Extreme Rainfall Frequency Analysis for Louisiana

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A comparative study of five popular frequency distributions and three parameter estimation methods was conducted on the rainfall data from 92 stations in Louisiana. Computed results showed that the log-Pearson Type 3 (LPEAR3) distribution along with the method of moments was the best choice for the Louisiana rainfall data. Maximum annual 24-hr rainfall maps for return periods of 2, 5, 10, 25, 50, and 100 years were developed by using hourly precipitation data. These new isohyetal maps were compared with the U.S. Weather Bureau Technical Paper 40 maps based on the performance indexes of the standardized mean square error (*MSE*) and the standardized bias (BIAS). On the average, the new maps reduced the *MSE* by 58 percent and the BIAS by 80 percent. A first-order error analysis was performed on the parameters of the LPEAR3 distribution. Computed results showed that the predicted quantiles of the LPEAR3 distribution were most sensitive to the parameter of population mean and least sensitive to the coefficient of skewness.

Many times in hydrologic studies, flood discharges and hydrographs must be estimated for ungauged sites. The accuracy of estimated flood discharges from a rainfall-runoff model depends heavily on the accuracy of the estimated rainfall values. The first extended rainfall frequency study in the United States was made by Yarnell (*1*) and was presented in the form of maps for several combinations of return periods and durations for the continental United States. The only published report with regard to precipitation in Louisiana was found to be *Louisiana Rainfall* (*2*), published by the Louisiana Department of Public Works in 1952.

The U.S. Weather Bureau updated the rainfall maps with additional data and published them as TP-40 (*3*) in 1961. To date, TP-40 is the most widely used source of rainfall information. This rainfall atlas contains 50 maps of the United States with contour lines of rainfall amounts for durations varying from 30 min to 24 hr and return periods from 2 to 100 years. The accuracy and resolution of TP-40 maps are limited because of the small number of rain gauges available at the time of preparation and the short period of records at each gauge station, and the TP-40 maps have wide contour intervals and lack the detail necessary for the accurate design of drainage structures in a particular watershed.

A supplement to TP-40, HYDRO-35 (*4*), was published by the National Oceanic and Atmospheric Administration of the National Weather Service in 1977. HYDRO-35 provides rainfall contour maps for 5- to 60-min durations and 2-, 10-, and 100-year return periods for the eastern and central United States. This set of maps is a useful addition to TP-40 for estimating design rainfalls of short durations or developing intensity-duration-frequency (I-D-F) charts. Other studies have been undertaken along similar lines by Pennsylvania State University for the Pennsylvania Department of Transportation (*5*) and the Arizona Department of Transportation (*6*). However, results of these studies are only applicable to those two states.

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One of the major objectives of rainfall frequency analysis is to estimate the magnitude of extreme rainfall or the rainfall intensity for a given duration and return period. The rainfall quantiles, in conjunction with a rainfall-runoff model, are used to compute flood quantiles of a stream. Several studies have been reported in the literature to compare the performance of various distributions with various parameter estimation methods (*7–10*). However, there is no general consensus on either the performance of a specific distribution or a specific parameter estimation method. For example, Arora and Singh (*10*) concluded, on the basis of their Monte Carlo simulation results, that the LPEAR3 with the method of moments (MOM) performed poorly and suggested a revision of the recommendation by the U.S. Water Resources Council (*11*) of using LPEAR3-MOM; others, however, found that LPEAR3-MOM gave consistent and efficient estimates (*11,12*).

In this paper the authors will make a comparative evaluation of five distributions and three parameter estimation methods for the Louisiana rainfall data and discuss the development of maximum annual 24-hr rainfall maps for return periods of 2, 5, 10, 25, 50, and 100 years.

ANALYSIS OF RAINFALL DATA

Hourly precipitation data were obtained from the National Climatic Data Center of the National Weather Service, U.S. Department of Commerce. The raw data contain records of 92 rain gauges in Louisiana. The average record length of the 92 stations is 18 years with minimum and maximum values of 2 and 40 years. However, almost every station had periods of missing records. Records at stations within a 10-mi radius were combined when no single station had a continuous record of sufficient length to provide a complete data set for a reliable statistical analysis. This grouping of rain gauges provided 26 synthesized (representative) stations, which are shown in Figure 1. Mass curves were developed for the entire period of records for all 26 representative stations to check for consistency of the records. These curves showed that no further adjustment to records was needed. The average record length of the 26 synthesized stations is 38 years; minimum and maximum values are 30 and 40 years. The group of stations that composed a synthesized station were designated as primary

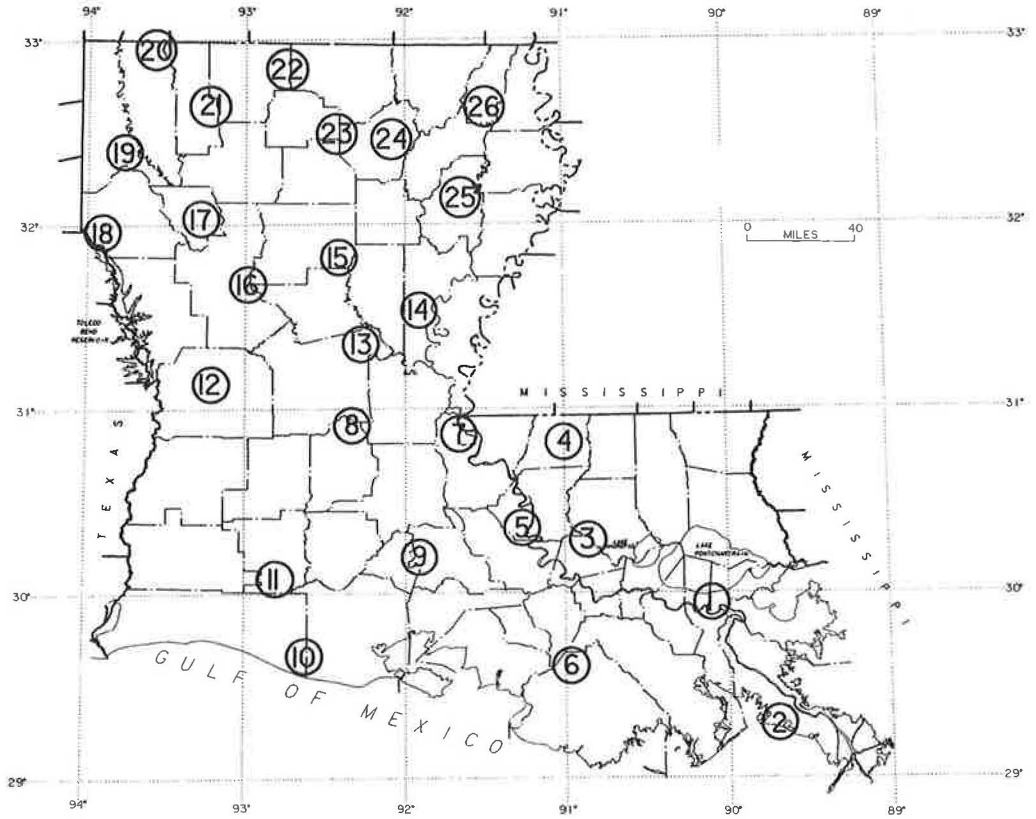

FIGURE 1 Representative rain stations in Louisiana.

stations. The stations outside the group of primary stations were designated as secondary stations. Missing data were directly substituted from the primary stations, and the inverse-distance-squared method was used to fill the data gaps from the nearby secondary stations.

The complete data sets for the synthesized stations were examined for possible errors in the filling of data gaps by checking the annual precipitation between the synthesized and primary stations. When the annual precipitation of the synthesized stations exceeded the annual precipitation of the primary stations by more than 15 percent, the selection of secondary stations was modified. The suitability of the synthesis was further examined using correlation plots of monthly rainfall between primary, secondary, and synthesized stations. Use of these plots ensured that the homogeneity property was not severely violated when filling the data gaps.

With 26 complete data sets representing the 26 synthesized stations, the annual maximum 1-, 3-, 6-, 12-, 24-, 36-, 48-, 60-, 72-, and 96-hr rainfall depth series at each synthesized station was calculated. This was done by making the rainfall data continuous by inserting zero values for non-rainfall hours, then scanning the continuous data and finding the maximum XX-hr annual rainfall depth.

SELECTION OF DISTRIBUTIONS AND PARAMETER ESTIMATION METHODS

A quantitative and practical approach for the selection of an appropriate distribution and a parameter estimation method for the available data is to test some of the most frequently used distributions in applied hydrology along with the most popular parameter estimation methods. By using such statistical criteria as mean square error (*MSE*) and bias, the descriptive performances of different combinations of the distributions and estimation methods were compared and then the best combination was selected (*13*). In this study, five popular distributions and three parameter estimation methods widely used in applied hydrology were considered for a comparative evaluation. The five probability distributions are

- Two-parameter log-normal (LNO2),
- Three-parameter log-normal (LNO3),
- Pearson Type 3 (PEAR3),
- Log-Pearson Type 3 (LPEAR3), and
- Extreme-value Type 1 (GUMBEL).

The three parameter estimation methods are MOM, maximum-likelihood estimate (MLE), and principle of maximum entropy (POME).

These five distributions and three estimation methods have been discussed by various investigators (*7,8,10,13*). Therefore, they are not discussed here.

The descriptive performance indexes for the evaluation of different combinations of distributions and estimation methods, using observed data, are the standardized *MSE* and the standardized bias (BIAS) (*14*). The *MSE* at each station for a selected rainfall duration is defined as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{x_c(i) - x_o(i)}{\bar{x}}\right]^2 \quad (1)$$

where

$x_c(i)$ = computed rainfall values for ith plotting position, ranked in descending order;
$x_o(i)$ = observed rainfall values for ith plotting position, ranked in descending order; and
$\bar{x}$ = observed sample mean at same station.

Similarly, the standardized bias (BIAS) at a station for a selected rainfall duration is defined as

$$\text{BIAS} = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{x_c(i) - x_o(i)}{\bar{x}} \right] \quad (2)$$

A computer program was developed to estimate the parameters of the five distributions by three estimation methods, using the annual maximum rainfall series at each of the 26 synthesized stations, and to compute the *MSE* and BIAS for all combinations of the selected distributions and estimation methods at each station. The average *MSE* and BIAS were calculated for all combinations of distributions and methods over 26 stations. The average *MSE* and BIAS values for the 26 stations are given in Tables 1 and 2. Table 1 indicates that in terms of the average *MSE* for 26 stations, LPEAR3-MOM is the preferred combination of distribution and estimation method for Louisiana rainfall data. On the other hand, Table 2 indicates that LNO3-MLE gave the least average BIAS for the 26 stations. In practice, however, *MSE* is considered to be a more preferable performance index than BIAS when the corresponding BIAS is not excessively large. Since the LPEAR3-MOM has the smallest average *MSE* with the corresponding BIAS comparable to other methods, LPEAR3-MOM was selected as the most appropriate combination of distribution and estimation method for the Louisiana rainfall data.

DEVELOPMENT OF 24-hr ISOHYETAL MAPS

Quantiles of the 24-hr maximum rainfall for the 26 stations were computed by using LPEAR3 distribution with the MOM estimation method. The computed quantiles often change sharply from one station to another. Therefore, several rules were devised to make the isohyetal drawings meaningful. First, the means of the quantile values were computed from each 1-degree quadrangle of latitude and longitude to filter out possible random errors. The "initial" 24-hr isohyetal curves for various return periods were based on these average values. However, various types of errors exist that render the initial isohyetal curves unacceptable. To improve the initial curves, the following rules were applied:

1. If a station quantile in a 1-degree quadrangle deviates from its mean by three standard deviations, that quantile is eliminated from the computed data set.
2. If only one or two stations exist in a 1-degree quadrangle, adjacent station values are used to compute the mean value.

TABLE 1 Average *MSE* for 26 Stations for 24-hr Annual Maximum Rainfall Series

		LNO2	LNO3	PEAR3	LPEAR3	GUMBEL
	MAX	0.06289	0.05298	0.05722	0.05388	0.06815
MOM	AVG	0.01001	0.00872	0.00843	**0.00780**	0.01055
	MIN	0.00146	0.00124	0.00119	0.00104	0.00144
	MAX	0.07293	0.08230	0.08704	0.06683	0.07452
MLE	AVG	0.01234	0.01721	0.01885	0.01126	0.01434
	MIN	0.00118	0.00171	0.00195	0.00106	0.00088
	MAX	0.07293	0.08677	0.08946	0.07058	0.07262
POME	AVG	0.01234	0.01859	0.01900	0.01095	0.01255
	MIN	0.00118	0.00129	0.00174	0.00112	0.00098

TABLE 2 Average BIAS for 26 Stations for 24-hr Annual Maximum Rainfall Series

		LNO2	LNO3	PEAR3	LPEAR3	GUMBEL
	MAX	-0.00373	0.00071	0.00069	-0.00003	-0.00772
MOM	AVG	-0.00855	-0.00918	-0.00924	-0.01181	-0.01033
	MIN	-0.01481	-0.02654	-0.01821	-0.02674	-0.01316
	MAX	-0.00352	0.00090	0.00415	-0.00307	-0.00230
MLE	AVG	-0.00992	**0.00029**	0.00255	-0.01169	-0.01750
	MIN	-0.01997	-0.00160	0.00151	-0.02310	-0.03728
	MAX	-0.00351	0.00413	0.00499	-0.00375	-0.00802
POME	AVG	-0.00992	0.00232	0.00370	-0.00978	-0.00964
	MIN	-0.01996	-0.00476	0.00206	-0.01841	-0.01157

3. If a station is located between two adjacent 1-degree quadrangles, the quantile at that station is used in computations by both adjacent 1-degree quadrangles.
4. In the corner quadrangles where the trend of the isohyetal lines is not clear, nearby individual station values are given higher importance than average values.
5. When the isohyetal curves change sharply in a small local area, the curve is modified on the basis of the nearby curve pattern, geographical and climatological conditions, or the reliability of the nearby station data. This condition is necessary to provide smooth transitions for the isohyetal curves.

The final 24-hr isohyetal maps for the return periods $T = 2$, 5, 10, 25, 50, and 100 years were based on these rules. Figure 2 shows an example of the isohyetal map for the 50-year return period. A similar analysis was also conducted for rainfall durations of 1, 3, 6, 12, 36, 48, 60, 72, 84, and 96 hr (*13*).

VERIFICATION OF RESULTS

A comparison was made between the new isohyetal maps ("NewMaps") and the TP-40 maps for the return periods of 2, 5, 10, 25, 50, and 100 years. The standardized *MSE* and standardized bias (BIAS) were used as the performance indexes for the evaluation. *MSE* and BIAS were computed by using values from each map at the corresponding stations. The "observed" values are the predicted quantiles from the observed data by using the log-Pearson Type 3 distribution with MOM. Table 3 gives the average *MSE* and BIAS for six return periods. Table 4 gives the predicted quantiles from the TP-40 maps, the NewMaps, and LPEAR3-MOM at five typical stations for six return periods. On the average, for return periods of less than or equal to 25 years, the NewMaps are superior to the TP-40 maps in terms of both *MSE* and BIAS. For return periods of 50 and 100 years, the NewMaps are superior to the TP-40 maps in terms of *MSE* but have slightly larger values of BIAS. On the average, for all of the 26 synthesized stations corresponding to six return periods, the NewMaps reduced the *MSE* by 58 percent and the BIAS by 80 percent, compared with the TP-40 maps.

SENSITIVITY OF PREDICTED QUANTILES TO ERROR IN MODEL PARAMETERS

An error analysis was performed to evaluate effects of errors in estimated parameters on predicted rainfall quantiles by using the LPEAR3 distribution with MOM for parameter estimation. The procedure of first-order analysis outlined by Singh and Yu was followed (*15*). The parameters of the LP3 distribution for the annual maximum 24-hr rainfall at Station 1 for a return period of 50 years were estimated using MOM. The estimated parameters were $\overline{y} = 1.558$, $S_y = 0.3772$, and $G_y = 0.5366$, and the estimated 50-year quantile was 29.06 cm (11.44 in.). The coefficient of variation of the 50-year quantile was calculated by changing the coefficient of variation of each parameter from -1 to $+1$. Computed results

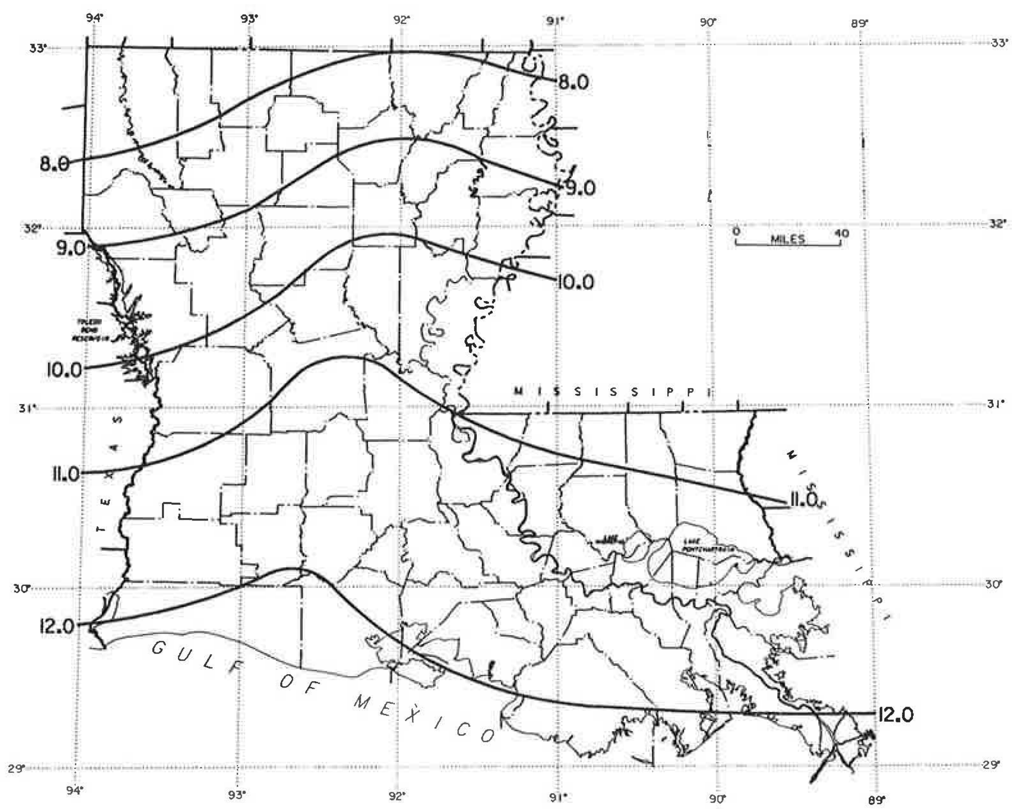

FIGURE 2 24-hr rainfall (inches) for 50-year return period in Louisiana.

TABLE 3 Average *MSE* and BIAS for TP-40 and New Maps

Return Period	TP-40 MAPS		NewMaps	
	MSE	BIAS	MSE	BIAS
2	0.035	0.167	0.004	0.005
5	0.024	0.120	0.004	0.016
10	0.021	0.102	0.004	0.006
25	0.018	0.050	0.010	0.010
50	0.022	0.013	0.016	0.030
100	0.034	-0.008	0.029	0.024

TABLE 4 Predicted Quantiles for Five Typical Stations

Station Number	Method	Return Period (year)					
		2	5	10	25	50	100
	TP-40	13.34	18.03	21.21	24.64	27.18	30.99
5	NewMaps	11.94	16.64	19.3	24.38	28.7	33.15
	LPEAR3	10.97	15.47	18.95	24.03	28.32	33.1
	TP-40	14.48	18.8	22.35	26.42	30.48	33.78
10	NewMaps	12.95	18.29	21.59	26.92	31.12	35.56
	LPEAR3	12.6	18.06	22.17	27.91	32.64	37.72
	TP-40	12.19	15.75	18.29	21.34	23.37	26.42
15	NewMaps	10.92	14.99	17.78	21.59	25.53	28.19
	LPEAR3	11.46	15.37	17.91	21.06	23.37	25.65
	TP-40	11.43	14.73	17.53	20.32	22.35	25.4
20	NewMaps	8.38	11.68	14.22	16.76	19.56	22.61
	LPEAR3	8.36	11.68	14.4	18.44	21.95	25.91
	TP-40	11.94	14.99	17.53	20.32	22.23	25.15
25	NewMaps	10.67	13.97	16.51	20.57	24.13	27.56
	LPEAR3	9.55	13.00	16.18	21.36	26.24	32.13

are plotted and shown in Figure 3. It is clearly demonstrated by this figure that the output error is most sensitive to errors in parameter $\bar{y}$, less sensitive to parameter S_y, and least sensitive to parameter G_y. Fortunately, the ranks of accuracy in estimating these three parameters are in reverse order of the sensitivity analysis. That is why the moment method of parameter estimation, based on the log-transformed data, often yields satisfactory results.

CONCLUSIONS

From this study, the following conclusions are drawn:

1. The log-Pearson Type 3 distribution along with MOM is the best combination of distribution and estimation method for the 26 synthesized Louisiana rainfall data sets.
2. For return periods of less than or equal to 25 years, the newly developed isohyetal maps are superior to the TP-40 maps in terms of both *MSE* and BIAS. For return periods of 50 and 100 years, the newly developed maps are superior to the TP-40 maps in terms of *MSE* but have a slightly larger BIAS. On the average, for all 26 synthesized stations corresponding to six return periods, the new maps reduced the *MSE* by 58 percent and the BIAS by 80 percent, as compared with the TP-40 maps. Thus, the NewMaps greatly improved the accuracy of the TP-40 maps on the basis of the available observed station data.
3. Estimated rainfall quantiles are most sensitive to the error in the estimated parameter $\bar{y}$ and least sensitive to error in the estimated parameter of skewness.
4. Results of this research are expected to enhance the accuracy of the predicted rainfall quantiles in the Louisiana region.

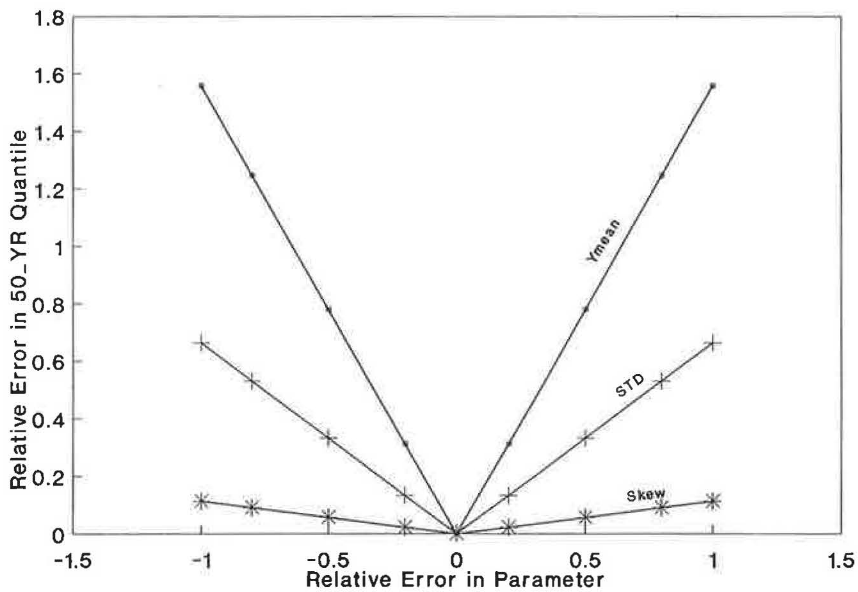

FIGURE 3 First-order analysis.

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