Design Considerations for Highway Reverse Curves

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Existing sight distance models of highway horizontal (vertical) curves are applicable only to simple circular (parabolic) curves. These simple models may greatly overestimate the lateral clearance needs for reverse horizontal curves and the curve length requirements for reverse vertical curves. No available analytical model quantifies the effects of the alignment reversal on sight distance characteristics. The geometric and sight distance characteristics of reverse horizontal and vertical curves are presented. For reverse horizontal curves the available sight distance is related to the parameters of the two circular arcs, the length of the common tangent, the locations of the driver and object, and the location of the vision-limiting obstacle. The obstacle may be located within the circular arcs or the common tangent. For reverse vertical curves the available sight distance is related to the parameters of the crest and sag arcs, the length of the common tangent, the locations of the driver and object, and their heights. The sight-hidden zone (dip) that may exist on a reverse horizontal (vertical) curve is examined. The sight distance profiles and minimum sight distances of reverse and simple curves are compared.

Sight distance is one of the basic elements in highway geometric design. The highway alignment should be designed so that the available sight distance is always equal to or greater than the required sight distance. Design values for required stopping, passing, and decision sight distances are presented by AASHTO (*1–5*), Neuman (*6*), Harwood and Glennon (*7*), and McGee (*8*). The available sight distance depends on the geometric elements of the highway. This paper addresses the geometric and sight distance characteristics of reverse horizontal and vertical curves. It will be useful first to review the previous work related to other types of highway curves and to describe the basic features of reverse curves.

PREVIOUS WORK

Besides reverse curves highway curves may be simple or compound curves. The simple curve consists of a single circular arc in horizontal alignment or a single parabolic arc (sag or crest) in vertical alignment. For simple horizontal curves, only the $S_m \leq L$ case is considered by AASHTO (*5*), where S_m is the minimum sight distance and L is the curve length. Waissi and Cleveland (*9*), on the basis of the NCHRP report by Olson et al. (*10*), addressed the $S_m > L$ case and developed approximate relationships for the available sight distance given an obstacle on the inside of the curve. The NCHRP model was extended, and exact sight distance relationships were developed (*11*). For simple vertical curves, sight distance models for sag and crest curves can be found in AASHTO (*5*) and Hickerson (*12*). Both $S_m \leq L$ and $S_m > L$ cases are considered. Sight distance on a simple sag curve with a noncentered overpass has been analyzed (*13*).

Compound horizontal curves consist of two circular arcs located on the same side of a common tangent (*12*). The lateral clearance needs on these curves have been established (*14*). Compound vertical curves (called *unsymmetrical curves*) consist of two parabolic arcs with a common tangent at the point of vertical intersection. The use of these curves may be required on certain occasions because of critical clearance and other controls (*4,5*). Sight distance models for unsymmetrical crest and sag curves were developed (*15,16*). The geometric characteristics of all types of horizontal curves and all types of vertical curves have been unified (*17,18*).

FEATURES OF REVERSE CURVES

For a given obstacle on the inside of an arc of a reverse horizontal curve, the available sight distance and lateral clearance needs could be found by using existing simple curve models. However these models will generally overestimate the lateral clearance needs because the alignment reversal reduces the needed lateral clearance. A reverse curve may exhibit a sight-hidden zone that affects traffic safety.

Reverse vertical curves are advantageous in hilly and mountainous terrains. Their use is also often necessary on interchange ramps (*19*). The geometries of reverse vertical curves (without intermediate tangents) are presented by Hickerson (*12*). The design of the crest arc of a reverse vertical curve by using simple crest curve models may greatly overestimate the required length. This is because the alignment reversal of the sag arc improves the sight distance and consequently reduces length requirements. A reverse vertical curve may exhibit a sight-hidden dip that affects traffic safety.

REVERSE HORIZONTAL CURVE

Geometric Characteristics

A reverse horizontal curve consists of two circular arcs, AB and CD, lying on the opposite sides of a common tangent (Figure 1). The radii of the two arcs are R_1 and R_2, and their central angles are I and J. The first and second tangents intersect at E, and their lengths are T_a and T_b, respectively. The intersection angle K is:

$$K = J - I \tag{1}$$

The relationships between the parameters of the reverse curve can be established by selecting arbitrary x and y coordinate axes

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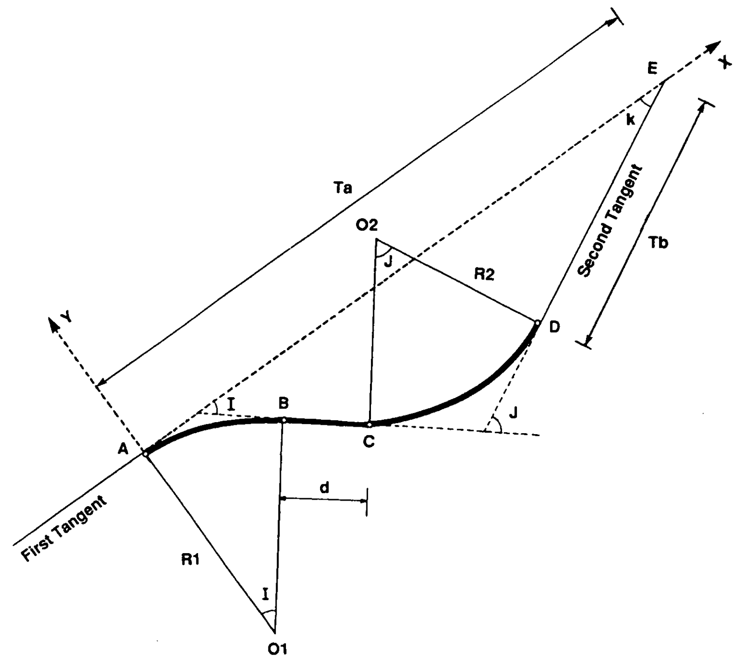

FIGURE 1 Geometry of a reverse horizontal curve.

at A, where the x-axis lies along the first tangent AE. Consider the closed traverse AO_1BCO_2DEA. Since the sum of the latitudes of the traverse must equal zero, then

$$-R_1 - d \sin I + (R_1 + R_2) \cos I - R_2 \cos K + T_b \sin K = 0 \quad (2)$$

Similarly, since the sum of the departures must equal zero, then

$$(R_1 + R_2) \sin I + d \cos I + R_2 \sin K + T_b \cos K - T_a = 0 \quad (3)$$

Equations 1 to 3 contain eight parameters: R_1, R_2, I, J, K, d, T_a, and T_b. When five of these parameters are known, including an angle, the equations can be solved to find the other three unknowns. For $K = 0$ similar relationships can easily be obtained.

Available Sight Distance

Consider an obstacle located on the inside of arc AB (Figure 2). The lateral clearance between the centerline of the inside lane and the obstacle is m_1. The angle between the obstacle and A is I_1. The available sight distance, S, depends on whether the driver is on the first tangent or on arc AB and on the location of the obstacle. There are five cases of the object location (*20*):

Case 1: Object on arc AB,
Case 2: Object on common tangent BC,
Case 3: Object on arc CD before the tangent point t,
Case 4: Object on arc CD beyond the tangent point t, and
Case 5: Object on the second tangent.

The tangent point t is the point at which the line of sight becomes tangent to arc CD. Clearly beyond this point the obstacle on arc AB has no effect and the available sight distance may then be controlled by another obstacle within the common tangent or arc CD. The obstacle may lie within the first arc or the common tangent. As an illustration, the relationships for Case 3 (with a driver on the first tangent) are derived next.

In this case the driver lies on the first tangent at a distance x from A and the object lies on arc CD before tangent point t (Figure 2). Then from triangle aAo_1,

$$\overline{ao_1} = (x^2 + R_1^2)^{1/2} \quad (4)$$

$$I_2 = \tan^{-1}(x/R_1) \quad (5)$$

The angle at o_1 between the driver and obstacle and the distance between them, S_d, are:

$$\angle\, ao_1e = I_1 + I_2 \quad (6)$$

$$L_{11} = RI_1\pi/180 \text{ degrees} \quad (7)$$

$$S_d = L_{11} + x \quad (8)$$

The angle θ, $\overline{qo_1}$, $\overline{qf}$, and $\overline{go_2}$ are:

$$\theta = \beta - (I - I_1) \quad (9)$$

$$\overline{qo_1} = (R_1 - m_1) \sin (180 \text{ degrees} - \beta)/\sin \theta \quad (10)$$

$$\overline{qf} = d/\tan \theta \quad (11)$$

$$\overline{go_2} = (R_1 + R_2) - (\overline{qf} + \overline{qo_1}) \quad (12)$$

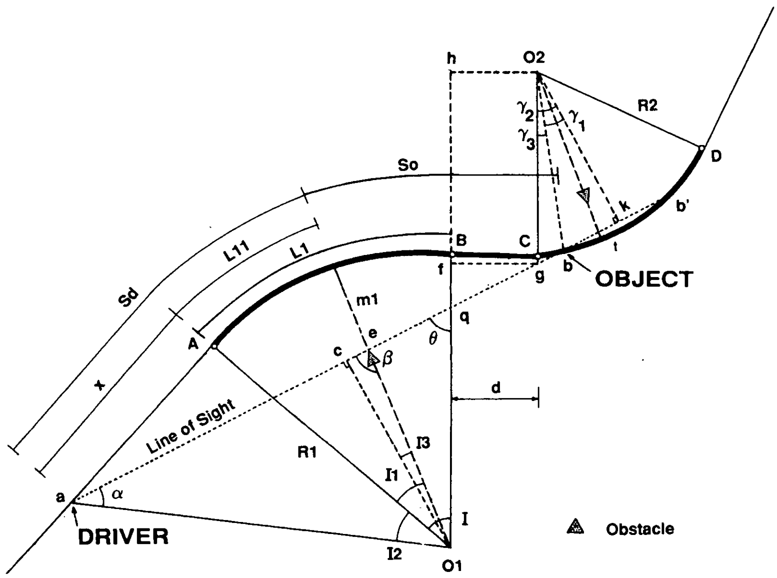

FIGURE 2 Case 3: Object on arc *CD* before tangent point *t* (example of a driver on the first tangent).

The perpendicular distance from o_2 to the line of sight, $\overline{ko_2}$, and the angles γ_1 and γ_2 are:

$$\overline{ko_2} = \overline{go_2} \sin \theta \quad (13)$$

$$\gamma_1 = \cos^{-1} (\overline{ko_2}/R_2) \quad (14)$$

$$\gamma_2 = 90 \text{ degrees} - \theta \quad (15)$$

Then γ_3 and the distance $\overline{Cb}$ are:

$$\gamma_3 = \gamma_2 - \gamma_1 \quad (16)$$

$$\overline{Cb} = R_2\gamma_3\pi/180 \text{ degrees} \quad (17)$$

The sight distance component S_o is:

$$S_o = (L_1 - L_{11}) + d + \overline{Cb} \quad (18)$$

where $L_1 = RI\pi/180$ degrees, and L_{11} is given by Equation 7. The available sight distance, S, equals $S_d + S_o$.

Finding Minimum Sight Distance

The minimum sight distance, S_m, along the reverse horizontal curve is found by computing the available sight distance, S, for successive locations of the driver until the minimum value is reached. The search starts with a large value of x and an increment, Δx. For Cases 2 and 3 the obstacle may lie below tangent *BC*, and for Cases 4 and 5 the obstacle may lie above tangent *BC*.

Practical Aspects

Sight-Hidden Zone

For Cases 1 to 3 of the object a sight-hidden zone (SHZ) is formed as shown by the zone from *b* to *c* in Figure 3(*a*). The line of sight intersects the second tangent or arc *CD* when the angle between the line of sight and the first tangent, ϕ, is greater than $-K$ (Equation 1). If K is positive (as in Figure 1) the line of sight intersects the second tangent for any location of the driver. If K is negative the line of sight intersects the second tangent when $\phi > -K$. In any case the SHZ starts when $\phi > -K$ and the intersection point c lies within the actual length of the second tangent T_2 or within arc *CD*. The length of the SHZ, L_{SHZ}, equals the distance on the road between b and c, $L_{SHZ} = S_{end} - S$, where S_{end} is the available sight distance from the driver to the end of the SHZ, and S is the available sight distance computed for Cases 1 to 3. An SHZ is undesirable because it makes the highway discontinuous and may affect the driver's perception of direction at night.

Figure 3(*b*) shows the variations of L_{SHZ} as the driver travels on the first tangent and arc *AB*. The obstacle lies within arc *AB*, where $I_1/I = 0.8$ and m_1 varies from 20 m (66 ft) to 40 m (131 ft). The driver location, X, is measured from A and is considered negative if the driver lies on the first tangent and positive if the driver lies on arc *AB*. Since K is positive (+10 degrees), the SHZ exists when the driver lies anywhere on the first tangent. For m_1 = 20 m (66 ft) the SHZ length is 273 m (896 ft) when $X = -300$ m (−984 ft) and the zone diminishes when $X = -7$ m (−23 ft). The SHZ length can be reduced (or avoided) by increasing the lateral clearance as shown in Figure 3(*b*).

Sight Distance Profile

The second arc of the reverse curve reduces the lateral clearance needs on the first arc. Figure 4 shows the sight distance profile of a reverse horizontal curve. There are two obstacles, one within arc *AB* and the other within arc *CD*, and two values of S_m, one for each obstacle. When the driver is at $X = -60$ m (−197 ft), the obstacle on arc *AB* no longer controls (SHZ diminishes). At this point the sight distance suddenly increases because it is controlled by the obstacle on arc *CD* and then gradually decreases to a second minimum value.

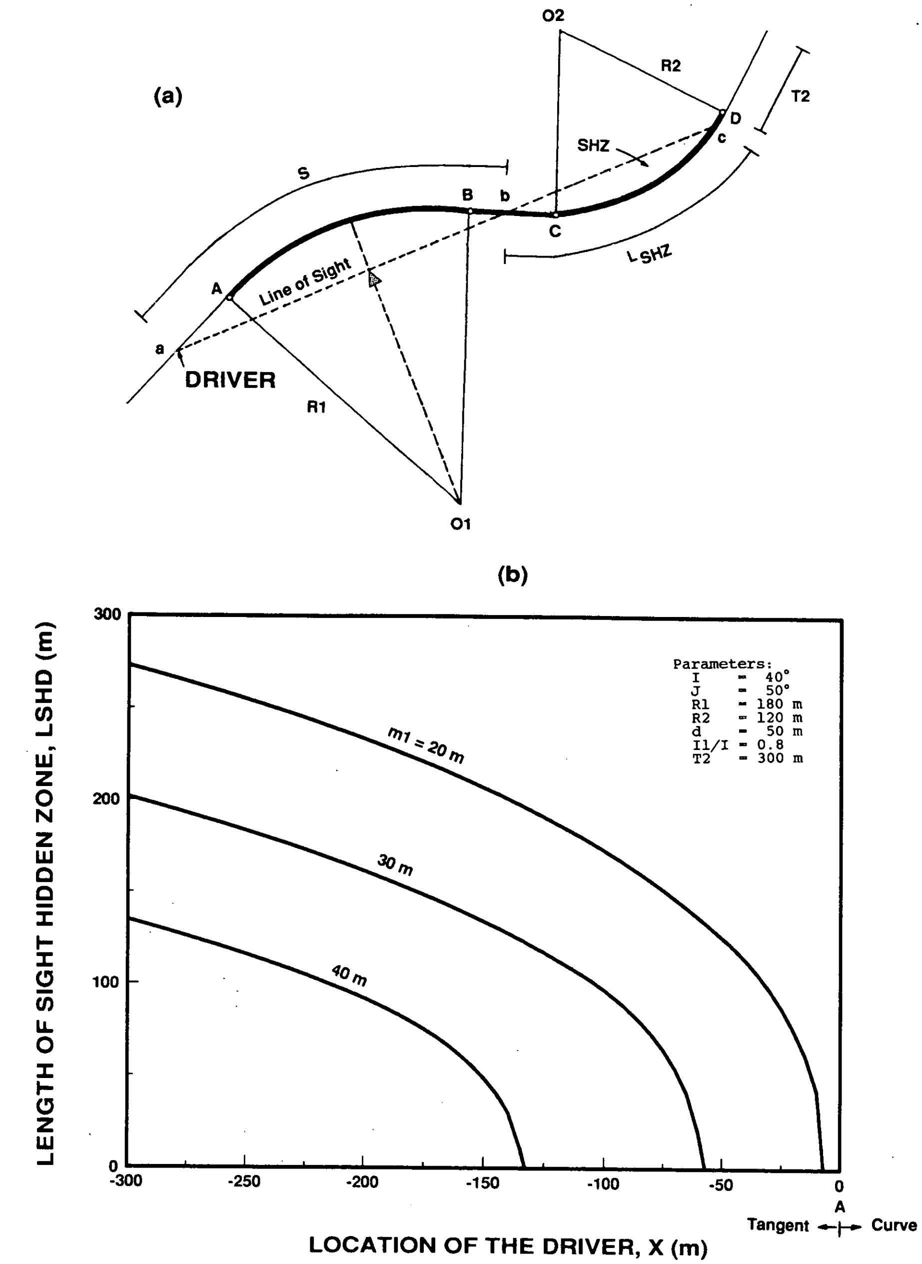

FIGURE 3 SHZ of reverse horizontal curve: (*a*) geometry of SHZ; (*b*) effect of lateral clearance m_1 on SHZ.

The sight distance profiles of a reverse curve and simple curve *AB* with m_1 = 40 m (131 ft) and I_1/I = 0.8 are shown in Figure 5. The profile of the simple curve is obtained by setting *d* equal to a very large value. S_m on arc *AB* is improved by the alignment reversal. For the reverse curve, S_m = 351 m (1,152 ft), and for the simple curve, S_m = 311 m (1,020 ft), a difference of +13 percent. The difference in S_m is large when (a) the lateral clearance is large, (b) the obstacle lies close to the common tangent, (c) the common tangent is short, and (d) R_2/R_1 is small.

Evaluation and Design Values

Table 1 gives the minimum sight distance for R_2/R_1 = 0.5, 1.0, and 100; an obstacle located within arc *AB*; and *d* = 50 m (164 ft). For R_2/R_1 = 0.5 and 1.0, the S_m values are applicable for only $J \geq 26$ and 18 degrees, respectively. For smaller *J*, S_m will be smaller than the values shown. The minimum sight distances for R_2/R_1 = 100 are about the same as those for a simple curve. The value of S_m becomes larger as R_2/R_1 becomes smaller, as expected. It is assumed that no obstacle exists on arc *CD*. If there are two obstacles, one within each arc, S_m can be found for each obstacle separately (assuming that the other obstacle does not exist). Then S_m on the reverse curve is the lesser of the two values. Note that for certain parameter values the sight distance is unlimited. In this case the obstacle is located such that the line of sight does not intersect the reverse curve when the driver lies on arc *AB* or on the first tangent within 500 m (1,640 ft) from *A*.

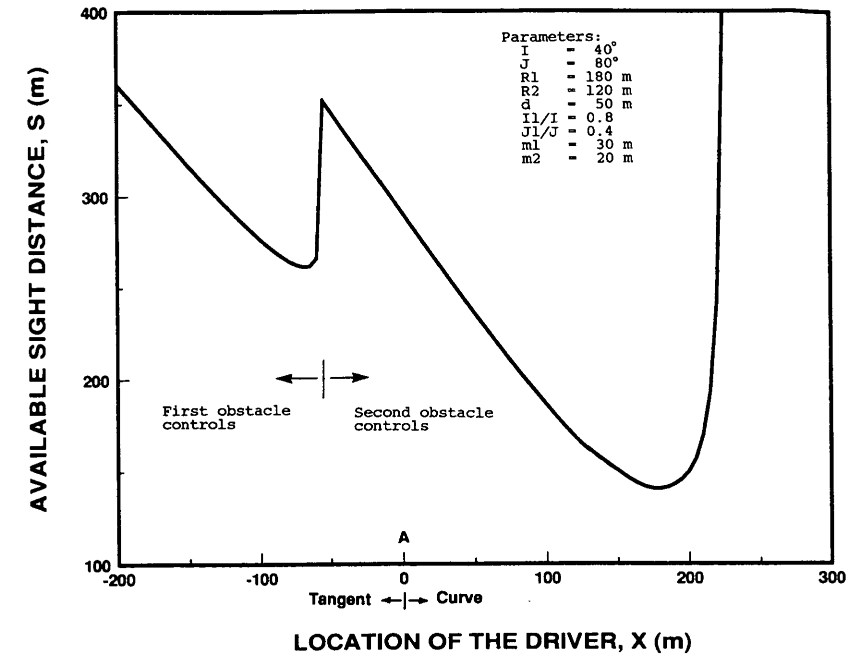

FIGURE 4 Sight distance profile of reverse horizontal curve.

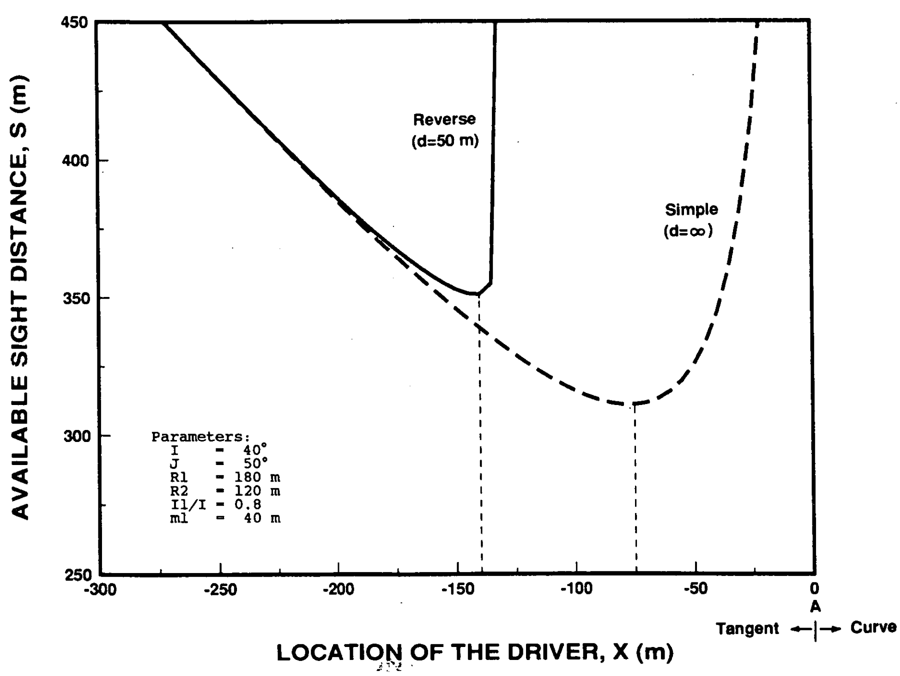

FIGURE 5 Comparison of sight distance profiles of reverse and simple horizontal curves (obstacle on first arc).

TABLE 1 Minimum Sight Distance on Reverse Horizontal Curve with Obstacle on First Arc (d = 50 m)

R2/R1	Cent. Angle I	Obst. Ratio (I1/I)	Radius of the First Arc, R1 (m)								
			100			200			300		
			m[a]=20	25	30	20	25	30	20	25	30
0.5	20	0.0	413	u[b]	u	330	504	u	336	438	605
		0.5	442	u	u	316	544	u	302	422	649
		1.0	u	u	u	u	u	u	598	u	u
	30	0.0	201	258	349	244	286	335	287	327	369
		0.5	184	245	349	206	249	304	232	271	315
		1.0	234	354	u	283	381	543	329	419	545
	40	0.0	175	203	236	231	261	291	283	316	347
		0.5	151	181	216	186	215	244	220	249	279
		1.0	180	223	281	243	289	344	300	349	404
1.0	20	0.0	352	u	u	316	434	649	332	415	526
		0.5	357	u	u	295	430	u	294	384	513
		1.0	577	u	u	444	u	u	434	658	u
	30	0.0	201	251	320	244	285	330	287	327	368
		0.5	183	236	311	206	248	296	232	271	313
		1.0	220	300	430	266	332	417	309	372	445
	40	0.0	175	203	234	231	261	291	283	316	347
		0.5	151	181	214	186	215	244	220	249	279
		1.0	178	216	260	238	277	320	293	334	377
100	20	0.0	262	320	378	293	351	409	324	382	441
		0.5	248	307	366	265	323	382	282	340	398
		1.0	262	320	379	294	352	410	325	383	442
	30	0.0	199	237	275	244	283	322	287	327	367
		0.5	180	219	258	206	245	284	232	271	310
		1.0	199	237	276	245	284	323	287	327	367
	40	0.0	175	203	231	231	261	291	283	316	347
		0.5	151	180	210	186	215	244	220	249	279
		1.0	175	203	232	231	262	291	283	316	347

a Lateral clearance in meters.
b Unlimited sight distance is available within 500 m from the start of the first arc.
Note: Minimum sight distances are expressed in meters.

For example find the required lateral clearance on a reverse curve with $R_1 = R_2 = 300$ m, $I = 20$ degrees, and $I_1/I = 1.0$ to satisfy a passing sight distance of 458 m (1,500 ft). From Table 1 $S_m = 434$ m (1,424 ft) and 658 m (2,159 ft) for $m = 20$ m (66 ft) and 25 m (82 ft), respectively. By interpolation the required lateral clearance is about 21 m (69 ft). By comparison the required lateral clearance for a simple curve is about 31 m (102 ft), as noted in Table 1 for $R_2/R_1 = 100$.

REVERSE VERTICAL CURVE

Geometric Characteristics

A general reverse vertical curve is shown in Figure 6. The curve consists of two parabolic arcs, CD and EF, with lengths L_1 and L_2 and algebraic differences in grade A_1 and A_2, respectively. The rates of change of grades are r_1 and r_2, respectively. The arcs are separated by a tangent distance d. The grades of the first and second tangents are g_1 and g_2, respectively. The grade is considered positive if it is upward to the right and negative if it is downward to the right. The grade of the common tangent DE is g_c. For $d = 0$, D and E coincide resulting in a point of reverse curvature. The algebraic difference in grade of the reverse curve, $A = g_2 - g_1$, is

$$A = A_1 + A_2 \tag{19}$$

Note that A_1 and r_1 of the crest arc are negative and A_2 and r_2 of the sag arc are positive. It is desirable to design reverse curves with intermediate tangents to provide a separation between the reverse curvatures of the two parabolic arcs (*21*). In Figure 6 the two arcs have the same slope at D and E,

$$g_1 + r_1 L_1 = g_2 - r_2 L_2 \tag{20}$$

The elevations of D and E and their relationship are:

$$\text{Elev}_D = g_1 L_1 + r_1 L_1^2/2 \tag{21}$$

$$\text{Elev}_E = \text{Elev}_F - g_2 L_2 + r_2 L_2^2/2 \tag{22}$$

$$\text{Elev}_E = \text{Elev}_D + d(g_1 + r_1 L_1) \tag{23}$$

Substituting for Elev_D and Elev_E from Equations 21 and 22 in Equation 23 and noting that $A = g_2 - g_1$, the following basic relationships for r_1 and r_2 are obtained:

$$r_1 = (AL_2 - 2W - 2dg_1)/L_1(L + d) \tag{24}$$

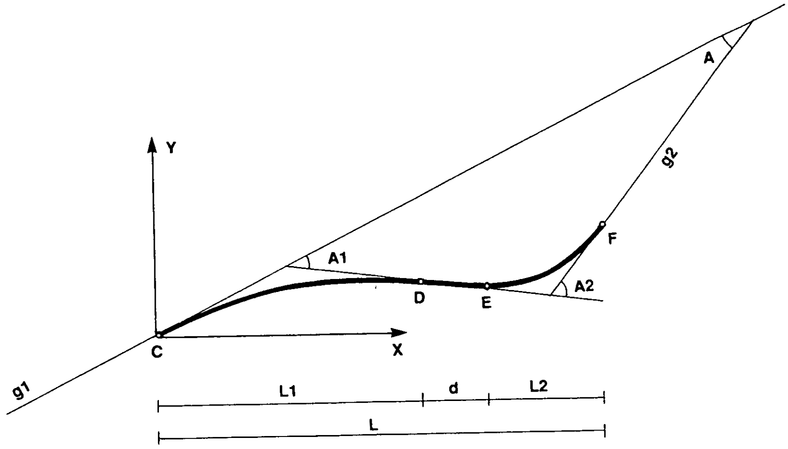

FIGURE 6 Geometry of reverse vertical curve.

$$r_2 = (AL_1 + 2W + 2dg_2)/L_2(L + d) \quad (25)$$

where

$$L = L_1 + L_2 + d \quad (26)$$

$$W = g_1L_1 + g_2L_2 - \text{Elev}_F \quad (27)$$

The sign of r_1 or r_2 will be positive if the respective arc is crest and negative if it is sag. For parallel tangents ($g_1 = g_2$) set $A = 0$ in Equations 24 and 25. It is interesting to note that Equations 24 and 25 are also applicable to symmetrical and unsymmetrical curves when W and d equal zero (for symmetrical curves $L_1 = L_2 = L/2$). The elevation of various points on the curve can easily be obtained.

Available Sight Distance

The available sight distance on the reverse curve, S, depends on the direction of travel. The relationships for S are developed for traveling from the crest to the sag arc. The relationships are also applicable to the other travel direction by exchanging the driver's eye height h_1 and the object height h_2. The available sight distance is considered for the crest curve (daytime conditions). There are two cases for the location of the driver: driver on the first tangent (Case A) and driver on the crest arc (Case B). For each of these locations there are four cases of the object location (*20*):

Case 1: Object on the crest arc,
Case 2: Object on the common tangent,
Case 3: Object on the sag arc, and
Case 4: Object on the second tangent.

The geometry of the available sight distance for these cases is shown in Figure 7. The letters in circles refer to the driver location, and the numbers in circles refer to the object location. In all cases the available sight distance, S, equals $S_d + S_o$, where S_d and S_o are the distances from the driver and object, respectively, to the tangent point a. S_d depends on the two cases of the driver, and S_o depends on the four cases of the object.

Case A: Driver on First Tangent

In Case A the driver lies on the first tangent at a distance T from the start of the crest arc. On the basis of the property of a parabola

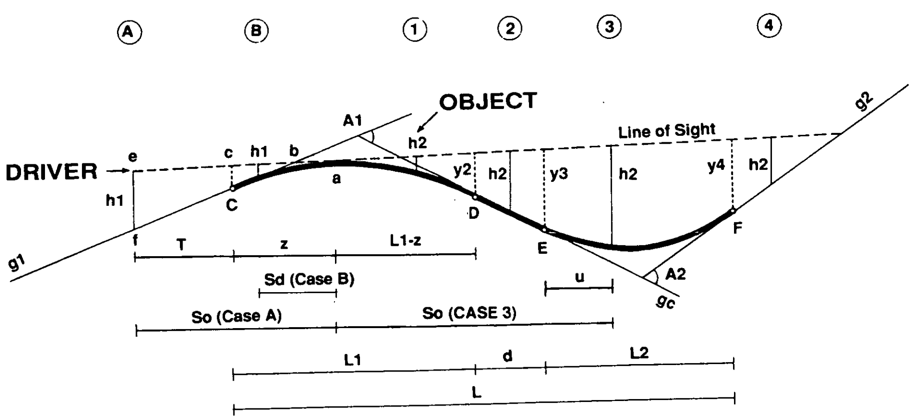

FIGURE 7 Geometry of sight distance on crest arc of reverse vertical curve.

and the similarity of triangles bcC and bef,

$$cC = -r_1 z^2/2 \quad (28)$$

$$z = T + [T^2 + (-2 h_1/r_1)]^{1/2} \quad (29)$$

where T is negative when the driver lies on the first tangent and positive when the driver lies on the crest arc. The sight distance component S_d equals $z - T$.

Case B: Driver on Crest Arc

In Case B the driver lies on the crest arc at a distance T from C. On the basis of the property of a parabola the distances v and z are (Figure 7)

$$v = (-2h_1/r_1)^{1/2} \quad (30)$$

$$z = T + (-2h_1/r_1)^{1/2} \quad (31)$$

The sight distance component S_d equals $z - T$. The sight distance component S_o depends on the object location. As an illustration the relationships for Case 3 are derived next.

Derivation of S_o *for Case 3*

In Case 3 the object lies on the sag arc and the driver lies on the first tangent or the crest arc. The vertical distances y_2 and y_3 are (Figure 7)

$$y_2 = -r_1(L_1 - z)^2/2 \quad (32)$$

$$y_3 = y_2 - r_1(L_1 - z)d \quad (33)$$

The vertical distance between the line of sight and the common tangent is

$$(h_2 + r_2u^2/2) = y_3 - r_1(L_1 - z)u \quad (34)$$

Equation 34 is quadratic in u, and its solution is

$$u_1 = [-r_1(L_1 - z) - G^{1/2}]/r_2 \quad (35)$$

$$u_2 = [-r_1(L_1 - z) + G^{1/2}]/r_2 \quad (36)$$

$$G = r_1^2(L_1 - z)^2 - 2r_2(h_2 - y_3) \quad (37)$$

If both roots of Equations 35 and 36 are positive the object can be seen by the driver at two locations on the sag curve. Between these locations a dip hidden from the driver's sight exists. For this reason only the smaller value of u is considered in computing the available sight distance. Thus the sight distance component S_o equals $L_1 - z + d + u_1$, where u_1 is given by Equation 35.

Finding Minimum Sight Distance

The minimum sight distance is found by exhaustive search. The search starts with a large (negative) value of T and an increment ΔT. For each value the available sight distance S is compared with the value of the previous iteration, S'. The search continues until $S > S'$ and then $S_m = S'$. For each iteration the cases of the driver and object are found by comparing h_2 with y_2, y_3, and y_4. The available sight distance for locations beyond the location of S_m is also obtained. This information is used to plot the sight distance profile on the reverse curve. The minimum sight distance varies with the travel direction.

Practical Aspects

Sight-Hidden Dip

A sight-hidden dip (SHD) is a safety concern on two-lane highways when a sag curve follows a crest curve [Figure 8(*a*)]. An SHD is defined as the portion of the road ahead of the driver within which an opposing vehicle will be hidden from the driver's view (*22*). In Figure 8(*a*), for a driver traveling on the first tangent with an eye height h_1, the line of sight is tangent to the crest curve at a. The hidden dip extends from a to b. However considering an opposing vehicle with a roof height h_2, the SHD extends from e to f. At these points the distance between the pavement and the line of sight is exactly h_2. Within the SHD this distance is greater than h_2, and outside the SHD this distance is less than h_2. Therefore an opposing vehicle within ae or fb will be visible to the driver.

To avoid the SHD the shaded area in Figure 8(*a*) should diminish so that a vehicle in the hidden dip is always visible to the driver. Practically, however, a smaller height h_2' should be used so that a portion of the top of the opposing vehicle is visible to the driver when it is at the lowest point of the dip [$h_2' = (1 - f_v)h_2$, where f_v is a visibility factor]. This is especially important for flat sag curves, where the increase in the visible portion of the opposing vehicle as it travels is small.

Figure 8(*b*) shows the effects on the SHD width of the algebraic difference in grades. The limits of each curve represent the roadway locations where the SHD starts and ends (SHD range). For example the SHD for A_2 = 6 percent starts when the driver lies on the first tangent at $T = -19$ m (−62 ft) and ends when the driver lies on the crest arc at $T = +6$ m (+20 ft). Therefore the SHD range is 25 m (82 ft). The SHD width at the start is 242 m (794 ft), and at the end the width is zero. By decreasing A_2 to 4 percent the SHD range is reduced to 7 m (23 ft). In fact for A_2 = 2 percent the SHD does not exist.

The results show that the AASHTO minimum rates of vertical curvature, on the basis of the SSD for the crest and sag curves, produce an SHD range as large as 1 km (0.62 mi). For small A_2 and small design speeds, however, the SHD range is negligible or does not exist. The AASHTO minimum rates of vertical curvature on the basis of the PSD for the crest curve and the SSD for the sag curve present no safety concerns.

Sight Distance Profile

The sight distance profile and minimum sight distance of a simple crest curve can be obtained by setting d equal to a very large value. The sight distance profiles of reverse and simple crest curves are shown in Figure 9. The reverse curve improves the minimum sight distance because the reversal elevates the driver

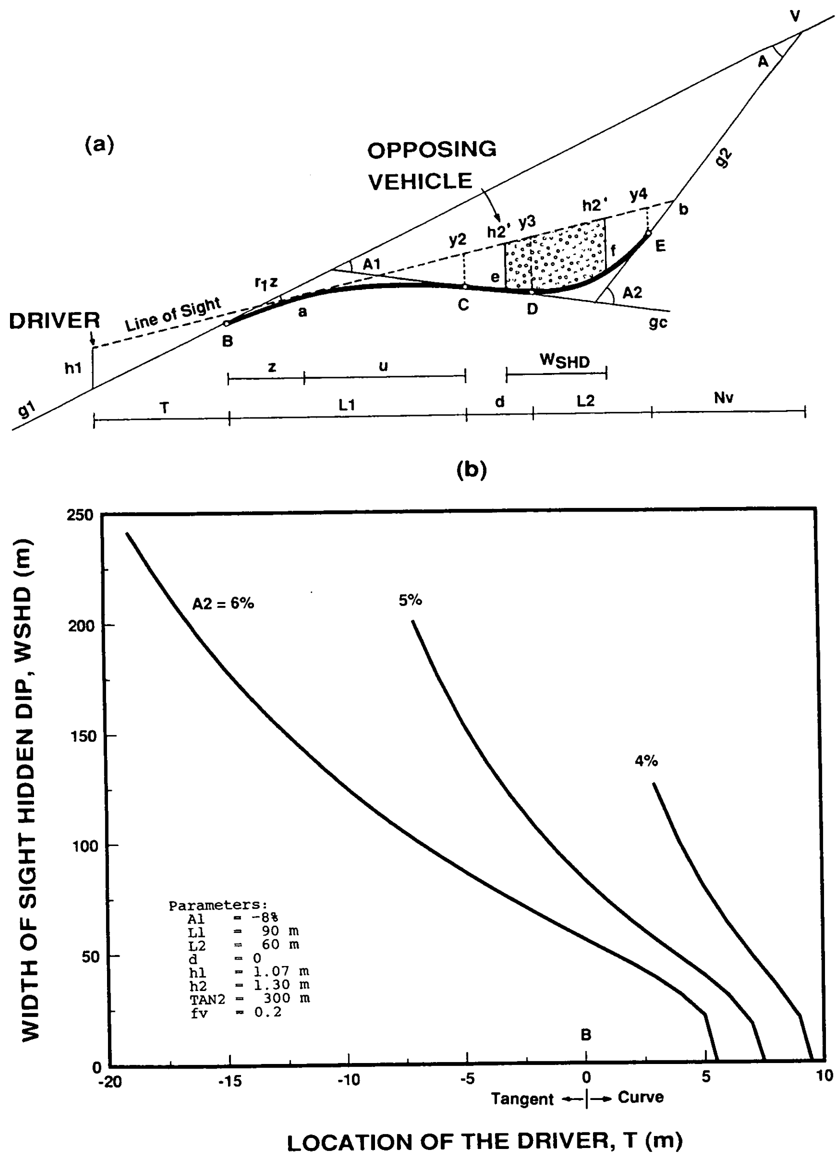

FIGURE 8 SHD of reverse vertical curve: (*a*) geometry of SHD; (*b*) effect of algebraic difference in grades, A_2.

or object. As the algebraic difference in grades of the sag curve increases, S_m increases, as expected.

The sight distance profiles of reverse and crest curves are quite different. For the reverse curve (A_2 = 16 percent) S_m occurs when the driver lies at $T = -33$ m (−108 ft). At $T = -29$ m (−95 ft) the sight distance is unlimited because the object lies above the line of sight. For a simple crest curve, however, the sight distance becomes unlimited only when the driver lies on the crest curve at $T = +15$ m (+49 ft) (not shown in Figure 9). The considerable difference in the shape of the sight distance profiles of reverse and crest curves affects the cost-effectiveness analysis of sites with restricted sight distances (*23,24*). For example if the required sight distance for the crest curve is 130 m (427 ft), crest curve models would predict that the length of the road with restricted sight distance is 35 m (118 ft), as shown in Figure 9. However the corresponding length on the reverse curve is only 8 m (26 ft).

S_m *Comparison with Simple Curves*

For SSD, the difference between the S_m of reverse and simple curves is large when L_1 is small and A_2 is large. The sight distance provided by a reverse curve is unlimited for small L_1. For PSD S_m is unlimited for a wider range of roadway parameters.

Length Requirements of Crest Arc

The length requirements of the crest arc of a reverse vertical curve were examined for $d = 0$ on the basis of the SSD and PSD needs of AASHTO (*5*). The lengths of the sag arc correspond to the minimum rates of vertical curvature for headlight control (upper range) on the basis of the SSD.

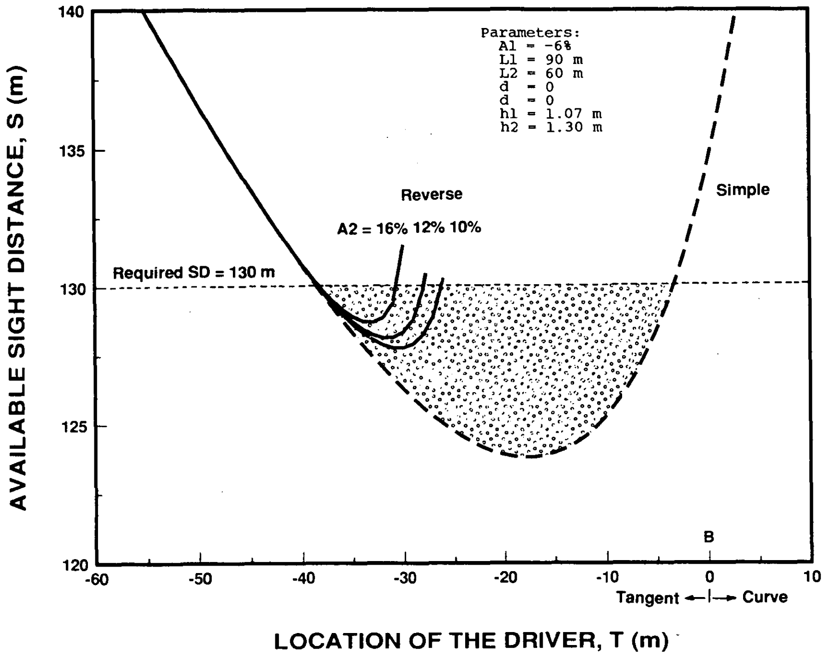

FIGURE 9 Sight distance (SD) profiles of reverse and simple crest curves.

The length requirements of the crest arc of a reverse curve on the basis of SSD are generally the same as those of a simple crest curve. The length requirements of the crest arc decrease only for limited conditions, for example, when $A_2 = -2$ percent and the design speed (DS) = 80 km/hr (50 mph). The length requirements on the basis of PSD of AASHTO are reduced to the minimum length criterion for many combinations of parameters.

What is interesting in the results is that for some cases the length requirements on the basis of PSD are less than those on the basis of SSD. This occurs, for example, for $A_1 = -2$ percent and DS = 80 km/hr (50 mph). In this case the length requirements on the basis of SSD should obviously be used, and PSD will consequently be available.

Computer Program

A computer program for the analysis of reverse horizontal and vertical curves has been developed. The program, which operates on a UNIX computer system, plots a sight distance profile, finds S_m along the reverse curve, and provides the characteristics of an SHZ or SHD. At present the program analyzes one set of curves at a time. The program will be modified so that an entire highway segment can be analyzed and sections with restricted sight distances are flagged. For reverse horizontal curves the program does not handle transition curves that affect sight distance. For simple circular curves transition curves have been found to reduce the required lateral clearance by a maximum value of 1.22 m (4 ft) (*10*).

SUMMARY AND CONCLUSIONS

No analytical method for evaluating the sight distance on highway reverse curves is available. This paper presented the geometric characteristics and available sight distances of reverse horizontal and vertical curves. Procedures for finding the minimum sight distance and analyzing the SHZ and SHD were addressed.

For reverse horizontal curves the SHZ length is related to the curve and obstacle parameters and to the length of the first and second tangents. Thus the analyst can examine the effects of different factors on the SHZ and find the lateral clearance required for avoiding the SHZ. The lateral clearance needs of reverse horizontal curves are generally less than those of comparable simple curves. This is due to the alignment reversal that improves the sight distance in comparison with that for a continuous tangent.

For reverse vertical curves the alignment reversal improves the sight distance and consequently reduces the required length of the crest arc. The results indicate that the length requirements of the crest arc are considerably less than those of a simple crest curve for certain ranges of roadway parameters. What is interesting is that, for certain combinations of parameters, the sight distance is unlimited and only a length equal to the minimum length criterion of vertical curves is needed.

This paper was concerned with the sight distance characteristics on the crest arc of a reverse vertical curve. The sight distance on the sag arc (nighttime conditions), which is also affected by the presence of the crest arc, needs to be explored. Another interesting area for future research is sight distance in three dimensions, which occurs when both horizontal and vertical curves exist at the highway location.

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