Arterial Signal Optimization Considering Left-Turn Control

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Arterial signal synchronization is the most cost-effective method for reducing vehicle operating costs and controlling fuel emissions on the urban street network. From a modeling perspective this process requires the determination of optimum signal control parameters with the objective of minimizing system disutility or maximizing the progression bandwidth. Some compromise approaches have also been suggested. This work focuses on the bandwidth approach. The objective of this research was to develop a traffic model with a capability for simultaneously optimizing all signal control variables including left-turn treatment, including protected, permissive, or protected-permissive phasing. For evaluation purposes a four-intersection arterial was analyzed by using the proposed (ZMODEL) and an alternative PASSER-II bandwidth model. The TRANSYT-7F model was subsequently used to evaluate the signal settings. As a result of the added flexibility in left-turn control, the model generally produced lower system cycle lengths and wider bandwidths for the artery. On the other hand and on the basis of limited experiments, the overall system utility may actually degrade. The trade-off between bandwidth efficiency and delays (for artery and all movements) is clearly demonstrated.

The need for competent traffic control systems on existing facilities is evident in urban/suburban areas as a result of an increase in travel demand, changes in trip origins and destinations, and financial constraints on major additions to physical capacity in urban areas.

The objective of a traffic control system is to improve traffic mobility subject to resource constraints such as safety, environmental impacts, capital, energy, and other societal concerns. Improvements in traffic management have a direct impact on the economic welfare of the community overall and specifically concerned individuals. Some of these economic factors would appear in the form of savings in travel time, greater driving comfort, less energy waste, greater safety, reductions in noise and air pollution, and reductions in wear and tear on the roadway system in general (*1*).

To implement a continuing, comprehensive, and coordinated traffic control strategy, optimal use of available network capacity should be ensured. Improving the traffic signal coordination system is one of the most effective countermeasures to enhance the overall traffic signal system capacity.

The main purpose of signal coordination is to discharge the maximum amount of main street traffic without enforced halts while allowing for adequate capacity for cross-street traffic. Two traffic signal system coordination approaches are available: (a) a bandwidth system that permits continuous movement in a progression bandwidth and (b) a disutility system that is aimed at minimizing travel cost (delays, stops, fuel emissions, etc.) along an arterial or through an area.

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Computer codes have historically been developed to determine the best signal coordination schemes. Some of these are based on bandwidth maximization such as MAXBAND (*2–4*) and PASSER-II (*5*) or minimizing system disutility such as TRANSYT-7F (*6–8*). Some compromise approaches have also been suggested (*9–11*). However, none of the methods considers the entire spectrum of traffic and signal timing elements simultaneously. A critical discussion of existing models can be found in a work by Khatib (*12*). Table 1 provides a summary comparison of selected arterial models in the United States.

BACKGROUND

In the bandwidth approach efficiency and attainability are two measures that are useful in assessing the utility of a coordination scheme (*5*).

The *efficiency* of a progressive system is defined as the average ratio of bandwidth to cycle length. The efficiency is a measure of how much of the cycle length has been used by the bandwidth. Thus, the closer the efficiency is to 100 percent, the larger the bandwidth.

Attainability is the ratio of bandwidth to the arterial minimum through green in each direction. An attainability of 100 percent implies that bandwidth can be found for the given splits. Thus, the attainability is a measure of the progression's ability to use the available greens of the intersections within the artery.

The principal features of the proposed formulation are highlighted and contrasted with MAXBAND's:

1. Phase lengths are globally optimized on the artery. By contrast, MAXBAND computes splits locally and applies them as constraints on the bandwidth size at each intersection.
2. Secondary flows (midblock volumes and turning volumes from the upstream intersection) are considered in the traffic flow model for optimizing queue clearance time. Thus, queue clearance time is added as a decision variable to be determined explicitly through the optimization.
3. Left-turn treatments cover protected, permissive, and protected-permissive phasing.
4. Slack green time outside the bandwidth is allocated to the favored bandwidth direction to reduce overall delay.
5. Phase length times are constrained by the minimum phase lengths as input by the user.

METHODOLOGY

The arterial synchronization problem is formulated from macroscopic traffic flow theory. It uses binary mixed integer linear pro-

TABLE 1 Summary Comparison of Selected Arterial Models in the United States

Control Variable	MAXBAND	PASSER-II	TRANSYT-7F	ZMODEL
Cycle Length	Opt[a]	Range[b]	Range[c]	Opt
Splits	Com[d]	Com[e]	Opt	Opt
Number of Phases	Input[f]	Input	Input	Opt
Phases Sequence	Opt[g]	Opt[g]	Input	Opt
Offsets	Opt	Opt	Opt	Opt
Left-Turn Treatment	Protected Only	Input	Input	Opt
Queue Clearance Time	Input	Input	Opt	Opt
Slack Time	NA	NA[h]	Opt	Opt
Secondary Flow	NA	NA[h]	Opt	Opt
Traffic Flow Model	Simple	Simple	Detailed	Simple
Optimal Solution	Global	Local	Local	Global

Opt[a] Optimized as a decision variable in the formulation.
[b] Range of cycle lengths is examined and the one that maximizes the progression bandwidth is selected *after* phase sequence and offsets have been optimized.
[c] Range of cycle lengths is examined and the one yielding minimum PI is selected *after* splits and offsets have been optimized.
Com[d] Computed to equalize degree of saturation for critical movements.
Com[e] Computed for each cycle length based on modified Webster's method.
[f] Input by the user.
[g] Select one sequence from a given set of possible phase sequences to maximize the progression bandwidth. Both MAXBAND and PASSER-II can only analyze *one* phase sequence for each cross street.
[h] Considered in delay computations *after* the optimization procedure has been completed.

gramming techniques to optimize the following signal control variables: cycle length, phase lengths, number and sequence of phases, offsets, and left-turn treatments. Consideration is given to secondary flows and queue clearance times. The objective is to maximize a weighted bandwidth in both directions of the arterial. This approach guarantees a global optimum solution to the problem. The proposed model (ZMODEL) takes the following general form:

Maximize: weighted bandwidth.
Subject to: network constraints, segment constraints, intersection constraints.

The ZMODEL formulation has some similarities to MAXBAND's (*13*). In the interest of clarity, variables and symbols similar to those used in MAXBAND are used in the ZMODEL formulation. In this paper only the most influential constraints on the formulation are discussed, including left-turn treatment, capacity, queue clearance time, and slack green time. Detailed derivations of the ZMODEL formulation can be found elsewhere (*12*). A summary of ZMODEL formulation is listed later in the paper.

Offset Selection

A complexity that arises in developing a general progressive system is in locating efficient offsets in both directions for optimal coordination. Offsets usually are referenced to one intersection (e.g., the first intersection in the system). For a fixed cycle length the sum of offsets in two directions (outbound and inbound) must be equal to an integer number of cycles.

Offset settings attempt to accommodate traffic movements through several signals without stops or delays. Progressive systems are based on the ideal offset, which is defined as a function of the block length, vehicle cruise speed, queue length, discharge headway, and lost time for vehicles at the first intersection (*14*).

Left-Turn Treatment

Left-turn treatment refers to the right-of-way designation to left-turn vehicles. Inadequate treatment can cause excessive delay on the approach and conceivably for the entire intersection (*15*). Three potential left-turn treatments are considered:

- *Protected,* in which left-turning vehicles have exclusive right-of-way during the phase;
- *Permissive,* left-turning vehicles have no exclusive phase, crossing the intersection in gaps of the opposing traffic stream; and
- *Protected-permissive,* a sequenced combination of the previous two treatments.

In an urban traffic network joint decisions should be made regarding signal parameters and left-turn treatment. These decisions directly affect the capacity and level of service for traffic movements. For example, when permissive phasing is allowed additional green time may be available to through traffic, thus enhancing the size of the bandwidth available for progression and reducing delays and stops over the network.

Capacity

The proposed model (ZMODEL) selects among permissive or protected-permissive left-turn treatments at each intersection. The permissive green time is implicitly optimized to maximize the permissive capacity. Total capacity is maximized by assigning green time to nonpriority flow on the following bases:

- To cross street through movements by using
  –maximum design volume-to-capacity ratios (V/C), and
  –minimum green time assigned to each movement as input;
- To all left-turn movements by using
  –maximum design V/C ratios,
  –the number of vehicles discharged during the clearance interval,
  –permissive left turns, and
  –minimum green time assigned to each movement as input.

The remaining green time is then fully assigned to the artery through movements.

Permissive Left Turns

The effective green time available for left-turn traffic is dependent on the phasing sequence for the opposing traffic. For example, consider the northbound left-turn movement in Figure 1. In Case 1 opposing traffic is serviced in a lead, which is followed by a through phase; therefore, the effective green will be equal to the length of the lead and through phases. In Case 2 the effective green is equivalent to the through phase time only. Other examples can be demonstrated, the point being that the capacity of permissive left-turn movement is intimately tied to the phasing sequence in effect.

In the program formulation the permissive green time is evaluated according to the following constraints:

The unsaturated green time cannot exceed the length of the through phase,

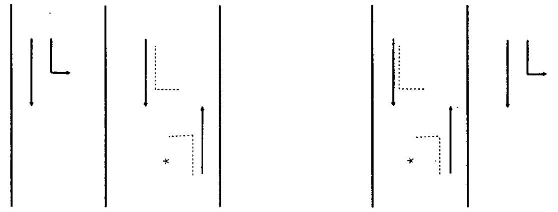

FIGURE 1 Phase sequence and left-turn capacity.

$$gu_{pi} \leq (\lambda_{ji} - l \times Z) \quad (1)$$

The opposing phase sequence determines the value of the opposing effective green time,

$$go_{pi} - (\lambda_{ji} + \lambda_{j'i} - l \times Z) \leq 1 - W_{jj'i} \quad (2)$$

$$go_{pi} - (\lambda_{ji} - l \times Z) \leq W_{jj'i} \quad (3)$$

The permissive left-turn capacity [cLT_{pi}(perm)] is then expressed as

$$cLT_{pi}(\text{perm}) = sLT_{pi}(\text{perm}) \times gu_{pi}/C \quad (4)$$

The opposed left-turn saturation flow [sLT_{pi}(perm)] is usually expressed as a function of the opposing through volume (*16,17*); it can be generalized as a second-degree polynomial:

$$sLT_{pi}(\text{perm}) = \alpha_1 + \alpha_2 \times vo_{pi} + \alpha_3 \times (vo_{pi})^2 \quad (5)$$

where α_1, α_2, and α_3 are coefficients defined by the user (e.g., $\alpha_1 = 1{,}400$, $\alpha_2 = -1$, and $\alpha_3 = 0$ in the 1985 Highway Capacity Manual).

Total Movement Capacity

The capacity for a traffic movement is defined as the product of the saturation flow rate and the effective green ratio. The proposed model considers only through and left-turn movements, with right turns assumed to be equivalent to through traffic.

Left-turn capacity consists of vehicles discharging in the protected phase [cLT_{pi}(prot)], the permissive phase [cLT_{pi}(perm)], and during the clearance interval (*SNH*). The latter is expressed as

$$SNH = 3{,}600 \times SNC \times Z \quad (6)$$

Thus, for each approach p at intersection i the total capacity for a left-turn movement [cLT_{pi}(tot)] is

$$cLT_{pi}(\text{tot}) = cLT_{pi}(\text{prot}) + cLT_{pi}(\text{perm}) + SNH \quad (7)$$

To ensure that each movement operates below its potential capacity, a certain amount of green slack time is introduced into the formulation. This produces a reserve capacity for each critical movement to accommodate random variations in demand and capacity. Thus, for left turns

$$XLT_{pi} \times cLT_{pi}(\text{prot}) + XLT_{pi} \times cLT_{pi}(\text{perm}) + SNH \geq vLT_{pi} \quad (8)$$

and for through movements

$$XTH_{pi} \times cTH_{pi}(\text{prot}) \geq vTH_{pi} \quad (9)$$

The final capacity constraints at intersection i are now summarized. There are four left-turn movement constraints and two through movement constraints. The through movement constraints apply only to cross-street traffic, whereas green for the artery is allotted all excessive green time. Therefore, for left-turns

$$XLT_{pi} \times sLT_{pi}(\text{prot}) \times (\Sigma_j aL_{jpi} \times \lambda_{ji} - l \times Z)$$
$$+ XLT_{pi} \times sLT_{pi}(\text{perm}) \times gu_{pi} + SNH \geq vLT_{pi} \quad (10)$$

and the capacity constraint for through movements is

$$XTH_{pi} \times sTH_{pi}(\text{prot}) \times (\Sigma_j aT_{jpi} \times \lambda_{ji} - l \times Z) \geq vTH_{pi} \quad (11)$$

Queue Clearance Time

Queue clearance time is defined as the time necessary to clear the accumulated queue on the approach before the main platoon reaches the downstream intersection. The accumulated vehicles consist of mid-block flows ($MidV_{pi}$) and turning vehicles from an upstream intersection (LT_{pi}, RT_{pi}).

$$Qs_{pi} = MidV_{pi} + LT_{pi} + RT_{pi} \quad (12)$$

On the artery the through band can be advanced by a queue clearance time to discharge secondary flow queues. The model can optimize this time to allow for smooth progression along the artery. The queue clearance time τ_{pi} in direction p at an intersection i should be sufficient to clear all secondary flows. It is related to flow variables as follows:

$$Qs_{pi}\,(r_{pi} + \tau_{pi}) = sTH_{pi} \times \tau_{pi} \quad (13)$$

or

$$\tau_{pi} = Qs_{pi} \times r_{pi}/(sTH_{pi} - Qs_{pi}) \quad (14)$$

Thus, the queue clearance time should not exceed τ_{pi}, nor should it be negative. Thus,

$$\tau_{pi} = \text{Max}\,[0, Qs_{pi} \times r_{pi}/(sTH_{pi} - Qs_{pi})] \quad (15)$$

which generates the following constraint at intersection i for approach p:

$$\tau_{pi} \leq [Qs_{pi}/(sTH_{pi} - Qs_{pi})]\, r_{pi} \quad (16)$$

Slack Green Time

Slack green time is defined as the excess green time outside the progression band at each intersection. It is the difference between through green time and the bandwidth (minus the queue clearance time, if any). This green slack time could be used for reducing intersection delay (*18*). Therefore, the slack green time is

$$SL_{pi} = g_{pi} - B_p - \tau_{pi} \quad (17)$$

ZMODEL allocates the green slack time in proportion to its main inbound/outbound flow at each intersection (or other priority criterion). Thus,

$$(1 - K_i)\, SL_{1i} - (1 - K_i) \times K_i \times SL_{2i} \geq 0 \quad (18)$$

Note that by definition SL_{pi} is equal to 0 for the critical intersection(s).

Summary of ZMODEL Formulation

Offset constraints apply to each intersection or link pair. Each intersection is denoted by i, and a link is defined as connecting intersections i and $i + 1$. Every approach of the intersection is denoted by p. Two consecutive phases are denoted by j and j'. The complete model formulation is now presented:

Objective function

$$\text{MAX}\,(B_1 + KB_2)$$

This objective function is identical to that of MAXBAND's subject to

- Bandwidth weight (similar to MAXBAND's):

$$(1 - K)\, B_2 - (1 - K)\, KB_1 \geq 0$$

- Cycle length range (similar to MAXBAND's):

$$1/C_2 \leq Z \leq 1/C_1$$

- Geometric constraint (similar to MAXBAND's), $i = 1, \ldots, n - 1$:

$$(w_{1i} + w_{2i}) - (w_{1i+1} + w_{2i+1}) + (t_{1ii} + t_{2ii}) + \Delta_i - \Delta_{i+1}$$
$$+ (1/2)\,(r_{1i} + r_{2i}) - (1/2)(r_{1i+1} + r_{2i+1}) - (\tau_{2i} + \tau_{1i+1}) - m_i = 0$$

- Speed constraints (similar to MAXBAND's), $i = 1, \ldots, n - 1$:

$$(d_{pii}/f_{pii})\, Z \leq t_{pii} \leq (d_{pii}/e_{pii})\, Z$$

- Change in speed on two consecutive links (similar to MAXBAND's), $i = 1, \ldots, n - 2$:

$$(d_{pii}/h_{pii})\, Z \leq (d_{pii}/d_{pii+1})t_{pii+1} - t_{pii} \leq (d_{pii}/g_{pii})\, Z$$

- Common cycle length (modified from MAXBAND's), $i = 1, \ldots, n$:

$$w_{pi} + B_p + r_{pi} + l \times Z \leq 1$$

$$r_{pi} + g_{pi} = 1$$

All remaining constraints are new to MAXBAND.

- Starting and ending phase times for phase j at intersection i, $i = 1, \ldots, n; j = 2, 3, 4, 6, 7, 8$:

$$V_{ji} - U_{ji} = \lambda_{ji}$$

- Starting and ending effective red times, $i = 1, \ldots, n$:

$$V_{pi} - U_{pi} = r_{pi}$$

- Phase sequence, $i = 1, \ldots, n; j, j' = 2, 3, 4, 6, 7, 8$:

$$V_{ji} - U_{j'i} \leq W_{jj'i}$$

$$V_{j'i} - U_{ji} \leq 1 - W_{jj'i}$$

- Delta constraint, $i = 1, \ldots, n$:

$$\Delta_{ji} - 0.5\, r_{1i} - U_{1i} + 0.5\, r_{2i} + U_{2i} = 0$$

- Minimum green time constraints, $i = 1, \ldots, n; j = 1, \ldots, 8$:

$$\lambda_{ji} \leq I_{ji}$$

$$\lambda_{ji} \geq \lambda_{ji}(\text{min}) \cdot I_{ji}$$

- Constraints to choosing at most three of four phases, $i = 1, \ldots, n; j = 1, \ldots, 8$:

$$\textstyle\sum_{ji} I_{ji} \leq 3$$

- Phase summation check, $i = 1, \ldots, n; j = 1, \ldots, 8$:

$$\textstyle\sum_{ji} \lambda_{ji} = 1$$

- Queue clearance time, $i = 1, \ldots, n$:

$$\tau_{pi} \leq [Qs_{pi} / (sTH_{pi} - Qs_{pi})] r_{pi}$$

- Slack green time, $i = 1, \ldots, n$:

$$(1 - K_i)\, SL_{1i} - (1 - K_i) \times K_i \times SL_{2i} \geq 0$$

- Capacity constraints, $i = 1, \ldots, n; j = 1, \ldots, 8; p = 1, \ldots, 4$:

  –For through movements:

$$XTH_{pi} \times sTH_{pi}(\text{prot}) \times (\textstyle\sum_j aT_{jpi} \times \lambda_{ji} - l \times Z) \geq vTH_{pi}$$

  –For left turns:

$$XLT_{pi} \times sLT_{pi}(\text{prot}) \times (\textstyle\sum_j aL_{jpi} \times \lambda_{ji} - l \times Z) + SNH$$
$$+ XLT_{pi} \times sLT_{pi}(\text{perm}) \times gu_{pi} \geq vLT_{pi}$$

- Permissive green time constraints, $i = 1, \ldots, n; j, j' = 1, \ldots, 8$:

$$gu_{pi} \leq \lambda_{ji} - l \times Z$$

$$go_{pi} - (\lambda_{ji} + \lambda_{j'i} - l \times Z) \leq 1 - W_{jj'i}$$

$$go_{pi} - (\lambda_{ji} - l \times Z) \leq W_{jj'i}$$

$$gu_{pi} \leq g_{pi}/(1 - Yo_{pi}) - [Yo_{pi}/(1 - Yo_{pi})]$$

- Variable constraints, $i = 1, \ldots, n; j, j' = 1, \ldots, 8$:

  –general integer: m_i
  –binary variables (0/1): I_{ji}, $W_{jj'i}$, aT_{jpi}, and aL_{jpi}
  –unrestricted in sign: Δ_i
  –upper bound of 1: $V_{ji} \leq 1$

Computational Features

ZMODEL uses a mixed integer linear programming problem formulation. The code is interfaced with the IBM Mathematical Programming System Extended/370 (MPSX/370) (*19*) as the optimization tool. MPSX/370 has been selected for its capability for solving large mixed integer linear programming problems with integer variables (0/1), general integer variables (0, 1, 2, . . .), and unrestricted-in-sign variables. The size of the proposed model depends on the scope of the arterial investigated. The model can handle up to 20 intersections on an arterial. The numbers of decision variables and constraints depend on the size of the arterial. For a set of n intersections, the following applies:

- Number of constraints = $(86\, n - 11)$
- Number of decision variables = $(61\, n)$ categorized as

  –$(17\, n)$ binary variables
  –$(n - 1)$ general integer variables
  –(n) real unrestricted in sign variables
  –$(42\, n + 1)$ continuous variables

where $2 \leq n \leq 20$.

EVALUATION

The MAXBAND (*13*) and PASSER-II (*5*) computer models can be used to optimize signal settings on the basis of the maximum bandwidth concept. MAXBAND can only handle protected turns (i.e., left-turn saturation rates are independent of opposing flows), whereas PASSER-II is able to design for protected-only as well as protected-permissive left-turn treatments. Because ZMODEL is designed to optimize left-turn treatments, PASSER-II was the logical comparative model.

To evaluate the performance of the signal settings, traffic disutility is introduced as a measure of traffic operation quality. Disutility is defined by a performance index (PI), which is a function of delays and stops (*20*). TRANSYT-7F was selected for performance evaluation because it has a realistic traffic model, is capable of modeling multiphase arterials, and can be constrained to maintaining a fixed bandwidth size. TRANSYT-7F simulation outputs resulting from both ZMODEL and PASSER-II settings were analyzed to compare the PI values obtained by both methods.

The evaluation procedure was accomplished in a series of sensitivity runs. All runs were applied to a four-intersection arterial segment. The test arterial is a segment on Skillman Avenue in Dallas,

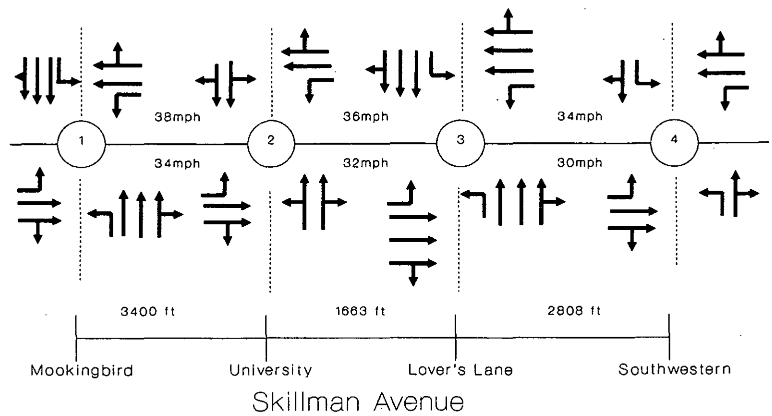

FIGURE 2 Test arterial.

Texas, which was used in the documentation of TRANSYT-7F (*20*) and PASSER-II (*5*) and which is shown in Figure 2.

Test Arterial Data

Saturation flow rates were estimated at 1,750 vphg per lane for through traffic lanes and 1,700 vphg per lane for left turns, except for the cross street at Intersection 2, where the estimated saturation flow rates are 2,600 vphg for each approach. Progression speeds and intersection spacing are shown in Figure 2. A ±2-mph variation in speeds and in changes of speeds between two consecutive links is allowed. Base traffic volumes are summarized in Table 2.

Traffic Parameters

In all proposed evaluations the following parameters were used:

- Cycle range of 60 to 110 sec,
- Lost time of 3 sec per movement,
- Bandwidth weights proportional to directional arterial through volumes, and
- Minimum phase lengths equal to 10 percent of the cycle length for through phases and 5 percent for all other phases.

Run Control Parameters

Because of computer execution time and memory limitations the following run control parameters were implemented in ZMODEL:

- Execution time of the MPSX/370 code was limited to 30 min,
- Memory workspace was limited to 5,000 cells (compared with the default value of 50), and
- Size of the branch-and-bound tree was increased to 4,000 nodes, where the default is

$$e = \text{Min}\ (500, 4 \cdot \text{number of integer variables}).$$

Thus, if no optimal solution is reached within the limitations given previously, the best integer solution to date is reported instead.

Results

A preliminary evaluation of the conditions described in Figure 2 and Table 2 is presented. Both ZMODEL and PASSER-II were used to generate optimal signal settings for protected-only as well as protected-permissive left-turn treatments. The following critical parameters are applied:

- Threshold V/C ratio, X = 90 percent,
- Permissive left-turn saturation flow model = $1{,}700 - Vo_{pi}$; and
- Number of vehicles discharging during the clearance interval = 2 vehicles/cycle.

Initial observations of the results depicted in Table 3 indicate that PASSER-II timings are virtually identical for the two left-turn treatments. The cycle length is identical, as are the artery splits. The

TABLE 2 Base Traffic Volumes for Test Arterial (in vph)

	Approach (p)							
	---- 1 ----		---- 2 ----		----- 3 -----		---- 4 ----	
Intersection	Outbound		Inbound		Cross street			
No.	T	L	T	L	T	L	T	L
1	287	88	1114	51	568	48	1560	240
2	369	58	1479	11	112	0	330	0
3	407	70	2052	24	227	100	877	54
4	468	26	1392	14	138	84	400	77

TABLE 3 Signal Setting Results for Test Arterial

	Source Model			
	PASSER-II		ZMODEL	
LT Treatment:	P[a]	PP[b]	P	PP
C, sec	110	110	110	62
$B_{1\,(2)}$ %	39[c] (45)[d]	43 (45)	50 (47)	57 (57)
Efficiency, %	38	41	48	57
Attainability, %	100	100	95	92
Prog. Speed, mph	31 (35)	31 (35)	32 (35)	33 (38)
Artery Splits, % at Intersection:				
1	50	50	58	62
2	79	79	84	76
3	65	66	66	68
4	59	61	61	70
Artery Phasing Pattern[e] at Intersection:				
1	Pat3	Pat3	Pat2	Pat0
2	Pat3	Pat1	Pat3	Pat2
3	Pat1	Pat1	Pat2	Pat1
4	Pat1	Pat1	Pat2	Pat0
v/c for arterial through movements at Intersection:				
1	.25(.83)	.22(.82)	.16(.67)	.14(.55)
2	.15(.63)	.15(.62)	.14(.57)	.14(.63)
3	.15(.73)	.14(.71)	.16(.69)	.12(.66)
4	.27(.78)	.25(.75)	.25(.74)	.20(.61)

[a]P = Protected only
[b]PP = Protected/permissive or permissive-only left-turns
[c]xx [d](xx) = Outbound (Inbound)
[e] Phasing pattern:
Pat1 = Outbound Lead
Pat2 = Inbound Lead
Pat3 = Dual Left Lead
Pat4 = Dual Left Lag
Pat0 = Permissive-Only Left Turns

bandwidth shows a slight improvement in the protected-permissive pattern for the outbound direction, whereas in the inbound (critical) direction the bandwidth remains unchanged.

In ZMODEL the effect of allowing permissive left turns was evident by a significant reduction in both cycle length and number of phases. As a result the outbound (inbound) bandwidth increased by 14 percent (21 percent) and the bandwidth efficiency increased by 18.75 percent.

Green splits for the arterial at the critical intersection (1) are virtually identical in PASSER-II runs; however, the same split went up from 58 to 62 percent in ZMODEL as a result of allowing permissive left turns. This increase is due to the ZMODEL structure in which a minimum green time will be assigned to every secondary movement on the basis of capacity and minimum green time constraints. The balance of the green is then allocated to the artery through movements. Hence, in protected-only treatments every left-turn movement must operate in at least one protected phase, whereas in protected-permissive treatments left-turn movements may operate in permissive-only phasing. Consequently, the required green times for the secondary movements are longer in protected-only than in protected-permissive phasing.

To assess system performance on the basis of traditional traffic network performance measures, settings derived in PASSER-II and ZMODEL were entered into the TRANSYT-7F model. The results are shown in Table 4.

The similarity in TRANSYT-7F results for protected-only phasing from both ZMODEL and PASSER-II settings was primarily due to the identical optimal cycle lengths derived from both methods. With protected-permissive treatment ZMODEL settings yielded improved network performance measures compared with those yielded by PASSER-II settings. For instance, the individual average vehicle delay is reduced by more than 56 percent and the per-

TABLE 4 TRANSYT-7F Results for Test Arterial

		Signal Settings Derived from				Units
		PASSER-II		ZMODEL		
LT Treatment:		P	PP	P	PP	
Travel Time	A[a]	105.25	98.61	106.90	83.11	veh-hr/hr
	N[b]	162.42	149.46	182.14	145.80	
Average Delay	A	12.99	12.46	14.11	5.39	sec/veh
	N	23.82	21.72	29.60	20.87	
Uniform Stops	A	3707	4605	4268	3180	veh/hr
	N	8163	8918	8946	7500	
Fuel Consum.	A	150.55	157.31	156.29	134.18	gal/hr
	N	226.43	227.48	247.07	213.10	
Performance Index	A	47.51	57.83	53.70	35.13	PI
	N	122.48	127.29	143.65	113.29	
Speed	A	28.56	30.33	27.44	33.51	mph
	N	27.69	29.83	25.65	33.11	

A[a] Arterial through traffic only
N[b] Network

formance index is reduced by 39 percent. The latter is expressed as a linear combination of delays and stops. Comparison of overall network performance by PASSER-II and ZMODEL was less promising with the latter model. This observation is not surprising; in fact, it is rooted in the optimization logic for ZMODEL as explained in the following.

In PASSER-II the phase lengths are calculated on the basis of equalizing the degree of saturation for critical conflicting movements, subject to minimum phase length constraints. Consequently, the cross-street movements are considered in the PI calculation and in the determination of splits at the same priority level as arterial traffic.

In ZMODEL cross-street traffic is designed to operate at or below a prespecified V/C ratio ($X = 0.9$, in the test run), thus permitting the arterial through movements to operate at the lowest possible X ratio. Because the strong nonlinearity in the delay-V/C ratio relationship is nonlinear at a high ratio, overall system delay may actually degrade in some cases.

The comparison of TRANSYT-7F results for protected-only treatment versus those for protected-permissive treatment indicates that the effect of permissive treatment is much more pronounced in ZMODEL than in PASSER-II. The PI with ZMODEL signal settings dropped by 21 percent when permissive left turns were allowed. In PASSER-II the PI increased by 3.9 percent. Finally, the average vehicle delay decreased by almost 30 percent with ZMODEL compared with about 9 percent in PASSER-II settings.

Sensitivity Analysis

Overall model evaluation is performed by means of a sensitivity analysis of critical parameters such as

- Left-turn volume (low = base, medium = 1.4 × base, high = 1.8 × base),
- Design degree of saturation ($X = 0.7$, 0.8, 0.9, and 1.0),
- Permissive left-turn saturation flow model ($1{,}700 - vo_{pi}$ and $1{,}400 - vo_{pi}$), and
- Number of discharged vehicles during the clearance interval (two and three vehicles/cycle).

A detailed sensitivity analysis of 48 ZMODEL evaluation runs was executed. However, only 12 PASSER-II runs were performed because the X ratios in PASSER-II are calculated implicitly. All evaluations were performed through repeated macroscopic simulations of the TRANSYT-7F signal network model.

It was found that PASSER-II settings were insensitive to the permissive left-turn capacity model. Signal performance indicators such as bandwidth efficiency, arterial split, V/C ratio for the critical artery movement, and performance index were virtually unchanged with the two left-turn model forms. In contrast, ZMODEL settings were much more dependent on the left-turn model; for example, an increase in the permissive left-turn saturation flow rate of 300 vphg per lane resulted in the following increases: bandwidth efficiency by 5 percent, artery split by 1 percent, secondary movement capacity by 10.3 percent, and network performance index by 12 percent.

In terms of design V/C ratios, PASSER-II settings could not be evaluated since the critical movement analysis determines a unique set of V/C ratios that are equalized for all primary and secondary movements on the artery. In ZMODEL, however, key performance indicators were found to be quite sensitive to the design X ratios. An increase in X from 0.7 to 1.0 resulted in improved bandwidth efficiency by 18.7 percent, increased artery splits by 11.4 percent, reduced capacity for secondary flows by 18.4 percent, and reduced artery performance index by 10.4 percent.

Improved efficiency for artery flows is invariably achieved at the expense of secondary movement performance, as expressed in terms of delays and queues. Not surprisingly, bandwidth efficiency

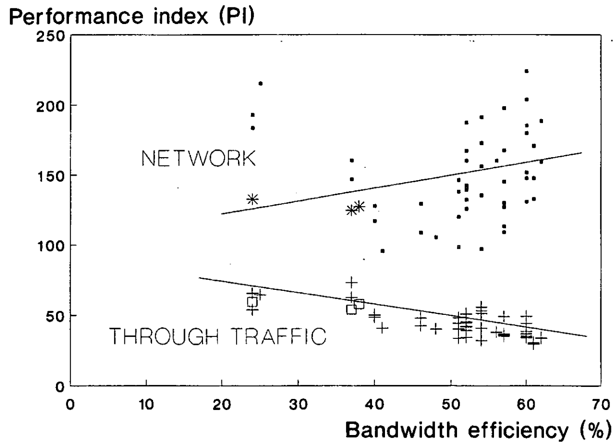

FIGURE 3 Bandwidth efficiency versus PI.

and the artery performance index are inversely correlated. In ZMODEL this correlation relationship exhibited a more oscillatory pattern when compared with that in PASSER-II.

The trade-off between PI and *B* is given in Figure 3 for selected ZMODEL runs. Trend lines indicate that the artery PI decreases as the bandwidth efficiency increases. Conversely, the system PI increases with an increase in efficiency. PASSER-II settings exhibited much less sensitivity in the relationship between PI and efficiency, as shown in Figure 3. Compared with ZMODEL the overall artery PI with PASSER-II settings is higher, whereas the system PI is lower.

Computational Performance

In the course of performing the evaluation runs in ZMODEL, MPSX/370 generated optimal solutions 77 percent of the time (35 runs) and the best feasible solutions 23 percent of the time (13 runs). The latter occurred because of constraints on central processing unit (CPU) time or workspace.

A statistical summary of the MPSX-generated solutions, optimal and the best feasible solutions, and CPU time is provided in Table 5.

SUMMARY

The focus of the study described here was to develop a tool for simultaneously optimizing all traffic and signal control parameters along an arterial. The model is formulated as a large mixed integer linear programming problem and is interfaced with the IBM MPSX/370 (*19*) as the optimization tool. Because of workspace and CPU limitations and because of the preponderance of binary variables (17 per intersection), the algorithm may not always converge to the optimal solution. In this case the best feasible solution achieved to date is reported. In the course of our experiments MPSX generated optimal solutions 77 percent of the time.

The principal contributions of ZMODEL can be summarized as follows:

1. The model specifies an optimal left-turn treatment for each intersection approach in the artery and cross street so as to maximize the progression bandwidth. The user has control over which treatments to consider. If this option is not exercised, the model recommends one of the following left-turn treatments: strictly permissive, strictly protected, or a combination of both. In the latter the influence of the opposing flow phase sequence on left-turn capacity is explicitly addressed. In all cases the user may specify the maximum number of vehicles that can discharge in the clearance interval each cycle;
2. The model specifies that all secondary flows (left turns on the artery and cross-street flows) operate at or below a prespecified V/C ratio. This concept successfully emulates semiactuated control in which the nonactuated (main street) phase receives the entire slack green time (*21*);
3. The model explicitly considers the additional arterial green needed to clear secondary queues as a decision variable that enters into the offset selection process. This is similar in concept to the work of Tsay and Lin (*22*); and

TABLE 5 Summary Statistics of MPSX-Generated Solutions

	Objective Cont.[a]	Optimal/ Feasible	Diff.[b] (%)	CPU Time (sec)
MPSX Optimal Integer Solutions: (35 runs)				
MAX	3.20	3.14	14.04	1377.16
MIN	2.42	2.17	0.00	9.42
AVG	2.85	2.68	5.91	490.03
STD	0.25	0.24	5.12	341.33
MPSX Best Integer Feasible Solutions: (13 runs)				
MAX	3.20	3.15	7.65	2100.63
MIN	2.58	2.38	0.62	341.90
AVG	3.00	2.89	3.83	1170.37
STD	0.18	0.22	2.45	716.47

[a] Optimal continuous solution of objective function - weighted B in both directions
[b] (Optimal continuous solution - Optimal/Best Integer solution) x 100 / (Optimal continuous solution)

4. The model reallocates the main street green at noncritical intersections in proportion to the artery directional flows.

CONCLUSIONS

The following conclusions can be drawn from this work:

1. When left-turn movements on arterials operate in a combination of protected and permissive phases as opposed to protected-only phasing, a reduction in the average through traffic delay along the artery is consistently observed, regardless of the model considered.
2. Signal settings derived from PASSER-II were only slightly sensitive to the permissive left-turn capacity model. By contrast ZMODEL settings were much more dependent on the left-turn model.
3. Key performance indicators of ZMODEL were found to be sensitive to the design X ratios. An increase in X results in improved bandwidth efficiency, increased artery splits, reduced capacity for secondary flows, and reduced artery performance index. In PASSER-II, however, the V/C ratios are fixed and calculated implicitly by the critical movement analysis method.
4. There appears to be a negative correlation between bandwidth efficiency and the artery performance index. However, the system performance index appears to be positively correlated with bandwidth efficiency because of the lower priority given to secondary movements, at least for the X ratios considered in the study.

GLOSSARY

The following symbols are used throughout this paper

Indexes

i = intersection (signal) index, $i = 1, \ldots, n$;
j = phase number index, $j = 1, 2, 3$, and 4 for the artery, $j = 5, 6, 7$, and 8 for the cross street;
j' = consecutive phase number index, $j' = 1, 2, 3$, and 4 for the artery, $j' = 5, 6, 7$, and 8 for the cross street; and
p = approach number index (direction), 1 = outbound, 2 = inbound, 3 (4) = 90 degrees clockwise from direction 1 (2).

Input Constants

C_1, C_2 = lower and upper bounds on cycle length (1/sec);
d_{pii} = distance between intersections i and $i + 1$ in the p direction (ft);
e_{pii}, f_{pii} = lower and upper speed limits, respectively, in the p direction at segment between intersections i and $i + 1$ (ft/sec);
$1/g_{pii}, 1/h_{pii}$ = upper and lower limits, respectively, on the allowable change in reciprocal speed in the p direction at segment between intersections i and $i + 1$ (1/ft/sec);
l = lost time per movement (sec);
LT_{pi} = left-turning vehicles approaching intersection i in the p direction from upstream intersection [vehicles per hr (vph)];
$MidV_{pi}$ = midblock volume approaching intersection i from p direction (vph);
RT_{pi} = right-turning vehicles approaching intersection i in the p direction from upstream intersection (vph);
SNC (SNH) = number of vehicles discharged during the clearance interval each cycle (per hr);
sLT_{pi}(prot) = protected left-turn saturation flow for approach p at intersection i (vph);
sLT_{pi}(perm) = permissive left-turn saturation flow for approach p at intersection i (vph);
sTH_{pi} = through saturation flow for approach p at intersection i (vph);
vo_{pi} = opposing flow rate of approach p at intersection i (vph);
vLT_{pi} = left-turn demand on approach p at intersection i (vph);
vTH_{pi} = through demand on approach p at intersection i (vph);
XLT_{pi} = threshold degree of saturation for left-turn movements for approach p at intersection i; and
XTH_{pi} = threshold degree of saturation for through movements for approach p at intersection i.

Binary Variables

aL_{jpi} = binary variable equal to 1 if left-turn movement on approach p at intersection i is serviced in phase j, 0 otherwise ($p = 1, 2, 3, 4$);
aT_{jpi} = binary variable equal to 1 if through movement on approach p at intersection i is serviced in phase j, 0 otherwise ($p = 3, 4$);
I_{ji} = binary variable equal to 1 when phase j is optimal at intersection i, 0 otherwise; and
$W_{jj'i}$ = binary variable equal to 1 when phase j' precedes phase j at intersection i, 0 otherwise.

Continuous Variables

cLT_{pi}(perm) = permissive left-turn capacity for approach p at intersection i (vph);
cLT_{pi}(prot) = protected left-turn capacity for approach p at intersection i (vph);
cLT_{pi}(tot) = total left-turn capacity for approach p at intersection i (vph); and
Yo_{pi} = volume to saturation flow ratio of the opposing through movement for approach p at intersection i; $Yo_{pi} = vo_{pi}/sTH_{pi}$.

Decision Variables

B_p = bandwidth in direction p, $p = 1$ outbound, $p = 2$ inbound (cycles);
C = cycle length (signal period) (sec);
g_{pi} = effective green time for approach p at intersection i (cycles);
go_{pi} = effective green on the opposing approach of p at intersection i (cycles);
gq_{pi} = portion of green phase blocked to left-turning vehicles by the clearing of an opposing queue of vehicles for approach p at intersection i (cycles);
gu_{pi} = portion of green not blocked by the clearing of an opposing queue of vehicles (cycles), where $gu_{pi} = g_{pi} - gq_{pi}$, or permissive green time for left-turn movement at intersection i of each approach p (cycles);
K = target ratio of total inbound to outbound through volume for the artery (bandwidth);
K_i = inbound to outbound through volume ratio at intersection i;

Qs_{pi} = secondary flow entering intersection i on approach p (vph);
r_{pi} = effective red time for approach p at intersection i (cycles);
SL_{pi} = slack green time on approach p at intersection i (cycles);
t_{pii} = travel time between intersections i and $i + 1$ in the p direction (cycles);
U_{ji} = starting time of phase j at intersection i (cycles);
U_{pi} = starting time of approach p effective red for intersection i (cycles);
V_{ji} = ending time of phase j at intersection i (cycles);
V_{pi} = ending time of approach p effective red for intersection i (cycles);
w_{pi} = time from right (left) side of red at intersection i to left (right) edge of outbound, $p = 1$ (inbound, $p = 2$), green band (cycles);
Z = signal frequency, $Z = 1/C$, (cycles/sec);
τ_{pi} = queue clearance time for approach p at intersection i, an advance of the bandwidth upon leaving intersection i (cycles);
λ_{ji} = length of phase j at intersection i (cycles);
$\lambda_{ji}(\min)$ = minimum length of phase j at intersection i (cycles); and
Δ_i = time from center of inbound effective red r_{2i} to nearest center of outbound effective red r_{1i}, positive if center of r_{1i} is to right of center of r_{2i} (cycles), unrestricted in sign variable.

General Integer Variables

m_i = real integer variable, $m = 1, 2, \ldots$, and
n = real integer variable, $i = 1, 2, \ldots, n$.

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