehand.

Multiple-Parameter Models

Because of the uncertainties inherent in the pseudo-coupled concept, there is a need for a subgrade model that incorporates spring coupling in the model's mathematics and is easy to implement and use in routine practice. Of all such models investigated, the most promising appear to be in the general category of multiple-parameter models. In addition to including spring coupling, these models are attractive because they are surface-element (two-dimensional) models whose governing equations do not require explicit consideration of the subgrade depth. The depth effects are built into the derivation of the equation defining the behavior of the subgrade model. A detailed discussion of multiple-parameter subgrade models identified to date is provided by Horvath (*2*).

The most accurate multiple-parameter model that has been identified and studied in detail to date is the Reissner Simplified Continuum (RSC) (*8*). Its governing equation is

$$p(x) - C_{R_1}\frac{d^2p(x)}{dx^2} = C_{R_2}w(x) - C_{R_3}\frac{d^2w(x)}{dx^2} \tag{7}$$

where C_{R_1}, C_{R_2}, and C_{R_3} are constants. Spring coupling comes from the terms involving the second derivatives of $w(x)$ and $p(x)$. Unfortunately, implementing this model into practice has been slow. The primary reason is that the solution requires consideration of subgrade boundary conditions that involve the first derivatives of both $w(x)$ and $p(x)$. The structural analysis software now used in practice cannot accommodate the latter condition directly. Work is in progress to develop an indirect method for implementing the RSC subgrade model within the capabilities of existing software. As an interim improvement, a recent suggestion (*9*) is to use a multiple-parameter model that is intermediate in accuracy between Winkler's hypothesis and the RSC. This model is usually referred to as Pasternak's hypothesis. Its governing equation is

$$p(x) = C_{P_1}w(x) - C_{P_2}\frac{d^2w(x)}{dx^2} \tag{8}$$

where C_{P_1} and C_{P_2} are constants. Spring coupling comes from the term involving the second derivative of $w(x)$. Of importance here is that not only is there no boundary condition involving the first derivative of $p(x)$, but $p(x)$ itself can be eliminated as a variable by combining Equations 5 and 8 so that the behavior of the combined base slab and Pasternak subgrade is defined by

$$D\frac{d^4w(x)}{dx^4} - C_{P_2}\frac{d^2w(x)}{dx^2} + C_{P_1}w(x) = q(x) \tag{9}$$

This is recognized as the equation defining the behavior of a beam-column with constant column tension of magnitude C_{P_2} supported on independent vertical springs of stiffness C_{P_1}. Therefore, the combined model of the base slab plus Pasternak subgrade will be referred to as the "beam-column analogy." The spring coupling inherent in Pasternak's hypothesis can be visualized as a pseudo-column tension that, from a structural-behavior perspective, effectively increases the base-slab stiffness. Most important, the modeling simplicity of a foundation supported on independent springs is preserved. Because structural-analysis software can model a spring-supported beam-column under constant tension, implementing the beam-column analogy into practice is simple. General recommendations for evaluating the coefficients C_{P_1} and C_{P_2} are presented in detail by Horvath (*9*). An example is included in this paper.

True Continua

Two categories of subgrade models are more accurate than multiple-parameter models. The first is boundary-element solutions of an elastic continuum in which the subsurface conditions are not modeled explicitly. The depth effects are built into a two-dimensional equation applicable only over the surface of the continuum. This is identical conceptually to the multiple-parameter surface-element models discussed previously, but the boundary-element method is more accurate because it involves more rigorous (and complex) mathematics.

The other category is explicit modeling of the continuum depth. As summarized by Horvath (*8*), closed-form elastic solutions have very limited use because of the large number of variables involved; therefore true continuum modeling is generally limited to using the finite-element method. In this case, a variety of soil models might be used, including linear-elastic and hyperbolic stress-strain models. Because of the effort involved in developing and debugging finite-element meshes, using this method, although perhaps offering the closest analytical match to reality, is generally impractical on a routine basis.

CASE HISTORY: BACKGROUND DATA

Introduction

A goal of this paper is to illustrate the accuracy of various types of subgrade models using a case history. Because of the dearth of suitable published data for cut-and-cover tunnels, the case history used in this paper involves a mat foundation for a building. However, the geometric and physical conditions for this mat are almost identical to those of a typical tunnel section, including the application of an uplift water pressure on the base slab. Consequently, use of this case history is relevant to cut-and-cover tunnels.

The building is the Whitaker Laboratory, which was constructed at the Massachusetts Institute of Technology campus in Cambridge, Massachusetts, in the 1960s. A detailed compilation of relevant structural and geotechnical data and observed settlements was published previously by DeSimone and Gould (*10*).

Structural Analyses and Properties

The Whitaker Laboratory mat is significantly longer than it is wide and orders of magnitude stiffer at foundation level in the longitudinal direction compared with the transverse direction. As a result, mat flexure was essentially limited to the transverse direction, which is identical to the behavior of a typical tunnel base slab. This restriction was taken advantage of in the current study by performing only a plane-strain analysis in the transverse direction. This also simplifies the presentation of results and the assessment of the accuracy of the subgrade models considered.

A transverse section through the Whitaker Laboratory mat is shown in Figure 3. It is 1.14 m (3.75 ft) thick. Note the great physical similarity to a center-wall box tunnel. Also shown are the service loads from an analysis of the superstructure frame neglecting any differential settlement. The moment loading along the exterior below-grade walls, which coincide with Column Lines A and C, is the result of the lateral earth and water pressures on these walls. Not shown is the 292-kN/m (20-kip/ft) axial compressive force per unit width on the mat from these loads. The flexural stiffness of the mat alone in the transverse direction was given as 2.58 $\times$ 10^6 kN-m^2/m (1.9 $\times$ 10^6 kip-ft^2/ft). The superstructure stiffness, using the simple additive method discussed by the American Concrete Institute (*11*), was 2.31 $\times$ 10^5 kN-m^2/m (1.7 $\times$ 10^5 kip-ft^2/ft). Note that the mat is relatively much stiffer than the superstructure. Because the settlement data used to compare actual and calculated mat behavior were obtained about 6½ years after the superstructure was poured, one-third of these stiffness values was used to account for assumed time-dependent effects on the Young's modulus of the concrete. Cracked-section behavior of the mat was considered and modeled using Branson's equation. The cracking moment for this mat was estimated to be 667 kN-m/m (150 kip-ft/ft).

Subsurface Conditions and Subgrade Model Parameters

The key soil parameter in tunnel base slab analysis is generally the compressibility (Young's modulus). Below the foundation level for this mat there is approximately 21 m (70 ft) of Boston blue clay. This is underlain by glacial outwash sands and till that were assumed to act as a rigid base. When dealing with a fine-grained soil (i.e., clay) as at this site, there are two limiting cases of soil behavior: the immediate (undrained) condition and the long-term (drained) condition. Only the drained condition was studied because the most complete settlement data published were for a time well after primary

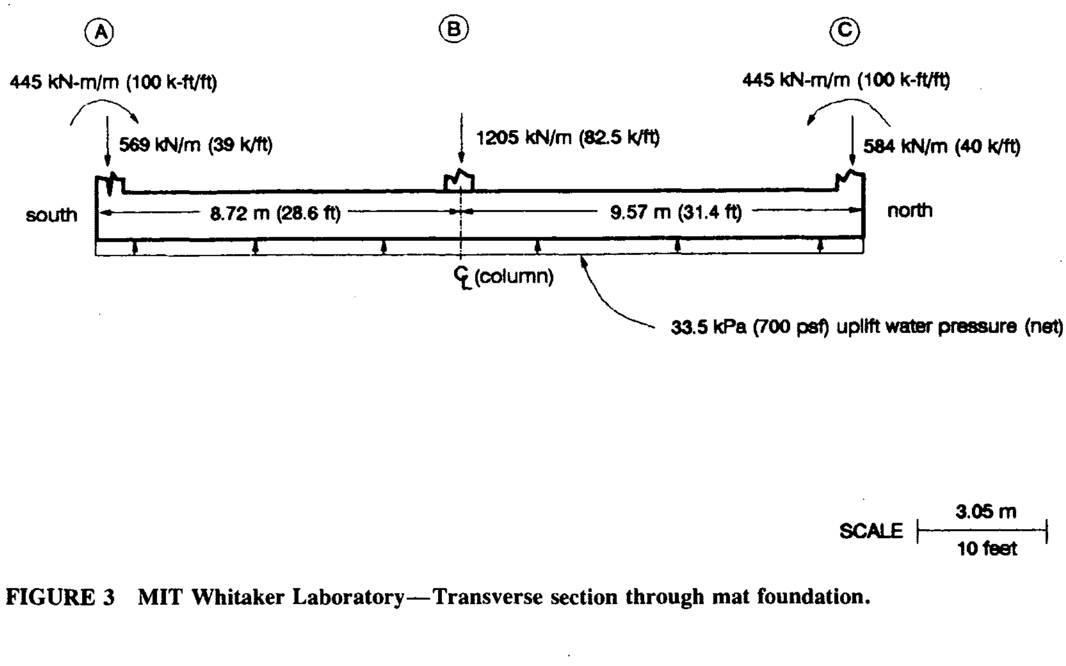

FIGURE 3 MIT Whitaker Laboratory—Transverse section through mat foundation.

consolidation was complete. This is not necessarily the more critical condition in terms of bending moments in the mat.

The procedure followed to arrive at a value of Young's modulus of the soil is detailed by Horvath (*12*). The equivalent average drained Young's modulus was calculated to be 38 MPa (800 ksf). The drained Poisson's ratio was assumed to be 0.25. The shear modulus was calculated to be 15 MPa (320 ksf). Using these elastic parameters, the coefficients for the various subgrade models compared in this study were calculated as follows:

- *Winkler's hypothesis/constant coefficient of subgrade reaction.* The Winkler-type simplified continuum has been found to be the most consistent method for calculating a constant value of Winkler's coefficient of subgrade reaction, k_{w_0} (*4*). Horvath (*13*) derived the following equation for an isotropic, homogeneous layer:

$$k_{w_0} = \frac{E_s}{H} \tag{10}$$

where E_s is the Young's modulus of the layer [38 MPa (800 ksf)] and H is the layer thickness [21.3 m (70 ft)].

- *Winkler's hypothesis/variable coefficient of subgrade reaction (pseudo-coupled concept) using generic solution.* The simplest generic pseudo-coupled solution is to use the value from Equation 10 everywhere except to double its value along the edges of the base slab.
- *Winkler's hypothesis/variable coefficient of subgrade reaction using Liao's method.* Liao's results provide a uniform value for $k_w(x)$ within the middle 60 percent of the foundation, with increasing values of $k_w(w)$ between there and the edges of the base slab. The elastic parameters and problem geometry were used to estimate the appropriate values from charts and tables by Liao (*1*);
- *Beam-column analogy.* As discussed by Horvath (*2*), there are at least five different ways to interpret the coefficients in Equation 8, which defines the behavior of this model. Of these, the Pasternak-Type Simplified Continuum appears to be the most logical. Horvath (*14*) derived the following for an isotropic, homogeneous layer:

$$C_{P_1} = \frac{E_s}{H} \tag{11}$$

$$C_{P_2} = \frac{G_s H}{2} \tag{12}$$

Equation 11 is identical to Equation 10 and represents the compression (spring) component of the subgrade. Equation 12 represents the shear (spring coupling) component, which is visualized as a fictitious tensile column force.

- *Reissner Simplified Continuum.* Horvath (*8*) derived the following for an isotropic, homogeneous layer:

$$C_{R_1} = \frac{G_s H^2}{12 E_s} \tag{13}$$

$$C_{R_2} = \frac{E_s}{H} \tag{14}$$

$$C_{R_3} = \frac{G_s H}{3} \tag{15}$$

Equation 14 is identical to Equations 10 and 11 and represents the compression component of the subgrade. Equations 13 and 15 represent the shear effects (Equation 15 is a pseudo beam-tension similar to Equation 12).

PRESENTATION AND DISCUSSION OF RESULTS

Introduction

For all parameters studied, it was found that using a constant Winkler coefficient of subgrade reaction and the simplest variable Winkler (pseudo-coupled) analysis where the coefficient of subgrade reaction was simply doubled along the edges produced nearly identical results. Consequently, the results for the latter analysis have been omitted from figures for clarity. In all figures, the results labeled "Winkler/variable" are those for Liao's pseudo-coupled solution.

Settlements

The comparison of calculated with average observed settlements is shown in Figure 4. The mat exhibited a slight overall dishing, although it was relatively stiff, even in the transverse direction, and exhibited only modest differential settlement. The RSC model provided good agreement with observed behavior and the best of all models considered. The results from Liao's pseudo-coupled solution were also good. The results from the beam-column analogy (Pasternak subgrade) and using a constant Winkler coefficient of subgrade reaction were similar and compared less well with observed behavior.

Bending Moments

The comparison of calculated moments is shown in Figure 5. Also included here are the results for the traditional method of assuming a rigid mat. Because of the known theoretical accuracy of the RSC model and the good comparison of observed settlements with values calculated using the RSC model, the RSC results were assumed to be correct for relative comparisons. As is typical, the relative range in calculated moments is less than that in settlements. Of particular note is the excellent agreement between the RSC and Liao methods as well as the significant underestimation of positive moments near the center using the rigid method. It is of interest to note that the theoretical cracking moment [667 kN-m/m (150 k-ft/ft)] would be exceeded only near the center of the mat.

Coefficient of Subgrade Reaction

As defined in Equation 1, the coefficient of subgrade reaction, $k(x)$, is the ratio of subgrade reaction to settlement. It is an observed result. However, using a subgrade model as simple as Winkler's hypothesis (with either a constant or variable Winkler coefficient of subgrade reaction) requires knowing

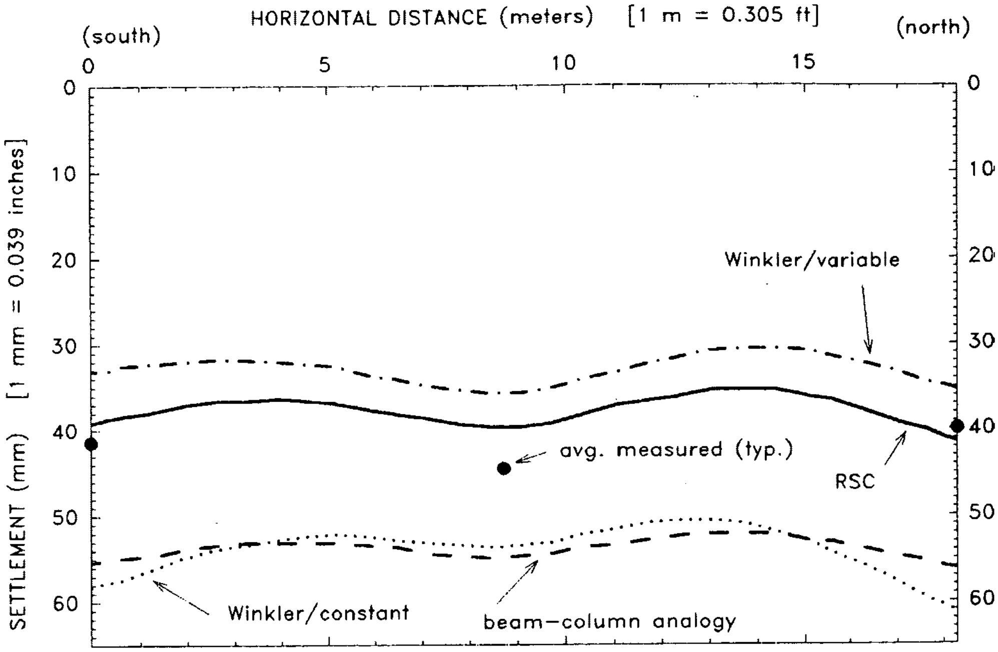

FIGURE 4 Whitaker Laboratory—Settlement comparison.

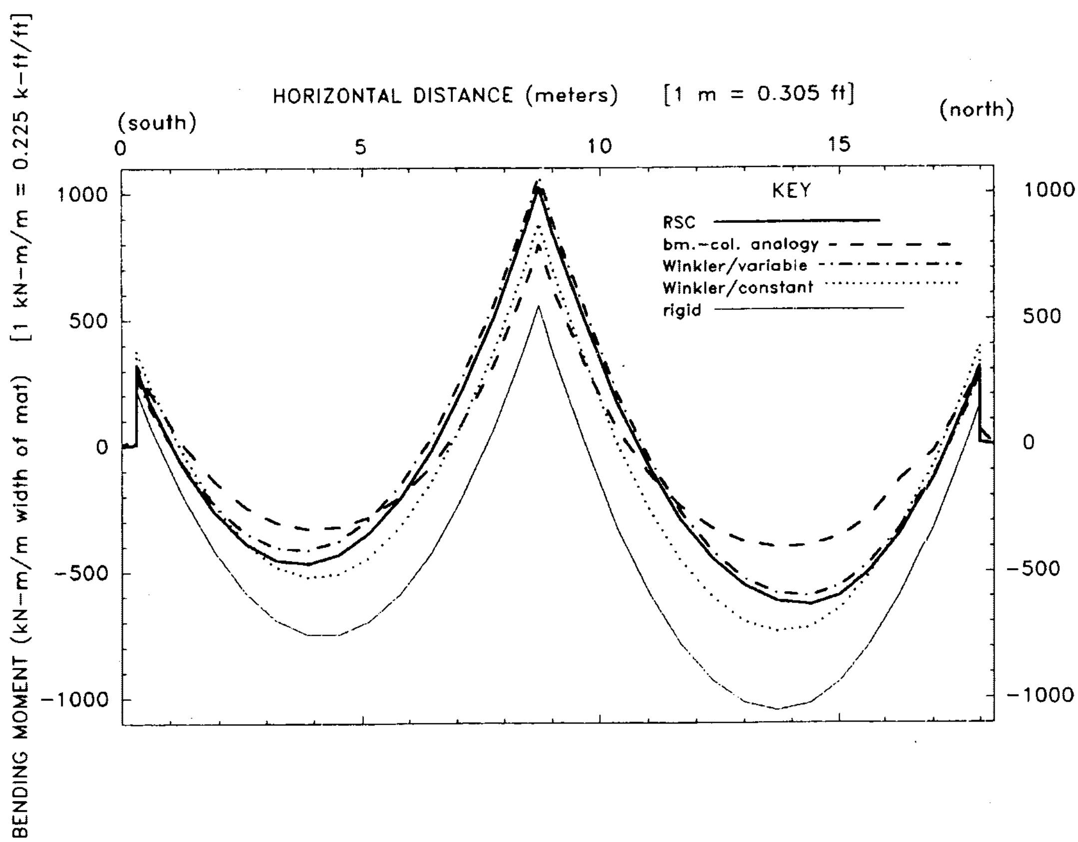

FIGURE 5 Whitaker Laboratory—Calculated bending moments in mat.

the $k(x)$ input beforehand. Therefore, one way to evaluate the accuracy of analyses performed using Winkler's hypothesis is to compare both the magnitude and variation of that assumed with the actual. Because contact stresses were not measured for this mat, the actual value is unknown. However, the value calculated using the RSC model was considered to be correct. Because the RSC model inherently incorporates spring coupling, the coefficient of subgrade reaction does not have to be assumed beforehand when using this subgrade model.

The comparison is shown in Figure 6. The generally good agreement between the values assumed in Liao's pseudo-coupled method and the RSC model explain why Liao's method produced good estimates of settlements and moments as shown previously. Similarly, the fact that the "actual" (RSC model) coefficient of subgrade reaction is not constant (nor does simply doubling the edge values provide significant improvement) explains why the traditional use of Winkler's hypothesis with a constant coefficient of subgrade reaction (or simply doubling the edge values) does not provide as good an estimate of settlement and moments.

CONCLUSIONS

Although only one case history was illustrated, the results are consistent with theoretical work (*8*); parametric studies involving foundation elements of numerous combinations of thickness, loading, and subgrade stiffness (*8*); and other case histories (*12,15*). Therefore, the following conclusions with respect to subgrade models are based on a collective evaluation of all work and not just the one case history:

- The Conventional Method of Static Equilibrium, which implies a perfectly rigid base slab, provides poor approximation of observed behavior. Virtually all base slabs exhibit some flexibility relative to the subgrade. Because moments of both signs will occur in base slabs, moments calculated assuming mat rigidity can be unconservatively in error in at least one sign.
- Using Winkler's hypothesis with a constant coefficient of subgrade reaction does not produce accurate estimates of both moments and settlements from a single value of Winkler's coefficient of subgrade reaction.
- The accuracy of results from using a variable Winkler coefficient of subgrade reaction (pseudo-coupled concept) are variable. The simplest approach of doubling the otherwise-constant value of k_{w_0} along the edges of the mat produced very little difference compared to using Winkler's hypothesis with a constant value for k_{w_0}. On the other hand, Liao's method produced results that agreed very well with the observed behavior and those calculated using the RSC subgrade model.
- The accuracy of results from Winkler's hypothesis is, in general, directly related to how well the assumed magnitude and variation of Winkler's coefficient of subgrade reaction matches the actual.
- The accuracy of results of the recently suggested beam-column analogy (which incorporates the Pasternak subgrade model) is also variable. In the case history shown, the results were only slightly better than those using Winkler's hypothesis with a constant coefficient of subgrade reaction. For other case histories (*9*), the improvement was considerably better. Based on work performed to date, the degree of improvement offered by the beam-column analogy appears to depend on the stiffness of the foundation relative to the subgrade. This is because the spring coupling represented by the pseudo column tension simply adds to the flexural stiffness of the foundation element. If, as with the case history in this paper, the foundation is already quite stiff, the improvement will be modest. For cases where the foundation element is relatively

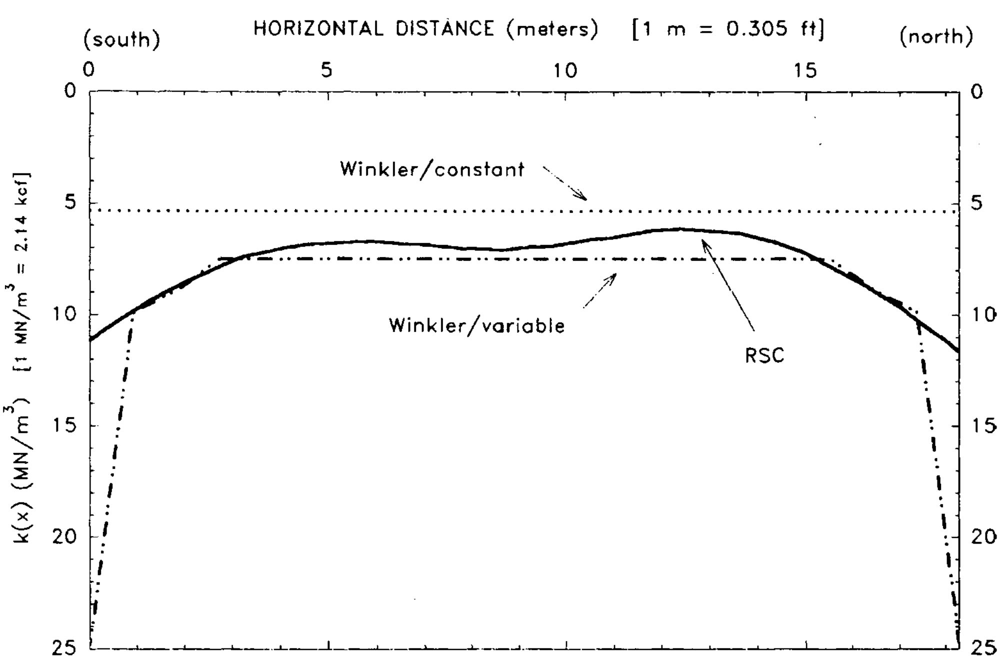

FIGURE 6 Whitaker Laboratory—Coefficient of subgrade reaction comparison.

more flexible (either a thinner element, stiffer subgrade, or both), the improvement would be greater.

• Of the subgrade models considered, the RSC provided the best agreement between observed and calculated settlements.

RECOMMENDATIONS

Subgrade Models

As with the above conclusions, the following recommendations are based on work in addition to the results presented in this paper. The overall recommendation is that a single subgrade model is used to calculate all parameters of interest in the design of a cut-and-cover tunnel (moments, settlements, etc.). Within this context, the following specific recommendations are made:

• Use of the Conventional Method of Static Equilibrium ("rigid method") should be discontinued.

• Use of the traditional form of Winkler's hypothesis with a constant coefficient of subgrade reaction should be discontinued.

• The general form of Winkler's hypothesis with a variable coefficient of subgrade reaction (the pseudo-coupled concept) can produce acceptable results provided that the reference analysis used to produce the values of $k_w(x)$ matches the problem of interest in terms of geometry, loading, and foundation stiffness. Thus the simple methods of doubling the Winkler coefficient of subgrade reaction at the edges or using a generic variation based on an elastic solution should not be used. Liao's method appears to be quite promising, and it is expected that the charts and tables provided by Liao (*1*) will be published soon so that designers of cut-and-cover tunnels will have access to this useful method [Liao has provided limited plots (*6*)]. The Discrete Area Method consistently produces good results, but it appears to be too cumbersome for routine practice, especially on smaller projects. This will likely continue to limit its use to those who are familiar and comfortable with the method.

• As an interim general-purpose method, the beam-column analogy should be used because it incorporates the Pasternak multiple-parameter subgrade model, which is fundamentally more accurate than Winkler's hypothesis. However, a boundary condition involving $w'(x)$ [the first derivative of $w(x)$] at the edges of the base slab must be dealt with. This issue is discussed by Horvath (*9*). Based on a limited study of this model to date, it is recommended that the continuity of $w'(x)$ be assumed. This can be achieved by specifying a zero-column-tension boundary condition at the edges of the base slab. It is also recommended that zero-horizontal-deformation boundary conditions be imposed at each edge of the base slab. This is to prevent calculation of fictitious horizontal deformations of the mat of a very large magnitude.

• Recommendations 3 and 4 should be considered only interim measures until such time that consistently more accurate subgrade models, such as the Reissner Simplified Continuum, can be implemented using structural analysis software that is available to practicing engineers. As stated previously, methods for accomplishing this are already under development.

• On large projects, at least one comprehensive model of the entire structure and subgrade should be developed and analyzed using a finite-element program such as *SOIL-STRUCT* (*14*), which can model all stages of excavation, construction, and loading. The results from such an analysis, if conducted in the beginning of the project and, ideally, in conjunction with an instrumented test section, can be used to calibrate simpler models for project-specific conditions.

It is important to note that all of the recommended methods require knowledge of Young's modulus for the subgrade materials as well as the depth to an effective "rigid base" beneath the tunnel. Because there is considerable judgment involved in determining both parameters, any problem in practice will require studying the sensitivity of the calculated results to some reasonable range in both parameters. In addition, the final structural design should be based on an evaluation of the range in calculated bending moments.

Structural Analysis

Although this paper has focused on subgrade models, other structural aspects that should be considered include the following:

• The well-known behavioral aspects of the tunnel concrete, such as modulus reduction, with time and cracked section behavior;

• Seasonal temperature variations of tunnel roofs that can cause lateral expansion of the roof and increased lateral earth pressures on the side walls. As a result, this will influence the loading on the base slab (*15*). Geothermal analyses should be performed to evaluate the thermal variation to determine whether they are significant (*16*). If they are, as an alternative to designing a heavier tunnel section to withstand the higher stresses, an economic evaluation of more-modern design strategies should be performed. Specifically, a newly-identified geosynthetic product, "geofoam," could be used to provide several functions (*17*). For example, geofoam could be used above the tunnel roof as thermal insulation or along the exterior side walls to cushion the effects of expansion and contraction.

• The use of "thick" elements (in which the effect of shear on flexural stiffness is considered) for modeling the foundation element versus the usual "thin" elements based on simple beam theory (Horvilleur and Patel, unpublished data). The conclusion of these studies suggest that shear effects are significant in some cases. Therefore, it would appear to be prudent to always model a base slab using thick elements if the analysis software used has this capability.

REFERENCES

1. Liao, S. S. C. *Estimating the Coefficient of Subgrade Reaction for Tunnel Design*. Parsons Brinckerhoff, Inc., 1991.
2. Horvath, J. S. Subgrade Models for Soil-Structure Interaction Analysis. In *Foundation Engineering: Current Principles and Practices*, ASCE, New York City, 1989, pp. 599–612.
3. Terzaghi, K. Evaluation of Coefficients of Subgrade Reaction. *Géotechnique*, Vol. 5, No. 4, 1955, pp. 297–326.
4. Horvath, J. S. *Further Evaluation of the Coefficient, or Modulus, of Subgrade Reaction, k, Using an Extension of Reissner's Sim-*

plified Elastic Continuum Concept. Research Report CE/GE–88–2. Manhattan College, Bronx, N.Y., 1988.
5. Bowles, J. E. *Foundation Analysis and Design*, 4th ed. McGraw-Hill Book Company, New York, 1988.
6. Liao, S. S. C. Using Finite Element Analysis to Derive the Coefficient of Subgrade Reaction. *Proc., Fifth International Conference on Computing in Civil Engineering*, Anaheim, Calif., 1993.
7. Ulrich, E. J. Subgrade Reaction in Mat Foundation Design. *Concrete International*, Apr. 1991, pp. 41–50.
8. Horvath, J. S. New Subgrade Model Applied to Mat Foundations. *Journal of Geotechnical Engineering*, ASCE, Vol. 109, No. 12, 1983, pp. 1567–1587.
9. Horvath, J. S. Beam-Column-Analogy Model for Soil-Structure Interaction Analysis. *Journal of Geotechnical Engineering*, ASCE, Vol. 119, No. 2, 1993, pp. 358–364.
10. DeSimone, S. V., and J. P. Gould. Performance of Two Mat Foundations on Boston Blue Clay. *Proc., Specialty Conference on Performance of Earth and Earth-Supported Structures*, 1972, pp. 953–980.
11. American Concrete Institute Committee 336. Suggested Analysis and Design Procedures for Combined Footings and Mats. *ACI Structural Journal*, Vol. 85, No. 3, 1988, pp. 304–324.
12. Horvath, J. S. Mat Foundation Analysis: A Review and Critique Based on Case Histories. Preprint paper, ACI Spring Convention, 1992.
13. Horvath, J. S. Modulus of Subgrade Reaction: New Perspective. *Journal of Geotechnical Engineering*, ASCE, Vol. 109, No. 12, 1983, pp. 1591–1596.
14. Horvath, J. S. *A Study of Analytical Methods for Determining the Response of Mat Foundations to Static Loads*. Ph.D. thesis. Polytechnic Institute of New York (currently Polytechnic University), Brooklyn, N.Y., 1979.
15. Horvath, J. S. *Base Pressure and Earth Pressure Measurements; Nürnberg Subway*. Research Report CE/GE–89–1. Manhattan College, Bronx, N.Y., 1989.
16. Horvath, J. S. Computer Software for Load-Deformation and Geothermal Analyses in Problems Involving Geosynthetics. *Geotextiles and Geomembranes*, Vol. 12, No, 5, 1993, pp. 425–433.
17. Horvath, J. S. Dark, No Sugar: A Well-Known Material Enters the Geosynthetic Mainstream. *Geotechnical Fabrics Report*, Oct. 1992, pp. 18–23.

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