Drainability of Granular Bases for Highway Pavements

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The best measure of the drainability of a granular base is the minimum degree of saturation that can be achieved through gravity drainage in the field. The amount of water that can drain from a base course depends not only on the physical properties of the material, but also on the cross-sectional geometry of the pavement system. A finegrained base may remain fully saturated under the largest suction that can be developed through gravity drainage. A formula for the minimum degree of saturation in the granular base is developed from Brooks and Corey's formula for water retention in unsaturated porous media. The relationship for drainable porosity in the FHWA subdrainage design manual tends to overestimate the amount of drainage from fine-grained bases and greatly underestimate the amount of drainage from coarse-grained bases. If the minimum degree of saturation for a granular base is sufficiently low, it will drain fairly quickly. The recommended method for the estimation of drainage times is a onedimensional analysis of the saturated flow below the phreatic surface. This analysis accounts for the nonuniform spatial distribution of drainable porosity. Casagrande and Shannon's procedure, which is recommended by FHWA, tends to underestimate drainage times, particularly for base courses that are relatively thin. The recommended procedures for subdrainage analysis have been implemented in the SUBDRAIN computer program of the Kansas Department of Transportation.

Pavements with inadequate subsurface drainage deteriorate much faster than well-drained pavements. If the base course of the pavement is saturated or nearly saturated, wheel loads can cause water and base material to be pumped out through joints and cracks and at pavement edges, which eventually undermines the pavement. Because it is virtually impossible to keep water from entering pavements through joints and cracks over the long run, good drainage is essential for pavement longevity.

The AASHTO procedure for pavement design (1) incorporates a drainage coefficient as a key input. The value of this coefficient depends on the quality of drainage of the pavement system and the percentage of time that the road bed is exposed to moisture levels near saturation. The AASHTO design guide relates the quality of drainage to the time required for the removal of water from the base course, but it does not specify what degree of drainage or level of saturation constitutes "removal." In FHWA's computer program (2), the drainability of the base is measured by time required for a saturated base to drain to water content equal to 85 percent of the water content at saturation.

One measure of the drainability of a base course is its coefficient of permeability (Darcy permeability), k. The coefficient of permeability depends upon the intrinsic permeability of the granular material and the specific weight and viscosity of the fluid.

The relationship is

$$k = \frac{K\gamma}{\mu} \tag{1}$$

in which K is the intrinsic permeability of the granular material and γ and μ are the specific weight and viscosity of the fluid, respectively. The coefficient of permeability, k, has dimensions of L/T (length/time). The intrinsic permeability, K, has dimensions of L^2 .

Another measure of drainability is the lowest degree of saturation that can be achieved through gravity drainage in the field. The degree of saturation, s, is defined as the ratio θ/n , in which θ is the volumetric water content and n is the porosity (the volumetric water content at complete saturation). The lowest degree of saturation that can be achieved in the field through gravity drainage is denoted s_{\min} . The difference between the water content at saturation and the lowest water content that can be achieved in the field through gravity drainage is termed the drainable porosity, n_d . The porosity, the drainable porosity, and s_{\min} are related as follows:

$$S_{\min} = 1 - \frac{n_d}{n} \tag{2}$$

In current practice, the drainable porosity of the base material is usually estimated from the coefficient of permeability by means of a relationship that appears in graphical form in FHWA's report *Highway Subdrainage Design (3)*. The algebraic form of this relationship is

$$n_{\rm d} = 0.0355k^{0.235} \tag{3}$$

for k in meters per day. The corresponding formula for the minimum degree of saturation is

$$s_{\min} = 1 - \frac{0.0355}{n} k^{0.235} \tag{4}$$

for k in meters per day. Equation 3, which is strictly empirical, was fitted to measured values of the coefficient of permeability and the drainable porosity for soils of varied gradations and densities. The report states that it "should be used with caution, particularly at the extremities where data were lacking or were quite scattered." Despite this warning, FHWA's DAMP program (2) obtains the drainable porosity of the granular base from Equation 3 exclusively. It does not allow the user to enter another value for the drainable porosity.

Drainage times are normally estimated, directly or indirectly, from formulas published by Casagrande and Shannon (4) in 1951.

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The methods for estimation of drainage times in FHWA's subdrainage design manual (3) and the DAMP program (2) are based on these formulas. The basic forms of Casagrande and Shannon's relationships were derived through a simplified one-dimensional analysis in which the phreatic surface (water table) was considered planar at all times. To compensate for the error introduced by this approximation, they incorporated an undetermined coefficient in their analysis as a correction factor and used experimental data to determine its values for various conditions.

This paper presents a new analysis of the drainage of a saturated granular base. Starting from basic principles of water retention and flow in porous media, this analysis leads to some new methods for the estimation of minimum degrees of saturation, drainable porosities, and drainage times. It also provides a basis for the evaluation of the current methods. Two example problems illustrate the practical application of the recommended procedures.

MINIMUM DEGREE OF SATURATION

Theory

Any granular material has a characteristic drainage curve that relates the degree of saturation to the pore-water suction head (negative pressure head), ψ . The drainage curve is best determined from measurements of the water content at equilibrium for successively larger suction heads in the laboratory. An approximate drainage curve can be computed from grain-size distribution and bulk density data (5). The drainage curves of most granular materials can be approximated closely by the formula of Brooks and Corey (6),

$$s = \begin{cases} 1 & \psi \leq \psi_a \\ s_r + (1 - s_r) \left(\frac{\psi_a}{\psi}\right)^{\lambda} & \psi > \psi_a \end{cases}$$
(5)

The terms s_r , ψ_a , and λ in Equation 5 are constants for a particular material. The values of these constants are determined by fitting Equation 5 to the data that make up the drainage curve. The constant s_r is termed the residual saturation. It is the degree of saturation that is approached asymptotically at very large suction heads. The constant ψ_a is termed the air-entry head. It is the suction head below which the material remains fully saturated. The dimensionless constant λ is termed the pore-size distribution index. The more uniform the material, the larger the value of λ .

Laliberte et al. (7) showed that the values of s_r , ψ_a , and λ are related to the porosity and intrinsic permeability of the granular material and the specific weight, viscosity, and surface tension of the fluid according to the formula

$$\frac{(1-s_r)n\sigma^2}{K\psi_a^2\gamma^2}\frac{\lambda}{\lambda+2}\approx 5$$
(6)

in which K is the intrinsic permeability of the granular material and γ , μ , and σ are the specific weight, viscosity, and surface tension of the fluid, respectively. The form of this relationship has a theoretical basis. The value of the constant on the right-hand side was determined experimentally.

The drainable porosity of the base course of a pavement depends not only on the physical properties of the material, but also on the cross-sectional geometry of the pavement system. The geometry of the pavement section determines the maximum porewater suction at any point in the base. Figure 1 shows a pavement section with a granular base and edge drains. The subgrade is considered impervious. If the granular base is saturated and then allowed to drain under the force of gravity with no further inflow and no evaporation, drainage will eventually cease. In this state of static equilibrium, the suction head at any point is equal to its height above the water table in the edge drain ($\psi = w + z$) and, from Equation 5, the corresponding minimum degree of saturation at this level, $s_{min}(z)$, is

$$s_{\min}(z) = \begin{cases} 1 & z \leq \psi_a - w \\ s_r + (1 - s_r) \left(\frac{\psi_a}{w + z}\right)^{\lambda} & z > \psi_a - w \end{cases}$$
(7)

At elevations $z \le \psi_a - w$, the granular base will not drain at all. The drainable porosity at any level is the difference between the porosity and the minimum water content at that level:

$$n_d(z) = n \left[1 - s_{\min}(z) \right]$$
(8)

Equation 8 follows from Equation 2. The average minimum saturation at a distance x from the edge drain, $\bar{s}_{\min}(x)$, is the average of $s_{\min}(z)$ over the thickness of the granular base:

$$\bar{s}_{\min}(x) = \frac{1}{d} \int_{mx}^{mx} s_{\min}(z) dz$$
(9)

The evaluation of the right-hand side of Equation 9 leads to an algebraic formula for $\bar{s}_{\min}(x)$:

$$\tilde{s}_{\min}(x) = \begin{cases} 1 & x \leq x_{1} \\ s_{r} + (1 - s_{r}) \left\{ \frac{\psi_{a} - w - mx}{d} \\ + \frac{\psi_{a}^{\lambda}}{d(1 - \lambda)} \cdot [(w + d + mx)^{1 - \lambda} \\ - \psi_{a}^{1 - \lambda}] \right\} & x_{1} < x \leq x_{2} \\ s_{r} + (1 - s_{r}) \frac{\psi_{a}^{\lambda}}{d(1 - \lambda)} [(w + d + mx)^{1 - \lambda} \\ - (w + mx)^{1 - \lambda}] & x > x_{2} \end{cases}$$
(10)



FIGURE 1 Cross section of pavement with granular base and edge drain.

in which

$$x_{1} = \begin{cases} 0 & \frac{\psi_{a} - w - d}{m} < 0\\ \frac{\psi_{a} - w - d}{m} & 0 \le \frac{\psi_{a} - w - d}{w} < L\\ L & \frac{\psi_{a} - w - d}{m} > L \end{cases}$$
(11)

$$x_{2} = \begin{cases} 0 & \frac{\psi_{a} - w}{m} < 0\\ \frac{\psi_{a} - w}{m} & 0 \le \frac{\psi_{a} - w}{m} < L\\ L & \frac{\psi_{a} - w}{m} > L \end{cases}$$
(12)

The average drainable porosity at a distance x from the edge drain, $n_d(x)$, can be determined from the porosity and the average minimum water content at this location as follows:

$$\overline{n}_d(x) = n \left[1 - \overline{s}_{\min}(x) \right] \tag{13}$$

The spatially averaged minimum saturation for the entire base course, S_{\min} , is the average of $\bar{s}_{\min}(x)$ from x = 0 to x = L:

$$S_{\min} = \frac{1}{L} \int_0^L \bar{s}_{\min}(x) dx$$
(14)

The evaluation of the right-hand side of Equation 14 leads to an algebraic formula for S_{min} :

$$S_{\min} = s_r + (1 - s_r) \left\{ \frac{x_1}{L} + \frac{(\psi_a - w) (x_2 - x_1)}{dL} - \frac{m(x_2^2 - x_1^2)}{2dL} - \frac{\psi_a^{\lambda}}{mdL(1 - \lambda) (2 - \lambda)} \cdot \left[(w + d + mL)^{2 - \lambda} - (w + d + mx_1)^{2 - \lambda} - (w + mL)^{2 - \lambda} + (w + mx_2)^{2 - \lambda} \right] - \frac{\psi_a (x_2 - x_1)}{dL(1 - \lambda)} \right\}$$
(15)



FIGURE 2 Minimum degree of saturation versus coefficient of permeability for Example 1.

The average drainable porosity for the entire base course, N_d , can be determined from the porosity and the spatially averaged minimum saturation as follows:

$$N_d = n(1 - S_{\min}) \tag{16}$$

For the base to drain at all, the air-entry head of the granular material must be less than the elevation difference between the top of the base at the crown and the water table in the edge drain $(\psi_a < w + d + mL)$.

The following example illustrates the relationship between the coefficient of permeability of the base material and the minimum degree of saturation. It also provides a comparison of Equations 15 and 16 and Equation 3.

Example 1: Minimum Degree of Saturation for Typical Pavement Section

Problem: The base course of a pavement is to have a slope of 0.02 m/m, a thickness of 0.1 m (3.9 in.), and a half-width (L in Figure 1) of 7.0 m (23.0 ft). The bottom of the drainpipe is to be 0.1 m (3.9 in.) below the bottom of the base (w = 0.1 m). The base is to be constructed of a well-sorted granular material. This type of material would have a porosity of about 0.40, a residual saturation on the order of 0.1, and a pore-size distribution index on the order of 4. The objective is to determine the (spatially averaged) minimum degree of saturation of the base for materials with coefficients of permeability from 10 to 1000 m/day (33 to 3,300 ft/day).

Solution: According to Equation 15, the minimum degree of saturation of the base is determined by four geometric variables (d, w, L, and m) and three properties of the material $(s_r, \psi_a, \text{ and } \lambda)$. Equation 6 provides an estimate of the air-entry head based on other properties of the material and the fluid. The solid curve in Figure 2 shows S_{\min} from Equation 15 for d = 0.1 m (3.9 in.), w = 0.1 m (3.9 in.), m = 0.02 m/m, L = 7.0 m (23.0 ft), $s_r = 0.10$, and $\psi_a = 1.333K^{-1/2}$ for ψ_a in meters and k in meters per day. The formula for ψ_a , from Equation 6, is based on n = 0.40, $\gamma = 9810$ N/m³, $\mu = 0.00131$ N s/m², and $\sigma = 0.0742$ N/m (water at 10°C).

The foregoing example demonstrates that a granular base must be fairly coarse to drain well. In this example, materials with coefficients of permeability less than 15 m/day do not drain at all because their air-entry heads are too large ($\psi_a > w + d + mL$). Materials with coefficients of permeability below 33 m/day (110 ft/day) will remain more than 85 percent saturated. A minimum saturation of 50 percent requires a coefficient of permeability of 64 m/day (210 ft/day). On the other hand, nearly all of the pore water will drain by gravity if the material is very coarse. Materials with coefficients of permeability above 150 m/day (490 ft/day) will drain to below 20 percent of saturation.

Figure 2 also shows Equation 3, the relationship incorporated in FHWA's DAMP program (2). This formula appears to overestimate the amount of drainage from fine-grained material and to greatly underestimate the amount of drainage from coarse-grained materials.

DYNAMICS OF DRAINAGE

One-Dimensional Analysis with Spatially Varied Drainable Porosity

Figure 1 shows the drainage of a granular base with no inflow or outflow through the pavement or subgrade. Drainage starts from

an initial condition of complete saturation at time t = 0. The objective of the analysis is to determine the degree of saturation of **Porosity** the base at times t > 0. The primary direction of flow is downslope

parallel to top of the subgrade. The vertical distribution of porewater pressure is essentially hydrostatic everywhere except over the edge drain and very near x = 0, where the vertical curvature of the streamlines is significant. Most of the flow occurs in the zone of positive pore-water pressures below the phreatic surface (the surface of atmospheric pressure).

The drainage of the base course is analyzed as a problem of one-dimensional unconfined saturated flow in the zone of positive pressures below the phreatic surface, with a spatially varied drainable porosity. The degree of saturation above the phreatic surface at a distance x from the edge drain is assumed to be $\bar{s}_{\min}(x)$. The corresponding water content above the phreatic surface is $n - \bar{n}_d(x)$.

The continuity equation for the flow in the zone below the phreatic surface is

$$\overline{n}_{d}(x) \frac{\partial h(x,t)}{\partial t} - \frac{\partial q(x,t)}{\partial x} = 0$$
(17)

in which h(x,t) is the elevation of the phreatic surface and q(x,t) is the discharge (per unit width) in the -x direction. The equation of motion is the Dupuit discharge formula for unconfined seepage over a sloping bed:

$$q(x,t) = K [h(x,t) - mx] \frac{\partial h(x,t)}{\partial x}$$
(18)

The substitution of the right-hand side of Equation 18 for q(x,t) in Equation 17 yields the governing equation with h(x,t) as the dependent variable:

$$\overline{n}_{a}(x) \frac{\partial h(x,t)}{\partial t} - K \frac{\partial}{\partial x} \left\{ [h(x,t) - mx] \frac{\partial h(x,t)}{\partial x} \right\} = 0$$
(19)

The initial condition is h(x,0) = mx + d, which represents complete saturation with no excess pressure. The lower boundary, x = 0, is the brink of the edge drain. The appropriate boundary condition at this location is a hydraulic gradient of unity (8). The upper boundary, x = L, is the crown of the road. Symmetry requires that no flow cross this boundary. This requirement is satisfied by a horizontal phreatic surface (hydraulic gradient of zero) until h becomes zero. The average degree of saturation at any time can be determined from the phreatic-surface profile:

$$S(t) = \frac{1}{L} \int_0^L \left\{ \bar{s}_{\min}(x) + [1 - \bar{s}_{\min}(x)] \frac{h(x,t) - mx}{d} \right\} dx$$
(20)

This mathematical model of the drainage process has been implemented in the SUBDRAIN computer program of the Kansas Department of Transportation. This program solves Equation 19 for the stated initial and boundary conditions by a nonlinear implicit finite-difference scheme. The program returns the average degree of saturation at the end of each time step. It also returns the time to 85 percent saturation (S = 0.85) and the time to 50 percent drainage ($S = S_{min} + 0.5N_d/n$).

One-Dimensional Analysis with Constant Drainable Porosity

A more approximate analysis with a constant drainable porosity leads to a simple algebraic formula for the time to 50 percent drainage. In this analysis, the spatially averaged drainable porosity, N_d , is substituted for the local drainable porosity, $\overline{n}_d(x)$, in the governing differential equation. With this simplification, the problem can be stated in terms of the dimensionless variables

$$T = \frac{Kmt}{LN_d} \tag{21}$$

$$X = \frac{x}{L}$$
(22)

$$H = \frac{h}{mL}$$
(23)

$$D = \frac{d}{mL} \tag{24}$$

The governing equation is

$$\frac{\partial H(X,T)}{\partial T} - \frac{\partial}{\partial X} \left\{ [H(X,T) - X] \frac{\partial H(X,T)}{\partial X} \right\} = 0$$
(25)

and the initial condition is H = D + X. The boundary conditions can also be stated in terms of these dimensionless variables. H(X,T) is determined entirely by D, the dimensionless thickness of the base.

The degree of drainage at any dimensionless time, U, is the fraction of drainable pore space that has been drained:

$$U = \frac{1 - S}{1 - S_{\min}}$$
(26)

Its value at any time can be calculated from the dimensionless phreatic-surface profile:

$$U(T) = \int_{0}^{1} [H(X,T) - X] dx \qquad (27)$$

Because D determines H(X,T), it also determines U(T). Figure 3 shows the relationship between the dimensionless time to 50 percent drainage, T_{50} , and the dimensionless thickness of the base, D, as determined from numerical solutions of the governing equation for many values of D. These numerical results are fitted closely by the simple empirical formula

$$T_{50} = \frac{0.63}{D + 1.3} \tag{28}$$

In dimensional form, this formula is

$$T_{50} = 0.63 \, \frac{LN_d}{Km} \left(\frac{d}{mL} + 1.3\right)^{-1} \tag{29}$$

Formulas for times to other degrees of drainage could be developed in the same way.

Simplified One-Dimensional Analysis by Casagrande and Shannon

Casagrande and Shannon's model of the drainage process (4) can be expressed concisely in terms of the dimensionless variables T, U, and D as

$$T = C(D) \cdot f(D, U) \tag{30}$$

in which

$$f(D, U) = \begin{cases} 2U - D \ln \frac{D + 2U}{D} & U \le 0.5\\ 1 + \ln \frac{2D - 2UD + 1}{(2 - 2U)(D + 1)} & (31)\\ - D \ln \frac{D + 1}{D} & U > 0.5 \end{cases}$$

and

$$C(D) = 2.45 - 0.8D^{-1/3} \tag{32}$$

The function f(D, U) is the solution for T from their simplified one-dimensional analysis with a planar phreatic surface. The function C(D) is a correction factor that was introduced to better fit the results of some laboratory and field experiments. A formula for the time to 50 percent drainage can be obtained by the substitution of 0.5 for U in Equation 30. This formula is

$$T_{50} = (1.225 - 0.4D^{-1/3}) \left(1 - D \ln \frac{D+1}{D}\right)$$
(33)

Equation 33 is plotted in Figure 3. For large values of D, Equation 33 closely approximates the numerical results from the complete one-dimensional analysis with a constant drainable porosity. For small values of D, Equation 33 appears to underestimate T_{50} considerably.

The following example illustrates the relationship between the coefficient of permeability of the base material and two measures of the drainage time for a typical pavement section. It also provides a comparison of three different methods for estimating these drainage times.



FIGURE 3 Comparison of two approximate methods for time to 50 percent drainage.



FIGURE 4 Time to 85 percent of saturation versus coefficient of permeability for Example 2.

Example 2: Drainage Times for Typical Granular Base

Problem: As in the previous example, the base course of a pavement is to have a slope of 0.02 m/m, a thickness of 0.1 m (3.9 in.), and a half-width (L in Figure 1) of 7.0 m (23.0 ft). The bottom of the drainpipe is to be 0.1 m (3.9 in.) below the bottom of the base (w = 0.1 m). The base is to be constructed of a well-sorted granular material. This type of material would have a porosity of about 0.40, a residual saturation on the order of 0.1, and a pore-size distribution index on the order of 4. The objective is to determine the times to 85 percent saturation (S = 0.85) and the times to 50 percent drainage (U = 0.50) for base materials with coefficients of permeability from 10 to 1000 m/day (33 to 3,300 ft/day).

Solution: The drainage times are estimated by three different methods:

1. Complete one-dimensional analysis with spatially varied drainable porosity from Equations 10-13 (SUBDRAIN program),

2. Complete one-dimensional analysis with a constant, spatially averaged drainable porosity from Equations 15–16 (modified SUBDRAIN program), and

3. Simplified one-dimensional analysis by Casagrande and Shannon (4) with constant drainable porosity from Equation 3 (DAMP program).

Figure 4 compares the results for the time to 85 percent of saturation. These results for Methods 1 and 3 do not differ greatly except for k < 40 m/day (130 ft/day). A comparison of the results for Methods 1 and 2 shows that the more approximate method yields considerably shorter estimates of drainage times for materials with permeabilities less than about 100 m/day (330 ft/day). The drainable porosity is actually larger near the centerline of the road than near the sides because of the difference in elevation. Most of the water that drains from the base must travel a distance greater than L/2 to reach the edge drain. This is why Method 1, which uses spatially varied drainable porosities, yields longer drainage times than Method 2, which uses a spatially averaged drainable porosity. The two methods yield nearly identical drainage times for very coarse materials because the spatial variability of drainable porosity is very small for these materials in this system.

Figure 5 compares the results for the time to 50 percent drainage. The drainage times for Method 3 are based on different drain-



FIGURE 5 Time to 50 percent drainage versus coefficient of permeability for Example 2.

able porosities, and therefore different volumes of drainage, than the drainage times for Methods 1 and 2. Method 2 underestimates the time to 50 percent drainage for materials with permeabilities less than about 100 m/day (330 ft/day) because the spatial variability of the drainable porosity is relatively large for these materials in this system.

CONCLUSIONS

The best measure of the drainability of a granular base is the minimum degree of saturation that can be achieved through gravity drainage in the field. The amount of water that can drain from a base course depends not only on the physical properties of the material, but also on the cross-sectional geometry of the pavement system. The geometry of the pavement section limits the amount of suction that gravity can exert on the pore water. A granular base must be fairly coarse to drain adequately. A fine-grained base may remain fully saturated under the largest suction that can be developed through gravity drainage.

Equation 15 can provide a good estimate of the minimum degree of saturation for a granular base. It incorporates both the water-retention properties of the drainage material and the crosssectional geometry of the pavement. Equation 3, which appears in FHWA's subdrainage design manual (3) and the DAMP computer program (2), tends to overestimate the amount of drainage from fine-grained bases and to greatly underestimate the amount of drainage from coarse-grained bases. Equation 3 does not account for the cross-sectional geometry of the pavement. The recommended method for estimation of drainage times is a one-dimensional analysis of the saturated flow below the phreatic surface, with the local drainable porosities determined from Equation 10. The equation that governs the drainage process is solved numerically by a finite-difference method. If the spatial variability of the drainable porosity is neglected, drainage times are underestimated. The formulas of Casagrande and Shannon (4) underestimate drainage times, particularly for systems in which d/mL << 1.

The recommended procedures for subdrainage analysis have been implemented in the SUBDRAIN computer program of the Kansas Department of Transportation. These procedures could easily be incorporated into FHWA's DAMP program and other similar programs.

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