From Single-Stage to Two-Stage Facility Location Model of Connecticut’s Highway Maintenance System

CHRISTIAN F. DAVIS AND GERARD M. CAMPBELL

The development of two facility location models for use in optimizing the locations of maintenance facilities for the Connecticut Department of Transportation (ConnDOT) is described. In the ConnDOT system vehicles and other equipment are stored at 55 roadway garages and are maintained at 13 repair facilities. In the single-stage model the optimum number of repair facilities is sought without changing the existing configuration of 55 roadway garages. In the two-stage model the optimum numbers of both repair facilities and roadway garages are sought. In both cases the solutions are achieved by using a mixed-integer programing formulation. The objective is to minimize the sum of annual transportation costs plus annualized overhead and expansion costs while maintaining acceptable levels of service. On the basis of the results from the single-stage model and external considerations, two options were recommended to ConnDOT management. The first option closes three repair facilities with an estimated net present value of savings of $5.0 million. The second option closes two repair facilities and has an estimated net present value of savings of $3.1 million. In addition to an explanation of how the single-stage model is built on to attain the two-stage model, a detailed description of the data gathering required to achieve this extension is given.

The Connecticut Department of Transportation (ConnDOT) operates 55 garages serving 8900 centerline-km (5,500 centerline-mi) of roadway. The vehicles and other equipment stationed at these garages and at 21 other locations are maintained at 13 repair facilities. This paper describes the development of two related optimization models to represent the maintenance system. The first model considers consolidation of repair facilities without changing the existing configuration of 55 roadway garages. Details of the solutions and the consequent recommendations made on the basis of this single-stage model are given. Then the modifications required to build on the single-stage model to develop the two-stage model are described. In the two-stage model consolidation of the 13 repair facilities and reconfiguration of the 55 roadway garages are both allowed.

SINGLE-STAGE MODEL

Existing System

In 1990 a report issued by a special state commission (1) suggested that economies might be realized by a consolidation of ConnDOT’s 13 vehicle repair facilities. In response to that suggestion ConnDOT commissioned a study by the Transportation Institute of the University of Connecticut to determine the optimum configuration of vehicle repair facilities. A detailed description of that study and the resulting recommendations have been given by Campbell and Davis (2). This section summarizes that work.

The assignment of roadway garages to repair facilities in the existing highway maintenance system is shown in Figure 1. Note that the repair facilities also serve vehicles housed at various locations (shown by triangles) other than the garages, such as vehicles used for electrical repair and bridge repair. All repair facilities are assumed to perform essentially all types of repairs.

A wide variety of equipment is housed at the garages represented in Figure 1. Overall there are approximately 700 heavy trucks, more than 500 light trucks, almost 800 pieces of heavy equipment such as tractors and sweepers, approximately 1,000 pieces of light equipment such as hand mowers and chain saws, and more than 2,000 other pieces of equipment requiring some maintenance.

Single-Stage Formulation

The model used for the present study is a version of what Aikens (3) refers to as the “static, capacitated facility location problem.” The model’s key decision variables relate to whether or not facilities are kept open. Also included are sets of variables that allow for the capacity expansions at open facilities that are necessitated by the reassignment of equipment. In the single-stage mixed-integer programing model shown here, f indexes over repair facilities and i indexes over roadway garages.

Minimize

\[ \sum_{i=1}^{13} \sum_{f=1}^{55} C_{if} X_{if} + \sum_{f=1}^{13} (K_f Y_f + MA_f + NB_f) \] (1)

Subject to

\[ \sum_{i=1}^{55} D_{if} X_{if} = (P_f Y_f + 2RA_f RB_f) \quad (f = 1 \text{ to } 13) \] (2)

\[ \sum_{f=1}^{13} X_{if} = 1 \quad (i = 1 \text{ to } 76) \] (3)

\[ A_f \leq Y_f \quad (f = 1 \text{ to } 13) \] (4)

\[ B_f \leq 8A_f \quad (f = 1 \text{ to } 13) \] (5)
FIGURE 1 Assignment of roadway garages to repair facilities in ConnDOT's existing highway maintenance system.

\[ B_i, X_{ij} \geq 0 \quad (i = 1 \text{ to } 76, \ f = 1 \text{ to } 13) \]  \
\[ Y_f, A_f \in \{0, 1\} \quad (f = 1 \text{ to } 13) \]

where

- \( Y_f = 1 \) if facility \( f \) is kept open and 0 otherwise;
- \( A_f = 1 \) if two work bays are added at facility \( f \) and 0 otherwise;
- \( B_f \) = number of work bays added, above two, at facility \( f \);
- \( X_{ij} \) = the proportion of garage \( i \)'s repair requirements satisfied by facility \( f \);
- \( C_{if} \) = total annual cost of transporting all equipment from garage \( i \) to facility \( f \) for repairs;
- \( K_f \) = annualized cost of keeping facility \( f \) open;
- \( M \) = annualized cost of adding two work bays at a facility;
- \( N \) = annualized cost of adding each work bay above two at a facility;
- \( D_i \) = total equipment repair requirements at garage \( i \);
- \( P_f \) = repair capacity, in terms of equipment serviceable, if facility \( f \) is kept open; and
- \( R \) = repair capacity added by adding a work bay at a facility.

Expansion costs also have one-time and annual components. The one-time component is the cost of building new work bays, and the annual part reflects additional building expenses resulting from expansion. When the total costs for expansions of various sizes were estimated, it was found that they could be accurately generalized by a function corresponding to the portion of the objective function that includes the \( A_f \) and \( B_f \) decision variables.

Transportation costs are the third type of cost included in the model's objective function. The total annual costs incurred if all equipment from roadway garage \( i \) is sent to repair facility \( f \) for repair, \( C_{if} \), were estimated by using cost functions that consider equipment quantities, travel distances, travel times, and expected numbers of visits per year.

**Constraints**

Before explaining the problem's constraints, a brief description of the quantification of repair capacities and requirements is in order.

The capacity of a repair facility, in terms of equipment serviceable, is limited primarily by its number of work bays and number of mechanics. Since the total equipment repair requirements do not change when equipment is reassigned to different repair facilities, the systemwide number of mechanics remains fixed and capacities are based strictly on numbers of work bays.

Figure 2 shows the equipment served versus the number of work bays for each active repair facility. There are dozens of different types of equipment within the system, and each has different maintenance requirements. To quantify the service requirements, six equipment categories were defined on the basis of similarities of maintenance requirements. Each "equipment served" value shown in Figure 2 represents the weighted average quantity of equipment served by a repair facility.

From Figure 2 it is apparent that excess capacity exists at some repair facilities. By choosing as benchmarks the repair facilities that serve large amounts of equipment for their numbers of bays,
an efficient frontier was established. For each repair facility not on the efficient frontier, the frontier was used to find the value of "equipment serviceable" associated with the number of work bays at the facility, as illustrated in Figure 3.

Constraint Set 2 (Equation 2) states that for each repair facility the total repair requirements served cannot exceed the available capacity in terms of equipment serviceable. Constraint Set 3 specifies that all repair requirements must be satisfied for each roadway garage.

Constraint Sets 4 and 5 pertain to expansions at repair facilities. According to Constraint Set 4 an expansion of two work bays cannot be done unless the repair facility is kept open. Constraint Set 5 ensures that an expansion beyond two work bays cannot be done unless the initial two bays are added, and it also limits the number of bays added above two to a maximum of eight.

Constraint Sets 6 and 7 are standard nonnegativity and binary variable constraints, respectively.

**Single-Stage Results**

The optimal solution of the single-stage problem called for elimination of 6 of 13 repair facilities, with a net present value of savings of $7.1 million. Such a dramatic change from the existing system was unexpected, particularly since many of the model’s underlying assumptions are conservative and favor the status quo.

In practical terms the closing of almost half of the repair facilities would cause considerable disruption. Therefore a set of runs to investigate the savings offered by solutions with more than seven repair facilities was designed by adding a constraint to the model that specified an allowable number of repair facilities. The problem was then resolved with the allowable number of repair facilities fixed at levels ranging from 8 to 13. The results of these runs are shown in Figure 4.

**External Considerations**

Consideration of factors outside the context of the model was important because the model could not capture all aspects of the real problem. The model's objective function, with its net present value criterion, reflects an emphasis on minimizing costs. This was practical, given the impetus for the study, but it must be recognized that other criteria are important as well. Ease of implementation, the levels of service provided, and robustness in the face of uncertainties are all factors not reflected in the model’s objective function. Nevertheless such factors had to be considered as recommendations were being developed.

**Single-Stage Recommendations**

On the basis of the results from the single-stage model and external considerations, two options were recommended to ConnDOT. The first option consisted of closing three repair facilities for an estimated savings of $5.0 million. Note from Figure 4 that this solution captures about 70 percent of the savings offered by the seven-facility solution, although only half as many repair facilities would be closed. In addition each of the closed facilities could be sold, because there are no other major operations on those sites. However this option has two disadvantages. First it calls for closing East Haven, which is one of the finest facilities in the system. Second it calls for expansion at Wethersfield, which would be extremely difficult because of its suburban location. The second option retains East Haven and calls for no expansion at Wethersfield, but saves only $3.1 million.
FIGURE 3 Use of "efficient frontier" to determine repair facility capacity.

FIGURE 4 Savings offered by solutions with seven or more repair facilities.
Implementation

From the outset great care was taken to involve ConnDOT management in the investigation. Their assistance was vital in a number of areas, including cost and parameter estimation and input regarding issues external to the mathematical model. Nevertheless at the beginning of the single-stage study ConnDOT was in a position of having to react to the initial commission recommendations within a limited time frame. At the conclusion of the single-stage study it was obvious that a study that is more proactive and that at the same time recognizes the potential for consolidating roadway garages would have been more desirable. Therefore rather than proceeding directly to implementing the results of the single-stage study a second study was undertaken to examine the two-stage problem.

TWO-STAGE MODEL

The two-stage facility location model builds on the single-stage model to allow for consolidation of the roadway garages as well as the equipment repair facilities. In this section the formulation of the two-stage model is described.

The two stages in the newly developed model correspond to the two types of facilities mentioned above—that is, equipment repair facilities and roadway maintenance garages. As before all repair facilities are assumed to have the same repair capabilities in terms of the types of repairs that they perform. Similarly all roadway garages are assumed to provide the same types of roadway maintenance.

Besides the two stages represented by the repair facilities and roadway garages, a third level exists within the maintenance system, namely the roadways that require state-provided maintenance. Predefined snow removal "runs," which number about 300, were adopted as the basic roadway sectors for the study.

Previous Work

The prior study most relevant to the location of roadway garages is that of Bell and Rainer (4). That study dealt with the location of roadway garages for the state of Alabama. Although they used a single-stage model, their methods are relevant to the current study because they modeled garages serving roadway sectors, which is the key new feature of the two-stage optimization model.

A description of prior research that has been done to extend single-stage facility location models to represent multiple stages is included in the survey by Aikens (3). In that survey two multistage facility location formulations are presented for systems without capacity constraints. The formulation by Kaufman et al. (5) is based on triple-subscripted variables \( x_{ijk} \) that correspond to the amount of demand in zone \( k \) served by plant \( i \) through warehouse \( j \). The formulation by Tcha and Lee (6) extends the formulation given by Kaufman et al. (5) to allow for any number of stages in the system. The emphasis of both of these multistage studies is on efficient solution procedures, not necessarily on modeling any specific application.

The formulation described below takes certain elements of its structure from previous multistage models, but it also adds some key new features of its own. The way that it simultaneously allows for facility closings, capacity expansions, garage and sector reassignments, and equipment transfers takes it a step beyond previously developed formulations.

Two-Stage Formulation

The two-stage mathematical programing model of ConnDOT’s maintenance facility location problem is shown below. As in the single-stage formulation, \( f \) indexes over repair facilities and \( i \) indexes over roadway garages. The \( j \) subscript, which is new in this formulation, indexes over roadway sectors. The way that it simultaneously allows for facility closings, capacity expansions, garage and sector reassignments, and equipment transfers takes it a step beyond previously developed formulations.

Minimize

\[
\sum_{f=1}^{13} (K_f Y_f + MA_f + NB_f) + \sum_{i=1}^{55} (L_i Y_G + MG AG_i + NG BG_i)
\]

\[
+ \sum_{f=1}^{13} \sum_{i=1}^{55} C_p X_p + \sum_{f=1}^{13} \sum_{i=1}^{55} U_i (T_p - TP_i)
\]

\[
+ \sum_{i=1}^{55} \sum_{j=1}^{300} TCOST_{p} X_S_{ij}
\]

Subject to

\[
\sum_{f=1}^{13} DX_f + \sum_{i=1}^{55} (T_p - TP_i) \leq (P_f Y_f + 2RA_f + RB_f)
\]

\[
\begin{align*}
-15 & \leq TP_i \leq 55, & (f = 1 \text{ to } 13) \\
D_i & \leq \left[ D_i + \sum_{f=1}^{13} (T_p - TP_i) \right], & (i = 1 \text{ to } 55) \\
\sum_{f=1}^{13} T_p & \leq (2RG AG_i + RG BG_i + XSPACE_i Y_G_i) & (i = 1 \text{ to } 55)
\end{align*}
\]

\[
\sum_{f=1}^{13} X_f = 1, & (i = 1 \text{ to } 76)
\]

\[
\sum_{i=1}^{55} X_S_i = 1, & (j = 1 \text{ to } 300)
\]

\[
\sum_{j=1}^{300} X_S_i = 20 Y_G_i, & (i = 1 \text{ to } 55)
\]

\[
A_f \leq Y_f, & (f = 1 \text{ to } 13)
\]

\[
B_f \leq 8A_f, & (f = 1 \text{ to } 13)
\]

\[
AG_i \leq Y_G, & (i = 1 \text{ to } 55)
\]

\[
BG_i \leq 8AG_i, & (i = 1 \text{ to } 55)
\]

The two-stage formulation is shown below.
The objective function (Equation 8) represents a minimization of all relevant costs. The cost components of the objective function are of three types: (a) the costs of keeping the repair facilities and roadway garages open, (b) the costs of expanding repair facilities and roadway garages, and (c) the costs of providing service. The cost components added beyond the single-stage model are elaborated upon below in the discussion of the input data requirements for the two-stage model.

**Constraints**

Supply constraints ensure that the total service being provided from a location does not exceed that location's capacity. Constraint Set 9 considers the capacity of repair facilities. The left-hand side of the inequality is the total equipment being serviced by a repair facility, and the right-hand side is the repair facility's capacity on the basis of the existing number of work bays and the numbers of work bays, if any, that are being added. Constraint Set 10 ensures that the service provided by each roadway garage is less than or equal to the capacity of the equipment at that garage, including any equipment that is being added to (or subtracted from) the garage. Constraint Set 11 ensures that the amount of equipment being added to a roadway garage does not exceed the garage's supply of available space.

Demand constraints are included in the model to ensure that all service requirements are met. Constraint Set 3, which is the same as that in the single-stage model, handles the repair of equipment from the roadway garages, and Constraint Set 12 handles roadway maintenance for all sectors. Constraint Set 13 states that a sector must be served by a roadway garage that is open. In that constraint set a value of 20 represents the maximum number of sectors that can be assigned to a single garage. This value was chosen because it appears to be large enough to be nonrestrictive for any realistic solution.

Constraint Sets 4, 5, 14, and 15 enforce logical relationships among expansion variables. As in the single-stage formulation, Constraint Set 4 states that a repair facility cannot be expanded unless it is kept open, and Constraint Set 5 ensures that expansions by more than two bays cannot happen unless the initial two-bay expansion occurs. Constraint Sets 14 and 15 represent similar constraints for the roadway garages.

Constraint Set 16 is included in the model to ensure that the amount of equipment being subtracted from a repair assignment does not exceed the amount that was assigned. Constraint Set 17 balances the additions and subtractions associated with reassigned equipment.

The last two constraints in the model, Constraint Sets 18 and 19, are nonnegativity and binary variable constraints, respectively.

**Obtaining Required Input Data**

The two-stage model presented above was run successfully by using artificial input data. In addition many of the actual data required for the two-stage model were collected for the single-stage model. On the basis of those data 8 of the 15 parameter types required to build the two-stage model have been estimated: $C_o, D_o, U_o, K_o, M, N, P_o$, and $R$. The estimation of the remaining seven parameter types is discussed below.

**Demand in Sector $j$**

Demand in Sector $j$, $D_{SR}$, is expressed in terms of the equipment required to serve that sector. It can be calculated as follows:

$$D_{SR} = \sum_{i \in j} SR_i$$

where

- $D_i$ = total equipment at roadway garage $i$;
- $SR_i$ = amount of snow removal equipment required for sector $j$;
- $\sum_{i \in j} SR_i$ = total amount of snow removal equipment for all sectors assigned to roadway garage $i$ in existing system; and
\{S_i\} = \text{set of sectors served by roadway garage } i \text{ in existing system.}

As mentioned earlier, \(D_i\) values were already estimated for the previous study. Data pertaining to the \(SR_i\) and \(\{S_i\}\) values are readily available on the basis of information contained in the "snow books" produced by ConnDOT's Office of Highway Operations. Therefore, estimation of \(DS_i\) values should present little difficulty.

**Total Cost of Servicing All Sector \(j\) Requirements from Garage \(i\)**

The total cost of servicing all sector \(j\) requirements from garage \(i\), \(TCOST_{ij}\), is the largest set of parameters in the model. There are 55 existing roadway garages and hundreds of roadway sectors, but fortunately an estimate is not required for all possible garage-sector pairs. On the basis of the rules of thumb provided by the Office of Highway Operations, a set of candidate roadway garages will be defined for each roadway sector, so that \(TCOST_{ij}\) values need to be estimated only for garage-sector pairs that correspond to such sets.

Another piece of the transportation cost function will be the number of trips made between a roadway sector and a roadway garage. On the basis of discussions regarding this aspect of the cost function, it was concluded that estimates can be provided by those most familiar with roadway maintenance operations. However, these estimates will be subject to some degree of error. It will therefore be important for the second phase of the study to include a sensitivity analysis that investigates the effects of errors in the \(TCOST_{ij}\) estimates.

**Annualized Cost of Keeping a Roadway Garage Open**

The annualized cost of keeping a roadway garage open, \(L_i\), will be estimated for 25 to 40 roadway garages that will be candidates for closing. These candidates will represent those roadway garages that have not been identified as "untouchable" by ConnDOT management. There are three major components to the \(L_i\) values: (a) salvage values of land and buildings, (b) savings in building expenses such as for heat and electricity, and (c) savings in salaries for support personnel.

Estimation of the salvage values of land and buildings for the roadway garages is a complicated matter. Three estimation alternatives have been identified: (a) field appraisal of each site, (b) estimates made on the basis of accounting records, or (c) not including property salvage values in the \(L_i\) estimates. The third alternative offers advantages in that it is the most conservative, requires the least amount of data gathering, and is consistent with the methods used in the study by Bell and Rainer (4).

Regarding the second major component of the \(L_i\) values, detailed building expense records for roadway garages are readily obtainable. For the third major component the Office of Highway Operations will be consulted regarding which positions could be eliminated if a roadway garage were closed and what the estimated cost savings would be.

**Capacity-Related Parameters**

Capacity-related parameters include \(RG_i\), the capacity added by adding a bay at a roadway garage; and \(XSPACE_i\), the extra space available at garage \(i\). Although the estimation of values for these parameters may appear to be straightforward, it requires clarification of certain assumptions related to policy, such as whether or not each major piece of equipment requires its own storage bay.

**Annualized Costs of Adding Bays at a Roadway Garage**

The annualized costs of adding bays at a roadway garage are represented by \(MG\) and \(NG\). Because the Office of Highway Operations has cost estimates for expansions of various sizes, the development of estimates for these parameters should be straightforward. The model allows for a maximum of 10 bays added at each roadway garage. This simple type of constraint may be appropriate for a first-pass analysis, but it would be better if the model could be based on realistic site-specific limitations on expansions. The trade-off, of course, is that a significant amount of work is required to estimate the potential for expansion at 55 garage sites.

**Use of Two-Stage Model**

Once the data required to complete the two-stage model have been obtained the model will be used to develop specific recommendations regarding the following five types of decisions:

1. Which repair facilities to keep open and which to expand,
2. Assignments of equipment to repair facilities,
3. Which roadway garages to keep open and which to expand,
4. Assignments of roadway sectors to roadway garages, and
5. Assignments of equipment to roadway garages.

Given the close relationship between the two models, the results of the two-stage model are expected to be consistent with those of the single-stage model. However the recommendations made on the basis of the two-stage model will be more comprehensive. These recommendations are likely to include cost-saving measures related to the consolidation of the roadway garages that could not be identified by using the single-stage model.

**SUMMARY AND CONCLUSION**

Two related optimization models representing Connecticut's system of roadway maintenance and vehicle repair facilities have been described. The single-stage model was developed to investigate the possible benefits that would result from consolidation of vehicle repair facilities. On the basis of that model two options were recommended, with estimated savings of $5.0 million and $3.1 million, respectively. The extension of the model to a two-stage formulation has also been described, and the nature of the additional data required to estimate the parameters for the two-stage model has been presented. Considering the results of the single-stage study, the follow-up study done on the basis of the two-stage model is expected to result in significant savings for the state of Connecticut.

**ACKNOWLEDGMENTS**

The research described here was supported by the Cooperative Highway Research Program between the Connecticut Department of Transportation and the University of Connecticut. The authors...
also acknowledge, with thanks, the considerable assistance given by personnel of ConnDOT during the course of the study. Particular thanks are due to Charles E. Dougan and James M. Sime for their assistance in coordinating meetings and James E. Lewis, Jr., Louis R. Malerba, and Patrick F. Rogers for their thoughtful comments and insight regarding ConnDOT maintenance operations.

REFERENCES


Publication of this paper sponsored by Committee on Maintenance and Operations Management.