Vehicle Sizing Model for Bus Transit Networks

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An iterative procedure to select vehicle sizes for the routes of a bus system with a given network configuration and origin-destination demand matrix is presented. The procedure starts by assigning a set of initial route service frequencies to compute route-level descriptors through a transit trip assignment model. The vehicle size for each route is computed analytically by a mathematical model that minimizes the total cost (operator cost and user cost) of each individual bus route. Revised frequencies are determined by applying a maximum allowed load factor consistent with the calculated vehicle sizes. The procedure terminates when frequencies of two consecutive iterations converge. The model is illustrated through a case application to the existing transit system in Austin, Texas. The result confirms the potential benefits of using variable vehicle sizes on different routes. However, the number of vehicle sizes in a system should be limited to avoid operational complexity and associated maintenance costs. In general, it appears that smaller buses could be operated on most of the bus routes in most North American cities to provide better service quality and lower operator cost.

Although both vehicle size and route frequency are important elements of bus service plans, most previous bus network design procedures treat vehicle size as a fixed value and compute route frequency either to achieve a minimum total generalized cost or to provide the capacity needed during peak-hour operation. Examples of these models are given elsewhere (*1–3*). The use of a fixed vehicle size simplifies the network design procedure, but it precludes the simultaneous consideration of various vehicle sizes in the bus system design and thus may result in ineffective resource allocation.

Because of high labor costs, transit operators in both Europe and North America tend to use fewer and larger buses to provide the capacity required during peak-period operation. Although smaller buses cost more to operate per seat, their use may offer several advantages. Glaister (*4*) argued that using small vehicles favors the provision of higher service frequencies, lowering average wait times and increasing operation speed; the improved service levels can be expected to generate new demand for bus transit. Furthermore, smaller buses may be better suited to some types of service, such as low-demand, low-occupancy, high-quality, or special transit, as Oldfield and Bly suggest (*5*). Smaller vehicles are more acceptable to residents of certain low-density neighborhoods and tend to inflict less damage on city street surfaces. Other suggestions for using different vehicle sizes are given elsewhere (*6–9*). To the extent that a given service area includes zones of different demand densities, allowing different vehicle sizes to operate on different bus routes and offer various types of services provides the transit operator with an additional choice dimension in designing the service configuration to better meet user needs and desired service levels.

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Only in a few studies have vehicle sizes been computed explicitly. Glaister (*9*) developed a simulation model to compare system operation under two vehicle sizes, a large vehicle (88 seats) and a small vehicle (15 seats). Results of the simulation suggest that buses seating 35 to 45 riders are likely to be most suitable for service in Aberdeen. Its level of detail notwithstanding, the computer simulation model does not describe explicitly the relationship between bus size and factors such as level of demand, operator cost, and load factor. Analytic models for finding optimal vehicle sizes have been developed for this purpose.

Previous analytic models include those of Jansson (*10*), Walters (*11*), Oldfield and Bly (*5*), and Chang (*12*). Jansson argued that previous analyses overweighted the producers' costs and underestimated the users' cost. He presented a model that minimizes total social cost (operator cost, passenger waiting time, and passenger riding time), subject to a peak capacity constraint satisfying a maximum occupancy rate (the ratio of the mean passenger flow to the product of the vehicle size and the service frequency). Jansson concluded that the optimal bus size determined by minimizing social cost tends to be smaller than that under the current practice of using a given vehicle size and setting the number of buses to achieve an average occupancy rate at or below a given maximum value.

Walters presented a simpler model that examines the trade-off between waiting time and labor cost. He also suggested that bus size should be considerably smaller than typically is used in Western European and North American cities. Gwilliam et al. (*13*) and Oldfield and Bly (*5*) argued that the waiting time assumption in Walters' model is questionable and thus yields an implausible relationship between optimal bus size and demand. Oldfield and Bly's model assumes elastic demand and determines the optimal bus size by minimizing total social cost. In addition, the average passenger waiting time in their model accounts for situations in which passengers are unable to board the first bus to arrive because it is full. They concluded that the optimal size lies between 55 and 65 seats (70-seat buses are most existing systems in the United Kingdom). Current cost structures could be changed to favor operation of smaller buses, but the optimal size seems unlikely to fall below 40 seats. Chang (*12*) presented analytic models to compare vehicle size for fixed route conventional bus with that of a flexible route subscription bus system. He concluded that the optimal vehicle size is less sensitive to the demand density for flexible route service than for fixed route service.

All the previous analytic models focus on the optimization of vehicle size for an individual bus line, which is treated independently of other lines in the network. In other words, demand on a particular bus line will not be affected by the optimal bus sizes and associated route frequencies of other bus lines. This is an incorrect assumption because, in a bus system, passengers may have several paths on which to complete their trips. Changes in bus sizes alter the route frequencies and should lead to a redistribution of passenger flows on the bus network. Jansson's and Walters's models consider demand on each given route to be known and constant. Although Oldfield and Bly's as well as Chang's models consider demand to be elastic to account for the change in route demand resulting from changes in bus size, they do not consider the systemwide effects of changes in vehicle size.

This paper presents a vehicle sizing procedure in the context of a design procedure for bus networks and service plans. Instead of assuming the demand on each bus line to be known and given, as in all previous models, the model presented here solves for the route demands by assigning the trips in a given origin-destination (O-D) demand matrix, using a transit trip assignment model. The trip assignment model also computes the maximum link flow on each bus route. The resulting maximum link flow is more reliable than the value obtained as the product of a maximum occupancy rate and vehicle seating capacity. Both the route demand and the corresponding maximum link flow then form the basis for obtaining a set of optimal bus sizes and the associated route frequencies that minimize a generalized cost function.

OPTIMAL VEHICLE SIZE FOR SINGLE ROUTE WITH GIVEN DEMAND

The well-known square-root rule for setting frequencies on bus routes is based on the minimization of the sum of operator cost and passenger waiting time. Major weaknesses of the square-root formulation are that it does not account for bus capacity constraints and that it assumes demand is independent of service frequency. In the transit industry, the frequency of service on a bus route commonly is set to achieve an applicable maximum allowed load factor (*14*) and can be written as

$$f_k = \frac{(Q_k)_{\max}}{LF_{\max} S_k} \tag{1}$$

where

f_k = route frequency for route k,
$(Q_k)_{\max}$ = maximum hourly link flow of route k,
$LF_{\max}$ = maximum allowed load factor, and
S_k = vehicle size.

According to the frequency formulation, transit operators can select the desired load factor to meet operational considerations, such as comfort. Different load factors may be set for different subsets of bus routes, depending on the type of service provided, service area, and other special considerations reflecting local political preferences. Of course, when the frequency generated from the equation is unacceptably low because of low patronage, a minimum frequency policy commonly is applied in practice, as recognized in the design procedure developed by Baaj and Mahmassani (*15*).

The approach to determining the optimal vehicle size for each individual route is similar to the generalized cost approach used to obtain the square-root expression for frequency setting. However, instead of considering the frequency as the decision variable and the vehicle size as a constant, the vehicle size is taken as the decision variable, and the frequency is set as a function of the vehicle size consistent with Equation 1.

For a given demand level on a bus route k, the optimal vehicle size is obtained by minimizing the generalized cost C_k, which consists of the operator cost C_{ko} and the user cost C_{ku} (i.e., $C_k = C_{ko} + C_{ku}$). The derivation of the optimal vehicle size is based on peak-hour operation, which is the most critical period for determining the required fleet size of the system.

Oldfield and Bly (*5*) presented a reasonable and simple approximate formulation that expresses total operator costs as a linear function of vehicle size, as follows:

$$C_{ko} = a(1 + bS_k)VM_k \tag{2}$$

where

a = constant that adjusts the overall cost level,
b = constant that captures the relative rate of increase in cost with increasing vehicle size, and
VM_k = total vehicle miles per hour operated on route k.

The total vehicle miles per hour for each route k can be expressed as

$$VM_k = f_k RTM_k \tag{3}$$

where f_k is the frequency of service on route k and RTM_k is the round-trip miles for route k.

If the function f_k is set according to the equal peak-hour load factor rule (Equation 1), the operator's cost can be expressed as

$$C_{ko} = a(1 + bS_k)RTM_k \frac{(Q_k)_{\max}}{LF_{\max} S_k} \tag{4}$$

From the passengers' point of view, the total user cost, C_{ku}, for route k consists of three components: waiting cost (WC_k), in-vehicle travel cost ($IVTTC_k$), and access cost (AC_k), as proposed by Chang (*12*).

$$C_{ku} = WC_k + IVTTC_k + AC_k \tag{5}$$

Under the assumptions that (*a*) passengers arrive at random (uniformly), (*b*) passengers can always board the first available bus, and (*c*) vehicles arrive at constant headways, the average waiting time for passengers using route k is taken as half of the route's headway. Assuming that waiting time is valued linearly (an assumption that may be relaxed if alternative value functions are calibrated from empirical behavioral data), the total waiting time for passengers using route k can be expressed as

$$WC_k = wTPT_k \frac{1}{2f_k} = wTPT_k \frac{LF_{\max} S_k}{2(Q_k)_{\max}} \tag{6}$$

where w is the value of waiting time and TPT_k is the total passenger trips (demand) per hour using route k (which is computed in the trip assignment procedure).

The expected transit passenger waiting time in an actual system depends on both the reliability of the bus schedule and the distribution of the passenger arrival times. Under the assumption of uniformly distributed random passenger arrivals at bus stops, the average passenger waiting time increases as bus headways become less regular because more passengers on average arrive during longer intervals and fewer arrive during shorter intervals (*16,17*). However, passengers may not arrive randomly in all cases. Some transit users tend, to some extent, to coordinate their arrivals with published schedules, if available, especially for routes with long headways. Bowman and Turnquist (*18*) have derived an expression for the expected wait time when the population of users is a mixture of "scheduled timers" and "random arrivals." The resulting waiting time function is highly system dependent and should be calibrated for each system, possibly for each bus route. However, the effect of schedule timing is offset to some extent by schedule unreliability, making the half-headway assumption an acceptable compromise. More important, from a design and frequency-setting standpoint, although "scheduled timers" may not incur an actual physical wait time at the stop, they incur a schedule delay relative to the actual time they would have wanted to depart. From the user cost standpoint in a design procedure, it is this schedule delay cost that must be included in the objective function, not the actual time at the stop. Evaluating waiting time on the assumption that users time their arrivals to coincide with the schedule can seriously underestimate user costs and lead to designs that do not meet user needs. This study uses a constant waiting value, w, for different modes (e.g., at home, at bus stop, or in office). Nevertheless, the procedure presented herein can be adapted easily to any waiting cost function specified by the model user, should sufficient justification and empirical support be available.

The in-vehicle travel cost is assumed independent of vehicle size, primarily because in-vehicle travel cost savings from using smaller buses are insignificant compared with the waiting-time cost savings. In-vehicle travel cost reduction may arise mostly from the possibly different average speeds of vehicles of different sizes. Smaller buses may provide faster service for two reasons: they have better maneuverability and fewer people are getting on and off them. Because bus speed is highly dependent on traffic conditions along the route, any improvement in the in-vehicle travel time cost of smaller buses usually is limited and insignificant relative to the potential waiting-time cost savings.

Another consideration of the constant $IVTTC_k$ assumption is the difficulty and resulting complexity of incorporating $IVTTC_k$ as a function of vehicle size in the cost function. The relationship between vehicle speed and vehicle size is difficult to specify analytically, especially in light of vehicle speed variation under different traffic conditions. Furthermore, vehicles of the same size with different engines may have different acceleration and deceleration characteristics. Therefore, it hardly seems worth the effort to incorporate route-dependent and condition-dependent $IVTTC_k$.

Using the above results and assumptions, the generalized cost C_k can be rewritten as follows:

$$C_k = a(1 + bS_k)RTM_k \frac{(Q_k)_{\max}}{LF_{\max}S_k} + wTPT_k \frac{LF_{\max}S_k}{2(Q_k)_{\max}} + AC_k + IVTTC_k \qquad (7)$$

Note that AC_k and $IVTTC_k$ are independent of vehicle size. The optimal bus size S_k^* for given route demand levels can be obtained by setting $dC_k/dS_k = 0$, and it can be expressed as

$$S_k^* = \frac{(Q_k)_{\max}}{LF_{\max}} \sqrt{\frac{2aRTM_k}{wTPT_k}} \qquad (8)$$

The relation indicates that the optimal vehicle size for a given demand level on a route is proportional to the level of the maximum link flow, $(Q_k)_{\max}$, and varies as the square root of round-trip miles of the route, RTM_k. The optimal vehicle size is inversely proportional to the load factor, $LF_{\max}$, as well as the square root of the total number of passenger trips, TPT_k, and the value of waiting time, w.

In Equation 8, the total cost (and associated optimal vehicle size) for a given route depends on the flow level, TPT_k. However, the latter is itself the result of the users' path choice through the network, which is a function of the vehicle sizes and frequencies, not only on the given route k, but on all network routes, $k = 1, \ldots, K$. The flows, TPT_k, $k = 1, \ldots, K$ are given by an assignment procedure, reflecting a passenger path choice rule, which distributes a given peak-period O-D trip matrix to the various bus routes. In our procedure, the vehicle sizes on each route (and associated frequencies) are set on the basis of route flows that are consistent with the vehicle sizes and frequencies through the iterative application of an assignment algorithm along with the vehicle sizing formula developed in this paper. However, the vehicle sizes obtained by this procedure are not necessarily optimal for the network as a whole. In other words, we do not seek to explicitly minimize the systemwide cost, $C = \sum_{k=1}^{K} C_k$, subject to consistency with a given assignment rule. Because of the network-level interactions described earlier, the objective function is not separable on a route-by-route basis. The resulting problems would be formidable because the assignment procedure used cannot be expressed as a well-behaved mathematical formulation. Instead, we propose a practical procedure that achieves an internally consistent solution that improves on existing methods.

VEHICLE SIZING PROCEDURE

The vehicle sizing procedure starts by assigning an initial set of frequencies to the bus routes. The O-D trip demand matrix for the bus system is then assigned to the bus routes using a transit trip assignment model. The transit trip assignment model computes both TPT_k (total passenger trips per hour using route k) and $(Q_k)_{\max}$ (the highest hourly link volume of route k): TPT_k and $(Q_k)_{\max}$ are then applied in Equation 8 to determine the locally "optimal" bus size for each route. To ensure that the resulting vehicle size remains within the range of buses under consideration, minimum and maximum size constraints are imposed. The vehicle size is then used in Equation 1 to compute the route frequency for each route. Note that for less congested bus lines the peak load factor method may generate frequencies that are lower than what riders can reasonably expect. In that case, a minimum frequency policy that sets route frequencies to a preset minimum value would be used instead.

The transit trip assignment model used in this study is described by Baaj and Mahmassani (*19*) in their transit network analysis procedure, TRUST. The model considers two main criteria: the number of transfers necessary to reach the destination and the trip times incurred with alternative path choices. The transit passenger is assumed to attempt to reach his or her destination by following the path that involves the fewest possible transfers. If two or more feasible paths are available with the same number of transfers, passengers are assumed to consider only those alternatives with trip times within a particular range. A "frequency-share" rule is then applied to assign trips according to the relative frequencies of service on the alternative paths. A more detailed description of the model can be found elsewhere (*2*).

Because the frequencies change from the initial values to new values, the demand of the bus system needs to be reassigned consistently with the new frequencies, and the optimal vehicle sizes and route frequencies then need to be recomputed as well. This procedure continues until two consecutive sets of route frequencies converge. This heuristic has exhibited convergence in all test cases conducted to date. Figure 1 shows the flowchart of the bus sizing procedure.

In summary, the procedure consists of the following steps:

Step 0. Assign an initial set of route frequencies.

Step 1. Compute TPT_k and $(Q_k)_{max}$ using the trip assignment model.

Step 2. Determine vehicle sizes using Equation 8. If the optimal vehicle size is less than the minimum vehicle size, set the vehicle size equal to the minimum vehicle size.

Step 3. Set route frequencies using Equation 1. If the resulting frequencies are less than the minimum frequency, set the frequencies to the minimum frequency.

Step 4. Check whether two consecutive sets of route frequencies converge. If yes, stop; otherwise, go to Step 5.

Step 5. Use route frequencies determined in Step 3, and go to Step 1.

The vehicle sizing model has been implemented as part of AI-BUSNET, an artificial-intelligence–based bus network design computer program that initially was developed at the University of Texas at Austin by Baaj and Mahmassani (*19*). The program is written in Lisp because its list data structure capabilities provide an effective data representation to support extensive path search and enumeration in the bus network design problem. The program runs on an Apple Mac-II with MicroExplorer, a Lisp language compiler.

ILLUSTRATIVE APPLICATION

The transit network of the Austin, Texas, urban area was selected to illustrate the above vehicle sizing procedure. The transit network consists of 40 routes with fixed schedules, operated by the Capital Metropolitan Transit Authority (Capital Metro). Express routes, UT shuttle routes, and 'Dillo routes, which reflect different service and operations concepts, are not considered in this application. Buses with 37 or 43 seats are used by the system for peak-hour operation. Required inputs for this study include data on the network connectivity, nodal composition for each bus route, and a peak-hour transit O-D demand matrix. A total of 177 nodes are defined to describe the service area and associated network connectivity. Table 1 gives the numbers of the network routes, the node composition of each route, and the associated service frequency. The information is presented in list form as input to the analysis. The transit peak-hour demand matrix for the Austin area is generated using daily boarding and alighting data provided by Capital Metro. The resulting O-D trips are not necessarily those actually using the system. The system serves approximately 5,800 trips during peak-hour operation.

Table 2 summarizes the parameters used to determine optimal vehicle size and the associated route frequency for each bus route. (Values attached to the parameters are discussed later.) The coefficients, a and b, in the operator's cost function are derived from the operator costs associated with different bus sizes provided by Capital Metro. The operator cost parameters should be recomputed for other cities because wage rates and gasoline cost vary from city to city. The maximum load factor for peak-hour service is set at 1.25 (i.e., up to 10 standing passengers are allowed at any time if the bus seating capacity is 40 passengers), which is suggested by *NCHRP Synthesis of Highway Practice 69* (1980). The waiting cost coeffi-

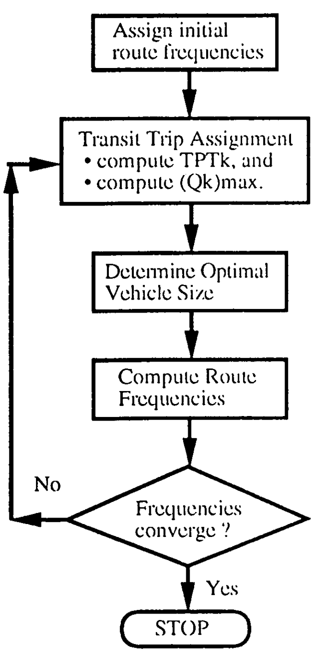

FIGURE 1 Vehicle sizing procedure.

TABLE 1 Bus Route Service Frequencies and Nodal Compositions

Route Name	Frequency	Nodal Composition
R1	7.5	(1 2 3 4 5 6 7 8 9)
R2	4.0	(13 12 11 10 1)
R3	4.0	(14 15 3 16 17 18 19)
R4	4.0	(25 24 23 22 15 21 10 20)
R5	3.0	(1 2 26 5 27 6 28 29 19)
R6	4.0	(1 2 10 30 31 32 33)
R7	4.0	(1 2 34 35 36 8 37 38 39)
R8	2.0	(40 41 42 43 44 45 46 36 8 19)
R9	1.82	(1 2 3 47 48 49 50 51)
R10	4.0	(10 1 52 53 54 55 56 57)
R11	1.5	(1 15 10 34 58 59 60 61)
R12	3.0	(2 64 65 66 67)
R13	4.0	(73 72 71 70 69 68 1 2 3)
R14	1.5	(78 77 76 75 63 21 10 74)
R15	4.0	(1 2 62 79 61 80 36)
R16	4.0	(84 83 82 81 1 2 62)
R17	4.0	(89 88 87 41 86 63 21 3 85)
R18	4.0	(90 10 20 91 43 92 93)
R19	1.5	(99 98 97 96 95 94 16 15 1)
R20	3.0	(1 15 62 59 100 101 102)
R21/22	2.14	(103 86 59 104 105 17 106 107 108)
R23	2.0	(19 109 110 111)
R25	2.0	(19 112 113 114 115)
R26	2.0	(119 118 25 117 116 1 2 26)
R27	4.0	(73 121 78 120 1 2 10 74)
R28	2.0	(78 69 54 83 66 122 123)
R29	1.5	(85 21 63 124 125)
R30	2.0	(123 128 127 126 52 2 74)
R31	1.33	(65 129 68 40 130)
R32	1.71	(36 80 131 42 132 133 134)
R33	2.0	(135 136 137 138 139 72 73)
R37	2.0	(140 21 10 141 142 45 101 143 144)
R38	2.0	(145 138 84 65 64 140 21 10 141)
R39	2.0	(36 146 46 147 148)
R40	1.71	(8 114 149 150)
R41	1.0	(151 152 153 9 8 5 2 154)
R42	2.0	(8 114 155 156 157 158)
R43	3.0	(159 160 67)
R44	1.0	(161 162 163 164 19)
R46	2.0	(19 165 166 167)

TABLE 2 Parameters Used in the Model and Values Assumed for the Application

Parameter	Definition	Value
a	coefficient on cost function	$ 2.96 / vehicle-mile
ACk	total passenger access cost of route k	-
b	relative gradient of cost function with vehicle size	0.0078
Ck	generalized cost for each route k	-
Cko	operator cost	-
Cku	user cost	-
fk	route frequency for route k	-
IVTTCk	total in-vehicle travel cost of route k	-
LFmax	the maximum allowed load factor	1.25
(Qk)max	the highest peak hour link volume on route k	-
RTMk	round trip miles for route k	-
Sk	vehicle size for route k	-
TPTk	total number of passengers using route k during the peak hour	-
vk	average bus speed for route k	12 mph
VMk	peak hour vehicle miles operated on route k	-
w	value of waiting time	$ 9, 12, 15/hour
WCk	total passenger waiting cost of route k	-

cient, *w*, is somewhat difficult to define. This application considers three values ($9/hr, $12/hr, and $15/hr). The minimum service frequency is set to be one bus per hour. A minimum vehicle size (10 seats) is selected when the calculated optimal vehicle size is less than that value.

Table 3 presents the resulting vehicle size and associated route frequency for each bus route in all three cases. Figure 2 shows the distribution of resulting vehicle sizes for all three waiting time values. In the case with the lowest waiting time value (w = $9/hr), 37 out of the 40 bus routes have an optimal bus size below 25 seats. Compared with the current bus system, this solution results in much lower passenger waiting cost ($7,616/hr versus $9,206/hr), with only slightly higher operator cost ($6,747/hr versus $6,6664/hr). For 15 out of 18 routes that currently operate with a frequency higher than two buses per hour, the model results provide for higher route frequencies relative to the current service. That is reasonable because the use of smaller buses usually requires higher service frequencies. Half of the 12 routes currently operating with a frequency of two buses per hour receive higher route frequencies. Only 1 of 10 routes currently operating with frequency less than two per hour receives a higher frequency in the model, suggesting that the current system uses higher minimum policy frequencies for routes with low passenger demand levels. In the two other cases with higher waiting time values, a smaller vehicle size is obtained for each bus route than in the case with the lowest waiting time value.

The result also demonstrates that the optimal vehicle sizes in the system are spread over a wide range. The use of a fixed vehicle size for the whole system is not an appropriate approach. Whereas it is infeasible to operate too many vehicle sizes in a system because of the resulting operational complexity and associated maintenance costs, meaningful benefits could be observed with a relatively small set of discrete values. For example, we reanalyzed the system under the assumption of only three commercially available vehicle sizes (37, 27, and 15 seats) and allocated those to each route using a simple nearest feasible integer heuristic. For the lowest waiting time value (w = $9/hr), the solution suggests that 32.2 percent of the savings in user costs (relative to the fixed size case) could be attained with just these three sizes. In addition, the operator cost in this case has been improved by a saving of $442/hr, as opposed to a loss of $83/hr in the optimal vehicle size case. Note that the operator cost savings are actually greater for the three sizes considered here than in the previous case because very small bus sizes are now avoided.

In general, bus systems operating a larger vehicle size have a lower operator cost. In Austin, larger vehicles (37 and 43 seats for peak-hour operation) are used on most bus routes. To provide a certain level of bus service, the bus system provides relatively high bus frequencies, resulting in higher operator cost. In addition, bus routes are operated with relatively low average load factors. Figure 3 shows that 34 out of 40 routes operate with load factors less than 0.75 in the Austin transit network. Similar situations are encountered in most North American cities. Capital Metro has recognized this fact and has been operating smaller buses on certain lower-demand suburban-oriented routes. For design purposes, vehicle sizes and service frequencies are selected to achieve the maximum allowed load factor. Therefore, only routes set at the minimum frequency because of low demand will have load factors below the maximum allowed.

CONCLUDING REMARKS

In this paper, an iterative procedure to determine the vehicle size accounting for the systemwide change in route demand resulting from changes in bus size is presented. An analytic formulation is derived to compute the locally optimal vehicle size by minimizing the total cost (operator cost and user cost) associated with each route. Bus route demand and the route's maximum link flow are critical to a determination of the optimal vehicle size. The demand and the maximum link flow are determined for each route by a trip assignment model that recognizes the operating characteristics associated with different vehicle sizes on each route.

The application shows that carefully selecting vehicle sizes will benefit both transit providers and users. In most North American cities, a large portion of routes is provided to low-demand areas to ensure users' mobility. Larger vehicles are still operated on these routes, resulting in either poor service quality or low vehicle occupancy rate. Clearly, a smaller vehicle size should be used on such routes. Because each bus system may include zones of different demand densities, various vehicle sizes should be used depending mainly on the bus route demand. However, the number of vehicle

TABLE 3 Optimal Bus Sizes and Associated Route Frequencies

Route Name	Case with w = \$9/hr		Case with w = \$12/hr		Case with w = \$15/hr	
	fk	Sk	fk	Sk	fk	Sk
R1	7.3	28	8.5	24	9.7	22
R2	5.0	12	5.5	11	6.0	10
R3	4.6	15	5.4	13	5.8	12
R4	4.8	14	5.5	13	6.5	11
R5	3.5	14	4.1	12	4.4	11
R6	3.4	10	3.4	10	3.4	10
R7	4.6	27	5.4	23	5.9	21
R8	4.4	19	5.2	16	5.6	15
R9	1.6	10	1.6	10	1.6	10
R10	4.4	13	5.1	11	6.2	10
R11	1.0	10	1.0	10	1.0	10
R12	4.3	12	5.2	10	5.2	10
R13	6.7	29	7.8	25	8.9	22
R14	1.0	10	1.0	10	1.0	10
R15	5.1	14	6.0	12	6.6	11
R16	4.4	14	5.1	12	5.5	11
R17	5.8	15	6.6	13	7.7	12
R18	3.4	10	3.4	10	3.4	10
R19	1.2	10	1.2	10	1.2	10
R20	3.8	11	4.2	10	4.2	10
R21/22	2.9	10	2.9	10	2.9	10
R23	1.0	10	1.0	10	1.0	10
R25	3.3	10	3.3	10	3.3	10
R26	3.2	15	3.7	13	4.0	12
R27	5.7	22	6.6	19	7.4	17
R28	1.0	10	1.0	10	1.0	10
R29	1.0	10	1.0	10	1.0	10
R30	1.6	10	1.6	10	1.6	10
R31	1.2	10	1.2	10	1.2	10
R32	1.4	10	1.4	10	1.4	10
R33	2.3	10	2.3	10	2.3	10
R37	3.2	14	3.7	13	4.1	11
R38	2.9	13	3.4	11	3.7	10
R39	1.0	10	1.0	10	1.0	10
R40	1.0	10	1.0	10	1.0	10
R41	1.0	11	1.0	10	1.0	10
R42	3.1	11	3.4	10	3.4	10
R43	1.0	10	1.0	10	1.0	10
R44	1.5	10	1.5	10	1.5	10
R46	1.0	10	1.0	10	1.0	10

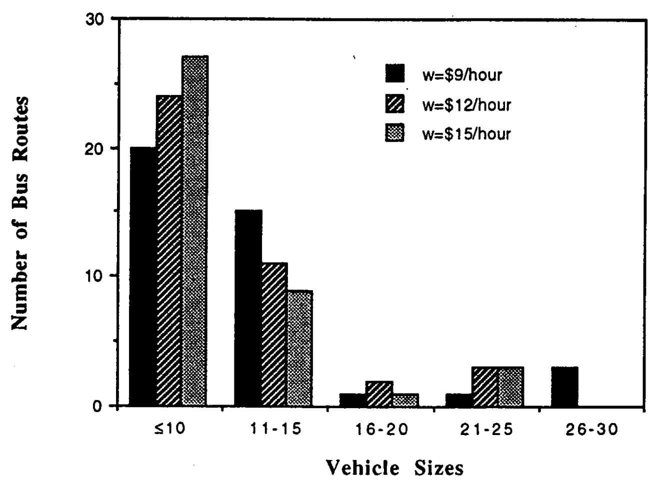

FIGURE 2 Distribution of vehicle sizes with different waiting time values.

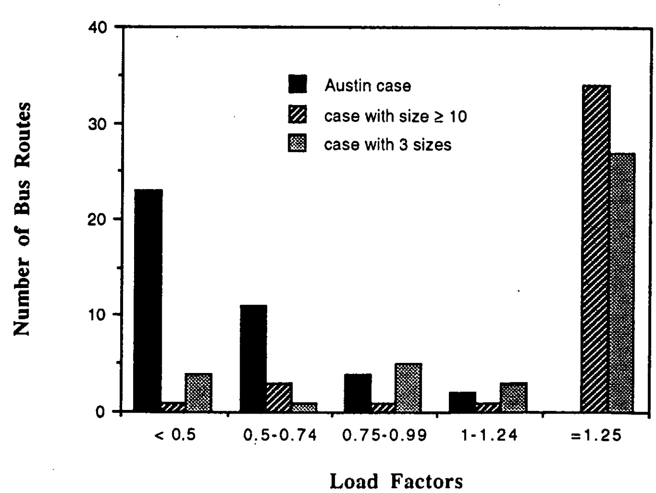

FIGURE 3 Distribution of load factors.

sizes in a system should be limited to avoid high maintenance cost and operational complexity. Incorporating variable vehicle sizes in the transit network design model will contribute to better and more realistic solutions to the problem. Although this paper demonstrates only the application of the vehicle sizing procedure to an existing bus system, the procedure has been implemented to enhance the network design procedure, AI-BUSNET.

Of course, the framework presented here incorporates a number of assumptions and relations that may be less applicable to certain locations than to others. These include the cost function components, such as relative waiting time and time cost of various operations. The methodology presented here provides a flexible framework to incorporate alternative assumptions and functional relations that may be tailored for specific cities. In future research, the passenger boarding and alighting time that affects the speed of different vehicle sizes will be considered. The optimal vehicle size that could be operated in different periods of the day will be investigated as well.

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