Pedestrian Speed-Flow-Density Relationships

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Understanding the relationships among pedestrian speed, flow, and density is essential for improving the design and operation of pedestrian facilities. Seven established models relating speed to density for vehicular flow were tested against a set of pedestrian data. The seven models were Greenshields (single-regime linear), May's bell-shaped curve, Underwood's transposed exponential curve, Greenberg's modified exponential curve, Edie's discontinuous exponential form, two-regime linear, and three-regime linear. The evaluation procedure closely follows that developed by Drake, Schofer, and May in 1967. The study site was near the entrance to a pedestrian tunnel that caused a single, extensive queue. The walkway portion closest to the tunnel had a capacity equal to or slightly greater than the tunnel. Pedestrian demand at the location increased from near zero to over capacity and then returned to near zero. Flow parameters were derived from videotape. The performance of each model is described both by the results of statistical tests and by visual examination of the flow-density-speed curves. The three-regime linear model was not found to be statistically significant. Of the three one-regime models, the bell-shaped was judged to be superior to the Greenshields and Underwood models because of its better predictions of optimum density and optimum speed. Of the three two-regime linear models, the Edie was judged best on the basis of statistical tests and predictions of flow parameters. Since two distinct regimes were found, the Edie model was deemed to be the best model for this data set.

A variety of mathematical relationships were examined to describe the relationships among speed, flow, and density in vehicular traffic flow. Pedestrian flow has usually been described by linear relationships between speed and density (*1–6*). At least one researcher has examined a multiregime linear model (*7*). A better understanding of the pedestrian speed-flow-density relationships can be useful to those involved in the design and operation of pedestrian facilities.

This study examined various means to describe pedestrian speed-flow-density relationships. Seven models often used to describe vehicular flow were tested against a pedestrian data set. The procedure closely follows that of Drake et al. for highway flow (*8*). The performance of each model is described by statistical tests and visual examination of the flow-density-speed curves.

SITE SELECTION AND DATA COLLECTION

A site providing data over the widest ranges of speed and density was desired. The site also had to provide an elevated point for video camera placement. The most desirable available site was a pedestrian tunnel entrance in Columbia, Missouri. Significant pedestrian volumes pass through the tunnel after University of Missouri football games, resulting in a single, extensive queue.

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The 30-m-long tunnel has a width of 8.5 m. The paved walkway approach narrows before entering the tunnel. The walkway portion closest to the tunnel was judged to have a capacity equal to or slightly greater than that of the tunnel itself.

The data were collected after a warm 1992 Saturday afternoon football game. The pedestrians had spent over 3 hr watching a narrow defeat of the home team. Pedestrian demand at the location increased from near zero to over capacity and then returned to near zero. A video camera was placed to view a 12-m length of the walkway, which narrows from 14 m to 8.5 m before the tunnel. The average widths of the four 3-m sections were 8.5, 10, 12, and 13 m.

Data were collected during 18.25 min of significant flow. Samples of speed were collected (using a stopwatch and the video image) over four 3-m lengths during 15-sec intervals. The 15-sec time span was deemed long enough to avoid unusual problems with extremely low or high flow characteristics but short enough to avoid a high percentage of time periods with varying flow characteristics within the time period. The number of pedestrians within each 3-m length was determined at the midpoint of each interval. Time mean speed was virtually identical to space mean speed because of the low variability of speed within each interval. Flow rate was derived from the product of speed and density. Data characteristics include the following:

Parameter	*Low*	*Mean*	*High*
Density (ped./m^2)	0.16	1.61	3.12
Speed (m/min)	11.5	37	73
Flow (ped./min/m of width)	9	46	75

Since the calibrated flow relationships were to use density as the independent variable, one potential problem was that some ranges of density were much more frequently represented than others. As expected, the least frequent density ranges were those near the likely critical density (*4*). To avoid biasing the regression analysis, a random sampling procedure similar to that described by Drake et al. (*8*) was used to provide equal representation from all density ranges. This procedure resulted in 15 data points for each 0.537-ped./m^2 increment of density, or 90 data points from the original 292.

ALTERNATIVE HYPOTHESES

The seven hypotheses relating speed to density examined by Drake et al. (*8*) for vehicles are examined here for pedestrians. The models are thoroughly described by Drake et al. Additional descriptions are available elsewhere (*1–4,9,10*).

DISCUSSION OF STATISTICAL ANALYSIS TECHNIQUES

The speed-density hypotheses were analyzed to verify the significance of the models and the ability of each model to predict the flow parameters. The procedure employed by Drake et al. (*8*), with some modifications, was used to compare the models.

Regression Analysis and Statistics

Linear regression was used to calibrate the models. Of the seven speed-density models, three were linear models and four were nonlinear models. Nonlinear models were reduced to linear forms using a transformation upon density or speed.

Discontinuous models were developed by minimizing the sum of squares about the regression line for each regime. The composite statistics for discontinuous regression were calculated by integrating the results of the separate regressions. For example, the single r^2-value for a multiregime model was based upon the total sum of squared errors from the mean speed of the entire sample and the sum of the residual errors in speed estimates.

Testing of Multiregime Hypotheses

Quandt (*11,12*) has recommended a maximum likelihood technique for estimating parameters of a linear regression system obeying two separate regimes. Quandt's technique, with some extensions developed by Drake et al. (*8*), is used here.

Other Statistical Tests

The t-test was employed to identify nonzero slopes. Significance of the entire regression was based upon F-values for the ratio of regression mean square to residual mean square.

ANALYSIS OF RESULTS

Break-Point Analysis

Quandt's break-point analysis (*8,11,12*) was performed for the four discontinuous hypotheses. The three two-regime models were investigated for 11 break points. The likelihood functions for the Edie and Greenberg hypotheses showed only one local peak and indicated a break point at density 1.075 ped/m^2. The two-regime linear hypothesis showed three local peaks and indicated a break point of 1.881 ped/m^2.

The three-regime model required analysis of 54 combinations of break points. The optimal break points were 1.075 and 2.15 ped/m^2.

Tests for Distinctly Separate Regimes

Test results for distinctly separate regimes are given in Table 1. F-tests were employed to investigate the existence of multiregimes. In the three-regime linear model, the two higher-density lines did not appear to be statistically different at the 95 percent confidence level. All the other models (the three two-regime models) showed significant differences between regimes.

Tests for Nonzero Slope and Entire Regression

The t-test (Table 2) indicated that all slopes were different from zero at the 0.05 level of significance. However, at the 0.01 level, the free flow regime of the three-regime linear model and the free flow regime of the Edie model were not shown to have slopes different from zero. All models showed high significance for the entire regression (F-test in Table 2).

INTERPRETATION

Results of the statistical tests must be tempered with judgment based upon knowledge from previous studies and from the data of this study. The results of the regression analyses are shown in Table 2. Flow parameters for each calibrated model are in Table 3, along with the authors' judgment of the probable ranges indicated by the data. The field data are shown against the flow models in Figures 1 through 7.

TABLE 1 Results of F-test for Distinctly Separate Regimes

	Test Parameters		
Hypothesis	Test	Calculated F	F-critical (α=0.05)
Greenberg	1 on 2	14.01	1.73
	2 on 1	37.44	1.74
Edie	1 on 2	2.46	1.65
	2 on 1	34.44	1.74
2-regime linear	1 on 2	3658	1.73
	2 on 1	26.43	1.74
3-regime linear	1 on 2	2.662	1.90
	2 on 1	8.533	1.88
	2 on 3	[a]1.745	1.90
	3 on 2	15.36	1.91

[a]2nd and 3rd lines of the 3-regime model do not differ from one single line at α=0.05. In all other hypotheses, all tests reveal significant differences indicating that separate regimes exist.

TABLE 2 Regression Analysis Summary

Hypothesis	Equation	r^2	S_e	${}^aF$-Test Value	${}^bt$ value for non zero slope	Signif. Diff. between regimes
Greenshields	S=63.97-17.12D	0.84	7.0	453	21.3	N.A.
Bell shape	S=55.6e^(-0.162D^2)	0.84	6.9	473	26.2	N.A.
Underwood	S=75.17e^(-D/4.166)	0.79	8.0	323	23.6	N.A.
Greenberg	S=58	0.83	7.1	439	N.A.	
	[D<1.07]					YES
	S=36.78ln(4.32/D)				20.1	
	[D≥1.07]					
Edie	S=60.83e^(-D/4,166)	0.84	6.8	474	1.8	
	[D<1.07]					YES
	S=36.78ln(4.32/D)				21.1	
	[D≥1.07]					
2-regime	S=62.81-15.34D	0.84	6.8	478	7.0	
linear	[D<0.188]					YES
	S=50.37-12.15D				7.6	
	[D≥0.188]					
3-regime	S=60.91-11.94D	0.85	6.7	499	1.8	
linear	[D<1.07]					NO
	S=72.06-21.53D				6.8	
	[1.07≤D<2.15]					YES
	S=40.35-8.56D				4.4	
	[D≥2.15]					

${}^aF_{critical} = 6.97$ at $\alpha = 0.99$ ${}^bt_{critical} = 2.37$ at $\alpha = 0.99$

$F_{critical} = 4.01$ at $\alpha = 0.95$ $t_{critical} = 1.66$ at $\alpha = 0.95$

Statistical Test Results

The results of the statistical tests can be summarized as follows.

1. All models satisfied the tests for significance of the entire regression.
2. All models satisfied tests for slopes different from zero at the 0.05 level of significance.
3. The three-regime linear model failed the test for three distinctly separate regimes.
4. The three two-regime models were each shown to identify separate regimes.
5. In each two-regime model there was a significant difference in standard error between the two regimes. The standard error was approximately 2.6 times larger in the free-flow regime than in the congested-flow regime.
6. The overall standard errors of the speed estimates ranged from 6.7 to 8.0 m/min. Excluding the Underwood model, the standard errors were in a narrow range from 6.7 to 7.1 m/min.
7. The r^2-values for the seven models ranged from 0.79 to 0.85. Excluding the Underwood model, the r^2-values were in a narrow range from 0.83 to 0.85.

Flow Parameter Results

A comparison of the model parameters with the field data and *Highway Capacity Manual* (HCM) parameters is presented below.

TABLE 3 Flow Parameter Summary

Hypothesis	Free flow speed, S_f (m/min)	Jam Density, D_j (ped/m^2)	Optimum Density, D_o (ped/m^2)	Optimum Speed, S_o (m/min)	Capacity (ped/m/min)
Data Set (subjective)	52-70	-	1.3-1.8	34-49	62-72
Greenshields	64	3.73	1.87	32	59
Bell shape	56	-	1.75	34	59
Underwood	54	-	1.89	28	52
Greenberg	54	4.32	1.59	37	59
Edie	61	4.32	1.59	37	59
2-regime linear	63	4.13	1.88	32	66
3-regime linear	61	4.71	1.68	37	62

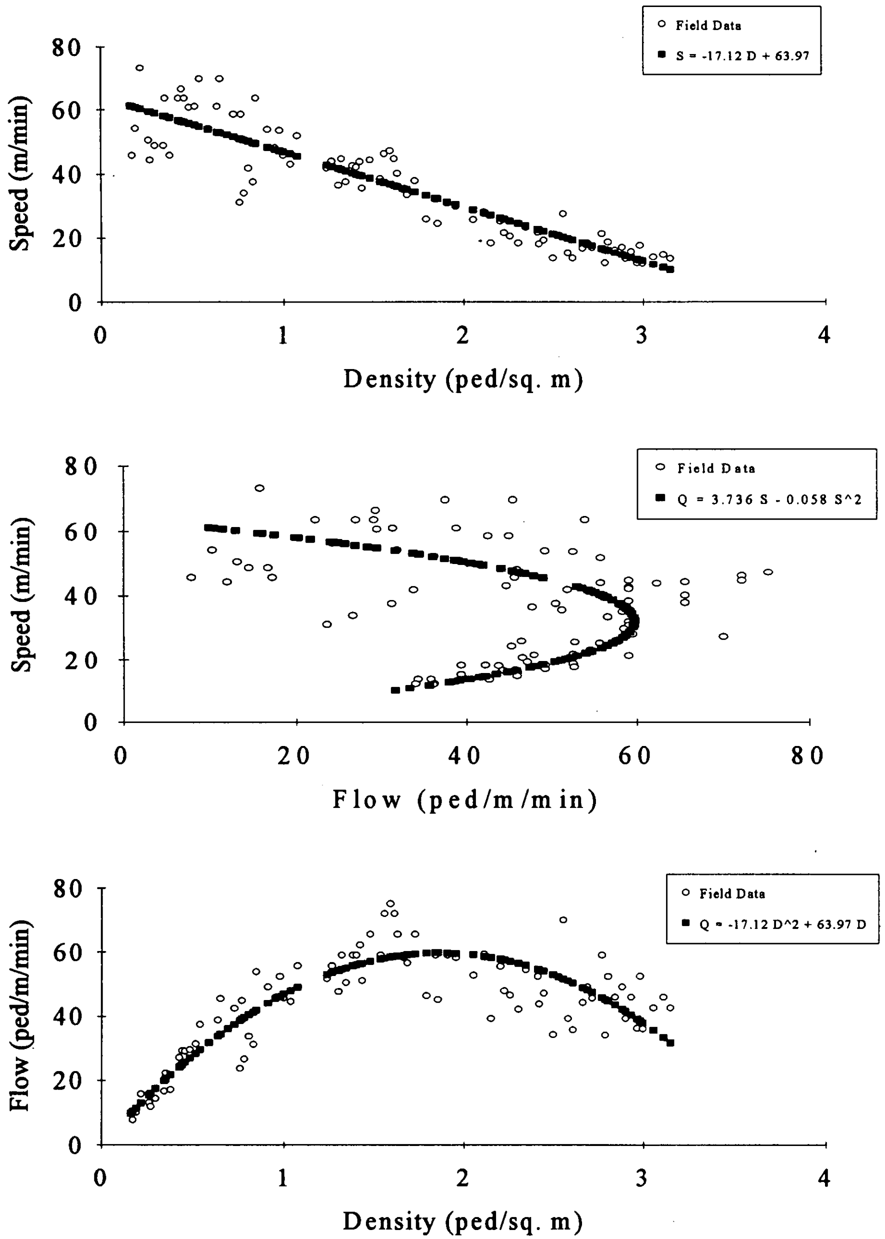

FIGURE 1 **Speed-density-flow plots for one-regime linear model.**

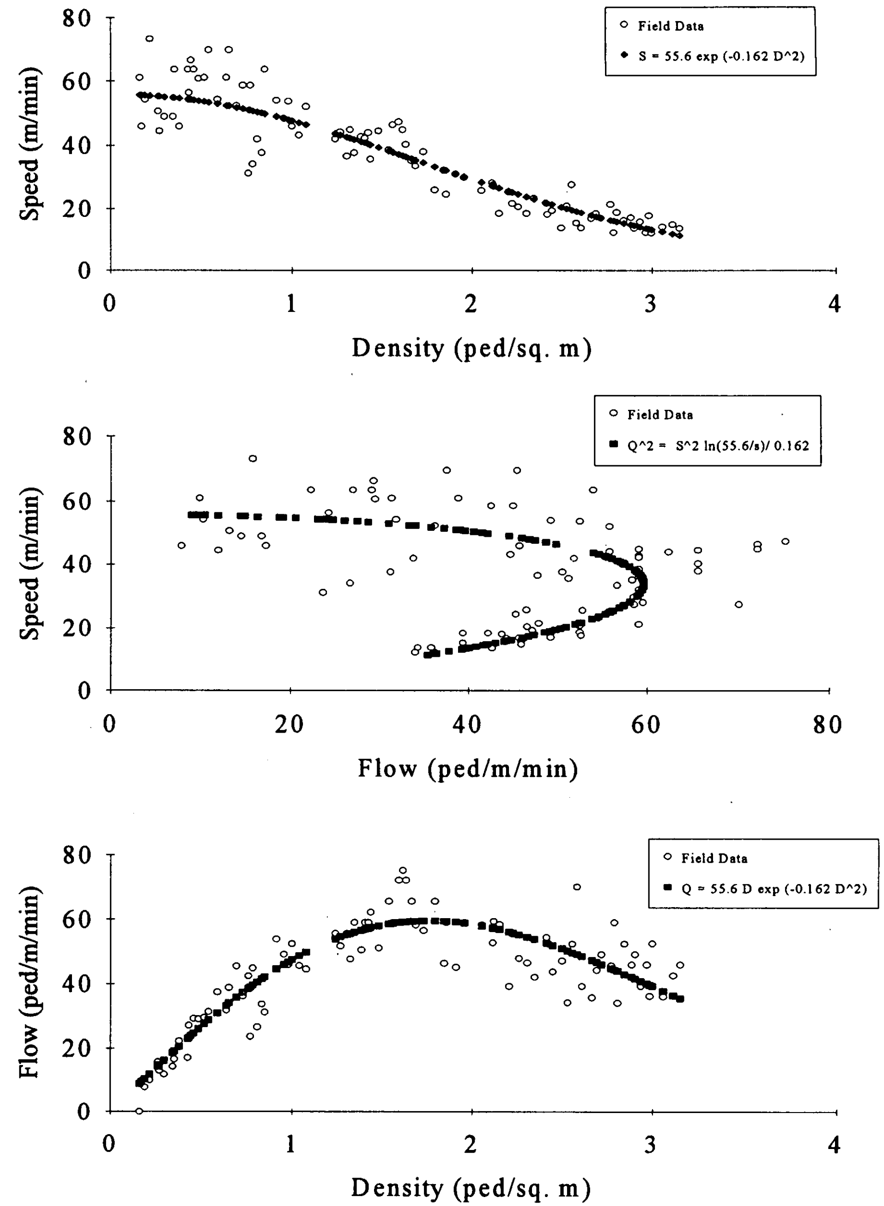

FIGURE 2 **Speed-density-flow plots for May's bell-shaped model.**

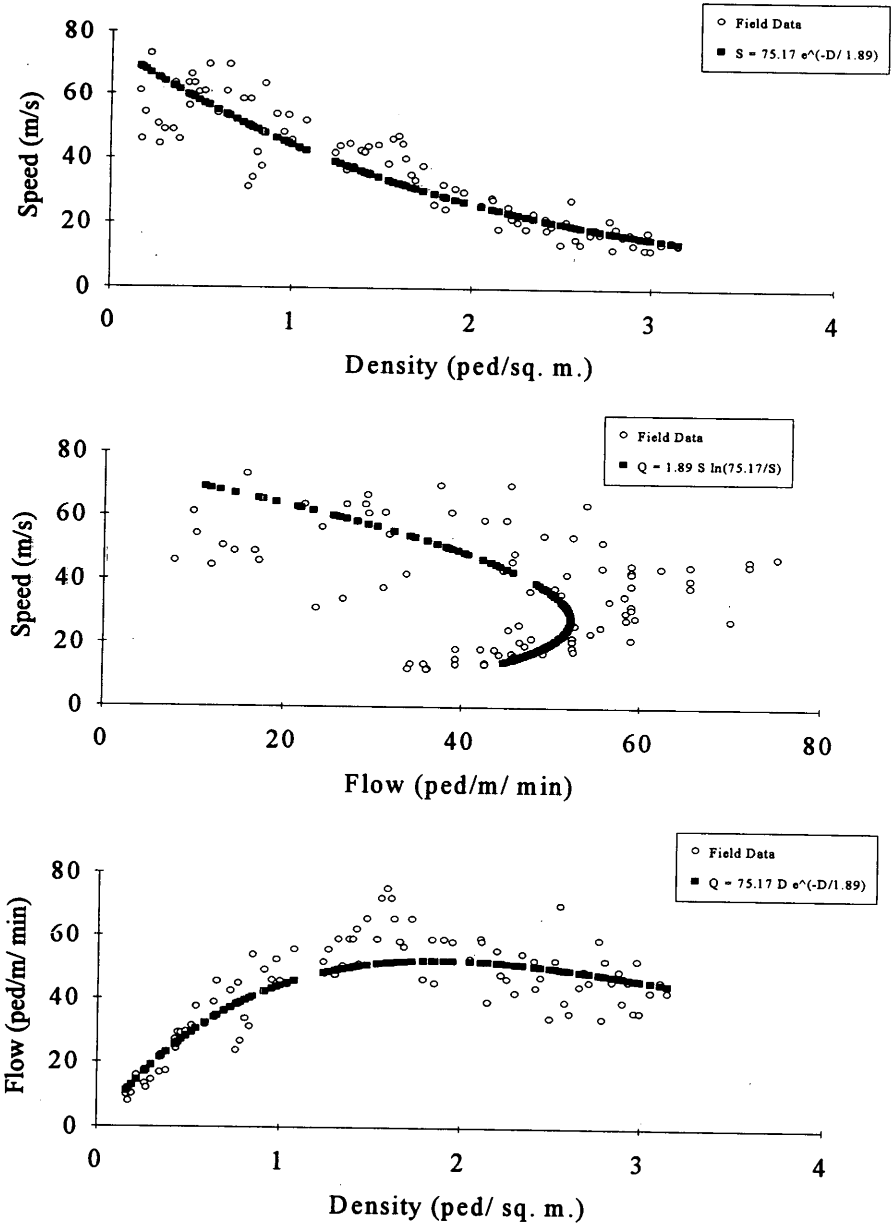

FIGURE 3 Speed-density-flow plots for Underwood model.

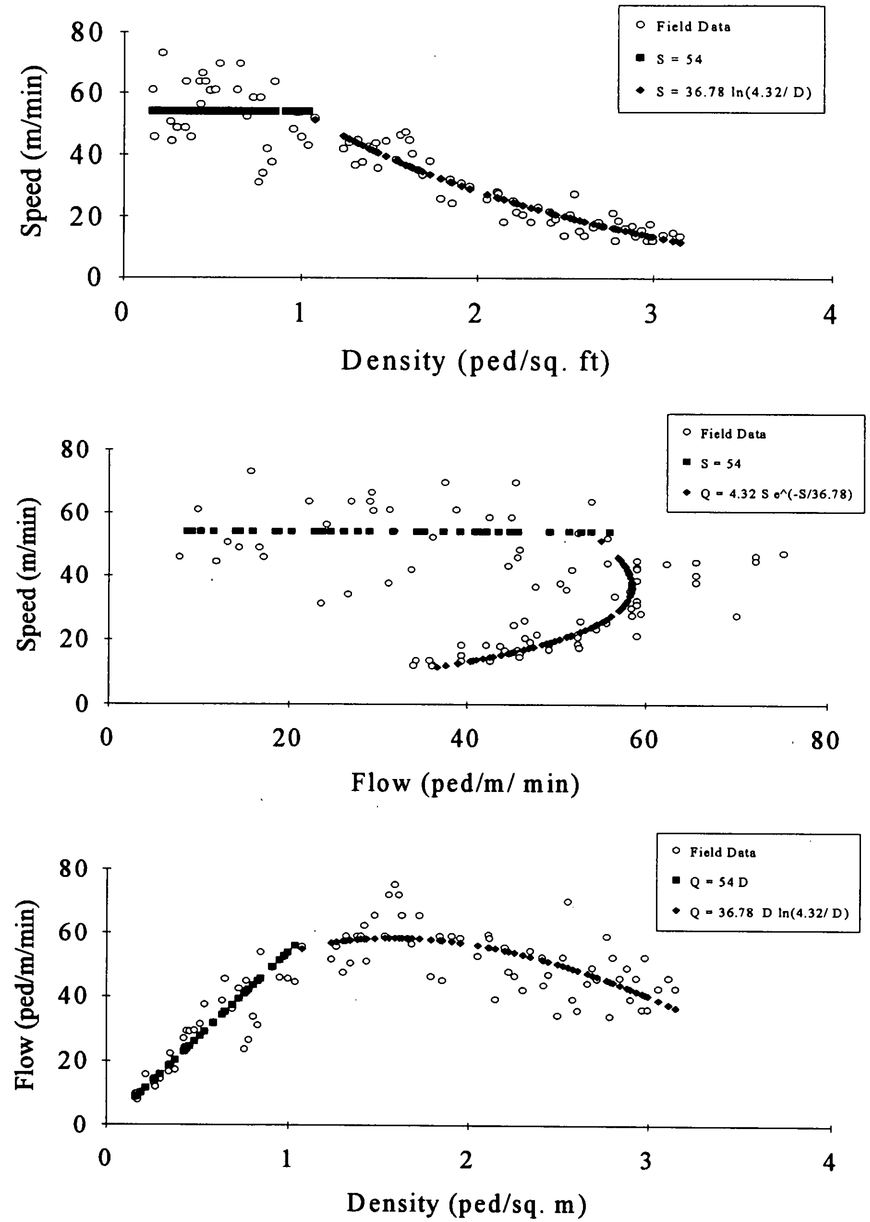

FIGURE 4 Speed-density-flow plots for Greenberg model.

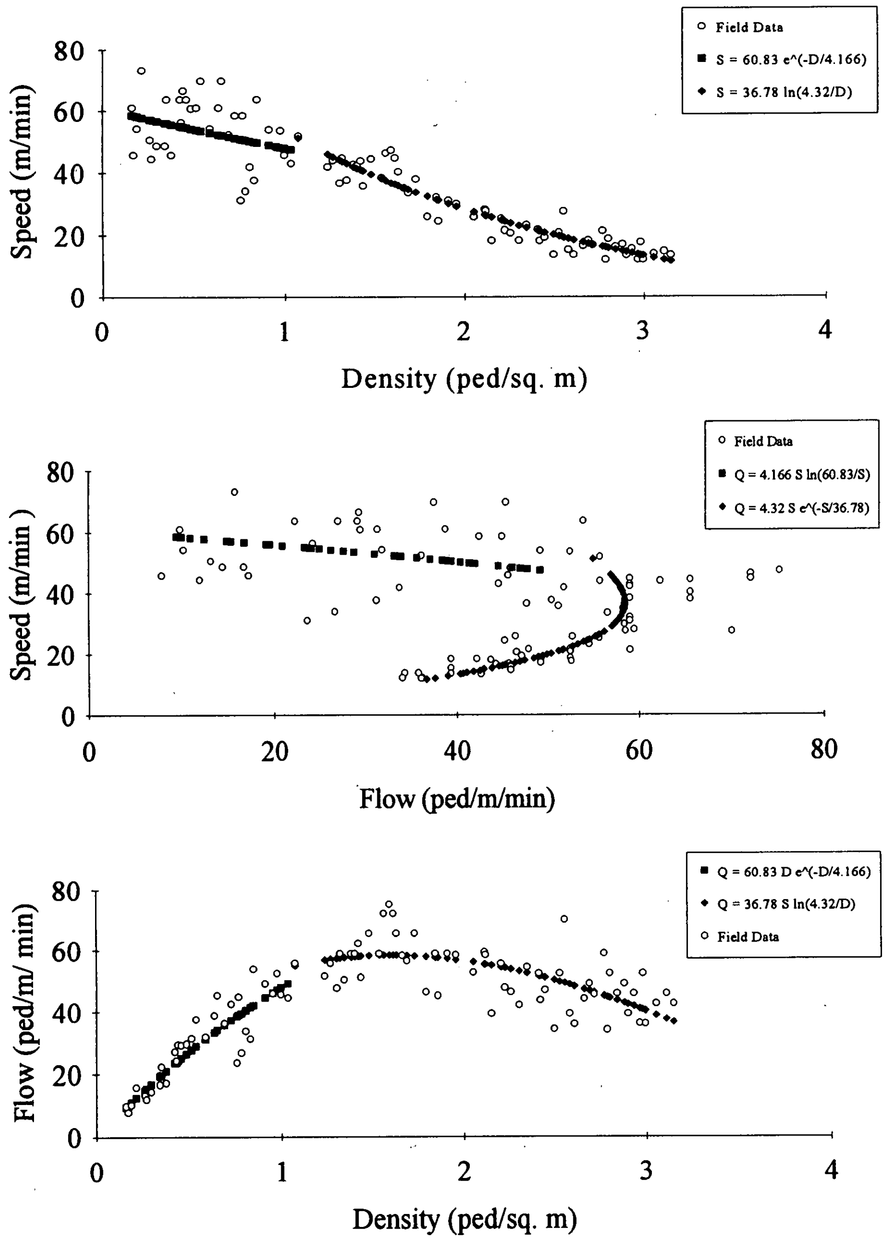

FIGURE 5 Speed-density-flow plots for Edie model.

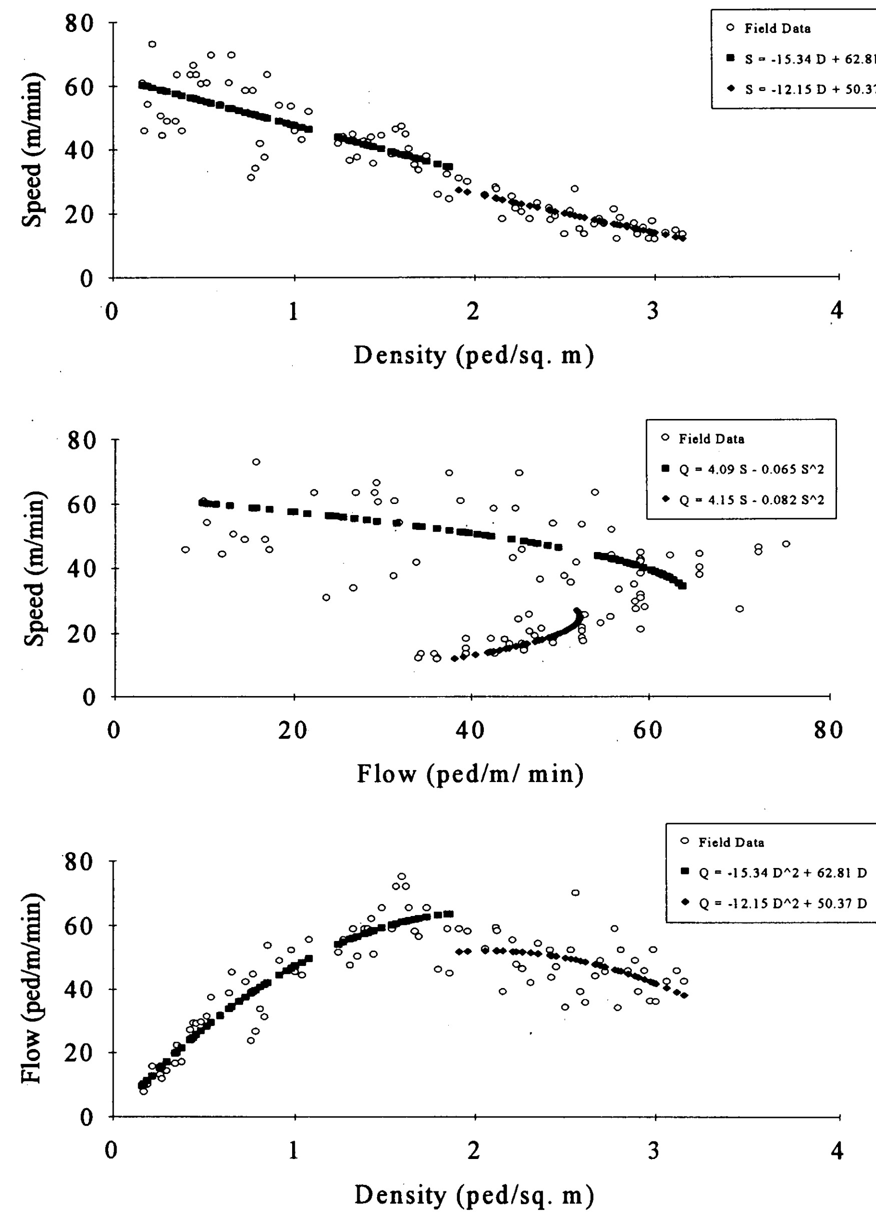

FIGURE 6 Speed-density-flow plots for two-regime linear model.

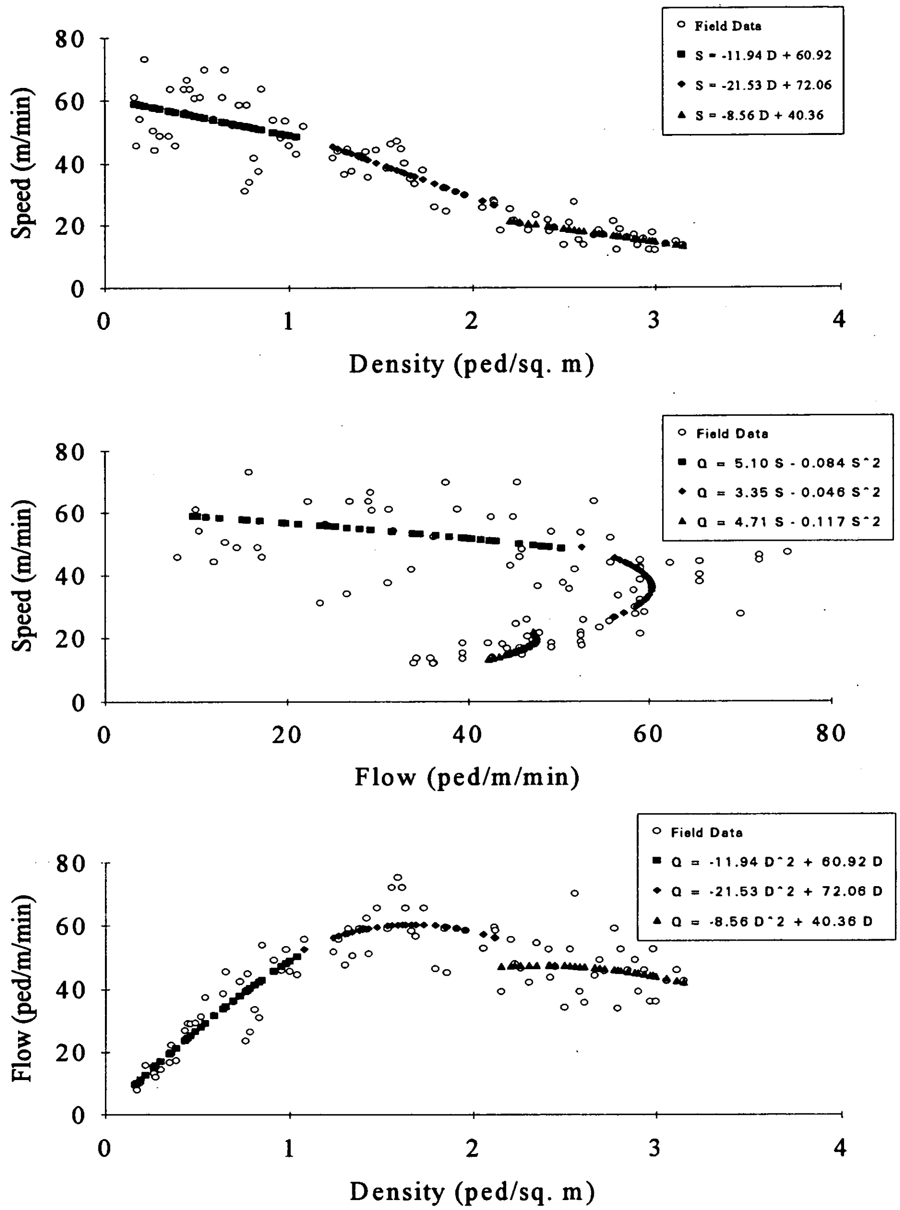

FIGURE 7 Speed-density-flow plots for three-regime linear model.

1. The two-regime and three-regime linear models gave fairly good estimates of maximum flow (66 and 62 ped/min/m, respectively) when compared with the field data. The Underwood model gave the lowest (and poorest) estimate (52 ped/min/m). The HCM (*1*) gives a capacity of 82 ped/min/m of walkway width.

2. Optimum densities predicted by the models ranged from 1.59 ped/m^2 to 1.89 ped/m^2. The data indicate an optimum density of about 1.3 to 1.8 ped/m^2. The HCM defines density at capacity as 1.8 ped/m^2.

3. The data indicate an optimum speed of around 35 to 50 m/min. The three-regime linear, Greenberg, and Edie models had optimum speeds within this range (37 m/min). All other models had lower optimum speeds.

4. The jam densities predicted by the models ranged from 3.73 to 4.32 ped/m^2. The highest recorded density was 3.149 ped/m^2, but the flow was not zero at this density.

5. The free-flow speeds predicted by the models (ranging from 54 to 64 m/min) appeared reasonable when compared with the data but appear low when compared with those of other studies (*1–7*). Perhaps sitting for over 3 hr on a warm, sunny day caused the pedestrians in this study to have relatively low speeds under free-flow conditions. The videotape also revealed that after the queue had dissipated, the pedestrians who approached the tunnel (i.e., the last to leave) seemed to walk at a slow pace for prevailing conditions. Perhaps these pedestrians were atypical of most pedestrians and biased the estimates of free-flow speed downward.

CONCLUSIONS

The three-regime linear model failed the test for significantly different equations for the free-flow and transitional-flow regimes.

The data do not support the theory that three separate regimes exist.

Of the three one-regime models, the Greenshields and May bell-shaped models had similar r^2-values and similar estimates of capacity. The Greenshields optimum density appeared to be high, whereas predicted optimum speed and capacity appeared to be low. May's bell-shaped curve provided better predictions of optimum density and optimum speed. Underwood's model had the worst r^2 and worst estimates of optimum density, optimum speed, and capacity. Of the three one-regime models, May's bell-shaped curve appears to be best for this data set.

The existence of two separate regimes was supported by the data. Significantly different curves apply to the two regimes, and the standard error is much larger in the free-flow regime than in the congested-flow regime. The three two-regime models had similar r^2-values. The two-regime linear model capacity estimate seemed reasonable, but its optimum density was too high and its optimum speed was too low. The Greenberg and Edie models gave slightly low estimates of capacity but provided good predictions of optimum density and optimum speed. An argument against the Greenberg model is that the data indicate a significantly negative slope for the range where the Greenberg model uses a constant speed. For the above reasons, the Edie model was judged to be best among the two-regime models.

Since the data were limited to one particular site, the results should not be viewed as universally applicable. However, the results indicate that further study is likely to lead to the conclusion that a multiregime (probably a two-regime) model is a better descriptor of flow on pedestrian facilities than the Greenshields one-regime linear model.

REFERENCES

1. *Special Report 209: Highway Capacity Manual.* TRB, National Research Council, Washington, D.C., 1985, Chapter 13.
2. Fruin, J. J. *Pedestrian Planning and Design.* Metropolitan Association of Urban Designers and Environmental Planners, New York, N.Y., 1971.
3. Pushkarev, B., and J. Zupan. *Urban Space for Pedestrians.* MIT Press, Cambridge, Mass., 1975.
4. May, A. D. *Traffic Flow Fundamentals.* Prentice Hall, Englewood Cliffs, N.J., 1990, pp. 315–317.
5. Tanaboriboon, Y., and J. Guyano. Level-of-Service Standards for Pedestrian Facilities in Bangkok: A Case Study. *ITE Journal,* Nov. 1989, pp. 39–41.
6. Tanaboriboon, Y., S. Hwa, and C. Chor. Pedestrian Characteristics Study in Singapore. *Journal of Transportation Engineering,* ASCE, Vol. 112, No. 3, 1986.
7. Polus, A., J. Schofer, and A. Ushpiz. Pedestrian Flow and Level of Service. *Journal of Transportation Engineering,* ASCE, Vol. 109, No. 1, pp. 46–56.
8. Drake, J. S., J. L. Schofer, and A. D. May. A Statistical Analysis of Speed Density Hypotheses. In *Highway Research Record 154,* HRB, National Research Council, Washington, D.C., 1967, pp. 53–87.
9. Greenshields, B. D. A Study of Traffic Capacity. *HRB Proc.,* Vol. 14, 1934, pp. 448–474.
10. McShane, W. R., and R. P. Roess. *Traffic Engineering.* Prentice Hall, Englewood Cliffs, N.J., 1990, pp. 290–299.
11. Quandt, R. E. The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes. *Journal of the American Statistical Association,* Vol. 53, 1958, pp. 873–880.
12. Quandt, R. E. Tests of the Hypothesis That a Linear Regression System Obeys Two Separate Regimes. *Journal of the American Statistical Association,* Vol. 55, 1960, pp. 324–330.

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