Rational Approach in Applying Reliability Theory to Pavement Structural Design

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A rigorous yet practical methodology for evaluating pavement design reliability is described. Pavement design reliability is expressed in terms of the probability that a pavement will withstand the actual number of load applications—that is, the traffic—on it during a selected design life while maintaining its structural integrity. Traffic is selected as the design element to which the reliability analysis should be applied because it is the only factor common to all pavement types. The methodology therefore provides a uniform basis for evaluating the reliability of alternative pavement designs with different pavement types. Alternative expressions of reliability (such as the probability of not exceeding a specified level of pavement distress) do not provide a proper comparison of alternative pavement types. A mathematical model that can be used to evaluate the reliability of alternative pavement designs and to calculate the design traffic for the selected design is presented. A systematic process for updating and improving initial estimates of the statistical parameters needed for the evaluation of reliability is also described.

The reliability theory provides a rational framework for addressing uncertainties in evaluating the projected performance of a facility during its intended life. In the context of pavements, pavement design reliability is defined as the probability that a pavement as designed will withstand the actual number of load applications on it during a selected design life while maintaining its structural integrity. Neither the actual traffic loading that will pass on a pavement during its design life nor the pavement's capacity to withstand traffic loading can be determined with certainty. Methods of reliability analysis formally address these uncertainties in the selection of a pavement design. The objective of reliability analysis is to provide a specified degree of (probabilistic) assurance that the pavement will perform satisfactorily while being subjected to the traffic and environmental conditions encountered during its design life.

A uniform method of evaluating pavement design reliability is essential in selecting the most appropriate pavement type for a given project location and in evaluating alternative pavement designs for a given pavement type. The choice of a pavement type may be made on the basis of estimated life-cycle costs. However, life-cycle costs of the selected designs for alternative pavement types cannot be directly compared unless the designs achieve the same level of reliability evaluated by a uniform method. Without a common definition of reliability and a uniform method of evaluating reliability, the comparison of life-cycle costs of alternative pavement types would be misleading and could result in the selection of a less cost-effective pavement type.

To provide uniformity pavement design reliability is defined as the probability that the pavement's traffic load capacity exceeds the cumulative traffic loading on the pavement during a selected design life. An alternative definition of reliability such as the probability of not exceeding a specified level of pavement distress is not useful for a direct comparison of the design reliabilities of different pavement types. This is because flexible and rigid pavements display different types of distresses. Traffic, however, provides a common basis because projected traffic at a given project location can be assumed to be independent of the pavement type.

This paper describes a mathematical model for evaluating pavement design reliability and discusses the estimation of model parameters by using the types of data that are generally available to highway agencies. The final section contains recommendations regarding the proper use of reliability theory in the design and selection of pavement type at a given project location. Although the focus of the paper is on initial pavement design, the same approach can also be used in evaluating rehabilitation alternatives for in-service pavements.

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MATHEMATICAL MODEL FOR EVALUATING PAVEMENT DESIGN RELIABILITY

The reliability R of a pavement design is defined as

$$R = \text{probability } [N > n] \tag{1}$$

where N is the number of traffic load applications that the pavement can withstand before losing its structural integrity, and n is the actual number of load applications on the pavement during a specified design life. If N exceeds n the pavement would maintain its structural integrity during its entire design life. Reliability is the probability that this condition would be met.

The evaluation of the probability in Equation 1 requires an assumption about the probability distributions of the two variables N and n. Field data suggest that the distributions of both variables are positively skewed (i.e., with longer tails to the right) and, consequently, assuming a lognormal probability distribution, is appropriate. Previous studies have commonly made this assumption (*1*).

With the assumption of lognormal probability distribution for N and n, Equation 1 can be rewritten as:

$$R = \text{probability } [\ln N > \ln n] = \text{probability } [\ln N - \ln n > 0] \tag{2}$$

where ln represents the natural logarithm of a variable. Since N and n are lognormally distributed, it follows that $\ln N$ and $\ln n$ would be normally distributed. Figure 1 is a schematic representation of the two distributions in which the overlap between the

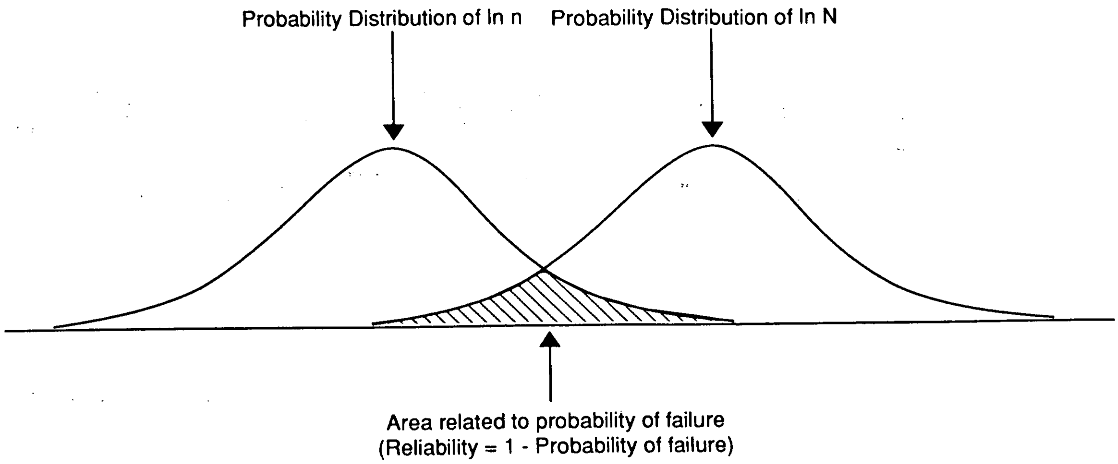

FIGURE 1 Probability distributions of ln *n* and ln *N*.

two distributions is related to the failure probability (i.e., the probability that ln N is less than ln n) and reliability is 1 minus the failure probability.

The greater the separation between ln N and ln n, the higher the design reliability would be. Thus, one may define the safety margin (SM) of a design as

$$\mathrm{SM} = \ln N - \ln n \tag{3}$$

A convenient measure of design reliability is the reliability index (β), defined as the ratio of the expected value of SM, denoted by $E[\mathrm{SM}]$, and the standard deviation of SM, denoted by $SD[\mathrm{SM}]$. Thus,

$$\beta = \frac{E[\mathrm{SM}]}{SD[\mathrm{SM}]} \tag{4}$$

$$= \frac{E[\ln N] - E[\ln n]}{\sqrt{\mathrm{var}[\ln N] + \mathrm{var}[\ln n]}} \tag{5}$$

in which $E[\bullet]$, $SD[\bullet]$, and var$[\bullet]$ represent the expected value, standard deviation, and variance, respectively, of a random variable.

The design reliability R can now be related to β as follows:

$$\begin{aligned} R &= \text{probability } [\ln N - \ln n > 0] \\ &= \text{probability } [\mathrm{SM} > 0] \\ &= 1 - F_U\left(\frac{0 - E[\mathrm{SM}]}{SD[\mathrm{SM}]}\right) \\ &= 1 - F_U(-\beta) \\ &= 1 - [1 - F_U(\beta)] \\ &= F_U(\beta) \end{aligned} \tag{6}$$

where F_U is the cumulative distribution function of a unit normal variate. Figure 2 illustrates the relationship between R and β.

The reliability index β thus expresses the mean safety margin (i.e., the mean separation between ln N and ln n) in terms of

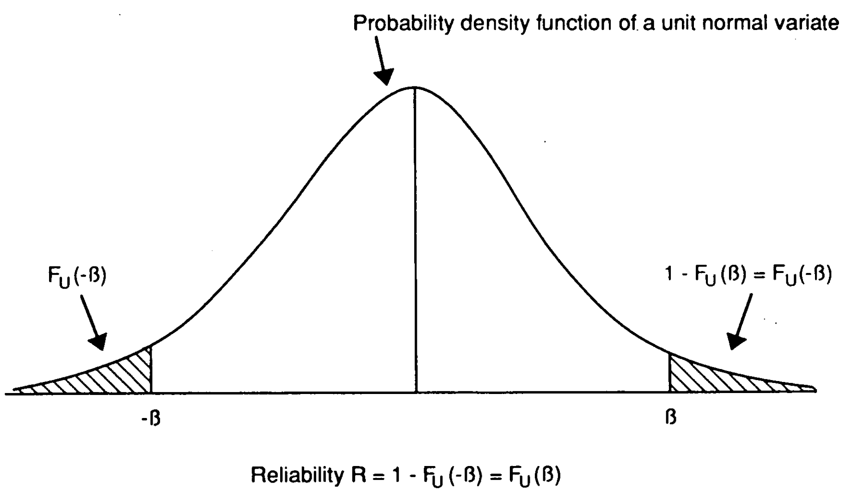

FIGURE 2 Reliability in terms of reliability index.

multiples of the standard deviation of the safety margin. Higher values of β are associated with greater levels of reliability. For given variances of ln N and ln n, β increases as the difference between the mean values of ln N and ln n increases. Similarly, for given mean values of ln N and ln n, β increases as the variances of ln N and ln n decrease. Thus, the reliability of a pavement design can be increased by increasing the average structural capacity (to withstand a greater number of load applications) or reducing the uncertainties in estimating the structural capacity and traffic loading.

The concept of the reliability index has been used for the design of structures (*2*), offshore platforms (*3*), and geotechnical facilities (*4*). The index provides a consistent and convenient basis for comparing the reliabilities of alternative designs of a given facility. With regard to pavement designs, life-cycle costs of alternative pavement types could be directly compared only if each type is designed to achieve the same reliability index.

ESTIMATION OF MODEL PARAMETERS

The input parameters needed to calculate the reliability index are

- The definition of "failure" criterion,
- Mean value and variance of ln N, and
- Mean value and variance of ln n.

The estimation of these parameters and the calculation of design traffic are discussed in this section.

Definition of Failure Criterion

In estimating a pavement's capacity to withstand traffic, one needs to define a "failure" criterion in a functional or structural mode. For mechanistic pavement designs the failure criterion may be stated in terms of a threshold level of pavement distress at which the pavement would be assumed to have lost it structural integrity. It should be noted that pavement structures do not fail catastrophically. A pavement failure is characterized by the development of a specific type of distress (such as fatigue cracking on a flexible pavement) of sufficient severity and extent at different points within a pavement section. This type of failure, sometimes referred to as a "stochastic failure," implies that a pavement section is not homogeneous in strength along its entire length. A pavement section designed and constructed the same way exhibits random variations in material properties and as-built characteristics. Locations at which several deficiencies coincide may fail, although the remaining section maintains its structural integrity.

Hence, the structural failure of a pavement may not necessarily imply that the pavement has fallen below an acceptable level of serviceability. Conversely, a pavement may fall below an acceptable level of serviceability prior to a structural failure. Thus, a time lag may exist between the structural failure and serviceability failure, as illustrated schematically in Figure 3.

In evaluating the reliabilities of alternative pavement types, the structural failure criteria should be selected such that this time lag to reach a given level of serviceability is similar for the alternative pavement types. That is, the degree of conservatism in the selected structural failure criteria as established from the analysis of actual performance data (e.g., nature and extent of failure manifestations, surface rideability, and maintenance records) and from judgment and experience should be the same for the alternative pavement types. Figure 3 illustrates the contrast between structural and serviceability failure.

Mean Value and Variance of ln N

It is assumed that a specific design procedure (equation) is used to estimate N, the number of load applications that a given design would withstand before reaching the threshold distress level. For mechanistic design the design equation is developed by using the pavement's structural response parameters (stresses, strains, deformations) and is validated by using data on past pavement failures.

Let the actual traffic load capacity of the pavement N be related to the estimated capacity $\hat{N}$, as follows:

$$N = \hat{N}\alpha_1$$

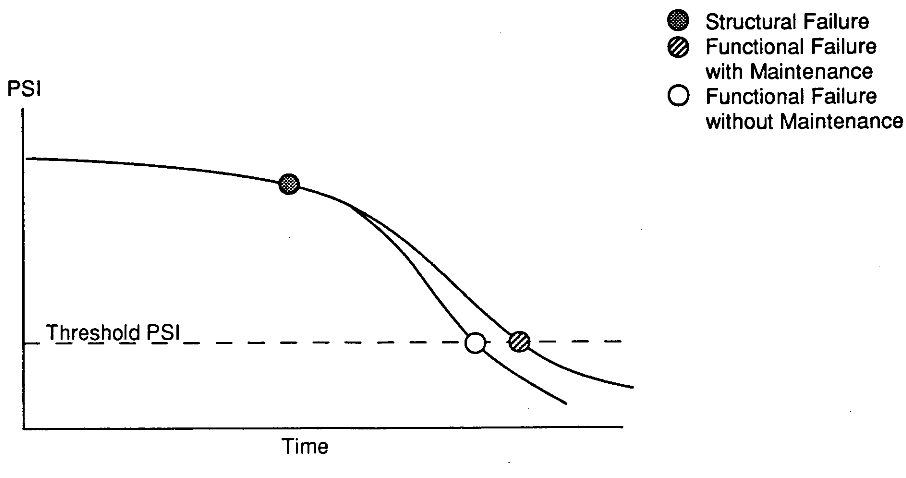

FIGURE 3 Present serviceability index (PSI) versus time showing functional and structural failures.

or

$$\ln N = \ln \hat{N} + \ln \alpha_1 \quad (7)$$

where $\ln \alpha_1$ is the deviation between the logs of actual and estimated traffic load capacity.

It is assumed that $\ln \alpha_1$ is a random error term that does not introduce any bias in the estimation procedure.

The mean and variance of $\ln N$ can be expressed as

$$E[\ln N] = \ln \hat{N} \quad (8)$$

$$\mathrm{Var}[\ln N] = \mathrm{var}[\ln \alpha_1] \quad (9)$$

Uncertainties in estimating traffic load capacity are captured in the variance of $\ln N$ (Equation 9). Thus, $\hat{N}$ should be the "best" (median) estimate of traffic load capacity that is obtained by using the median (rather than conservative) values of the design parameters such as material properties and structural response.

A number of sources of deviation could contribute to the overall deviation $\ln \alpha_1$, including

- Lack of fit of the design equation,
- Differences between design and as-built parameters,
- Construction variability, and
- Variability in the material properties.

The estimation of the variance of $\ln \alpha_1$ is an evolving process that begins with an initial estimate of $\ln \alpha_1$ on the basis of the available information from previous studies. This initial estimate is periodically updated as additional data specific to a given highway agency become available. The recommended steps in this process are described in the following:

1. Use the variance of $\ln \alpha_1$ derived in the AASHTO design guide (*1*) as an initial estimate. The AASHTO design guide estimates the variances of $\ln \alpha_1$ to be 0.194 and 0.114 for flexible and rigid pavements, respectively.
2. Compile pavement performance data over the past 10 years on sections of flexible and rigid pavements that have reached the applicable threshold distress levels. Although these sections may not have been designed by the current design methods, the percent variability of the deviation between the actual and estimated traffic load capacities can be expected to be similar for the recent and current design methods. This is because many components of the overall variability (such as construction practices, environmental conditions, and sources of raw materials) are likely to be similar.
3. Use the data compiled in Step 2 to estimate the traffic load capacity of each section. The cumulative traffic that has passed on the section is also estimated by using the available traffic data. The deviations between the log values of the actual cumulative traffic and the estimated traffic load capacity are then calculated and are used to estimate the variance of $\ln \alpha_1$. When data on a minimum of 10 pavement sections become available, the variance of $\ln \alpha_1$ calculated from these data should be used in place of the initial estimates in Step 1.
4. As pavement performance data from sections designed with the current design method become available, these sections should replace the older sections identified in Step 2. The analysis in Step 3 should then be updated with performance data compiled from the group of pavement sections designed by the current method.
5. The Long-Term Pavement Performance (LTPP) program initiated as a part of the Strategic Highway Research Program includes the collection of data on a large number of pavement sections nationwide. These sections have been selected by using a statistically based experimental design. This program is expected to accumulate sufficient information over the next few years for conducting a detailed analysis of the variability in pavement performance owing to variabilities in such factors as material properties, construction practice, design method, traffic projections, and environmental conditions. Formal statistical analysis of the data from LTPP should provide even more accurate estimates of the individual components of variance and the total variance of $\ln \alpha_1$. These estimates, when available, should then replace the prior estimates obtained in the preceding steps.

The steps outlined here will provide a reasonable initial estimate of the variance of $\ln \alpha_1$ and an evolving process to improve the accuracy of the initial estimate. The importance of this process is that it permits the application of reliability theory to pavement design even when data from statistically based road test programs are not available to estimate individual components of variability. The process yields reasonable estimates of the overall variability in traffic load capacity, N (and also in traffic loading, n). The estimates can be systematically updated and improved as additional data become available.

Mean Value and Variance of ln *n*

It is assumed that a specified traffic forecasting model is used to estimate the amount of cumulative traffic [in terms of number of equivalent single-axle loads (ESALs)] expected on a pavement. As with the traffic load capacity, the actual cumulative traffic n is related to the estimated cumulative traffic $\hat{n}$ as follows:

$$n = \hat{n}\, \alpha_2$$

or

$$\ln n = \ln \hat{n} + \ln \alpha_2 \quad (10)$$

It is again assumed that $\ln \alpha_2$ is a random error (without any bias). The mean and variance of $\ln n$ are then obtained from

$$E[\ln n] = \ln \hat{n} \quad (11)$$

$$\mathrm{Var}[\ln n] = \mathrm{var}[\ln \alpha_2] \quad (12)$$

As with the estimation of traffic load capacity, an evolving process can be used to estimate the variance of $\ln \alpha_2$. A reasonable initial estimate of this variance is 0.04, as derived in the AASHTO design guide (*1*). This estimate can be updated with traffic count data collected by the highway agency for a sample of pavement sections. The sample should include pavement sections constructed in the past 10 years for which actual traffic count data are available.

For example, using the current traffic forecasting model and the data that would have been available 10 years ago (but not using the traffic count data collected in the past 10 years), the total cumulative traffic (in terms of ESALs) during the past 10 years is estimated for each of the selected pavement sections. The deviations between the log values of both actual and estimated cumulative traffic are then used to estimate the variance of $\ln \alpha_2$ to replace the initial value of 0.04. This estimate can be further refined as additional data become available.

Calculation of Design Traffic To Achieve Desired Level of Reliability

For a specified level of design reliability, R^*, the corresponding reliability index can be calculated from Equation 6. Let this reliability index be denoted by β^*. Then the cumulative traffic for which the pavement should be designed is given by

$$\ln \hat{N} = \ln \hat{n} + \beta^* \sqrt{\text{var}[\ln \alpha_1] + \text{var}[\ln \alpha_2]} \tag{13}$$

Equation 13 can also be expressed as

$$\hat{N} = \hat{n}F \tag{14}$$

where F is defined as a traffic multiplier with the relationship to the reliability index β^* of

$$F = \exp\left(\beta^* \sqrt{\text{var}[\ln \alpha_1] + \text{var}[\ln \alpha_2]}\right) \tag{15}$$

Thus, if the pavement is designed for the cumulative traffic of $\hat{N}$ obtained from Equation 14, there is an R^* percent probability that the pavement would not fail (according to the defined failure criterion) before reaching the cumulative design traffic level.

SUMMARY AND RECOMMENDATIONS

A rigorous yet practical methodology for evaluating pavement design reliability was described in this paper. Pavement design reliability was expressed in terms of the probability that a pavement will withstand the actual number of load applications, that is, the traffic, on it during a selected design life while maintaining its structural integrity. Traffic was selected as the design element to which the reliability analysis should be applied because it is the only factor common to all pavement types. The methodology therefore provides a uniform basis for evaluating the reliability of alternative pavement designs with different pavement types. Alternative expressions of reliability (such as the probability of not exceeding a specified level of pavement distress) do not provide a proper comparison of alternative pavement types.

The methodology quantifies uncertainties in estimating a pavement's traffic load capacity and the actual cumulative traffic loading on the pavement during a specified design period. A reliability index was calculated for each alternative design, taking into account

1. The difference between the estimated traffic load capacity and the projected traffic loading,
2. The variance of the deviations between actual and projected traffic load capacities (caused by the variability in material properties and construction practices), and
3. The variances of the deviations between actual and estimated traffic loadings.

Procedures for systematically updating and improving initial estimates of the statistical parameters needed for the evaluation of reliability were described.

Finally, the reliability of a given design was calculated as a function of the reliability index, and appropriate probability distributions of the two variables ($\ln N$ and $\ln n$) were used to define this index. These probability distributions were selected on the basis of the statistical evaluation and interpretation of the available data. The design that achieves a desired level of reliability with the minimum life-cycle cost may be selected. The reliability index of the selected design is then used to calculate a traffic multiplier (see Equation 14). When this traffic multiplier is applied to the projected traffic loading (expressed, for example, in terms of 18,000-lb ESALS for any mix of traffic) it establishes the level of traffic for which the pavement should be designed to achieve the specified reliability.

This is a proven methodology that has been used to evaluate the design reliabilities of highways as well as such engineered systems as landfills, dams, buildings, bridges, and offshore platforms. Its use is recommended in evaluating the reliabilities of alternative pavement designs with different pavement types for a given project. The selection of the most cost-effective pavement type can be made only after ensuring that the selected designs of the alternative pavement types achieve the same level of reliability and that a consistent procedure is used to evaluate design reliability so that the reliability levels calculated for the different pavement types are directly comparable to one another.

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