

Modeling Performance of Highway Pavements

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A highway performance prediction model predicts the performance of highway pavements as a function of explanatory variables such as pavement characteristics, ambient climate, usage of the system, and so on. However, there is no unambiguous approach that can be used to directly measure the performance of the highway pavement. Performance is considered to be unobservable (latent). The problems with developing performance deterioration models include the definition of the aforementioned unobservable performance in terms of the observed or measurable distress measures of the system and simultaneously relating the performance to the explanatory variables. Previous research is extended by exploring the existence of a two-component performance measure for highway pavements: a latent variable to represent functional performance and another variable to represent the structural integrity or structural performance of the pavement. A case study is conducted on a data set from Brazil compiled by the World Bank.

Why are performance deterioration models necessary? The determination of cost-effective maintenance actions requires information on:

- Current condition (obtained from an inspection of the facility), and
- Anticipated conditions for different maintenance and rehabilitation actions (obtained from performance deterioration models).

The emergence of a large variety of automated technologies (such as video, laser, radar, and infrared technologies) that can be used to collect information on pavement conditions has made available large quantities of data for the analysis of pavement performance. These new technologies require new methods for processing their nascent outputs to a manageable size meaningful for decision making. On the other hand existing approaches to pavement performance analysis are based on subjective indexes that use a predetermined set of distress measures selected at a time when less developed data collection technologies were used [e.g., measures like present serviceability index (PSI) (1) and pavement condition index (2)]. There is a need for an improved performance analysis methodology to exploit these enhanced data collection capabilities. By using new methods for performance analysis, it should be possible to plan more cost-effective maintenance actions for highway pavements.

PAVEMENT MANAGEMENT PROCESS

This section outlines the different facets of the pavement management process. The reader is directed to Ben-Akiva et al. (3,4)

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for a more comprehensive treatment of the pavement management process. The pavement maintenance management problem consists of the allocation of limited resources for the maintenance and rehabilitation of different pavements over both spatial and temporal dimensions. The pavement management process can be divided into the following three task areas:

- Data collection and analysis (including inspection);
- Performance analysis and modeling; and
- Maintenance and rehabilitation (M&R) and inspection strategy selection.

These tasks are related in the manner shown in Figure 1. The facility condition data collected by using different technologies are used in two ways. First, they are one of the items used in the estimation of pavement performance models. Second, they provide the initial values in the prediction of future performance. In addition to M&R strategies, the third block includes models for the selection of future inspection strategies. This effect is represented by the feedback loop of Figure 1.

Data Collection and Analysis

Data on the extent of pavement damage (such as area or length and severity of damage), on the causal variables that affect deterioration such as usage, age, environmental conditions, and pavement type are collected. The data may be collected by visual inspection, through manual measurements, or through automated techniques. The data collected, after suitable processing, can be used as inputs to performance analysis. The data collected can also be used for monitoring purposes to validate model predictions and update a model system after it has been implemented.

Performance Analysis and Modeling

A deterioration model links a measure of the condition of the facility to a vector of explanatory variables. The condition measure in its simplest form is just the extent of damage; more complex indexes that combine the extent of different types of damage may also be used.

This paper addresses the issue of estimating pavement performance deterioration models. The framework presented by Ben-Akiva et al. (5,6) is used to estimate two-component performance deterioration models. Hence, this paper concentrates on this block of the pavement management process.

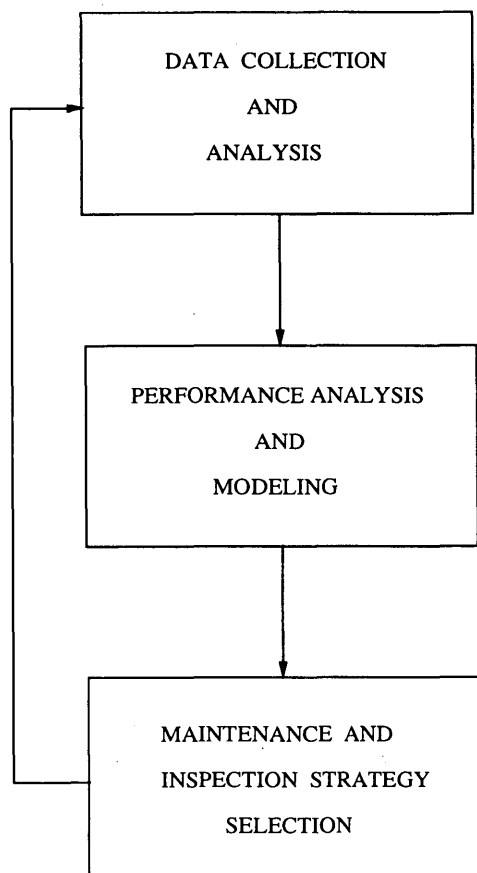


FIGURE 1 Highway pavement management process.

M&R and Inspection Strategy Selection

This block involves the choice of an optimal maintenance activity and inspection strategy that minimizes the total costs over the planning horizon subject to various resource and technical constraints. A methodology that recognizes the trade-offs between inspection costs (which increase with the accuracy of the measurement technology used) and the added costs of M&R (which decrease with the increased accuracy of the information provided by this technology) to address the inspection decisions in a systematic manner is presented by Madanat and Ben-Akiva (7).

FRAMEWORK OF MODELING PAVEMENT PERFORMANCE

The framework of modeling pavement performance is presented in Figure 2. The independent variables affecting the performance of the facility form X : ($K \times 1$). These factors can be classified broadly into the following categories:

1. Inherent factors (factors associated with the facility itself, such as facility type and construction quality), and
2. Extraneous factors (which include facility usage, maintenance actions performed, and environmental factors).

Since the facility performance is not directly observable, it is characterized by a latent variable vector, S : ($M \times 1$). The latent

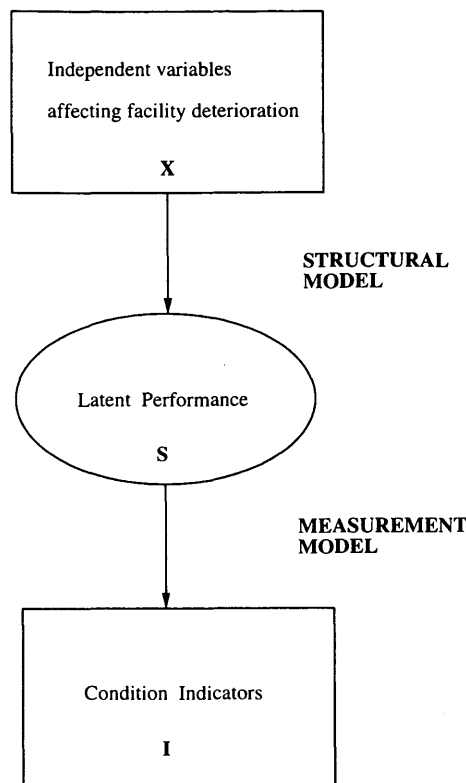


FIGURE 2 Pavement performance modeling framework.

performance of the facility manifests itself in the form of measurable indicators I : ($P \times 1$). The indicators of the latent pavement performance are roughness, cracking, rutting, surface patching, raveling, and so on. The relationships among the independent variables X and the performance vector S form the structural model (Equation 1). The mapping from the latent performance S to the indicators I form the measurement model (Equation 2). Using linear functional forms the model system is written as

$$S = \gamma X + \zeta \quad (1)$$

$$I = \Lambda S + \epsilon \quad (2)$$

where

γ : ($M \times K$) and Λ : ($P \times M$) = parameter matrices to be estimated,

ζ : ($M \times 1$) = error component in structural model, and

ϵ : ($P \times 1$) = error component in measurement model.

Without any loss of generality, all observed variables are written as deviations from their respective means, or else the intercepts must be included in both the structural and measurement models. A comprehensive treatment of the theory and estimation of latent variable models is found in work by Everitt (8) and Bollen (9).

CASE STUDY

Description of Data

The data used in the study described here were collected during World Bank road deterioration studies undertaken in Brazil from

TABLE 1 Descriptive Statistics of Selected Variables

Variable	Units	Mean	Std. dev	Minimum	Maximum
Age since rehabilitation	years	8.19	4.53	0.007	23.05
Equivalent Standard axles	80 kip load $\times 10^6$	2.226	3.2	0.00011	30.28
Cumulative precipitation	meter	2.58	1.92	0.00	7.22
Roughness	QI in counts/km	40.53	15.36	12.92	99.69
Cracking	% surface area	15.16	25.70	0.00	100.00
Rut depth	mm	3.71	2.19	0.00	15.75
Surface patching	% surface area	2.47	8.65	0.00	86.29
Ravelling	% surface area	7.61	19.70	0.00	100.00
Benkelman deflection	mm	0.65	0.36	0.12	2.03
Structural number	units	4.12	1.03	1.62	7.73

1975 to 1982 under the *Research on Interrelationships Between the Costs of Highway Construction and Utilization (10)*.

Surfacing Type

There were 3,149 observations [an average of 8.3 observations per highway section (one lane of 280 to 560 m in length) at approximately half-yearly intervals] of the pavement condition, cumulative traffic, environmental factors, maintenance status, and pavement strength at given dates as they evolved during the study period. There were four main types of pavement surfaces at the first observation dates:

1. Asphalt concrete, original (AC);
2. Double surface treatment, original (DST);
3. Asphalt concrete overlay on original asphalt concrete surfacing (ACOVL); and
4. Asphalt concrete overlay on original double surface treatment (DSTOV).

Explanatory Variables

The important explanatory variables, which affect performance deterioration, available in the data set are

1. Age since the most recent rehabilitation (AGER), measured in years.
2. Cumulative number of equivalent standard axles (ESAXL) since the most recent rehabilitation at the date of observation.
3. Cumulative precipitation since the most rehabilitation (CP), in meters.
4. Structural number (SNC), a measure of pavement strength, obtained from the thicknesses and stiffnesses of different pavement layers.

Condition Indicators

The indicators of pavement performance are roughness, cracking, rutting, surface patching, and raveling. Roughness (RQI), a mea-

sure of the longitudinal irregularity of the road surface, is measured in quarter-car index with units in counts per kilometer. As a measure of cracking, the area of indexed cracking (CRX) as a percentage of the surface area is also available. Rutting (RDMN) is measured as the mean rut depth under a 1.2-m straightedge across two wheelpaths and four test points per wheelpath. Surface patching (SPAT) is measured as the sum of the areas of surface patching expressed as a percentage of the surface area. Raveling (RAV) is measured as the sum of the areas raveled expressed as a percentage of the section area. The indicators of highway pavement strength are Benkelman deflection (DEFS) and rut depth. The descriptive statistics of selected variables are given in Table 1.

Model 0: Present Serviceability Index Type Deterioration Model

In the PSI type deterioration model, which uses the formula for PSI described previously (11), the PSI for each highway segment is fitted as the function of condition indicators. The fitted PSI variable is regressed on a set of variables, which include pavement age, cumulative axle loading, precipitation, and structural number, to obtain the deterioration model as shown here:

$$\widehat{\text{PSI}} = \alpha_0 + \alpha_1 \text{AGER/SNC} + \alpha_2 \text{ESAXL/SNC} + \alpha_3 \text{CP/SNC} + \epsilon \quad (3)$$

The estimated model is presented in Table 2. All of the estimated coefficients are significant and have the right signs. The fit of the

TABLE 2 Model 0: PSI-Type Deterioration Model

Independent variable	Estimate	t-statistic
Intercept	5.023	76.08
AGER/SNC	-0.388	-15.05
ESAXL/SNC	-0.117	-2.51
CP/SNC	-0.201	-3.18

model is poor, with an R^2 of 0.19 (number of observations = 1,571). Model 0 serves as the base model against which the estimated latent performance models are compared.

Estimated Latent Performance Models

In the next two sections latent performance deterioration models estimated on the basis of the data set are presented. The models are broadly classified into

- Models in which performance is characterized by a single latent variable.
- Models in which performance is characterized by two-component latent vector.

Comparisons of different models are primarily based on squared multiple correlation, a measure of goodness-of-fit similar to R^2 in a regression model, of each equation in the model, the "correctness" of the estimated parameters, and their significance. All of the models are estimated by using the unmaintained highway segments. Thus, the number of observations used in the analysis is less than the total number of observations in the data set. Furthermore, all observations are pooled under the assumption of no temporal dependence of the deterioration process for each highway segment.

The covariance matrix of the error term in the structural model is unconstrained (all the elements are free parameters that are estimated), whereas the covariance matrix of the error term in the measurement model is constrained to a diagonal matrix. This is a realistic assumption because the measurement processes for the different indicators are independent.

Single Latent Performance Models

In this section models are estimated on the basis of the hypothesis that the performance of the highway facility can be characterized by a single latent variable. The measured damage components are directly used as indicators of the underlying latent variable.

Model 1 In Model 1 the performance of the pavement is hypothesized to be affected by AGER, CP, and ESAXL, which represents traffic loading or usage of the highway. Thus the effects of facility usage, environmental conditions, and age of the pavement on performance are captured in the deterioration model.

In the measurement model, the observed damage measurements—RQI, CRX, RDMN, SPAT, and RAV—are used directly as indicators of the latent performance of the pavement segment because of the absence of a priori information regarding the manifestation process of the underlying performance variable. Since the latent variable is unobserved it does not have a definite scale. Hence it is necessary to fix one parameter in each column of the Λ matrix in the measurement model to unity. This defines the unit of measurement of the latent variable to be the same as that of the corresponding observed variable. In this model the scale of the performance variable is set equal to that of roughness. The structural and the measurement models are given in the following: for the structural model,

$$S = \gamma_1 \text{AGER} + \gamma_2 \text{ESAXL} + \gamma_3 \text{CP} + \zeta$$

TABLE 3 Structural Model, Model 1

Independent variable	Estimate	t-statistic
Age since rehabilitation	1.097	15.01
Equivalent standard axles	0.295	3.52
Cumulative precipitation	0.141	1.09

Coefficient of determination = 0.36

For the measurement model,

$$\text{RQI} = S + \epsilon_1$$

$$\text{CRX} = \lambda_2 S + \epsilon_2$$

$$\text{RDMN} = \lambda_3 S + \epsilon_3$$

$$\text{SPAT} = \lambda_4 S + \epsilon_4$$

$$\text{RAV} = \lambda_5 S + \epsilon_5$$

The estimated model system is presented in Tables 3 through 5. The coefficients of AGER and ESAXL are significant, whereas the coefficient of cumulative precipitation is insignificant, implying inconsequential effects of precipitation on pavement performance deterioration. In the measurement model the squared multiple correlation (SMC) is a measure of the variance of the indicator explained by the latent variable; a higher value of SMC for a particular measurement equation implies that the associated indicator is a better measure of the latent variable than an indicator with a lower value of SMC. In Model 1 roughness, cracking, and rut depth have relatively high SMCs (approximately 0.4), implying that these damage components are good indicators of performance. On the other hand patching and raveling appear to be poor measures of the latent performance of the pavement. The overall fit of the measurement model is 0.63, which is higher than the overall fit of the structural model (0.36). It must be noted that the fit of the structural model (0.36) is higher than the R^2 of 0.19 in Model 0.

In the equation for the estimated value of latent performance as a linear function of the indicators, the coefficients of the different indicators are calculated by using the full information extraction method described by Gopinath (12). The adjusted R^2 (a measure of the goodness-of-fit of the model) for this extracted latent performance model is 0.68, with all of the estimated coefficients being statistically significant. It can be seen that the t -statistics of roughness, cracking, and rut depth are more than those of patching and raveling, reiterating the relative importance of the former measures. This model can be used to estimate the performance given the distress measurements from an inspection of the facility.

TABLE 4 Measurement Model, Model 1

Indicator	Estimate	t-statistic	squared multiple correlation
Roughness	1.000	— ^a	0.35
Cracking	1.778	16.50	0.40
Rut depth	0.150	16.45	0.39
Surface patching	0.348	11.28	0.13
Raveling	0.431	6.53	0.04

^aFixed parameter.

Coefficient of determination = 0.63

TABLE 5 Extracted Latent Performance Model, Model 1

Indicator	Estimate	t-statistic
Roughness	0.145	14.33
Cracking	0.134	23.17
Rut depth	1.556	23.21
Surface patching	0.099	5.91
Ravelling	0.063	9.41

Adjusted $R^2 = 0.68$

Total number of observations = 1571

Model 2 Model 2 SNC, which is a measure of the strength of pavement on the basis of the thickness and stiffness of the different pavement layers, is used as an additional explanatory variable in the structural model corresponding to Model 1. This variable is expected to capture the effects of pavement strength on the deterioration process, and thus differentiate among different pavement types. One would expect a stronger pavement to have a slower rate of deterioration than a weaker pavement. Therefore, the explanatory variables AGER, ESAXL, and CP are divided by SNC to model these interaction effects in the structural model.

The estimated model system is presented in Tables 6 through 8. The fit of the structural model increased from 0.36 (in Model 1) to 0.61, implying that structural strength is an important explanatory variable affecting the deterioration process. In the measurement model the fit of the rut depth equation increased to 0.5 from 0.39 in Model 1. This observation is also reflected in the increase in the value of the rut depth coefficient in the extracted latent performance model. The surface patching coefficient is statistically insignificant in the extracted latent performance model. Thus, the addition of a strength-related explanatory variable suggests the presence of a performance variable related to the structural integrity of the pavement. Of the two single latent performance models presented so far, this model has good fits for the structural and the extracted latent performance models. Thus, it can be used to predict the future condition of the pavement as well as to calculate present performance if the facility is inspected (i.e., the indicators are measured) for the present period.

The observations made in Model 2 suggest the presence of another dimension to pavement performance—a pavement strength-related dimension as seen in the relative importance of the rut depth indicator (a strength-related variable)—in addition to a surface quality-related dimension. In the next section a two-component latent performance vector—a latent variable, S_{func} , to represent ride quality or functional performance and a latent variable, S_{struc} , to represent pavement strength or structural performance—is used to characterize the performance of the pavement.

Two-Component Latent Performance Model (Model 3)

In Model 3 it is hypothesized that the functional performance of the pavement is characterized by a latent variable, S_{func} , and the

TABLE 6 Structural Model, Model 2

Independent variable	Estimate	t-statistic
AGER/SNC	3.482	16.30
ESAXL/SNC	4.078	12.15
CP/SNC	1.503	3.75

Coefficient of determination = 0.61

TABLE 7 Measurement Model, Model 2

Indicator	Estimate	t-statistic	squared multiple correlation
Roughness	1.000	— ^a	0.356
Cracking	1.458	17.27	0.306
Rut depth	0.178	21.35	0.498
Surface patching	0.235	8.98	0.071
Ravelling	0.555	8.67	0.061

^aFixed parameter.

Coefficient of determination = 0.62

structural integrity of the pavement is characterized by another latent variable, S_{struc} . The functional aspect of pavement performance is related to the surface characteristics such as roughness, cracking, and raveling that affect user costs. The structural aspect of performance is of no perceptible concern to the user, whereas it is extremely important to the highway agency because the future functional performance may depend on the present structural condition. Furthermore, the maintenance actions could be chosen with the objective of improving the functional performance (crack filling, resurfacing, thin overlay) or improving the structural performance (preventive maintenance, reconstruction). The indicators of latent functional performance are those distress measurements that affect the user costs directly (usually surface distresses). The indicators of latent structural performance are those related to the strength of the pavement. The functional performance is hypothesized to be affected by the age of the pavement, traffic loading, precipitation, and pavement strength. Similarly, the structural performance is written as a function of the age of the pavement, traffic loading, and pavement strength.

In this specification the functional and structural performances are hypothesized to be affected by the age of the pavement, precipitation, traffic loading, and structural number. The explanatory variables are specified as in Model 2 to model the effects of pavement strength on the functional and structural performance deterioration processes. In the measurement model the scale of S_{func} performance is set equal to that of roughness, whereas the scale of S_{struc} performance is set equal to Benkelman deflection. The loadings of the indicators of functional and structural performance are chosen after extensive experiments. For the structural model,

$$S_{func} = \gamma_1 \text{AGER/SNC} + \gamma_2 \text{ESAXL/SNC} + \gamma_3 \text{CP/SNC} + \zeta_1$$

$$S_{struc} = \beta_1 \text{AGER/SNC} + \beta_2 \text{ESAXL/SNC} + \beta_3 \text{CP/SNC} + \zeta_2$$

TABLE 8 Extracted Latent Performance Model, Model 2

Indicator	Estimate	t-statistic
Roughness	0.171	16.89
Cracking	0.092	17.87
Rut depth	2.016	31.35
Surface patching	0.021	1.21
Ravelling	0.082	10.76

Adjusted $R^2 = 0.69$

Total number of observations = 1571

TABLE 9 Structural Model for Latent Functional Performance, Model 3

Independent variable	Estimate	t-statistic
AGER/SNC	4.322	16.25
ESAXL/SNC	0.401	0.88
CP/SNC	3.207	5.05

Squared multiple correlation= 0.47

For the measurement model,

$$RQI = S_{func} + \epsilon_1$$

$$CRX = \lambda_2 S_{func} + \epsilon_2$$

$$RDMN = \lambda_3 S_{struc} + \epsilon_3$$

$$SPAT = \lambda_3 S_{func} + \epsilon_4$$

$$RAV = \lambda_4 S_{func} + \epsilon_5$$

$$DEFS = S_{struc} + \epsilon_6$$

The estimated structural and measurement models are presented in Tables 9 through 11, and the extracted latent performance model is presented in Table 12. Both the functional and structural performance equations in the structural model have decent fits (0.47 and 0.43, respectively). The overall fit of the structural model is 0.68. The significant explanatory variables affecting functional performance are AGER/SNC and CP/SNC. This observation suggests that the age of the pavement and precipitation are important factors in the functional performance deterioration process, whereas traffic loading has inconsequential effects. On the other hand, as seen in the significance of the explanatory variables in the structural performance equation, the important variables affecting structural performance are traffic loading and the age of the pavement, whereas precipitation is found to be insignificant.

In the measurement model the SMC of rut depth equation is 0.86, indicating that rut depth is a good measure of structural performance. Furthermore, the SMC of the Benkelman deflection equation is 0.31, indicating that deflection is also a reasonable measure of structural performance. The SMC of the roughness measurement equation is 0.55, from which one can infer that roughness is a good measure for representing the functional performance of the pavement. The overall fit of the measurement model is 0.93. The fits of the structural and functional performance equations in the extracted latent performance model have good fits (0.87 and 0.95, respectively). All of the estimated parameters are significant.

TABLE 10 Structural Model for Latent Structural Performance, Model 3

Independent variable	Estimate	t-statistic
AGER/SNC	0.055	12.04
ESAXL/SNC	0.126	13.81
CP/SNC	0.000	0.17

Squared multiple correlation= 0.43

Total coefficient of determination of structural model= 0.68

TABLE 11 Measurement Model, Model 3

Indicator	Functional perf.		Structural perf.		sq. multiple correlation
	estimate	t-stat	estimate	t-stat	
Roughness	1.000	- ^a	-	-	0.55
Cracking	1.253	16.70	-	-	0.30
Rut depth	-	-	9.994	18.17	0.86
Surface patching	0.331	13.72	-	-	0.19
Ravelling	0.361	6.90	-	-	0.04
Benkelman defs.	-	-	1.000	-	0.31

^aParameter fixed or not estimated.

Total coefficient of determination of measurement model= 0.93

CONCLUSIONS

In this paper various highway pavement performance deterioration models were presented. Structural number, a pavement strength-related variable, is found to be an important explanatory variable affecting the performance deterioration process of the pavement. By including structural number, a measure of the thickness and stiffness of the different pavement layers, in the structural deterioration model the effects of pavement strength on the deterioration process are captured.

The hypothesis of the performance of the pavement characterized by a two-component latent performance vector—one latent variable that represents functional performance and another one that represents structural performance—was tested. The good fits for the structural and functional performance deterioration equations and the extracted latent performance model demonstrate the identification and effectiveness of two-component latent performance models. Roughness and cracking are found to be good indicators of functional performance, whereas Benkelman deflection and rut depth are the indicators of structural performance. Further work is needed to study the specific effects of different maintenance actions on the progression of structural and functional performance.

The primary motivation for modeling the functional and structural performance deterioration processes separately stems from the use of these deterioration models in the context of a pavement management system. M&R activities are undertaken to preserve the surface quality and structural integrity of the pavement. These activities consist of surface maintenance (routine maintenance, resurfacing, etc.) and structural maintenance (reconstruction). Also, in the case of preventive maintenance actions, one needs to model

TABLE 12 Extracted Latent Performance Model, Model 3

Indicator	Functional performance		Structural performance	
	estimate	t-stat	estimate	t-stat
Roughness	0.426	69.19	- ^a	-
Cracking	0.136	37.42	-	-
Rut depth	-	-	0.077	143.42
Surface patching	0.096	8.94	-	-
Ravelling	0.051	11.89	-	-
Benkelman defs.	-	-	0.068	21.14
Squared multiple correlation	0.87		0.95	

^aParameter not estimated.

Total coefficient of determination= 0.98

Total number of observations= 1543

the structural performance deterioration because the structural aspect is of no immediate concern to the users, whereas it is critical to the highway agency so that it can avoid more expensive corrective measures in the future.

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