Minimum Horizontal Curve Radius as Function of Grade Incurred by Vehicle Motion in Driving Mode

MARIOS KONTARATOS, BASIL PSARIANOS, AND APOSTOLOS YOTIS

An enriched "bicycle" model was developed to describe the driving mode during the motion of a passenger car. The bicycle model is contrary to the mass-point model, which is suitable for the examination of the braking mode of a passenger car. The analysis concludes that there is a strong relationship between the radius of the horizontal curve and grade, which is found by studying vehicle motion on a helical surface. In some cases values of the minimum radius of the horizontal curve derived from the relationship exceed those suggested by AASHTO (1990) or RAS (1984). This means that, in such cases, existing guidelines lead to underdesigned values, because they do not consider that the driving mode during vehicle motion is critical.

In modern road design theory, the determination of the minimum horizontal curve radius for a certain speed is carried out on the basis of the following assumptions:

1. The vehicle is reduced to a simple mass-point,
2. The configuration of the road is a plane curved surface (i.e., there is no grade), and
3. The motion of the vehicle is governed by side friction values recommended by design guidelines.

Although the first two assumptions represent globally accepted design practices, two distinctive approaches exist in the third assumption. According to the first approach accepted by AASHTO policy in 1990 (*1*) (AASHTO-1990), side friction values are established on the basis of the comfort of the driver while negotiating a curve. This actually means that curve design is directly related to the dynamic constraints imposed on the vehicle as it moves in the driving mode. On the contrary, the second approach, which is accepted, among others, by the German 1984 RAS policies (*2*) (RAS-1984), refers to the motion of the vehicle under braking conditions. Specifically, according to the latter approach, the maximum side friction values that are used are such that considerable reserves of friction are disposed at the longitudinal direction in the case of braking. Although this difference in approach is implied in various guidelines, accidentally similar factors for limiting the side friction values were established.

The adoption of these assumptions in current road design theory, although validated by empirical data, leads to the calculation of minimum radius (R_{MIN}) values as an element independent from other design elements that coexist at the same road segment or other design constraints as vehicle characteristics. An optimal design process, however, can be accomplished by synthesizing and quantifying the interactive relations that exist between design goals, design constraints, and design elements, as Glennon and Harwood (*3*) have clearly pointed out. A model stronger in its ability to describe the cornering motion can be established, giving reliable design criteria for horizontal curves.

This paper intends to contribute to the enhancement of the design modeling tool so that these interactions between design elements can be revealed. Thus a vehicle-road model is formulated in which road geometry is almost exact and the vehicle (passenger car) is defined as a rigid body moving in the driving mode complying with the AASHTO-1990 motion mode. This model is used to determine the value of the minimum horizontal curve radius. Thus, a full extension of all three classical assumptions of the vehicle-road model as mentioned above is accomplished. Finally, an enriched "bicycle" model is developed. The model describes a vehicle that has height, with the forces acting on the interior and exterior wheels being equal, whereas those acting on the front and rear wheels depend on the type of vehicle drive.

Attempts to extend the classical vehicle-road model by some of the three-dimensional road parameters and other operational features of the vehicle are found in the literature (*4–6*). Some of the conclusions reached, however, must be regarded with caution. For example, the finding that the combined effect of grade and cross-slope has no substantial influence on the value of the minimum horizontal curve radius was derived by assuming values of maximum sliding coefficient of friction of between 0.3 and 0.5, which are too high according to established road safety criteria (*4*). Furthermore, because the specific analysis was limited to the braking mode, the calculated values of R_{MIN} were not significantly different from those given by the mass-point model.

In the analysis of the extended vehicle-road model introduced in this paper, the numerical values of parameters used are those accepted by current road and automobile design policies or standards. It is pointed out, however, that no claim of completeness in the overall numerical analysis can be made. Before quantitative statements find their place in design policies, two efforts must be successfully accomplished. First, representative values of the parameters introduced in the present vehicle-road model must be estimated to fit best the prevailing local conditions (e.g., maximum coefficient of friction and representative values of vehicle characteristics). Second, a comparison must be made between the two vehicle operating modes (the braking and driving modes) to establish which one is critical in each combination of horizontal and vertical road geometry elements.

Finally, it should be pointed out that the human factor may impose additional restrictions on the maximum reserve of friction that can be used in the lateral direction (*7,8*). Therefore, the study

M. Kontaratos, TRENDS, 9 Kondylaki Street, GR-11141 Athens, Greece. B. Psarianos and A. Yotis, National Technical University of Athens, GR-15773 Athens-Zographou, Greece.

of the complete driver-vehicle-road system may lead to more unfavorable values of various road features (e.g., higher values of R_{MIN}) in comparison with those determined by the analysis presented in this paper.

DETERMINATION OF THE MINIMUM RADIUS

For a given design speed, the minimum curve radius R_{MIN} represents a crucial value for the design of the horizontal alignment. In this paper this value is calculated by the model developed in Appendix B:

$$R_{MIN} = \frac{V^2}{g(n_F f_{Y,MAX} + q)} \quad (1a)$$

$$R_{MIN} = \frac{V^2}{g(n_R f_{Y,MAX} + q)} \quad (1b)$$

for front and rear wheel drive vehicle, respectively, where $1 - f_{Y,MAX} \cdot q \simeq 1$, and the factors n_F and n_R are equal to

$$n_F = \sqrt{1 - \frac{f_{X,F}^2}{f_{X,MAX}^2}} \cdot \left[1 - \frac{h}{l_R} \cdot s - \frac{A_z}{m \cdot g} - \frac{A_x}{m \cdot g} \cdot \frac{h}{l_R}\right] \quad (2)$$

$$n_R = \sqrt{1 - \frac{f_{X,R}^2}{f_{X,MAX}^2}} \cdot \left[1 + \frac{h}{l_F} \cdot s - \frac{A_z}{m \cdot g} + \frac{A_x}{m \cdot g} \cdot \frac{h}{l_F}\right] \quad (3)$$

In the above expressions, $f_{X,MAX}$, which is a function of speed, is the maximum available tangential coefficient of friction. (The abbreviations for the other parameters are defined after Appendix B in the section Nomenclature.) To make use of Equations 1a and 1b, the value of $f_{X,MAX}$ must be greater than the sliding coefficient of friction $f_{X,SL}$ suggested by the current design policies. This is because the present model refers to the driving and not the sliding mode of vehicle movement. The factor by which this should be increased, however, is open to further discussion. Because the purpose of this paper is to introduce the unfavorable safety conditions that may arise in the selection of R_{MIN} if only the driving mode is considered, the peak value of friction coefficient as a function of slip $f_{X,MAX}$ was chosen (*9,10*). This value, being 10 to 40 percent higher than the sliding coefficient of friction, was selected here to remain constant at 30 percent, that is,

$$f_{X,MAX} = 1.3 \cdot f_{X,SL} \quad (4)$$

During the cornering process, apart from the tangential friction, a side friction must be available. The maximum value of the side friction coefficient ($f_{Y,MAX}$) may be considered to be identical to the sliding coefficient value in the longitudinal direction ($f_{X,SL}$). The distribution of friction in both directions as a vehicle undergoes a culvilinear motion is governed by the expression (*9*):

$$\left[\frac{f_Y}{f_{Y,MAX}}\right]^2 + \left[\frac{f_X}{f_{X,MAX}}\right]^2 \leq 1 \quad (5)$$

Equations 1 to 5 constitute the analytical tool for the operational design of a roadway alignment. By using those equations, the least value of a curve radius or the necessary superelevation for accomplishing safely the cornering motion at a given design speed [or 85th percentile operating speed (V_{85})] may be calculated. Inversely, the safe cornering speed can be determined for a curve with a given radius and superelevation rate. Those properties, which are already well known for the mass-point model and the braking mode at level alignments, are made available for the enriched bicycle vehicle model developed herein and the driving mode for a three-dimensional alignment.

QUANTITATIVE ANALYSIS

A quantitative analysis was carried out to determine the deviations that may be imposed on the design road parameters by the equations formulated above. In the following quantitative analysis, the numerical investigation is conducted by defining a representative car (representative values of various vehicle characteristics). Obviously, its characteristics may change from one country to another as well as over time. The characteristics of this car should be compatible with recent technological changes; however, the characteristics of the older cars that are still in use may mainly influence their prototype parameters. Furthermore, it should be stressed that such cars should have unfavorable characteristics to meet the safety criteria that are usually set up according to a considerably conservative threshold (*11*).

It has been proven that the characteristics that mainly influence vehicle performance in the cornering process are the vehicle drive, the vehicle mass (m), the aerodynamic drag coefficient (c), and the position of the vehicle's center of gravity along its longitudinal axis (l_R), whereas a minor influence is the height of the center of gravity above the pavement (h) (*1*). For the needs of the work described here, the following values are assigned to them: m = 1000 kgr, c = 0.4, $l_R/1$ = 0.4; and h/l = 0.25 (where l is the distance between the front and rear wheels; Front-wheel drive) (*12,13*). The present investigation is limited to curved segments with superelevation rate q = 0.07.

Comparison Between Design Policies with Different Safety Margins

The R_{MIN} values calculated for various alignments and design speeds are compared with the corresponding radii suggested by AASHTO-1990 and the German RAS-1984 geometric design policies for highways. The discrepancy in the numerical values is due to the different safety margins accepted by AASHTO-1990 and RAS-1984 in the values of available road-pavement friction. Specifically the RAS-1984 policies accept values considerably lower than accepted by ASSHTO-1990, which additionally decrease significantly with increasing speed [i.e., at (50 mph), $f_{X,SL}$ = 0.3 and 0.24 at 70 mph, $f_{X,SL}$ = 0.28 and 0.17 for AASHTO-1990 and RAS-1984, respectively]. It should be noted that the discrepancy between the maximum allowable tangential friction values found in the AASHTO-1990 policy and the RAS-1984 guidelines seems to originate from the pavement data inventory. Friction values depend on a variety of factors (type and condition of tires, type and condition of the pavement surface, weather conditions, vehicle and driver performance under driving or braking modes). The RAS-1984 friction values were determined on the basis of an extended data inventory (*10*) and correspond to the skid resistance values of 95 percent of new pavements in the Federal Republic of Germany. That means that only 5 percent of any new road surfaces may be invalid for the application of these values (*14*). On the contrary, the AASHTO-1990 policies admit that although all influencing factors should be incorporated in the determination of friction values, "*available* data are not fully detailed over the range for all those variables, and conclusions must be made in terms of the safest *reported* average values" (*1*).

Furthermore, an indication of an insufficient friction values inventory is that "American friction values . . . clearly contradict the worldwide research experience which shows that friction values should substantially decrease with increasing speed" (*15*).

Influence of Vehicle Speed

In the present analysis, the important aspect of vehicle speed is examined. Four major results are derived.

1. There is a relationship between the horizontal curve radius and the grade, whereas the latter is superimposed on the former at the same road segment (Figures 1 to 4). This finding opposes the classical approach in which these two elements are considered to be independent with respect to safety; thus, the horizontal configuration and the vertical profile are selected independently and without serious interaction.
2. The R_{MIN} increases with increasing grade; their relationship is described by a convex function (Figures 1 to 4). This finding appears to violate the intuition of the highway engineer, according to which the safety requirements become more critical as the vehicle moves downgrade. However this rule is valid when considering the braking mode, whereas for the vehicle motion in driving mode, the opposite is correct. In the latter case, as the vehicle moves upgrade, greater longitudinal forces act on it, demanding greater reserves of friction. Consequently, fewer reserves of friction remain to be used in the lateral direction (Tables 1 and 2).
3. The required R_{MIN} values increase dramatically with grade at higher vehicle speeds (Figures 3 and 4).
4. In a number of cases the values of R_{MIN} given by the existing guidelines are lower than those that are required. This means that they underdesign for these cases, because they do not consider the driving mode of the vehicle motion as being critical.

In general, drivers actually move at speeds higher than the design speed (*16,17*). The consistency of road alignment should not allow deviations of V_{85} more than 16 to 20 km/hr (10 to 12 mph) from the design speed. Even under favorable driving conditions, this criterion does not exclude the possibility of inadequate minimum horizontal curve radii. On the basis of a broad classification of highway alignment (very good, good, and fair), an investigation of the necessary R_{MIN} values for all three cases was carried out. This broad classification corresponds to an accepted deviation of V_{85} from design speeds of 0, 5, and 10 mph, respectively (*18*).

The result is shown in Figures 1 to 4 for design speeds 50 and 60 mph. The former reveals that AASHTO-1990 values for R_{MIN} are adequate for good geometric designs. For fair designs, however, values greater than the suggested values are needed for grades of more than 3 percent. Worse results are apparent in the case of RAS-1984 policies. The relatively conservative friction factors that are accepted can be exceeded even at a very good

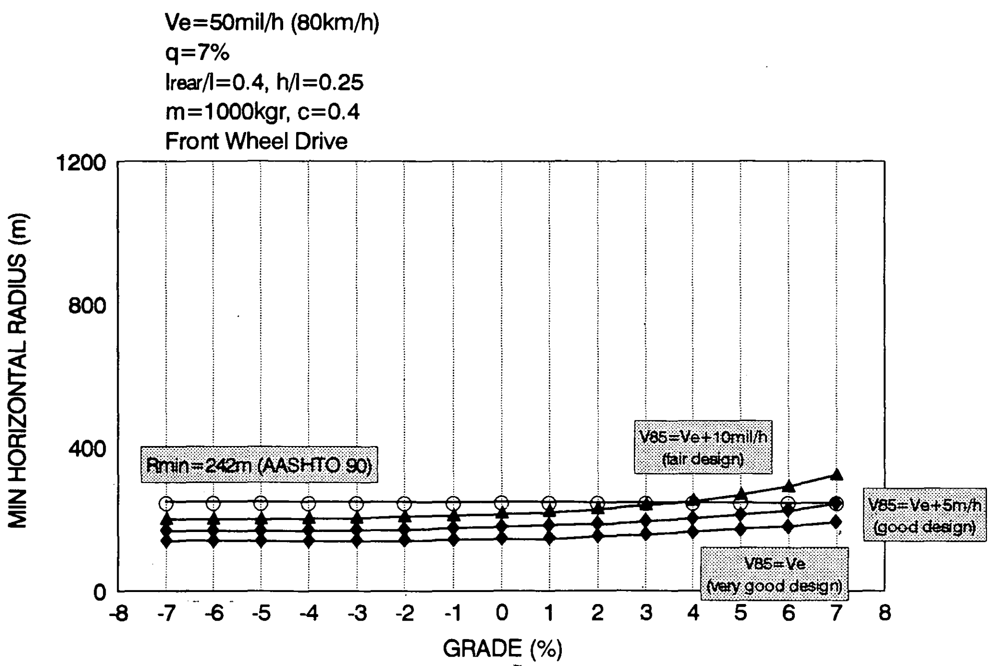

FIGURE 1 Influence of speed on R_{MIN} (V = 50 mph) values on the basis of AASHTO-1990.

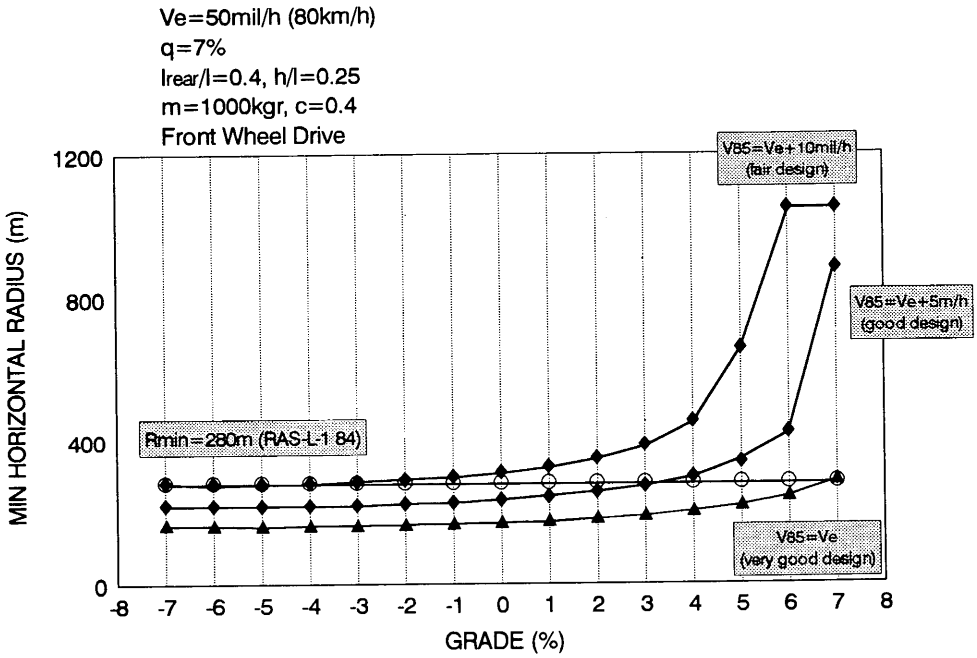

FIGURE 2 Influence of speed on R_{MIN} (V = 50 mph) values on the basis of RAS-1984.

geometric design when the operating speed equals the design speed (Figures 1 and 2). At a design speed of 60 mph, a fair geometric design worsens the situation dramatically. Figure 4 shows that at this speed the R_{MIN} values suggested by RAS-1984 lie well below the minimum values necessary in the driving mode at all grades.

The differences between design policies were derived, as mentioned above, because they assume different safety margins (*14*). Specifically, RAS-1984 assumes much worse pavement performance than that assumed by AASHTO-1990 as being representative, revealing in this way the more apparent underdesigning trend. In this sense, although it is stated that AASHTO-1990 leads

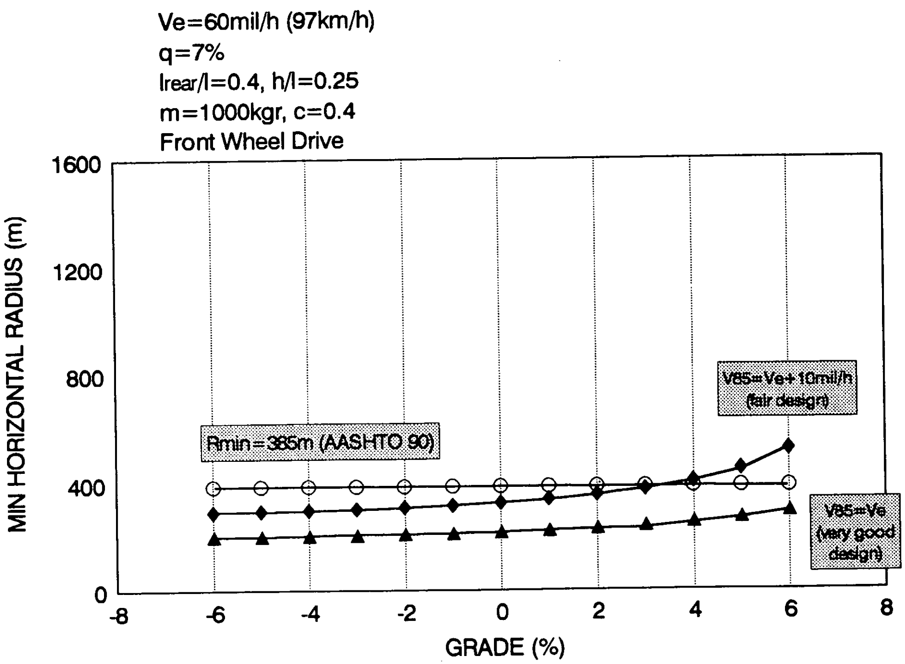

FIGURE 3 Influence of speed on R_{MIN} (V = 60 mph) values on the basis of AASHTO-1990.

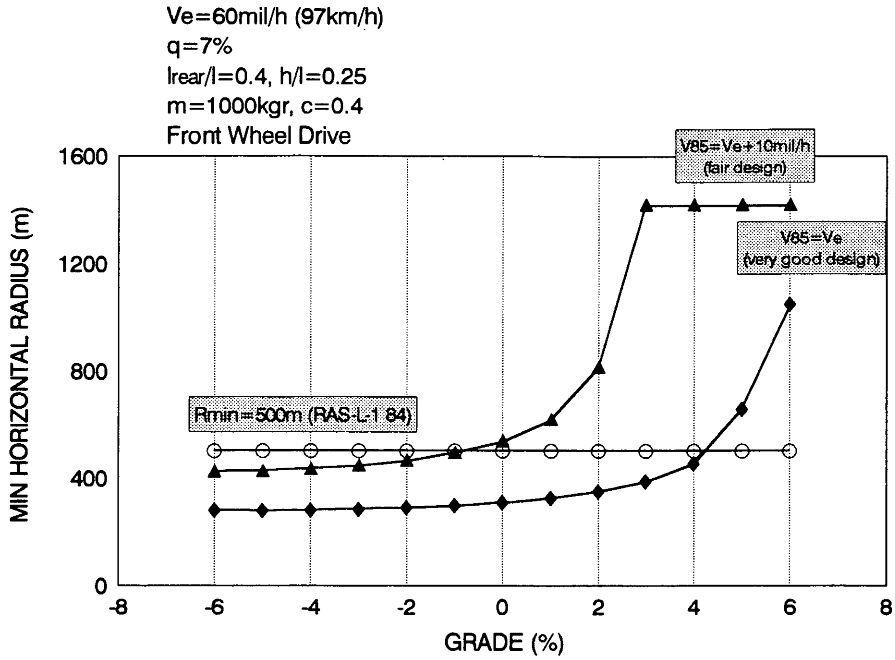

FIGURE 4 Influence of speed on R_{MIN} (V = 60 mph) values on the basis of RAS-1984.

TABLE 1 Demand Values of f_X for Various Speeds and Grades, and Remaining Friction Reserve To Be Used in Lateral Direction f_Y (Values Corresponding to AASHTO-1990 Policy); 1 mph = 1.61 km/hr)

Grade	V_{85} = 50mph		V_{85} = 60mph		V_{85} = 70mph	
	f_X	f_Y	f_X	f_Y	f_X	f_Y
0%	0.09	0.29	0.12	0.27	0.16	0.25
3%	0.17	0.27	0.20	0.25	0.24	0.21
6%	0.25	0.23	0.28	0.19	0.14	0.14

TABLE 2 Demand Values of f_X for Various Speeds and Grades and Remaining Friction Reserve To Be Used in Lateral Direction f_Y (Values Corresponding to RAS-1984 Policy)

Grade	V_{85} = 50mph		V_{85} = 60mph		V_{85} = 70mph	
	f_X	f_Y	f_X	f_Y	f_X	f_Y
0%	0.09	0.23	0.12	0.18	0.15	0.12
3%	0.17	0.20	0.19	0.13	0.23	0
6%	0.25	0.15	0.28	0	0.32	0

to underdesigns in fewer cases, this by no means should be interpreted as a safer policy standard than the RAS-1984 standard.

CONCLUSIONS AND RECOMMENDATIONS

The mass-point model and its inherent simplification in describing the cornering motion of a vehicle have deprived highway engineers of the ability to consider the phenomena governing the motion of a vehicle on a curve. In the preceding discussion, it was shown that neglect of the driving mode and the three-dimensional configuration of the roadway can lead to erroneous decisions concerning the selection of the appropriate horizontal curve radius. Design policies must recognize this fact. More striking results may be obtained and, consequently, a wider number of problematic situations may be identified if more conservative friction values are used. This is not a theoretical exercise, because recent as well as earlier researchers have pointed out (*19,20*) that in the driving mode the peak friction values cannot actually be used, let alone be used to determine safety criteria.

Furthermore, a new dynamic approach to the road design process must be introduced. The term *dynamic* is used in two ways. The first approach concerns the road design process itself, because the minimum horizontal curve radius does not remain constant along a roadway but changes with operating speed and grade of alignment. This can be termed the *internal dynamics* of the geometric design process. The second approach, *external dynamics*, refers to vehicle characteristics and pavement condition. Those characteristics generally vary from one country to another, thus implying different safety needs at curve sites. The deviating results obtained by using the AASHTO-1990 and the RAS-1984 values in the present analysis are a typical example of the different parameters of external dynamics in the road design process.

There is no doubt that the present analysis is deterministic. The intention of this paper, however, is not to give ready-to-use results but to point out critical safety situations that arise from the driving mode and the three-dimensional configuration of the road, which are not considered today. Further work is needed to integrate the stochastic dimension of the problem and satisfy the dynamic property of the road design process. A recent attempt applied to the mass-point model variables has been found in the literature (*21*); however, it may not be easy to repeat that work with the variables of the enriched model proposed in this paper. It may be difficult or even impossible to determine a probability distribution function of a variable such as R_{MIN}, depending on a number of stochastic independent variables like vehicle characteristics, speed, and pavement quality.

In addition, the following issues need to be investigated:

- The cornering performance of each individual vehicle type in the passenger car fleet must be examined and the results compared with current design policies.
- Because highways are built to serve the entire car fleet, an extension of the above investigation must be conducted to include trucks as well.
- On the basis of the two types of investigations needed as mentioned above, a comparison of the two vehicle modes, driving and braking, must be performed to determine which will govern the calculation of the critical value of a specific parameter for each combination of horizontal and vertical geometric elements.
- In the case of two-lane rural roads the operating speed has been proven to be a crucial design parameter beyond the design speed (*18,22*). Therefore, the analysis of the critical values of the design parameters must include the operating speed.

All these efforts are prerequisites before definite decisions for the minimum horizontal curve radius and other geometric features of the alignment can be made.

Finally, it should be pointed out that the analysis presented herein can be directly implemented in tort liability cases to determine the influence of vehicular parameters on the driving performance in a specific highway segment. This may prove to be of decisive importance under several circumstances.

APPENDIX A REPRESENTATION OF THREE-DIMENSIONAL ROAD SURFACE

The center of gravity of a vehicle (passenger car) is assumed to move on a space curve defined by its position vector **r**. To this curve (center of road line) is assigned a triplet of unit vectors (**t**, **n**, **b**), mutually orthogonal, composing the moving trihedron of the curve (*23*).

All forces and moments applied to a vehicle responsible for its movement in a trajectory, coinciding with the curve defined by the position vector **r**, may be expressed as a function of another triplet of unit vectors (t, ϵ, ζ) of the three-dimensional surface, with the curve given by **r** as its generator. Such surfaces are well known from differential geometry as *ruled surfaces*. The relationship that holds between the triplet (t, ϵ, ζ) and the conventional parameters that define a three-dimensional road surface, that is, cross-slope q and grade s, is in matrix form

$$(t, \epsilon, \zeta) = D(\hat{t}, \hat{\epsilon}, \hat{\zeta})^T \tag{A-1}$$

where D is the transformation matrix:

$$D = \begin{vmatrix} 1 + \alpha q s & -\alpha & \alpha q - s \\ \alpha - q s & 1 & -\alpha s - q \\ s & q & 1 \end{vmatrix} \tag{A-2}$$

and the unit vector triplet ($\hat{t}$, $\hat{\epsilon}$, $\hat{\zeta}$) represents the corresponding triplet to a plane surface (with no grade or cross-slope).

APPENDIX B VEHICLE DYNAMICS ON A THREE-DIMENSIONAL ROAD SURFACE

The motion of a passenger car on a road can be divided into three translatory movements, namely, longitudinal, lateral, and vertical, as well as three rotational movements, yaw, roll, and pitch. All of these individual movements occur along and around the vector triplet (t, ϵ, ζ). However, not all of them are important in terms of road design. Only the movements along and around the tangential vector t, that is, longitudinal movement and lateral movement, are critical for the formulation of road design criteria.

In considering the moving vehicle as the reference system, the forces imposed on it can be determined. These are illustrated in Figure B-1 (*24*).

1. The gross vehicle weight $\boldsymbol{G}$

$$G = mg\boldsymbol{\zeta} \tag{B-1}$$

2. The wheel-pavement contact forces analyzed to three components: the driving forces $\boldsymbol{X}$, the lateral forces $\boldsymbol{Y}$, and the vertical forces $\boldsymbol{Z}$, which for small slip angles are

$$\boldsymbol{X} = (X_F + X_R)\,\boldsymbol{t} \tag{B-2}$$

$$\boldsymbol{Y} = -\,(Y_F + Y_R)\,\boldsymbol{\epsilon} \tag{B-3}$$

$$\boldsymbol{Z} = -\,(Z_F + Z_R)\,\boldsymbol{\zeta} \tag{B-4}$$

3. The air resistance force $\boldsymbol{A}$

$$\boldsymbol{A} = -\,(A_X\boldsymbol{t} + A_Z\boldsymbol{\zeta}) \tag{B-5}$$

Under the influence of the above forces as well as of different moments (rolling resistance moments, bearing moments, etc.), the vehicle moves on the road surface, whereby the conservation laws of linear and angular momentum apply.

$$\sum_{\text{Forces}} = m\mathbf{r}'' = m\frac{dv}{dt}\boldsymbol{t} + m\frac{V^2}{R}\boldsymbol{n} + m\frac{V^2}{H}\boldsymbol{b} \tag{B-6}$$

and

$$\sum_{\text{Moments}} = D' \tag{B-7}$$

Introduction of Equations B-1 to B-5 into Equations B-6 and B-7 and successive multiplication by the vectors $\boldsymbol{t}$, $\boldsymbol{\epsilon}$, and $\boldsymbol{\zeta}$ result in an explicit expression of the acting forces on the moving vehicle. Not all of them, in fact, are of particular interest to the road designer, as mentioned above. Taking into consideration a nonaccelerated movement of the vehicle on a helical surface (i.e., a three-dimensional surface: horizontal curve and constant grade) in which all three geometric parameters remain constant, namely, grade s, superelevation q, and radius R

$$V = \text{constant} \tag{B-8}$$

$$s \simeq \tan(s) \simeq \sin(s) = \text{constant} \tag{B-9}$$

$$q \simeq \tan(q) \simeq \sin(q) = \text{constant} \tag{B-10}$$

$$R = \text{constant} \tag{B-11}$$

the expressions giving the forces acting on a vehicle with front-wheel drive are obtained as follows:

$$X_F = Gs + A_X + \frac{m^2}{4\delta}\left[\frac{V^2}{R} - gq\right]^2 + f_R Z_R \tag{B-12}$$

$$X_R = -\,f_R Z_F \tag{B-13}$$

Corresponding expressions may be derived for a vehicle with a rear-wheel drive.

$$Y_F = m\left[\frac{V^2}{R} - gq\right]\frac{l_R}{l} \tag{B-14}$$

$$Y_R = m\left[\frac{V^2}{R} - gq\right]\frac{l_F}{l} \tag{B-15}$$

$$Z_F = G\left[\frac{l_R}{l} - s\frac{h}{l}\right] - A_{Z,F} + mq\frac{V^2}{R}\frac{l_R}{l} \tag{B-16}$$

$$Z_R = G\left[\frac{l_F}{l} + s\frac{h}{l}\right] - A_{Z,R} + mq\frac{V^2}{R}\frac{l_F}{l} \tag{B-17}$$

$$A_{Z,F} = A_Z\frac{l_R}{l} - A_X\frac{h}{l} \tag{B-18}$$

$$A_{Z,R} = A_Z\frac{l_F}{l} + A_X\frac{h}{l} \tag{B-19}$$

To obtain the above equations, the assumption was made that the axes of the front and rear wheels and the center of gravity are tracing nearly parallel curves. Furthermore, the coefficient of tire stiffness δ is considered equal for all tires, whereas the lateral force is linearly related to the slip angle α.

From the above set of expressions, the friction coefficients in the longitudinal and lateral directions are readily available. The friction coefficients are given as

$$f_{X,F} = \frac{X_F}{Z_F} \tag{B-20}$$

$$f_{Y,F} = \frac{Y_F}{Z_F} \tag{B-21}$$

Similar expressions may be used for the friction coefficients for the rear wheels.

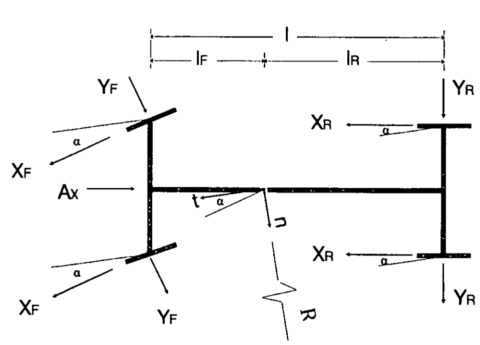

FIGURE B-1 Forces acting on a passenger car moving along a road segment.

NOMENCLATURE

A, A_X, A_Z, $A_{Z,F}$, $A_{Z,R}$ = Air resistance force and its components to respective axes and to respective wheels (F = front; R = rear)

$\boldsymbol{b}$ = Unit binormal vector

c = Aerodynamic drag coefficient
f_R = Rolling coefficient of friction
$f_{X,F}, f_{Y,F}, f_{X,R}, f_{Y,R}$ = Coefficients of friction to respective directions and wheels
$f_{X,\mathrm{MAX}}, f_{Y,\mathrm{MAX}}$ = Maximum coefficient of friction to respective directions
$f_{X,\mathrm{SL}}$ = Sliding coefficient of friction
H = Radius of vertical curve
h = Distance of center of gravity from the pavement
G = Vehicle weight
g = Gravity acceleration rate
l = Distance between front and rear wheels
l_F, l_R = Distance between the center of gravity and the respective wheels
m = Vehicle mass
$\mathbf{n}$ = Principal normal unit vector
n_F, n_R = Factors
q = Superelevation rate
R = Radius of horizontal curve
s = Grade
$\mathbf{t}$ = Unit tangent vector
V = Vehicle speed
V_{85} = 85th percentile of speed
X_F, X_R = Driving forces acting on respective wheels
Y_F, Y_R = Lateral forces acting on respective wheels
Z_F, Z_R = Vertical forces acting on respective wheels
α = Slip angle
δ = Tire stiffness coefficient
ϵ = Generating vector
ζ = Normal vector

REFERENCES

1. *A Policy on Geometric Design of Highways and Streets.* AASHTO, Washington, D.C., 1990.
2. *Richtlinien für die Anlage von Strasse, Teil: Linienfuehrung (RAS-L-1).* Forschungsgesellschaft für das Strassen and Verkehrswesen, Cologne, Germany, 1984.
3. Glennon, J. C., and D. W. Harwood. Highway Design Consistency and Systematic Design Related to Highway Safety. In *Transportation Research Record 681*, TRB, National Research Council, Washington, D.C., 1978, pp. 77–88.
4. Dunlap, D. F., P. S. Fancher, R. E. Scott, C. C. MacAdam, and L. Segel. *NCHRP Report 184: Influence of Combined Highway Grade and Horizontal Alignment on Skidding.* TRB, National Research Council, Washington, D.C., 1978.
5. Zuk, W. Instability Analysis of a Vehicle Negotiating a Curve with Downgrade Superelevation. In *Highway Research Record 390*, HRB, National Research Council, Washington, D.C., 1972, pp. 40–44.
6. Durth, W. *Heft 163: Ein Beitrag zur Erweiterung des Modells fuer Fahrer, Fahrzeug und Strasse in der Strassenplanung.* Strassenbau und Strassenverkehrstechnik, Bonn–Bad Godesberg, Germany, 1974.
7. Graus, J., and M. Livneh. Superelevation and Curvature of Horizontal Curves. In *Transportation Research Record 685*, TRB, National Research Council, Washington, D.C., 1978, pp. 7–13.
8. Macura, D. *Psychological Factor in Dimensioning of the Horizontal Road Curves.* Association of the Road Societies of Yugoslavia, Belgrade, 1984.
9. Bergman, W. Skid Resistance Properties of Tires and Their Influence on Vehicle Control. In *Transportation Research Record 621*, TRB, National Research Council, Washington, D.C., 1976, pp. 8–18.
10. Gauss, F. Vehicle Design and Skid Resistance. In *Transportation Research Record 621*, TRB, National Research Council, Washington, D.C., 1976, pp. 19–27.
11. Kontaratos, M. *Relationships Between the Roadway Geometrics Satisfying the Criteria of Safety and Fuel Consumption for the Driving Motion of a Passenger Vehicle.* Ph.D. thesis. National Technical University of Athens, Athens, Greece, 1990.
12. Flegl, H., T. Foeldi, and L. Witte. Handling Characteristics of Four-Wheel Drive Vehicles. In *The Dynamics of Vehicles on Road and on Railway Tracks. Proc., 8th IA VSD-Symposium*, Massachusetts Institute of Technology, Cambridge, 1983, pp. 15–19.
13. Cardon, M. H., G. B. Hickner, and R. W. Rothfusz. Development and Evaluation of Anti-Lock Brake Systems. In *Transportation Research Record 624*, TRB, National Research Council, Washington, D.C., 1976, pp. 1–14.
14. Lamm, R. Driving Considerations: A Comparison of German and American Friction Coefficients for Highway Design. In *Transportation Research Record 960*, TRB, National Research Council, Washington, D.C., 1984, pp. 13–20.
15. Lamm, R., E. M. Choueiri, P. B. Goyal, and T. Mailaender. Design Friction Factors of Different Countries Versus Actual Pavement Friction Inventories. In *Transportation Research Record 1260*, TRB, National Research Council, Washington, D.C., 1990, pp. 135–146.
16. Leisch, J. E., and J. P. Leisch. New Concepts in Design Speed Application. In *Transportation Research Record 631*, TRB, National Research Council, Washington, D.C., 1977, pp. 4–14.
17. McLean, J. Driver Speed Behaviour and Rural Alignment Design. *Traffic Engineering and Control*, Vol. 22, No. 4, April 1981, pp. 208–211.
18. Lamm, R., and E. M. Choueiri. *A Design Procedure to Determine Critical Dissimilarities in Horizontal Alignment and Enhance Traffic Safety by Appropriate Low-Cost or High-Cost Projects.* Final Report for the National Science Foundation (grant no. ECE-8414755). Clarkson University, Potsdam, N.Y., March 1987.
19. Olson, P. L., D. E. Cleveland, P. S. Fancher, L. P. Kostyniuk, and L. W. Schneider. *NCHRP Report 270: Parameters Affecting Stopping Sight Distance.* TRB, National Research Council, Washington, D.C., 1984.
20. Harwood, D. W., W. D. Glauz, and J. M. Mason, Jr. Stopping Sight Distance Design for Large Trucks. In *Transportation Research Record 1208*, TRB, National Research Council, Washington, D.C., 1989, pp. 36–46.
21. Hirsh, M., J. N. Prashker, and M. Ben-Akiva. New Approach to Geometric Design of Highways. In *Transportation Research Record 1100*, TRB, National Research Council, Washington, D.C., 1986, pp. 50–57.
22. Lamm, R., E. M. Choueiri, J. C. Hayward, and A. Paluri. Possible Design Procedure to Promote Design Consistency in Highway Geometric Design on Two-Lane Rural Roads. In *Transportation Research Record 1195*, TRB, National Research Council, Washington, D.C., 1988, pp. 111–122.
23. Psarianos, B. *Ein Beitrag zu der Entwicklung des raumlichen Trassierungsprozesses von Verkehrswegen und insbesondere von Strassen.* Dissertation. Hannover, Germany, 1982.
24. Mitschke, M. *Dynamik der Kraftfahrzeuge: Antrieb und Bremsug*, Band A. Springer-Verlag, Berlin, 1982.

Publication of this paper sponsored by Committee on Geometric Design.