Estimation of Standard Deviation of Predicted Performance of Flexible Pavements Using AASHTO Model

A. SAMY NOURELDIN, ESSAM SHARAF, ABDULRAHIM ARAFAH, AND FAISAL AL-SUGAIR

Explicit inclusion of the reliability factor in pavement design was one of the major changes included in the 1986 version of the AASHTO *Guide for Design of Pavement Structures*. The design procedures for flexible (and rigid) pavements provide a common method for incorporating reliability into the design process on the basis of a shift in the design traffic. The guide recommended various levels of reliability (1 − risk of failure) for each roadway functional classification and suggested a range of values for the standard deviation of the combined traffic prediction and performance prediction (S_o). The S_o range suggested for flexible pavements is 0.4 to 0.5, with a typical value of 0.49. The guide also suggested that the variance of the performance prediction (S_N^2) represents about 82 percent of S_o^2 and the variance of the traffic prediction S_w^2 represents about 18 percent of S_o^2, where $S_o^2 = S_N^2 + S_w^2$. This means that the standard deviation of performance can be estimated to be in the range 0.36 to 0.45, with a typical value of 0.44. The manner in which the values of the mean and the coefficient of variation (which is equal to the standard deviation/mean) of flexible pavement layer thicknesses and strength parameters can be used as inputs to estimate the standard deviation of predicted pavement performance (S_N) is described. These values can be estimated easily by experienced pavement engineers, sometimes as well as they can be measured. The analysis resulted in an estimated S_N (typical S_N of 0.47) almost identical to what is recommended by the AASHTO guide (typical S_N of 0.44). However, the sequence leading to that estimation is believed to be of interest to users of the guide. In addition, the process of identifying the variability in pavement performance because of the variability of the factors that control that performance is demonstrated. This process is applicable to any pavement or overlay design model.

Any pavement thickness (or overlay) design method is usually based on six specific design parameters. These design parameters are the traffic characteristics, the subgrade soil characteristics, the pavement layer characteristics, the climatic conditions, the failure criteria, and the variabilities of all of those parameters.

In the 1986 AASHTO *Guide for Design of Pavement Structures* (*1*) all of the above design parameters are represented. In the design equation for asphalt pavements (flexible pavements),

1. The traffic characteristics are represented by the sum of equivalent single axle loads of 8.154 metric tons (18,000 lb) during the design period (ESAL or W_{t18}),
2. The subgrade soil characteristics are represented by the resilient modulus *MR*,
3. The pavement layer characteristics (thickness and material properties) are represented by the structural number *SN*,
4. The climatic conditions are represented implicitly within the design parameters *MR* and *SN*,
5. The failure criteria are represented by the drop in serviceability during the design period (ΔPSI), and
6. The variabilities of all of the above design parameters are represented by the standard normal deviate (Z_R) and the combined standard deviation of the traffic prediction and performance prediction (S_o). The inclusion of these variability parameters comes from the fact that each design parameter is actually a random variable with a specific distribution during the design period.

The traffic prediction equation can be expressed as follows:

$$\log W_{t18} = \log (\text{ADT}\cdot P\cdot D_d\cdot L_d\cdot \text{TF}\cdot 365\cdot n\cdot \text{GF})$$

where

- W_{t18} = predicted traffic in terms of accumulated number of ESALs (lb) during the design period,
- ADT = average daily traffic,
- P = percent trucks,
- D_d = directional distribution,
- L_d = lane distribution,
- TF = truck factor, which is equal to the number of ESALs per truck,
- n = design period (in years), and
- GF = growth factor.

The standard deviation of the parameter $\log w_{t18}$ is termed S_w and is defined as the standard deviation of traffic prediction.

On the other hand the performance prediction equation is expressed as follows:

$$\log W_{t18} = 9.36 \log (SN + 1) - 0.2 + \frac{\log\left(\dfrac{\Delta PSI}{4.2 - 1.5}\right)}{0.4 + \left(\dfrac{1{,}094}{(SN + 1)^{5.19}}\right)} + 2.32 \log MR - 8.07$$

where

- W_{t18} = predicted performance in terms of accumulated ESALs, (lb).
- SN = structural number,

A. S. Noureldin, U.N. Development Program Experts Team, Ministry of Communications, Riyadh 11178, Saudi Arabia. E. Sharaf, A. Arafah, and F. Al-Sugair, Civil Engineering Department, King Saud University, Riyadh 11421, Saudi Arabia.

ΔPSI = drop in serviceability index, and
MR = resilient modulus.

The standard deviation of the parameter log W_{t18} is termed S_N and is defined as the standard deviation of performance prediction.

The AASHTO guide defines the combined standard deviation of traffic prediction and performance prediction (S_o) through the equation

$$S_o^2 = S_N^2 + S_w^2$$

where S_N is the standard deviation of the predicted performance of paving materials, and S_w is the standard deviation of the predicted amount of traffic that will use the facility.

This paper describes a simplified methodology for the estimation of S_N and the range of values of the associated design parameters.

RELIABILITY AND SOURCES OF MATERIAL VARIABILITY

Pavement thickness design for newly constructed roads (or overlay design for deteriorated roads) involves the selection of a specific value for each of the design factors. However, the design values used as inputs are seldom, if ever, unique or constant values. Every design value has some randomness in its measurement. The recognition of this stochastic or random nature of material properties has brought more attention to the explicit use of reliability concepts within the field of pavement thickness or overlay design.

Reliability is defined as the probability that the pavement system will perform its intended function over its design life and under the conditions (or environment) encountered during operation (*1*).

Material variability can be grouped into six main categories (*2,3*):

1. The inherent variability in the basic properties of materials known as *randomness*.
2. The variability in the properties of materials because of the lack of accuracy of the test method.
3. The variability that results from different laboratories and operators.
4. The variability in materials that results from circumstantial conditions during pavement service life (construction, climate, aging, traffic, etc.).
5. The modeling variability that results from inaccuracy in the design models, known as uncertainty.
6. The unexplained replication variability that still remains after considering all other categories of variability.

METHODS OF RELIABILITY ANALYSIS

Prediction of S_N can be done by several methods of reliability analysis. These methods have distinctive characteristics and assumptions and can be categorized into three distinct approaches (*3*):

1. Exact,
2. Approximation by using first-order second-moment (FOSM), and
3. Point estimate.

Exact Approach

Use of the exact approach requires the knowledge of the exact distribution (probability density function) for each of the independent variables associated with a random variable model (or function) (*3,4*). Suppose, for example, that the performance of a flexible pavement can be represented by the model

$$\log W_{t18} = 11.75 - 4.6 \log d_o$$

where W_{t18} represents the accumulated number of ESALs during the design period, and d_o represents the center deflection of a falling-weight deflectometer because of a 9,000-lb load at a pavement temperature of 20°C. The exact approach for the prediction of the standard deviation (S_N) of the random variable (log W_{t18}) requires the knowledge of the exact distribution of the random variable (d_o). In some instances the independent variable (d_o) can be assumed to be normally distributed, lognormally distributed, or uniform. After knowing (or assuming) the exact distributions of the independent variables, the exact distribution of the dependent variable can be obtained, and hence its standard deviation can be determined.

Approximation Approach

The approximation approach requires the knowledge of only the coefficients of variation (COVs) of the independent variables to predict the standard deviation of a dependent random variable (COV is the standard deviation divided by the mean). Cornell approximation (*4*) makes use of the first-order Taylor series expansion such that if F, for example, is a function of independent variables SN and MR given by the model

$$F = \phi\ (SN, MR)$$

then

$$\text{Mean of } F = \phi\ (\text{mean of } SN, \text{ mean of } MR)$$

$$\text{Variance of } F = \text{variance of } SN \cdot P_2 + \text{variance of } MR \cdot P_1$$

where

$$P_2 = \left(\frac{\partial F}{\partial SN} \text{ at mean of } SN\right)^2$$

= square of the partial derivative at mean SN

$$P_1 = \left(\frac{\partial F}{\partial MR} \text{ at mean of } MR\right)^2$$

= square of the partial derivative at mean MR

This approach is also called the FOSM approximation (*4*). Taking, for example, the model where

$$\log W_{t18} = 11.75 - 4.6 \log d_o$$

then

$$\mathrm{Var}(\log W_{t18}) \cong \mathrm{Var}(d_o) \cdot \left(\frac{4.6 \log e}{\bar{d}_o}\right)^2$$

and

$$S_N = \text{standard deviation of } \log W_{t18}$$
$$\cong \frac{\text{standard deviation of } d_o}{\bar{d}_o} \cdot 2$$

or

$$S_N = \text{standard deviation of } \log W_{t18}$$
$$\cong \mathrm{COV}(d_o) \cdot 2$$

It can be noted that the standard deviation of W_{t18} (standard deviation for pavement performance) can be estimated to be a constant value multiplied by the value of the COV for center deflection for that model. An experienced pavement engineer can provide an estimate for the COV of center deflection (as good as the one measured), and hence can provide a good estimate of the standard deviation of log W_{t18}.

Point Estimate Approach

The point estimate approach does not use the Taylor series but instead uses measured values of the independent variables at different points to estimate the random dependent variable. This method is known as the point estimate approach (*5*).

Referring back to the example in which the model representing pavement performance is

$$\log W_{t18} = 11.75 - 4.6 \log d_o$$

d_o can be measured at different points, and hence, log W_{t18} can be calculated at these points by using the model. The mean, variance, standard deviation, or COV of the random variable (log W_{t18}) can then be obtained by using these calculated values.

ESTIMATION OF AASHTO S_N

The AASHTO design model for the design of flexible pavements is (*1*)

$$\log_{10}(W_{18}) = Z_R \times S_o + 9.36 \log_{10}(SN + 1) - 0.2 + \frac{\log_{10}\left(\frac{\Delta PSI}{4.2 - 1.5}\right)}{0.4 + \left[\frac{1{,}094}{(SN + 1)^{5.19}}\right]} + 2.32 \log_{10} MR - 8.07$$

where

W_{18} = predicted number of 8,154-kg (18-kip) ESALs,
Z_R = standard normal deviate,
S_o = combined standard error of the traffic prediction (S_w) and performance prediction (S_N) where $S_o^2 = S_N^2 + S_w^2$,
MR = resilient modulus (psi), and
ΔPSI = drop in serviceability during the design period.

SN is equal to the structural number indicative of the total pavement thickness required:

$$SN = a_1D_1 + a_2D_2m_2 + a_3D_3m_3$$

where

a_i = ith layer coefficient,
D_i = ith layer thickness (in.), and
m_i = ith layer drainage coefficient.

Tables 1 and 2 presents the suggested levels of reliability for various functional classifications and the corresponding Z_R values, respectively (*1*).

The estimation of S_N by the approximation approach for reliability analysis (see above) requires

1. An estimate of the coefficient of variation of the structural number SN,
2. An estimate of the coefficient of variation of the resilient modulus MR, and
3. An estimate of the COV of the drop in serviceability ΔPSI.

The following sections present how these estimates can be obtained for a fixed drop in ΔPSI of 1.7 (ΔPSI = 4.2 − 2.5 = 1.7).

TABLE 1 Suggested Levels of Reliability for Various Functional Classifications (*1*)

Functional Classification	Recommended Level of Reliability	
	Urban	Rural
Interstate and other freeways	85 - 99.9	80 - 99.9
Principal Arterials	80 - 99	75 - 95
Collectors	80 - 95	75 - 95
Local	50 - 80	50 - 80

Note: Results based on a survey of the AASHTO Pavement Design Task Force

TABLE 2 Standard Normal Deviate (Z_R) Values Corresponding to Selected Levels of Reliability (*1*)

Reliability, R (percent)	Standard Normal Deviate, Z_R
50	-0.000
60	-0.253
70	-0.524
75	-0.674
80	-0.841
85	-1.037
90	-1.282
91	-1.340
92	-1.405
93	-1.476
94	-1.555
95	-1.645
96	-1.751
97	-1.881
98	-2.054
99	-2.327
99.9	-3.090

Estimation of COV for AASHTO Layer Coefficients

Figure 1 provides the relation between the layer coefficient of asphalt concrete layer (a_1) and its modulus (E_{AC}) suggested by the AASHTO guide (*1*) as a result of the research work reported in *NCHRP Report 128* (*6*). The average AASHO Road Test conditions suggest an average value of Marshall stability of 906 kg (2,000 lbs) and an average dynamic modulus, E_{AC} (axial loading), of 3103 MPa (450,000 psi) at 68°F (20°C), corresponding to a layer coefficient of 0.44 (*7*). It is imperative to indicate that the asphalt concrete layer coefficient (a_1) not only is dependent on the asphalt concrete modulus (or Marshall stability) but also is dependent on the thickness and material properties of the underlying paving layers. However, for the purposes of simplifying variability estimation, the relationship provided in Figure 1 can be utilized, since it is already usable for estimating the average value of a_1. The exact model for the relationship shown in Figure 1 does not need to be known to estimate the COV for the layer coefficient, which can be estimated as follows: Since a_1 is a function of E_{AC}

$$a_1 = \phi\,(E_{AC})$$

then

$$\text{Var}\,(a_1) \cong \text{Var}\,(E_{AC}) \cdot \left[\frac{\partial\phi(E_{AC})}{\partial E_{AC}}\right]^2$$

$$\frac{\text{Var}\,(a_1)}{\bar{a}_1^2\,\bar{E}_{AC}^2} \cong \frac{\text{Var}\,(E_{AC})}{\bar{a}_1^2\,\bar{E}_{AC}^2} \cdot (\text{slope of the relation at } \bar{E}_{AC})^2$$

where E_{AC} and a_1 are mean values

$$\overline{\text{COV}\,(a_1)}^2 \cong \frac{\bar{E}_{AC}^2}{\bar{a}_1^2} \cdot \overline{\text{COV}\,(E_{AC})}^2 \cdot \overline{\text{slope}}^2$$

$$\text{COV}\,(a_1) \cong \text{COV}\,(E_{AC}) \cdot \frac{450{,}000}{0.44} \cdot \frac{0.22}{450{,}000}$$

$$\text{COV}\,(a_1) \cong 0.5\ \text{COV}\,(E_{AC})$$

where the slope $\cong \dfrac{0.22}{450{,}000}$ (Figure 1)

When the model for thickness equivalency of Odmark (*8*) is considered, then

$$a_1 = 3\sqrt{\frac{E_{AC}}{E_{\text{reference}}}}$$

or

$$a_1 = \frac{(E_{AC})^{1/3}}{\text{constant}}$$

then

$$\log a_1 = 1/3 \log E_{AC} - \log \text{constant}$$

and hence

$$\text{Var}\,(\log a_1) = 1/9\ \text{var}\,(\log E_{AC})$$

can be derived by using the approximation approach that

$$\text{Var}\,(\log a_1) \cong \frac{\overline{\text{COV}\,(a_1)}^2}{5.3}$$

and hence

$$\therefore\ \overline{\text{COV}\,(a_1)}^2 \cong \frac{1}{9}\ \overline{\text{COV}\,(E_{AC})}^2$$

$$\therefore\ \text{COV}\,(a_1) \cong \frac{\text{COV}\,(E_{AC})}{3}$$

Rada and Witczak (*9*) developed a relationship for granular base materials such that

$$a_2 = 0.249 \log MR - 0.977$$

where MR is the resilient modulus of the base materials, and a_2 is its layer coefficient. This relationship is also used to estimate the layer coefficient of the base material in the AASHTO guide (*1*). It is also essential to indicate that a_2 is also dependent on other pavement layer moduli as well as thicknesses. However, for

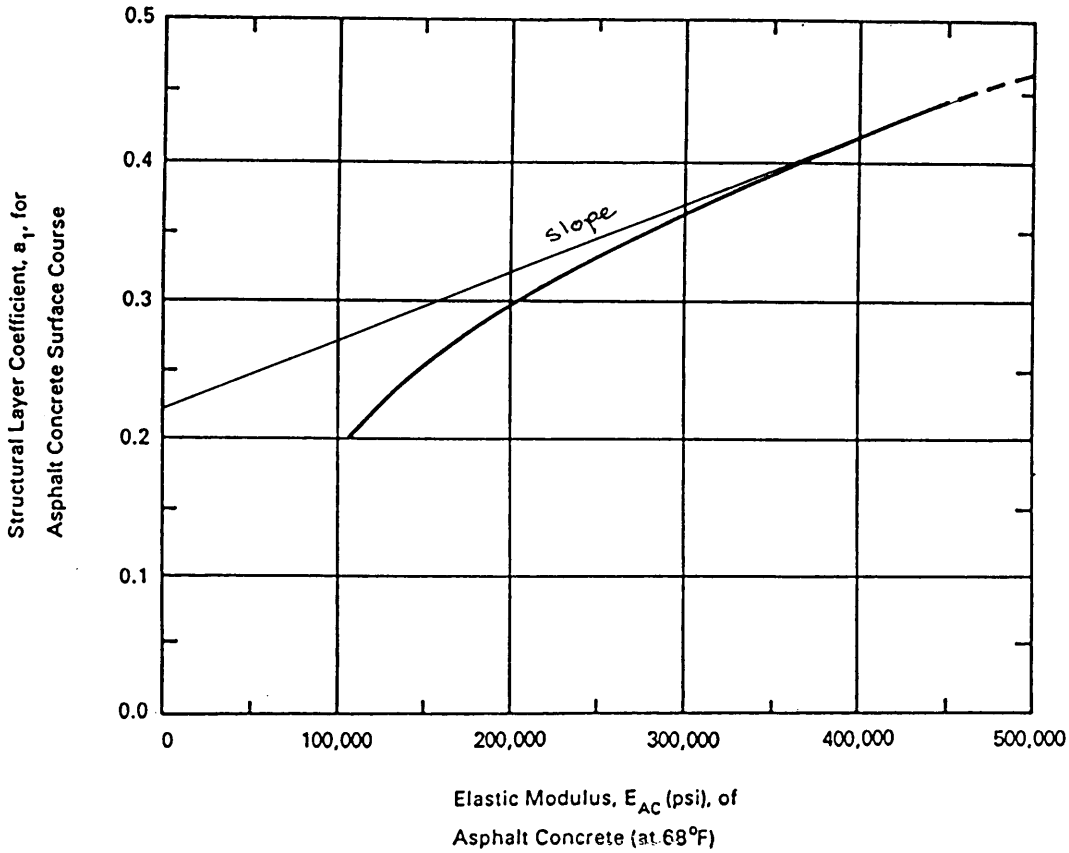

FIGURE 1 Chart for estimating structural layer coefficient of dense-graded asphalt concrete on the basis of elastic modulus (*1*).

purposes of simplifying the estimation of the COV for a_2, the above equation can be utilized as follows:

$$a_2 = \phi\ (MR)$$

$$\text{Var}\ (a_2) \cong \text{Var}\ (MR) \left[\frac{\partial\phi(MR)}{\partial MR}\right]^2$$

$$\text{Var}\ (a_2) \cong \text{Var}\ (MR) \left(\frac{0.249 \cdot 0.434}{\overline{MR}}\right)^2$$

$$\text{Var}\ (a_2) \cong \text{Var}\ (MR) \left(\frac{0.108139}{\overline{MR}}\right)^2$$

$$\frac{\text{Var}\ (a_2)}{\overline{a}_2^2} \cong \frac{\overline{MR}^2 \cdot \text{Var}\ (MR)}{\overline{a}_2^2 \cdot \overline{MR}^2} \left(\frac{0.108139}{\overline{MR}}\right)^2$$

$$\overline{\text{COV}\ (a_2)}^2 \cong \frac{\overline{MR}^2}{\overline{a}_2^2}\ \overline{\text{COV}\ (MR)}^2 \left(\frac{0.108139}{\overline{MR}}\right)^2$$

$$\text{COV}\ (a_2) \cong \frac{30{,}000}{0.14}\ \text{COV}\ (MR) \left(\frac{0.108139}{30{,}000}\right)$$

where the average modulus and layer coefficient of the base course during the AASHO road test were 207 MPa (30,000 psi) and 0.14, respectively.

Hence

$$\text{COV}\ (a_2) \cong 0.77\ \text{COV}\ (MR)$$

when using the model

$$a_2 = 3\sqrt{\frac{MR}{MR_{\text{reference}}}}$$

the COV for a_2 can be estimated as follows:

$$\text{COV}\ (a_2) \cong \frac{\text{COV}\ (MR)}{3}$$

Rada and Wiczak (*9*) established the following relationship between subbase modulus *MR* and layer coefficient a_3:

$$a_3 = 0.227 \log MR - 0.839$$

This equation is also being used in the AASHTO guide for estimating the layer coefficient a_3.

By using the same approach described previously, it can be shown that

$$\text{COV}\ (a_3) \cong 0.9\ \text{COV}\ (MR)$$

or

$$\text{COV }(a_3) \cong \frac{\text{COV }(MR)}{3}$$

for the model

$$a_3 = 3\sqrt{\frac{MR}{MR_{\text{reference}}}}$$

It can be concluded that the COVs for AASHTO layer coefficients can be estimated as follows:

$$\text{COV }(a_1) \cong (0.33 - 0.5) \text{ COV }(E_{AC})$$

$$\text{COV }(a_2) \cong (0.33 - 0.77) \text{ COV }(MR)$$

$$\text{COV }(a_3) \cong (0.33 - 0.9) \text{ COV }(MR)$$

Since many highway agencies in the United States and all over the world have not yet equipped their laboratories with the equipment necessary to characterize paving layers in terms of *MR*, COVs for the AASHTO layer coefficients can be estimated in a more simple manner as follows:

$$\text{COV }(a_1) \cong (0.33 - 0.5) \text{ COV of Marshall stability}$$

$$\text{COV }(a_2) \cong (0.33 - 0.77) \text{ COV of CBR}$$

$$\text{COV }(a_3) \cong (0.33 - 0.9) \text{ COV of CBR}$$

This is because *MR* can be modeled to be $MR \cong K \cdot \text{CBR}$, where *K* is a constant and hence

$$\text{Var }(MR) \cong K^2 \text{ Var (CBR)}$$

$$\frac{\text{Var } MR}{\overline{MR}^2} \cong K^2 \frac{\text{Var (CBR)}}{\overline{\text{CBR}}^2} \cdot \frac{\overline{\text{CBR}}^2}{\overline{MR}^2}$$

$$\overline{\text{COV }(MR)}^2 \cong K^2\, \overline{\text{COV (CBR)}}^2 \cdot \frac{\overline{\text{CBR}}^2}{K^2\, \overline{\text{CBR}}^2}$$

$$\text{COV }(MR) \cong \text{COV (CBR)}$$

Table 3 provides the COVs for some basic characteristics of pavement materials. The values in Table 3 are combined from previous reports (*2–4*) and from construction records in Saudi Arabia.

Table 4 summarizes the estimated COVs for the AASHTO layer coefficients on the basis of the values presented in Table 3.

Estimation of COVs for AASHTO Drainage Coefficients

Table 5 presents the drainage coefficients (m_i) recommended by AASHTO and as provided in the 1986 AASHTO guide (*1*). The ranges provided in Table 5 can be used to estimate the COVs employing the approach described by MS-17 of the Asphalt Institute (*10*) and reported by Yoder and Witczak (*2*), where

$$\text{COV} \cong \frac{\text{range} \cdot 0.3249}{\text{range midpoint}} \cdot 100$$

Using this equation it can be noted that the smallest estimated COV is for the condition of excellent drainage quality and less than 1 percent exposure to a moisture level approaching satura-

TABLE 3 Pavement Material COVs

Property	Coefficient of Variation, %	
	Range	Typical
Layer Thickness	3 - 12	7
Bituminous Surface	5 - 15	10
Bituminous Base	10 - 15	12
Granular Base	10 - 20	15
Granular Subbase		
Elastic Modulus		
Bituminous Layers	10 - 20	15
Granular Base	10 - 30	20
Granular Subbase	10 - 30	20
Subgrade	10 - 30	20
Marshall Stability	10 - 20	15
CBR		
Base	10 - 30	20
Subbase	10 - 30	20
Subgrade	10 - 30	20
Percent Compaction		
Surface	1 - 2	1.5
Base	2 - 3	2.5
Subbase	2 - 3	2.5
Subgrade	2 - 3	2.5
Maximum Deflection	10 - 30	20

TABLE 4 Estimated COVs of AASHTO Layer and Drainage Coefficients and AASHTO Structural Number

Parameter	Coefficient of Variation, %	
	Range	Typical
a_1	3 - 10	6
a_2	3 - 23	13
a_3	3 - 27	15
m_i	1.2 - 20	10
SN	5 - 18	11

tion. The COV at this condition can be estimated to be

$$\text{COV} \cong \frac{(1.4 - 1.35) \cdot 0.3249}{1.375} \cdot 100 \cong 1.2 \text{ percent}$$

On the other hand the largest estimated COV is for the condition of very poor drainage quality and 5 to 25 percent exposure to a moisture level approaching saturation. The COV at this condition can be estimated to be

$$\text{COV} \cong \frac{(0.75 - 0.4)\, 0.3249}{0.575} \cdot 100 \cong 20 \text{ percent}$$

In general the COVs for AASHTO drainage coefficients can be estimated to be in the range of 1.2 to 20 percent (Table 4).

Estimation of COV for AASHTO *SN*

The AASHTO structural number is defined by the equation (*1*)

$$SN = a_1D_1 + a_2m_2D_2 + a_3m_3D_3$$

where a_1, a_2, a_3, m_2, m_3, D_1, D_2, and D_3 are as defined above. By using the same approach described above, it can be concluded that

$$\begin{aligned}\text{Var}(SN) \cong{} & \bar{a}_1^2\, \text{Var}(D_1) + \bar{D}_1^2\, \text{Var}(a_1) \\ & + \bar{a}^2\, \bar{m}_2^2\, \text{Var}(D_2) + \bar{a}_2^2\, \text{Var}(m_2)\, \bar{D}_2^2 \\ & + \text{Var}(a_2)\, \bar{m}_2^2\, \bar{D}_2^2 + \bar{a}_3^2\, \bar{m}_3^2\, \text{Var}(D_3) \\ & + \bar{a}_3^2\, \text{Var}(m_3)\, \bar{D}_3^2 + \text{Var}(a_3)\, \bar{m}_3^2\, \bar{D}_3^2\end{aligned}$$

where a_1, a_2, a_3, m_2, m_3, D_1, D_2, and D_3 are the mean values, or

$$\begin{aligned}&\overline{SN}^2\, \overline{\text{COV}(SN)}^2 \\ &\cong \bar{a}_1^2\, \bar{d}_1^2 \left[\overline{\text{COV}(a_1)}^2 + \overline{\text{COV}(d_1)}^2\right] \\ &\quad + \bar{a}_2^2\, \bar{m}_2^2\, \bar{d}_2^2 \left[\overline{\text{COV}(a_2)}^2 + \overline{\text{COV}(m_2)}^2 \right.\\ &\quad \left. + \overline{\text{COV}(d_2)}^2\right] + \bar{a}_3^2\, \bar{m}_3^2\, \bar{d}_3^2 \left[\overline{\text{COV}(a_3)}^2 \right.\\ &\quad \left. + \overline{\text{COV}(m_3)}^2 + \overline{\text{COV}(d_3)}^2\right]\end{aligned}$$

TABLE 5 Recommended m_i Value for Modifying Structural Layer Coefficients of Untreated Base and Subbase Materials in Flexible Pavements (*1*)

Quality of Drainage	Percent of Time Pavement Structure is Exposed to Moisture Levels Approaching Saturation			
	Less Than 1%	1.5%	5.25%	Greater Than 25%
Excellent	1.40 - 1.35	1.35 - 1.30	1.30 - 1.20	1.20
Good	1.35 - 1.25	1.25 - 1.15	1.15 - 1.00	1.00
Fair	1.25 - 1.15	1.15 - 1.05	1.00 - 0.80	0.80
Poor	1.15 - 1.05	1.05 - 0.80	0.80 - 0.60	0.60
Very Poor	1.05 - 0.95	0.95 - 0.75	0.75 - 0.40	0.40

To illustrate the computation process consider a pavement section having

$\overline{D}_1$ = 6 cm (2.4 in.), $\overline{a}_1 = 0.44$

$\overline{D}_2$ = 18 cm (7.1 in.), $\overline{a}_2 = 0.14$

$\overline{D}_3$ = 23 cm (9.2 in.), $\overline{a}_3 = 0.11$

$\overline{m}_2 = 1.0$, $\overline{m}_3 = 1.0$, and $\overline{SN} = 3.06$,

by employing the lower and upper values of the estimated COVs to estimate the COV for the structural number, it can be noted that the COV for *SN* can be estimated to be in the range of 5 to 18 percent for this pavement section. It can also be noted that the changes in thickness combination do not significantly affect this range.

Estimation of S_N

The AASHTO performance prediction equation for a fixed drop in ΔPSI of 1.7 ($\Delta PSI = 4.2 - 2.5 = 1.7$) can be written as follows:

$$\log W_{t18} = \phi\ (MR, SN)$$

where *MR* is the resilient modulus and *SN* is the structural number. Then

$$S_N^2 = \text{Var}(\log W_{t18}) = \text{variance of performance prediction}$$

$$\cong \text{Var}(MR) \cdot P_1 + \text{Var}(SN) \cdot P_2$$

where P_1 is the variance component of *MR*

$$\cong \left(\frac{\partial \phi}{\partial\ MR} \text{ at mean } MR\right)^2$$

$$\cong \left(\frac{2.32 \log e}{\overline{MR}}\right)^2 \cong \frac{1}{\overline{MR}^2}$$

where $\overline{MR}$ is the mean of *MR*.

$$P_2 = \text{variance component of } SN$$

$$\cong \left(\frac{\partial \phi}{\partial SN} \text{ at mean } SN\right)^2$$

$$\cong \left[\frac{4.065}{\overline{SN}+1} - \frac{1135.57}{\left(0.4 + \dfrac{1{,}094}{(\overline{SN}+1)^{5.19}}\right)^2 (\overline{SN}+1)^{6.19}}\right]^2$$

where $\overline{SN}$ is the mean for *SN*.

$$S_N^2 = \overline{\text{COV}\ (MR)}^2 + P_2\ \overline{SN}^2 \cdot \overline{\text{COV}\ (SN)}^2$$

$$S_N = \sqrt{S_N^2}$$

Table 6 presents the P_2 values corresponding to the mean values of the structural number ($\overline{SN}$), and Figure 2 illustrates the relationship between P_2 and *SN*.

TABLE 6 Variance Component P_2 Corresponding to Mean Value of Structural Number *SN*

Mean Structural Number $\overline{SN}$	Variance Component P_2
1	4.065
2	1.634
3	0.763
4	0.475
5	0.370
6	0.299
7	0.242
8	0.197
9	0.162
10	0.135

$$P_2 = \left[\frac{4.065}{\overline{SN}+1} - \frac{1135.57}{\left(0.4 + \dfrac{1094}{(\overline{SN}+1)^{5.19}}\right)^2 (\overline{SN}+1)^{6.19}}\right]^2$$

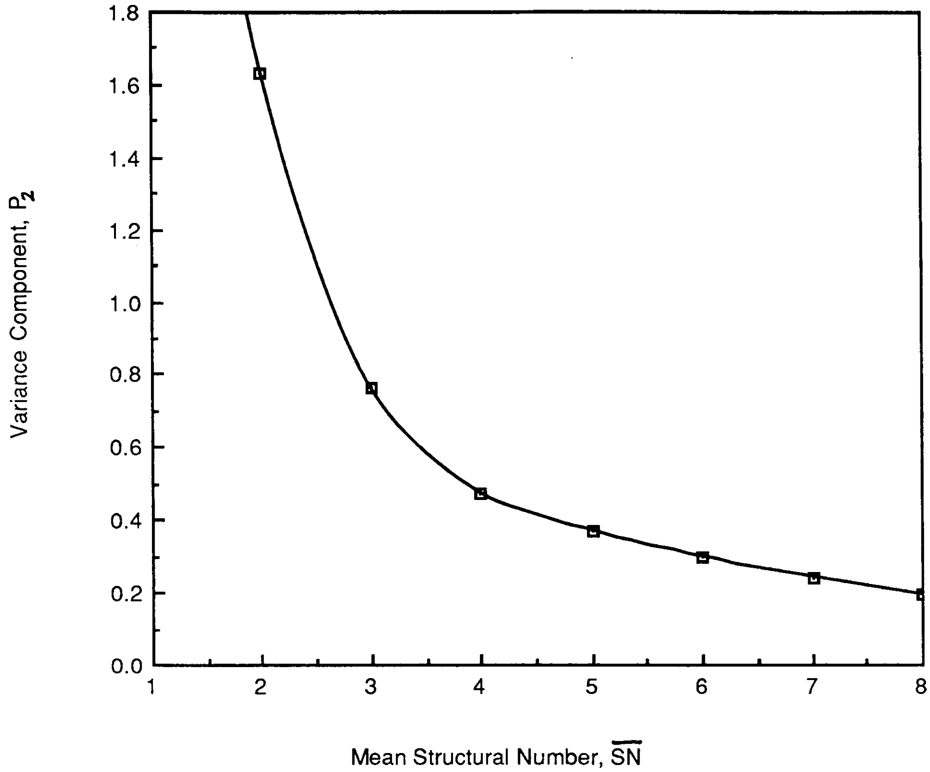

FIGURE 2 Relationship between mean structural number and variance component P_2.

By using the lower, upper, and typical values of the COV for the subgrade *MR* (10, 30, and 20 percent, respectively; Table 4) together with the lower, upper, and typical values of the COV for *SN* (5, 18, and 11 percent, respectively; Table 4); it can be shown that S_N is in the range of 0.17 to 0.58, with a typical value of 0.36 for a pavement with an $\bar{SN}$ of 4.0, for example.

When the lack-of-fit variance and unexplained replication variance of the AASHTO model itself [0.0763 + 0.0113 = 0.0876; Volume 2 of the guide (*11*)] are added to adjust the estimate for S_N, it can be computed that S_N is in the range of 0.34 to 0.66, with a typical value of 0.47, compared with the values recommended by AASHTO of 0.36 to 0.45, with a typical value of 0.44.

APPROXIMATION APPROACH VERSUS POINT ESTIMATE APPROACH

It is important to indicate that the presence of a correlation between independent variables of the performance prediction equation may result in an over- or underestimation of S_N when the approximation approach is used. If a negative correlation exists, for example, between *SN* and *MR*, the standard deviation estimated by the approximation approach will be overestimated. This case occurs for nonlinear "stress hardening" subgrades, in which the decrease in *SN* results in an increase in the stresses imposed on the subgrade, and hence an increase in *MR*. On the other hand if a positive correlation exists between *SN* and *MR*, the standard deviation estimated by the approximation approach will be underestimated. This case occurs for nonlinear "stress softening" subgrades, in which the decrease in SN results in an increase in the stresses imposed on the subgrade, and hence a decrease in *MR*.

In addition, the models used to estimate the coefficients of variation for the AASHTO layer coefficients (presented above) have their own uncertainties, and their added lack of fit variance may be significant so that it must be accounted for.

In those circumstances explained above the point estimate approach for reliability analysis may be preferred over the approximation approach, unless the covariances of the independent parameters can be quantified practically and the uncertainty in the models used is taken into consideration.

On the other hand usage of the point estimate approach, which requires measured values of the independent variables at different points to estimate the performance variable, may be not practical in comparison with the approximation approach.

However, in situ material characterization through back-calculation of paving layer moduli from deflection basins obtained during nondestructive testing of pavements can make the point estimate approach for reliability analysis as practical as the approximation approach.

The following example illustrates the fact that the two approaches may lead to almost identical estimations of S_N, regardless of the concerns expressed above.

Table 7 presents the back-calculated subgrade modulus and effective structural number for a 1,300-ft pavement segment [data are from Noureldin (*12*)]. The performance variable (log W_{t18}) was point estimated for each point by using the AASHTO model. When using the approximate approach S_N (without the inclusions

TABLE 7 Back-Calculated Subgrade Modulus (*MR*) Back-Calculated Effective Structural Number (*SN*), and Calculated Performance Variable (log W_{t18}) Along a 1,300-ft Pavement Segment

Distance Feet	Backcalculated Subgrade Modulus MR, PSI	Backcalculated Effective Structural Number SN_{eff}	Log W_{t18}
0	28,000	3.17	7.663
100	24,500	3.04	7.419
200	23,600	3.00	7.347
300	29,800	3.24	7.784
400	28,400	3.19	7.694
500	35,100	3.42	8.091
600	28,400	3.19	7.799
700	32,000	3.32	7.688
800	24,500	3.04	7.729
900	35,100	3.42	7.824
1000	26,000	3.09	7.843
1100	37,300	3.49	7.825
1200	24,200	3.02	7.517
1300	28,000	3.17	7.688
Mean	28,900	3.20	7.708
Std. Dev.	4,400	0.16	0.188
Coeff. of Var.	15.20%	5.00%	2.44%

of the unexplained and lack-of-fit variance of the AASHTO model) is estimated to be

$$S_N \cong \sqrt{\text{COV }(MR)^2 + P_2 \cdot \overline{SN}^2 \cdot \overline{\text{COV }(SN)}^2}$$

$$\cong \sqrt{0.152^2 + 0.675 \cdot \overline{3.2}^2 \cdot \overline{0.05}^2}$$

$$= 0.201$$

On the other hand, when using the point estimate approach, the standard deviation of performance parameter (log W_{t18}) is 0.188, as shown in Table 7. It can be noted that the two estimations are almost identical.

SUMMARY

The empirical or mechanistic pavement performance models usually used to predict the average performance of the pavement can also be used to predict the possible variations in that performance in terms of the statistical means, variances, standard deviations, or COVs.

The predicted average pavement performance requires knowledge of the predicted average of every independent variable affecting that performance and appearing in the design model. Likewise, the predicted variability in pavement performance requires the knowledge of the predicted variability of every independent variable affecting that performance. Rational quantification of these variabilities is essential for incorporating reliability into the design process.

Explicit quantification of these variabilities was presented in this paper in terms of the mean values and COVs of the independent variables that appear in the AASHTO pavement performance model. However, the concept presented is applicable for any other pavement performance model.

COVs of paving layer thicknesses and material properties for which ample data are available can be used by experienced pavement engineers to make realistic estimations that are almost as good as the values that can be measured; these were shown to be transferable into COVs for the independent variables of the AASHTO model.

The COVs for the independent variables of the AASHTO model were then shown to be a practical tool for quantifying the variability in flexible pavement performance and estimate the standard deviation of that performance.

ACKNOWLEDGMENTS

This paper was prepared through the study Adaptation of a Reliability Based Code for the Design of Flexible Pavement in the

Kingdom of Saudi Arabia, sponsored by King Abdel Aziz City for Science and Technology (KACST) under Grant AR-11-57. The authors appreciate the KACST sponsorship of the project and the Ministry of Communications, from which typical raw data were obtained.

REFERENCES

1. *Guide for Design of Pavement Structures*, Vol. I. AASHTO, Washington, D.C., 1986.
2. Yoder, E., and M. Witczak. *Principles of Pavement Design*, 2nd ed. John Wiley & Sons, Inc., New York, 1975.
3. Lytton, R. L., and D. Zollinger. Modeling Reliability in Pavement. Presented at 72nd Annual Meeting of the Transportation Research Board, Washington, D.C., January 1993.
4. Harr, M. E. *Reliability Based Design in Civil Engineering*. McGraw-Hill, New York, 1987.
5. Rosenblueth, E. Point Estimate for Probability Moments. *Proc., Mathematics*. National Academy of Sciences, Washington, D.C., 1975.
6. Van Til, C. J., F. McCullough, B. Vallerga, and G. Hicks. *NCHRP Report 128: Evaluation of AASHO Interim Guides for Design of Pavement Structures*. HRB, National Research Council, Washington, D.C., 1972.
7. *Special Report 61: The AASHO Road Test*. HRB, National Research Council, Washington, D.C., 1962.
8. Odmark, N. *Investigations as to the Elastic Properties of Soils and Design of Pavements According to the Theory of Elasticity*. Meddelande 77. Statens Vagenstitute, Stockholm, Sweden, 1949.
9. Rada, G., and M. Witczak. A Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Material. In *Transportation Research Record 810*, TRB, National Research Council, Washington, D.C., 1981.
10. *Asphalt Overlays and Pavement Rehabilitation*. MS-17. Asphalt Institute, 1969.
11. *Guide for Design of Pavement Structures*, Vol. 2. AASHTO, Washington, D.C., 1986.
12. Noureldin, A. S. New Scenario for the Process of Backcalculation of Layer Moduli for Flexible Pavement. Presented at 72nd Annual Meeting of the Transportation Research Board, Washington, D.C., January 1993.

The contents of this paper reflect the views of the authors, who are responsible for the accuracy of the data and the analysis presented herein. The contents do not necessarily represent the official views or policies of KACST or the Ministry of Communications. This paper does not constitute a standard, specification, or regulation.

Publication of this paper sponsored by Committee on Flexible Pavement Design.