Resilient and Plastic Behavior of Classifier Tailings and Fly Ash Mixtures

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The resilient modulus and plastic deformation of two materials, currently considered waste products, were studied. The first material is fly ash, which is a waste product of coal combustion. The second is classifier tailings, a fine-grained material that is a by-product of aggregate processing. Results from cyclic triaxial testing used to study the resilient and plastic response of fly ash, classifier tailings, and a mixture of the two materials are presented. By itself neither material exhibits sufficient stiffness to realize any advantage in pavement construction. However, the resilient modulus of a mixture of the two materials is higher and plastic deformation lower than that of either material considered alone. Since no previously published experience with this material mixture exists, it is considered a new type of geomaterial. Historically, because of difficulties associated with cyclic triaxial testing, empirical formulas have been used to estimate resilient modulus. Therefore, the usefulness of applying existing empirical relations to the materials investigated to estimate resilient modulus is explored. Mechanistic-based pavement analyses were performed to predict pavement lives for flexible pavements having a subgrade of fly ash, classifier tailings, or a mixture of the two. The benefits of the material mixture in improving the pavement performance over that of either of its constituents are demonstrated.

In this study a mixture of two fine-grained materials, currently disposed of as waste products, is investigated for use as a roadbed material. The first is a Type F fly ash, a waste product from coal combustion. The second is the result of aggregate processing in the Buffalo, New York, area. Gravel is processed containing traces of native limestone and dolomite. Construction specifications limit the amount of material passing the No. 200 sieve in processed aggregate. Therefore, fine material is separated from the aggregate sluiced to detention basins, dredged, and stockpiled in landfill operations. This fine material is referred to throughout the aggregate processing industry as classifier tailings.

The use of fly ash as roadbed material has been the subject of previous research (1–5). These studies provided valuable data on engineering properties of fly ash or fly ash mixtures such as shear strength, permeability, moisture-density relationships, and so on. However, more information is required for predicting the performance of fly ash or fly ash mixtures as a component of a pavement structure.

Head (6) investigated mixtures of fly ash and coal refuse for potential highway base course material. On the basis of material characteristics including CBR, pavement performance was predicted using VESYS II, a pavement performance simulation program. Results indicated that thickness requirements for base courses constructed with the fly ash/coal refuse mixture are less than those for base courses constructed with conventional crushed stone. Head’s study is prominent in the sense of introducing mechanistic-empirical pavement analysis to the evaluation of waste material as highway construction material. However, the evaluation of the fly ash/coal refuse mixture for potential base course material may be misleading. Estimations of resilient modulus for the material were based on empirical relationships that may not be appropriate for unusual materials such as waste product mixtures, which in many cases represent new types of geomaterials.

In this paper, resilient and plastic behavior of fly ash, classifier tailings, and mixtures of these two materials are studied from the standpoint of their potential for use as roadbed material within flexible pavement systems. The usefulness of applying existing empirical formulas for estimating resilient modulus of the investigated materials will be explored. Experience with roadbed soils in Korea demonstrating poor correlation between predictions of resilient modulus made with a popular empirical formula and actual measurements is also presented.

The benefits of using the material mixture to improve pavement performance are analyzed. Mechanistic pavement analyses are performed to predict pavement lives for flexible pavement having a subgrade of fly ash, classifier tailings, or a mixture of the two. For the purpose of comparison the pavement lives are predicted using both pseudoplastic analysis and layered elastic analysis with damage assessment of the pavement system. The latter involves the use of an empirical relationship to define pavement failure due to rutting, and its application to subgrade constructed with new geomaterials for which experience is limited is questioned.

RESILIENT AND PLASTIC BEHAVIOR

To investigate the potential use of fly ash, classifier tailings, or a mixture of the two materials in highway construction, a series of standard geotechnical tests was performed. Details of the tests and results are presented by Lee and Fishman (7), although salient details of the tests will be repeated here. The fly ash is nonplastic consisting of 24 percent sand, 48 percent silt, and 28 percent clay-size particles. The classifier tailings have a liquid limit of 31 percent and plastic limit of 17.5 percent consisting of 31 percent sand, 39 percent silt, and 30 percent clay-size particles. Mixtures of fly ash/classifier tailings exhibit increased permeability, increased unconfined
compressive strength, and decreased compressibility compared with either of its constituents and decreased maximum compacted density, increased optimum compacted water content, and decreased plasticity compared with classifier tailings.

Resilient and plastic behavior of the materials were studied via cyclic triaxial testing. Details of sample preparation, test procedures, determination of optimum mix ratio, and test data were reported by Lee and Fishman (7). A brief summary will be given here. The ratio of classifier tailings to fly ash used in the material mixture was 10:3 by weight. All testing was performed on materials compacted to maximum dry density as determined by the standard Proctor compaction test at a water content wet of optimum. A triaxial cell was used that can accommodate specimens 7.1 cm wide by 14.6 cm high, and water was used for confining fluid. Repeated axial loads were applied by pneumatic pump with loading time 0.25 sec and a load frequency of 0.5 Hz. Deformation and loads measured by an LVDT and load cell were collected by a computer-controlled data acquisition system every 0.05 sec. Samples were subjected to a range of maximum deviatoric stress from 6 to 110 kPa with confining pressure from 0 to 110 kPa. One thousand cycles of deviatoric stress was applied at a frequency of 0.5 Hz.

The resilient behavior of fly ash is similar to that of granular materials, which exhibit a dependency of resilient modulus, $M_r$, on the first invariant of the stress tensor, $J_1$. For fly ash $M_r$ may be expressed (in kPa) as

$$M_r = 92.02 \times J_1 + 8,943.2 \tag{1}$$

Figure 1 shows a plot of $M_r$ for fly ash versus $J_1$. On the basis of the Asphalt Institute's recommendation (8), the typical range of confining pressure within a flexible pavement subgrade is 6.9 to 34.5 kPa ($J_1$ is 20.7 to 137.8 kPa). As shown in Figure 1, $M_r$ of fly ash for this range is 11,000 to 21,000 kPa. On the basis of Asphalt Institute data (8), this condition indicates very poor roadbed soil.

The resilient modulus of classifier tailings was found to be dependent on the magnitude of deviatoric stress as shown in Figure 2. This behavior is typical for fine-grained materials.

Resilient modulus for the classifier tailings may be expressed using a bilinear model (in kPa) as follows:

$$M_r = \begin{cases} -2,346 \times (\sigma_d - \sigma_{db}) + 26,352 & \text{if } \sigma_d < \sigma_{db} \\ 51 \times (\sigma_d - \sigma_{db}) + 26,352 & \text{if } \sigma_d > \sigma_{db} \end{cases} \tag{2}$$

where $\sigma_d$ is deviatoric stress and $\sigma_{db}$ is deviatoric stress at breakpoint. For comparison, Figure 2 shows the relation for $M_r$ as a function of $\sigma_d$ for typical fine-grained materials as discussed by Thompson and Elliot (9). The use of classifier tailings as subgrade material may not have an advantage over the use of medium stiff naturally occurring subgrade clay materials, which may be readily available near a given site.

The observed resilient behavior of the mixture of fly ash and classifier tailings also demonstrates dependence on $J_1$ as shown in Figure 3. For the mixture $M_r$ may be expressed (in kPa) as

$$M_r = 196 \times J_1 + 24,287 \tag{3}$$

Quantitatively, $M_r$ values of the material mixtures are 31,000 to 48,000 kPa at states of stress of practical interest for subgrade material. On the basis of Asphalt Institute data (8) this range of $M_r$ represents good subgrade soil. It is hoped that this preliminary data will inspire further study and encourage use of fly ash/classifier tailing mixtures as subgrade materials in flexible pavement structures.

Plastic behavior of investigated materials was studied through statistical analysis. Statistical models of accumulated axial plastic strain were established by correlating confining pressure ($\sigma_c$), deviatoric stress ($\sigma_d$), and number of load applications ($N$). The following expressions were obtained by multivariate regression analysis:

$$\log \varepsilon_{pl} = -3.946 - 0.109 \log \sigma_c + 0.314 \log \sigma_d + 0.460 \log N \tag{4}$$

($R^2 = 0.8661 \quad S = 0.1285$)

$$\log \varepsilon_{pl} = -4.469 - 0.103 \log \sigma_c + 0.472 \log \sigma_d + 0.341 \log N \tag{5}$$

($R^2 = 0.7998 \quad S = 0.1747$)
log \( s_p \text{classifier} = - 4.579 - 0.058 \log \sigma_s + 0.782 \log \sigma_d + 0.381 \log N \)

\( R^2 = 0.6821 \quad S = 0.2335 \) \hspace{1cm} (6)

A comparison of accumulated plastic strain versus number of load repetitions for the investigated materials at a typical subgrade stress condition is shown in Figure 4. The accumulation of plastic strain for the fly ash/classifier tailing mixture is distinctly less than that of either constituent. Pavement life predictions presented later in the paper will quantify the increased resistance to rutting that is a manifestation of improved plastic behavior.

**ESTIMATION OF RESILIENT MODULUS \( (M_r) \) BY EMPIRICAL METHOD**

Because of difficulties associated with cyclic triaxial testing, various approximate methods have been suggested to estimate \( M_r \). In this paper, the ability of some approximate methods to predict \( M_r \) is investigated for fly ash, classifier tailings, and the fly ash/classifier tailings mixture. Results of grain size analysis indicate that materials are fine grained (7). For estimating \( M_r \) of fine-grained materials, the correlation proposed by Heukelom and Klomp (10) is the most popular method (the result is in kPa):

\[ M_r = A \times CBR \] \hspace{1cm} (7)

where \( A \) is a coefficient from linear regression. The range of \( A \) is from 5,167.5 to 20,670, and usually 10,335 is used. Use of Equation 7 has been recommended differently by AASHTO (11), the Asphalt Institute (12), and Klomp and Dormon (13). AASHTO (11) suggests that Equation 7 be used for soils with a soaked CBR of 10 or less, whereas the Asphalt Institute suggests the use of Equation 7 for soils up to CBR of 20 determined at a moisture content consistent with the field condition. Klomp and Dormon's suggestion is similar to the Asphalt Institute's and is also used for this study. CBRs de-
determined for the materials investigated are summarized in Table 1.

By using the CBRs reported in Table 1 for each material, values of \( M_r \) are estimated with Equation 7 and compared with measurements. Figure 5 shows that Equation 7 provides a reasonable estimate of \( M_r \) for the classifier tailings. However, Figure 1 and Figure 3 show that Equation 7 overestimates \( M_r \) compared with direct measurements performed on samples of fly ash and fly ash/classifier tailing mixture.

In Korea, research to establish simple estimation methods for \( M_r \) has been performed to facilitate the introduction of mechanics-based pavement design methods in the future. Since the CBR test is routinely performed as part of a site evaluation, the application of Equation 7 to commonly encountered Korean subgrade soils was studied. Woo et al. (14) concluded that Equation 7 is not appropriate for estimating \( M_r \) of Korean subgrade soils on the basis of a comparison, shown in Figure 6, with direct measurements of \( M_r \) from the results of cyclic triaxial testing. The usefulness of Equation 7 for Korean subgrade soils is questionable since the common type of Korean subgrade soil is silty sand or clayey sand, rather than clay-type soil. Because of the difficulties associated with cyclic triaxial testing, several simple tests or approximate methods for determination of \( M_r \) have been proposed, but no reliable method is established yet. This is one reason why mechanics-based pavement design is avoided by many Korean engineers.

The poor estimation of \( M_r \) for fly ash and the fly ash/classifier tailings mixture is expected since Equation 7 was developed on the basis of a perceived \( M_r\)-CBR relationship for fine-grained materials. A relationship for granular materials discussed by AASHTO may be more suitable for fly ash and the material mixture, since the \( M_r \) of each is related to \( J_t \). Correlations of \( M_r \) and CBR were provided by AASHTO (11) for granular materials including the effect of state of stress as follows:

\[
\begin{align*}
J_t \ (kPa) & \quad M_r \ (kPa) \\
689.0 & \quad 5,098.6 \times \text{CBR} \\
206.7 & \quad 3,031.6 \times \text{CBR} \\
137.8 & \quad 2,342.6 \times \text{CBR} \\
68.9 & \quad 1,722.5 \times \text{CBR}
\end{align*}
\]

As shown in Figures 1 and 3, the preceding table gives a reasonable estimation of \( M_r \) for fly ash and the fly ash/classifier tailings mixtures.

Recent research has attempted to estimate the \( M_r \) of natural fine-grained subgrade material, accounting for dependence of \( M_r \) on deviatoric stress. On the basis of typical resilient behavior of fine-grained soils, Thompson and Robnett (15) investigated factors affecting the resilient behavior of fine-grained soils. The study indicated that the degree of saturation, distribution of particle size, plastic index, unconfined compressive strength, and modulus of elasticity are factors related to \( M_r \). Similarly, Drumm et al. (16) proposed a hyperbolic model to describe the nonlinear relationship for the resilient modulus of fine-grained soils as a function of deviatoric stress as follows:

\[
M_r = \frac{(a + b \times \sigma_d)}{\sigma_d}
\]

TABLE 1 CBR Values Determined for the Materials Investigated

<table>
<thead>
<tr>
<th>Materials</th>
<th>CBR (Unsoaked)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fly Ash</td>
<td>7.0</td>
</tr>
<tr>
<td>Classifier Tailings</td>
<td>2.0</td>
</tr>
<tr>
<td>Material Mixture</td>
<td>12.6</td>
</tr>
</tbody>
</table>
where $a$ and $b$ are material coefficients and are functions of previously mentioned factors. The expression of $a$ and $b$ is cited by Drum et al. (16). Details of input data for the application of Drum et al.’s study to the estimation of $M$, for classifier tailings are presented by Lee and Fishman (7). For the classifier tailings $M$, values estimated by Equation 8 are compared with direct measurements of $M$, in Figure 5. As discussed by Drum et al. (16), this empirical equation does not provide a good approximation in the range of low deviatoric stress. However, it renders a reasonable approximation when the deviatoric stress is 34.5 kPa or more.

Drum et al. (16) also provide an empirical expression for $M$, at a breakpoint, a significant parameter for bilinear representation of $M$, versus $\sigma_d$. This expression is a function of the aforementioned factors included in Equation 8, and given data relative to the classifier tailings a breakpoint resilient modulus $M_{br}$ is estimated as 39.6 kPa. As shown in Figure 5, Drum et al.’s empirical equation for estimating $M_{br}$ gives a good approximation of $M_{br}$ of classifier tailings determined by direct measurement. Though fly ash and fly ash/classifier tailing mixtures are fine-grained soils, Drum et al.’s study is not applicable to these materials since $M$ does not depend on the magnitude of deviatoric stress.

**PAVEMENT LIFE PREDICTION**

The benefits of using the material mixture to improve pavement performance is demonstrated through mechanistic pavement analyses performed to predict pavement lives for flexible pavement having a subgrade of fly ash, classifier tailings, or the fly ash/classifier tailings mixture.

For comparison, the pavement lives are predicted using both pseudoplastic analysis (Method 1) and multilayered elastic analysis (Method 2) with damage assessments of the pavement system. Pavement systems modeled by Methods 1 and 2 are shown in Figures 7 and 8. The algorithm describing Method 1 is shown in Figure 9. Method 2 is performed using
DAMA, which is a multilayered elastic analysis program distributed by the Asphalt Institute used to estimate repetitions to failure where pavement condition is described in terms of rutting and surface failure. In the analysis performed herein only rutting failure is considered.

Pavement life predictions made with Method 1 incorporate plastic behavior, which is unique to a given material, directly as input. For pavement damage assessment models such as Shell (17) or MS-1 (8), failure criteria incorporated in Method 2 are empirical relations that attempt to correlate observed damage in pavements with the computed elastic compressive strain at the top of the subgrade layer. Thus, the improved plastic properties of the fly ash/classifier tailings mixture are not accounted for directly in Method 2. Results from both methods indicate increased life for pavements having subgrades constructed with a fly ash/classifier tailings mixture compared with those constructed of either fly ash or classifier tailings.

Pavement lives predicted by Method 1 are plotted in Figure 10 for pavements having subgrades constructed with fly ash, classifier tailings, or the fly ash/classifier tailings mixture. Pavement life realized by incorporating the fly ash/classifier tailings mixture in the subgrade is an order of magnitude higher than for pavements constructed with subgrades of fly ash or classifier tailings alone. The increased pavement life is due to the improved resilient and plastic behavior of the material mixture over that of its constituents.

Comparisons of pavement lives by Methods 1 and 2 for pavements having subgrade of fly ash, classifier tailings, and the fly/ash material mixture are plotted in Figures 11, 12, and 13, respectively. In all cases incorporating the Shell failure criterion with Method 2 renders a longer life prediction than if the MS-1 failure criterion is incorporated. Figure 11 indicates that for pavements having subgrades constructed with fly ash, the pavement life predicted with Method 1 lies between the predictions made with Method 2 incorporating the Shell or MS-1 failure criterion. Figure 12 indicates that for pavements having subgrade constructed with classifier tailings, the pavement life predicted with Method 1 is close to that predicted by Method 2 with the MS-1 failure criteria incorporated. Figure 13 indicates that for pavements having subgrades constructed with fly ash/classifier tailings mixture, the prediction pavement life with Method 1 is close to the...
FIGURE 9  Algorithm for pavement analysis (Method 1).

FIGURE 10  Predicted pavement life by Method 1.
FIGURE 11 Predicted pavement life for fly ash subgrade, Methods 1 and 2.

FIGURE 12 Predicted pavement life for classifier tailings subgrade, Methods 1 and 2.

FIGURE 13 Predicted pavement life for a material mixture subgrade, Methods 1 and 2.
prediction of Method 2 with the Shell failure criterion is incorporated. These data suggest that uncertainty exists in the use of limiting subgrade strain condition, such as the Shell or MS-1 criterion, for the prediction of pavement life. The use of Method 1 gives the analyst greater flexibility in the specification of individual material plastic behavior. Confidence in Method 1 is verified since results are within the range of field observations as described by the Shell and MS-1 failure criterion. For future pavement construction Method 1 is recommended for preliminary assessment of pavement performance.

**CONCLUSIONS**

1. The fly ash/classifier tailings material mixture demonstrates better resilient and plastic behavior than does either of its constituents.

2. Empirical expressions for estimation of $M_r$ were examined for the investigated materials. For classifier tailings the $M_r$-CBR relationship for fine-grained soils described by Equation 7 provides a reasonable approximation. Drumm et al.,’s empirical equation (Equation 8), also approximate for fine-grained materials, provides a better description of the dependence of $M_r$ on material properties and $\sigma_3$ than Equation 7; however, further research into functions of this type is required. On the basis of the experience in Korea, caution should be exercised in the sand or clayey sand, which may exhibit some behavior attributed to granular materials.

3. Neither Equation 7 nor Equation 8 is appropriate for estimating $M_r$ of fly ash or fly ash/classifier tailings mixture. Since fly ash and fly ash/classifier tailings mixture exhibit behaviors similar to granular materials, $M_r$-CBR correlation, which includes a dependence on confining pressure ($f_c$), appears to be more appropriate.

4. Pavement rutting life predictions indicate that pavement life improved by an order of magnitude for flexible pavements having subgrades constructed with the fly ash/classifier tailings mixture over that of subgrades constructed with fly ash or classifier tailings alone. Results obtained with pseudoplastic analysis are considered more reliable than those obtained by limiting subgrade strain criteria since plastic behavior, which is unique to a material, is input directly in the pseudoplastic analysis.

**REFERENCES**


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