# Demand for Aircraft Gates 

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#### Abstract

Demand for aircraft gates, which is defined as the number of aircraft expected to require the service provided at a terminal building at any given time during one day's operation, depends on flights scheduled and their actual behavior relative to those schedules. The schedules provide a deterministic element to the process of generating the actual number of aircraft at gates, and deviation from these schedules provides a stochastic element to the process. A model that incorporates these two elements has been developed to estimate gate requirements at airports. The results of applying the model to an actual operation of aircraft gates have demonstrated the ability of the model to describe gate occupancy as a function of time of day with reasonable accuracy. The results have also shown that a common gate use strategy (i.e., first-come, first-served discipline) requires fewer gates than strategies under which the use of gates is restricted to flights of a particular air carrier or sector. Furthermore, it has been demonstrated, given a scheduling practice involving bank operations, how the time interval between banks influences the requirement for gates.


An air terminal's ability to process aircraft and passengers depends to a large extent on the interface between the terminal building and the aircraft. This interface is carried out at aircraft gates. The term "gate" designates an aircraft parking space adjacent to an air terminal and used for the servicing, loading, and unloading of a single aircraft. A major problem facing operators at many large airports is that demand for gates at certain times of the day often exceeds the number of gates available. Previous work has shown that deviation of flights from their schedules tends to increase the number of aircraft gates required during the busy periods of the day (1). The objective of this research is to study demand for aircraft gates given an underlying flights' schedule and the variation in their actual behavior from this schedule. Demand is defined as the number of aircraft expected to require a connection with the terminal building at any time during the operation of 1 day. As part of the analysis it is possible to investigate the influence of different scheduling practices and assignment strategies on gate requirements at airports.

Most procedures used in the past for estimating gate requirements at airports have been based on either average performance figures or idealized stochastic models. Horonjeff and McKelvey (2) suggested the following deterministic formula for computing the required number of gates $(G)$ :

$$
\begin{equation*}
G=(C T) / U \tag{1}
\end{equation*}
$$

where
$C=$ design volume of arrivals or departures (aircraft/hr),
$T=$ weighted average gate occupancy time (hr), and
$U=$ gate utilization factor.
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The gate utilization factor represents the amount of time the gates are occupied in relation to the time available. This factor must be applied, because it is virtually impossible for all gates available at a terminal building to be used 100 percent of the time. The technique is a valid planning tool if the scheduling practices and aircraft servicing procedures are assumed to be fixed. Precise schedules, however, are rarely available during the planning stages of an air terminal. In addition, flights usually deviate from their schedules, and airport operators have to alter their service practices to cope with these deviations. To recognize the stochastic nature of flight arrivals and departures, Bandara and Wirasinghe (3) defined, for planning purposes, the number of gates as

$$
\begin{equation*}
G=R(T+S) \tag{2}
\end{equation*}
$$

where $R, T$, and $S$ are random variables that represent arrival rate, gate occupancy time, and separation time, respectively. The separation time, $S$, is selected as a substitution for the utilization factor in Equation 1 and defined as the time between departure from a gate and the next arrival. Data from Vancouver International Airport showed that the probability distribution of $G$ during peak periods can be approximated by Type I extreme value distribution.

A queueing model that assumes the arrival of flights to be homogeneous Poisson and gate occupancy times to be exponentially distributed has been proposed by Rallis (4). The Poisson process assumes the number of arrivals in nonoverlapping time intervals to be stationary. This is not the case in the arrival process of flights due to the influence of their schedules.

Many computer programs have been developed to simulate the daily assignment of aircraft to gates at airports $(5,6)$. In these programs a fixed schedule of flights has been used as an input source to a queueing system in which the service mechanism has been a gate assignment strategy and deviations from the schedule have not been allowed. To overcome this shortcoming, Gosling (7) has proposed a gate assignment expert system that, in addition to its ability to deal with gate assignment under normal situations, could be used to assign aircraft to gates under situations in which flight operations depart from a predefined plan. The system has been applied to a small operation of aircraft gates at Denver Stapleton Airport, and the ability of the system to deal with large-scale gate operations is yet to be investigated.

The actual arrival and departure times of flights in relation to their scheduled times, and the effect of this variation on the demand for aircraft gates, were first studied by Steuart, who developed a simple stochastic model based on timedependent Bernoulli trials to investigate how strategies of scheduling flights influence the demand for aircraft gates (1).

His work concentrated on bank operations where all flights in a bank were assumed to have the same behavior. The time dependent Bernoulli parameter $p(t)$ was defined in the model as the probability that a gate is occupied by an aircraft at a given time $t$, and all the moments of the resulting binomial distribution were determined from this parameter. Empirical data collected at O'Hare Airport, Chicago, were used to estimate the parameter $p(t)$.
The work presented in this paper is an expansion on Steuart's earlier work. The relationship between occupancy of aircraft gates and flights' behavior is established. A stochastic model, based on the assumption that each flight has a unique behavior relative to its scheduled arrival and departure times, is then developed to estimate the demand for gates as a function of time of day. Finally, an application of the model to an actual operation of aircraft gates and procedures to estimate gate requirements under different scheduling practices and assignment strategies are presented. Only operations of aircraft gates on regular days are considered in this study. Operations of gates on irregular days in which bad weather conditions or air traffic control failures, or both, occur were a subject of another study by Steuart (8) who showed the schedule to have little or no effect on the process of generating loads on gates on these days.

## FLIGHT BEHAVIOR AND OCCUPANCY OF AIRCRAFT GATES

Gate occupancy can be described by arrival and departure times of each flight. As a result, the measure of occupancy performance is taken to be the relationship between a flight's actual arrival and departure times relative to the schedule. The difference between a flight's scheduled arrival time and its actual arrival time is defined as an arrival lateness. A negative arrival lateness implies the flight arrived before its scheduled arrival time. Similarly, the difference between the flight's scheduled departure time and its actual departure time is defined as a departure lateness. Flights usually do not depart before their scheduled departure time, and a negative lateness is not expected. Figure 1 shows a scatter plot of the joint arrival and departure lateness of 750 flights that arrived at Terminal 2 of Toronto's Lester B. Pearson International Airport from October 16 to November 3, 1987. Points within 15 min of arrival lateness demonstrate considerable scatter, which would imply that departure lateness is independent of arrival


FIGURE 1 Joint arrival and departure lateness.
lateness within this range. As the arrival lateness becomes larger, the points show a correlation between arrival lateness and departure lateness.
Flights may experience departure lateness because of arrival lateness or other factors such as mechanical or security problems. To distinguish between the departure lateness caused by arrival lateness and that caused by other factors, the departure lateness of late arrivals is analyzed. Late arrivals are defined as those flights that arrive after their buffer times. The buffer time of a flight is the point of time defined by the end of the time interval that is the difference between the scheduled occupancy time of the flight and the time required to service the aircraft at a gate. It is measured in minutes after the scheduled arrival time of the flight.
One hundred seventy flights out of the 750 flights mentioned earlier were late arrivals. A scatter plot of the joint buffer and departure lateness of these flights is shown in Figure 2. The buffer lateness of a flight is defined as the difference between the flight's actual arrival time and its buffer time. The plot shows a strong correlation between departure lateness and buffer lateness. This would imply that if a flight arrives $t \mathrm{~min}$ after its buffer time, a departure lateness close to $t \mathrm{~min}$ will be the likely result. Figure 3 shows a scatter plot of scheduled and actual gate occupancy times of the same 170 flights. As can be seen from the plot, the actual occupancy times are significantly less than the scheduled occupancy times for most flights, which implies that, in the case of a late arrival, airport operators try to service the aircraft as quickly as possible to minimize delay, and consequently the actual gate occupancy time of the flight becomes disassociated from its scheduled occupancy time.


FIGURE 2 Joint buffer and departure lateness, late arrivals.


FIGURE 3 Actual versus scheduled occupancy, late arrivals.

## STOCHASTIC MODEL OF GATE OCCUPANCY

## Stochastic Derivations

Although flights are scheduled to arrive and depart at specific times, they usually deviate from these times. The reasons for deviation are numerous and unpredictable; therefore, deviations are assumed to be random phenomenon. For a given flight, let the scheduled arrival time define the time origin ( $t=0$ ). If $Y(t)$ is defined as a random variable whose value is determined by the event that the aircraft occupies a gate at time $t$, with $t$ measured in minutes relative to the scheduled arrival time, this is a time dependent Bernoulli trial with
$Y(t)= \begin{cases}1 & \text { if the aircraft occupies a gate at time } t \\ 0 & \text { otherwise }\end{cases}$
Therefore, if an aircraft occupies a gate time $t$, it must have arrived before time $t$ and left after time $t$, and the probability that the aircraft occupies a gate at time $t$ is

$$
\begin{equation*}
P[Y(t)=1]=P\left[(A \leq t) \cap\left(D+t_{d} \geq t\right)\right] \tag{4}
\end{equation*}
$$

or

$$
\begin{equation*}
P[Y(t)=1]=P\left[(A \leq t) \cap\left(D \geq t-t_{d}\right)\right] \tag{5}
\end{equation*}
$$

where
$A=$ random variable describing arrival lateness measured in minutes from scheduled arrival time,
$t_{d}=$ scheduled departure time in minutes after scheduled arrival time, and
$D=$ random variable describing departure lateness measured in minutes from scheduled departure time. Flights usually do not depart before their scheduled departure times; therefore $D \geq 0$.

As an illustration, the scheduled arrival time, the scheduled departure time, $t_{d}$, and buffer time measured in minutes after the scheduled arrival time, $t_{b}$, are plotted on a time axis in Figure 4.

Arrival times and departure times of flights that arrive before their buffer times can be assumed to be statistically independent. Arrival times and departure times of flights that arrive after their buffer times can be assumed to be correlated. In this context, two random events are defined: $B$ is the event that a flight arrives before its buffer time, and $B^{c}$ is the event


FIGURE 4 Representation of scheduled arrival and departure times in reference to buffer time.
that a flight arrives after its buffer time (complement of arriving before the buffer time). The probabilities of these two events can be expressed in terms of the cumulative distribution of the random variable $A$ as follows:

$$
\begin{align*}
& P(B)=P\left(A \leq t_{b}\right)=F_{A}\left(t_{b}\right)  \tag{6}\\
& P\left(B^{c}\right)=1-P(B)=1-F_{A}\left(t_{b}\right) \tag{7}
\end{align*}
$$

$B$ and $B^{c}$ are mutually exclusive, collectively exhaustive events. Therefore, the probability of another event, $[Y(t)=1]$, can be expressed in terms of those two events in the following manner

$$
\begin{align*}
P[Y(t)=1]= & P\{[Y(t)=1] \mid B\} P(B) \\
& +P\left\{[Y(t)=1] \mid B^{c}\right\} P\left(B^{c}\right) \tag{8}
\end{align*}
$$

The occupancy of a gate by an aircraft at time $t$, given the aircraft arrives before the buffer time, $\{[Y(t)=1] \mid B\}$, can be defined by the conditional joint event that the aircraft arrives before $t$ and departs after $t$ given arrival is before the buffer time. It follows that

$$
\begin{equation*}
P\{[Y(t)=1] \mid B\}=P\left\{\left[(A \leq t) \cap\left(D \geq t-t_{d}\right)\right] \mid B\right\} \tag{9}
\end{equation*}
$$

This expression can be evaluated by considering three nonoverlapping time intervals: $\left(t<t_{b}\right),\left(t_{b} \leq t \leq t_{d}\right)$ and $\left(t>t_{d}\right)$. For $t<t_{b}$, the probability that an aircraft occupies a gate is simply the probability it has arrived because, by definition, flights do not depart before their scheduled departure time $t_{d}$, and $t_{b}<t_{d}$. For $t_{b} \leq t \leq t_{d}$, given the aircraft arrives before the buffer time and, by definition, does not depart before its scheduled departure time, the probability that the aircraft occupies a gate at time $t$ equals unity. For $t>t_{d}$, given the aircraft arrives before the buffer time, it has therefore arrived before the scheduled departure time, and the probability that it occupies a gate at any time $t$ greater than $t_{d}$ is simply the probability it has not yet departed. In summary,
$P\{[Y(t)=1] \mid B\}=\left\{\begin{array}{l}F_{A \mid B}(t) \quad \text { for } t<t_{b} \\ 1 \quad \text { for } t_{b} \leq t \leq t_{d} \\ 1-F_{D \mid B}\left(t-t_{d}\right) \quad \text { for } t>t_{d}\end{array}\right.$
where

$$
\begin{aligned}
F_{A \mid B}(t) & =F_{A}(t) / F_{A}\left(t_{b}\right) \text { for } t<t_{b}, \\
& =1 \text { for } t \geq t_{b},
\end{aligned}
$$

and $F_{D \mid B}(t)$ can be estimated directly by considering the departure lateness of flights that arrive before their buffer times.

The occupancy of a gate by an aircraft at time $t$, given that the aircraft arrives after the buffer time $t_{b},\left\{[Y(t)=1] \mid B^{c}\right\}$, can be defined by the conditional joint event that the aircraft arrives before $t$ and departs after $t$ given arrival is after the buffer time. It follows that

$$
\begin{equation*}
P\left\{[Y(t)=1] \mid B^{c}\right\}=P\left\{\left[(A \leq t) \cap\left(D \geq t-t_{d}\right)\right] \mid B^{c}\right\} \tag{11}
\end{equation*}
$$

This expression can be evaluated by considering two nonoverlapping time intervals: $\left(t \leq t_{b}\right)$ and $\left(t>t_{b}\right)$. For $t \leq t_{b}$, given arrival is after the buffer time, the probability that an
aircraft occupies a gate is simply zero. For $t>t_{b}$, since, as explained earlier, the actual occupancy time of a late arriving flight (i.e., one that arrives after its buffer time) is disassociated from its scheduled occupancy time, the flight's actual departure time can be determined more accurately by its actual arrival and occupancy times than by its scheduled departure time and departure lateness. Let $H$ be a random variable defined as the actual occupancy time in minutes of a late arriving flight. The probability of a gate occupancy can then be defined as

$$
\begin{array}{r}
P\left\{[Y(t)=1] \mid B^{c}\right\}=P\left\{[(A \leq t) \cap(A+H \geq t)] \mid B^{c}\right\} \\
\text { for } t>\mathrm{t}_{b} \tag{12}
\end{array}
$$

where the right-hand side of the equation can be written as

$$
\begin{align*}
P\left[(A \leq t) \mid B^{c}\right] & +P\left[(A+H \geq t) \mid B^{c}\right] \\
& -P\left\{[(A \leq t) \cup(A+H \geq t)] \mid B^{c}\right\} \tag{13}
\end{align*}
$$

Since $H$ as defined cannot be negative, the last term of the expression must equal unity, therefore,

$$
\begin{align*}
P\{[Y(t) & \left.=1] \mid B^{c}\right\} \\
& =\left\{\begin{array}{l}
0 \quad \text { for } t \leq t_{b} \\
F_{A \mid B c}(t)-F_{A+H \mid B c}(t) \quad \text { for } t>t_{b}
\end{array}\right. \tag{14}
\end{align*}
$$

where

$$
\begin{aligned}
\mathrm{F}_{A \mid B C}(\mathrm{t}) & =0 \text { for } t \leq t_{b} \\
& =\left\{\left[F_{A}(t)-F_{A}\left(t_{b}\right)\right] /\left[1-F_{A}\left(t_{b}\right)\right]\right\} \text { for } t>t_{b}
\end{aligned}
$$

The cumulative distribution $F_{A+H \mid B C}(t)$ can be obtained at any time $t$ greater than $t_{b}$ by summing the probabilities of all pairs $\left[(a, h) \mid B^{c}\right]$ for which $(a+h) \mid B^{c} \leq t$. For all times less than the buffer time $t_{b}$, this cumulative distribution is zero by the definition of $A \mid B^{c}$ and $H \mid B^{c}$.

The approximate shape of the function $P[Y(t)=1]=p(t)$ is shown in Figure 5 along with the functions $P\{[Y(t)=1] \mid B\}$ and $P\left\{[Y(t)=1] \mid B^{c}\right\}$. This function of time increases from zero, starting at some point before the scheduled arrival time of the flight, and reaches a maximum near the scheduled departure time and then decreases.

## Special Cases

So far, only flights that have arrival, departure, and buffer times (i.e., turnaround flights) have been considered. The discussion can be extended to cover originating and terminating flights. At a given time $t$, the probability that a gate is occupied by an aircraft that stays at the gate overnight and departs in the morning (i.e., a morning originating flight) is 1 for $t \leq t_{d}$, and $\left[1-F_{D}\left(t-t_{d}\right)\right]$ for $t>t_{d}$. During the day, aircraft are brought from hangars to gates at some time before the scheduled departure times of originating flights. The time an aircraft is brought from the hangar to a gate before the flights' scheduled departure time depends on the type of aircraft (i.e., wide, narrow, or small body) and the flight sector (i.e., domestic, international). This time can be assumed to be a constant $t_{1}$ for each aircraft type and flight sector. It


FIGURE 5 Schematic development of random variable $Y(t)$ : (a) $P[Y(t)=1 \mid B]$, (b) $P\left[Y(t)=1 \mid B^{c}\right]$, (c) $P[Y(t)=1]$.
follows that the probability that a gate is occupied by an aircraft, when the flight is originated during the day, is zero for $t \leq t_{1}, 1$ for $t_{1}<t \leq t_{d}$ and $\left[1-F_{D}\left(t-t_{d}\right)\right]$ for $t>t_{d}$.

At a given time $t$, the probability that a gate is occupied by an aircraft that arrives last and stays at the gate overnight (i.e., an evening terminating flight) is $F_{A}(t)$ for all $t$. During the day, aircraft are towed off gates at some time after the arrival of terminating flights. This time can also be assumed a constant $t_{2}$ for each aircraft type and flight sector. The probability that a gate is occupied by an aircraft, when the flight is terminated during the day, can be calculated as [ $\left.F_{A}(t)-F_{A}\left(t-t_{2}\right)\right]$ for all $t$.

## Occupancy of Aircraft Gates by Time of Day

On any given day, each scheduled flight $i$ has a unique $Y_{i}(t)$, which depends on the flight's scheduled arrival and departure times, aircraft service time, and its behavior relative to these times. If $s_{i}$ is defined as the scheduled arrival time of the $i^{\text {th }}$ flight, the expectation of the random variable $Y_{i}(t)$, which is assumed to be Bernoulli at any given time $t$ relative to the scheduled arrival time, is
$E\left[Y_{i}(t)\right]=P\left[Y_{i}\left(t-s_{i}\right)=1\right]=p_{i}\left(t-s_{i}\right)$
and the variance of the random variable $Y_{i}(t)$ is

$$
\begin{equation*}
\operatorname{Var}\left[Y_{i}(t)\right]=p_{i}\left(t-s_{i}\right)\left[1-p_{i}\left(t-s_{i}\right)\right] \tag{16}
\end{equation*}
$$

The total number of aircraft occupying gates at time $t$ on a given day can now be defined as
$N(t)=\sum_{i} Y_{i}(t)$

The expectation of the random variable $N(t)$ is
$E[N(t)]=\sum_{i} E\left[Y_{i}(t)\right]$
The variance of the random variable $N(t)$ can be calculated as
$\operatorname{Var}[N(t)]=\sum_{i} \operatorname{Var}\left[Y_{i}(t)\right]+\sum_{i \neq j} \operatorname{cov}\left[Y_{i}(t), Y_{i}(t)\right]$

## Numerical Illustrations

As illustrations of how the model can be used to calculate the probability that an aircraft occupies a gate at time $t$ and the expected value and the variance of the number of aircraft occupying gates at time $t$ the following two examples are presented:

## Example 1

In Example 1, a flight is scheduled to arrive at an air terminal at 12:00 p.m. and to depart at 13:00. The aircraft service time is 45 min . The flight's buffer time can be calculated as [(13:00 $-12: 00)-45=15 \mathrm{~min}$ after the scheduled arrival time], that is, $12: 15$. The probability that the flight arrives before its buffer time is $F_{A}(15)$. The probability that the aircraft occupies a gate at 12:45 given it has arrived before the buffer time is 1 . The probability that the aircraft occupies a gate at 12:45 given it has arrived after the buffer time is $F_{A \mid B C}(45)-$ $F_{A+H \mid B c}(45)$. Data collected to analyze the flight's behavior relative to its schedule indicated that $F_{\dot{A}}(15)=0.820$ and $F_{A \mid B C}(45)-F_{A+H \mid B c}(45)=0.848$. The probability that the aircraft occupies a gate at $12: 45$ can now be calculated as
$P[Y(45)=1]=1 * 0.820+0.848 *(1-0.820)=0.973$

## Example 2

In Example 2, three flights are scheduled to be serviced by an air terminal during a given day. The probabilities that each one of these flights occupies a gate at 12:45 p.m. were calculated using the model as $p_{1}(12: 45)=0.973, p_{2}(12: 45)=$ 0.562 , and $p_{3}(12: 45)=0.310$. The expected value of the number of aircraft occupying gates at $12: 45$ is

$$
E[N(12: 45)]=0.973+0.562+0.310=1.845 \text { aircraft }
$$

If the flights are assumed to behave independently of each other, the variance of the number of aircraft occupying gates at $12: 45$ can be estimated as

$$
\begin{aligned}
\operatorname{Var}[N(12: 45)]=0.973 & (1-0.973)+0.562(1-0.562) \\
& +0.310(1-0.310)=0.486 \text { aircraft }^{2}
\end{aligned}
$$

## APPLICATIONS OF MODEL

## Empirical Realization of Model

Actual operation of aircraft gates at Terminal 2 of Toronto's Lester B. Pearson International Airport has been used to test the performance of the model. The number of gates available to accommodate passenger aircraft at Terminal 2 is 36 . Twentyeight of these gates can accommodate narrow-body aircraft (B727, B737, DC8, DC9) and wide-body aircraft (B747, B767, L1011, L15, DC10). The remaining eight gates can accommodate only small aircraft (CVR, DH8, B10, B99). The data used in the exercise were developed from summaries and logs of daily operations kept by Air Canada, the prime occupant and owner of Terminal 2 gates. Only passenger aircraft that have been assigned to terminal gates were included in the data. Aircraft that have been assigned nonterminal gates (cargo aircraft) were not included.

To put the proposed model in an operational form, it is necessary to describe each flight's behavior relative to its schedule and buffer time. Data limitations made this impractical; therefore, it was important to find, if possible, the common characteristics of some identifiable group of flights and assume that all flights within the group behave in a similar, though random, manner. Analysis of data collected at Terminal 2 showed that flights of different sectors and different aircraft types behave in different manners relative to their schedules. Accordingly, flights were categorized into five groups based on aircraft type and flight sector where transborder flights are those departing to or arriving from the United States:

1. Narrow-body aircraft, domestic flights;
2. Narrow-body aircraft, transborder flights;
3. Wide-body aircraft, domestic flights;
4. Wide-body aircraft, transborder and international flights; and
5. Small aircraft, domestic and transborder flights.

Scheduled arrival and departure times and actual arrival and departure times of flights in each of the five categories were recorded for 30 days of operation in October and November 1988. Arrival lateness, departure lateness, and actual occupancy time distributions were then developed for each category. As an example, Figure 6 shows arrival lateness distributions of the five categories of flights. Aircraft service and buffer times were determined from Air Canada's gate planning guidelines. Using these data, an estimate $\hat{p}_{i}(t)$ of the function $P\left[Y_{i}(t)\right]$ was obtained for each scheduled flight $i$ on Thursday, January 14,1988 , and estimates of $E[N(t)]$ were calculated, using the model, at $1-\mathrm{min}$ intervals. These estimates were compared to the sample mean $\bar{x}_{N(t)}$ calculated from gate occupancy data of 9 weekdays: Monday, January 11 to Friday, January 15 and Monday, January 18 to Thursday, January 21, 1988, and to the scheduled gate occupancy.

As shown on Figure 7, estimates of $E[N(t)]$ are in agreement with $\bar{x}_{N(T)}$ 's for most of the day. Both estimates differ significantly from the scheduled gate occupancy during busy periods (1:00 to 3:00 p.m. and 6:00 to 8:00 p.m.). During these periods, deviation of flights from the schedule causes the actual required number of gates to exceed the number


FIGURE 6 Arrival lateness distributions, all aircraft.


FIGURE 7 Gate occupancy.
specified by the schedule. The maximum scheduled gate occupancy from 1:00 to $3: 00 \mathrm{p} . \mathrm{m}$. is 20 aircraft. Both estimates of gate occupancy show higher values during this period.
The variance of the number of aircraft occupying gates at time $t, \operatorname{Var}[N(t)]$, depends on the degree of correlation existing between flights. One extreme assumption is that of uncorrelated flights. The variance then becomes
$\operatorname{Var}[N(t)]=\sum_{i} \operatorname{Var}\left[Y_{i}(t)\right]$
Another extreme is to assume flights to be perfectly correlated and the variance is

$$
\begin{align*}
\operatorname{Var}[N(t)]= & \sum_{i} \operatorname{Var}\left[Y_{i}(t)\right] \\
& +\sum_{i \neq j}\left(\operatorname{Var}\left[Y_{i}(t)\right]\right)^{1 / 2}\left(\operatorname{Var}\left[Y_{j}(t)\right]\right)^{1 / 2} \tag{21}
\end{align*}
$$

Figure 8 shows estimates of $\operatorname{Var}[N(t)]$ for both cases and the sample variance, $s_{N(t)}^{2}$. The estimate of $\operatorname{Var}[N(t)]$ under the assumption of no correlation is consistently smaller than the sample variance. This implies that some kind of positive correlation occurs between flights. However, this correlation is small as indicated by the magnitude of the sample variance relative to the estimate of $\operatorname{Var}[N(t)]$ under the assumption of perfect correlation.

## Gate Requirements at Airports

At a given time $t$, the random variable $N$ describing the number of gates occupied by aircraft can be thought of as the sum of a large number of independent, but not identically distributed, random variables each of which has a small effect on the sum. It follows from the central limit theorem that $N$ is


FIGURE 8 Gate occupancies, estimated variance.
asymptotically normal with mean $E(N)$ and variance $\operatorname{Var}(N)$. In this context, the number of aircraft gates required at time $t$ can be calculated such that the probability that the demand for gates exceeds this number is very small, say 0.05 . This is the value $n$ such that
$1-F_{N}(n)=0.05$
In terms of $Z$, the standardized normal $(0,1)$ distribution, where
$Z=[N-E(N)] / \sqrt{\operatorname{Var}(N)}$
one should calculate $n$ such that $F_{z}(z)=0.95$. It follows that $z=1.645$ and
$n=E(N)+1.645 \sqrt{\operatorname{Var}(N)}$
The maximum value of $n$ over time of day provides an estimate of the required number of aircraft gates.

Equation 24 assumes a common gate use strategy (firstcome, first-served discipline). Exclusive use of gates by particular air carriers or by flights of certain sectors is another gate use strategy used at many airports. Under such a strategy, Equation 24 can be rewritten for flights of each carrier or sector $k$, as

$$
\begin{equation*}
n_{k}=E\left(N_{k}\right)+1.645 \sqrt{\operatorname{Var}\left(N_{k}\right)} \tag{25}
\end{equation*}
$$

The total required number of gates can then be obtained by aggregating the maximum values of $n_{k}$ 's over time of day.

## Gate Requirements at Terminal 2 of Toronto Airport

A strategy of using gates exclusively for flights of a particular sector is currently adopted at Terminal 2, Lester B. Pearson

International Airport. Fifteen of the 28 available gates that can accommodate both wide- and narrow-body aircraft are dedicated to domestic flights: 8 to transborder flights, and 5 to international flights. Under this strategy and for the schedule of the previous section that represents a typical week-day operation, gate requirements estimated using the procedure described in the preceding comply with the available number of gates. This is expected because the schedule has presumably been designed to account for the available gates and the adopted strategy of their use. For the same schedule, a common use strategy (first-come, first-served discipline) results in a saving of four gates as compared with the existing strategy. An exclusive strategy under which a group of gates is dedicated to Air Canada flights and the remaining gates are used by other air carriers requires a total of 27 gates. Sixteen of these gates are required by Air Canada and the remaining 11 by the other carriers.

## Analysis of Bank Operations

Many airports are used by airlines as collection-distribution centers for their passengers. Flights are brought in and dispatched in banks (groups). The purpose is to facilitate transfer of passengers and baggage between flights in the same bank. If the banks are spaced apart such that interactions between flights of different banks are minimal, gate requirements can be estimated from the scheduled occupancies by taking the size of the largest bank plus one or two gates to account for variations. As the spacing between banks decreases, however, gate requirements increase because of deviation of flights from their schedules. The question is how close the banks could be scheduled without causing an excessive increase in gate requirements. Consider, for example, a schedule that consists of two banks of the same size, say, 25 flights each. Each flight has a scheduled occupancy time of 60 min and all flights are of the same sector and of the same aircraft type. Gate re-

TABLE 1 Gate Requirements under Bank Operations

| Spacing Between Banks <br> $(\mathrm{min})$ | Maximum <br> $\mathrm{E}[\mathrm{N}(\mathrm{t})]+1.645 \mathrm{~V} \operatorname{VAR}[\mathrm{~N}(\mathrm{t})]$ | Required Number of Gates |
| :--- | :--- | :--- |
| 60 | 35.88 | 36 |
| 70 | 29.18 | 30 |
| 80 | 27.21 | 28 |
| 90 | 26.09 | 27 |
| 100 | 25.95 | 26 |
| 110 | 25.85 | 26 |
| 120 | 25.71 | 26 |
| 130 | 25.66 | 26 |
| 140 | 25.66 | 26 |
| 150 | 25.66 | 26 |

quirements, under a common gate use strategy, for different time intervals between the two banks are shown on Table 1. When the flights of the second bank are scheduled to arrive 40 min or later after the departure of the flights of the first bank, 26 gates are required. As the spacing between the two banks decreases, the required number of gates starts to increase and reaches 36 when the departure of the flights of the first bank and the arrival of the flights of the second bank coincide.

## CONCLUSIONS

Two factors have been shown to influence demand for aircraft gates: the flights' schedules and their actual behavior relative to these schedules. The first factor provides a deterministic element to the process of generating demand and the second factor provides a stochastic element to this process. The stochastic gate occupancy model presented in this paper provides a plausible tool for estimating gate requirements at airports based on these two elements. However, improvements can be achieved by further investigation of the cause and the nature of correlation between flights.

The use of computer-generated nominal schedules for estimating loads on terminal facilities has become a common practice at many airports. These schedules are generated from annual forecasts by making assumptions about aircraft fleet mix and load factors. The procedures of the paper provide a tool for incorporating the stochastic nature of flight arrivals and departures in the process of estimating gate requirements using nominal schedules.

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