Estimation of Green Times and Cycle Time for Vehicle-Actuated Signals

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An analytical method for estimating average green times and cycle time at vehicle-actuated signals is presented. The examination is limited to the operation of a basic actuated controller that uses passage detectors and a fixed gap time setting. Both fully actuated and semiactuated control cases are discussed. The practical cycle and green time method for computing fixed-time signal settings is also outlined. A discussion of the arrival headway distributions is presented since the estimation of arrival headways is fundamental to the modeling of actuated signal timings. The method given provides essential information for predicting the performance characteristics (capacity, degree of saturation, delay, queue length, and stop rate) of intersections controlled by actuated signals and for investigating the optimization of actuated controller settings. Further work is needed to validate and calibrate the formulas given using real-life and simulation data.

This paper presents an analytical method for estimating average green times and cycle time at vehicle-actuated signals. This information is essential for the prediction of the performance characteristics (capacity, degree of saturation, delay, queue length, and stop rates) of intersections controlled by actuated signals. The method can be seen as an extension of the current Australian, U.S. *Highway Capacity Manual* (HCM), United Kingdom, and similar methods for the analysis of fixed-time (pretimed) signals (*1–4*). The practical cycle and green time method for computing fixed-time signal settings is also outlined (*1,2*).

This paper is limited to the operation of a basic actuated controller that uses passage detectors and a fixed gap time setting. Both fully actuated and semiactuated control cases are discussed. The author is preparing a more comprehensive report that discusses actuated signal controllers that use various gap reduction methods with passage and presence detectors (*5*).

The literature on actuated signal operations is limited compared with that on fixed-time signals. However, there are still many useful papers on actuated signals, mostly based on the use of simulation methods, and a few of them describe analytical techniques. A detailed literature review is outside the scope of this paper. The descriptions of actuated controller operations provided by Staunton (*6*) and the analytical methods provided by Lin (*7,8*) were used in the development of the work reported in this paper.

The method presented for the analysis of actuated signal operations can be implemented manually. However, implementation through computer software such as SIDRA (2) is useful for dealing with complex intersection geometry and phasing arrangements and for obtaining solutions that require iterations.

The arrival headway distributions are discussed first, since the estimation of arrival headways is fundamental to the modeling of actuated signal timings.

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ARRIVAL HEADWAY DISTRIBUTIONS

Accuracy in predicting small arrival headways (up to about 12 sec), rather than the whole range of headways, is particularly important in modeling actuated signal operations. A class of arrival headway distributions referred to as M1 (negative exponential), M2 (shifted negative exponential), and M3 (bunched exponential) is considered. The M3 model was proposed by Cowan (*9*) and used extensively by Troutbeck (*10–12*) for estimating capacity and performance of traffic circles and other unsignalized intersections. A special case of the M3 model has been used by Tanner (*13,14*) for unsignalized intersection analysis. The M1 and M2 models can be derived as special cases of the M3 model through simplifying assumptions about the bunching characteristics of the arrival stream.

The M1 and M2 models are more commonly used in the traffic analysis literature as models of random arrivals. However, the M3 model is found to be more representative of real-life arrival patterns. The more commonly used shifted negative exponential (M2) model is found to give poor predictions for the range of small headways, which is of particular interest when modeling actuated signal operations and gap acceptance at intersections.

This paper uses the bunched exponential (M3) model for deriving various formulas for the analysis of actuated signal operations. It is recommended that this model be used consistently for all urban traffic analysis (gap acceptance modeling at signalized and unsignalized traffic facilities, modeling of traffic performance, and so on). For a detailed discussion of the bunched exponential model of arrival headways, see a recent paper by Akçelik and Chung (*15*).

The cumulative distribution function, $F(t)$, for the bunched exponential distribution of arrival headways, representing the probability of a headway less than t sec, is

$$F(t) = \begin{cases} 1 - \phi \exp[-\lambda(t - \Delta)] & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \tag{1}$$

where

Δ = minimum headway in arrival stream (sec),
ϕ = proportion of free (unbunched) vehicles, and
λ = model parameter calculated from $\lambda = \phi q_t/(1 - \Delta q_t)$, where q_t is the total arrival flow (vehicles/sec).

The proportion of bunched vehicles in the arrival stream is $(1 - \phi)$. The free (unbunched) vehicles are those with headways greater than the minimum headway (Δ), and the proportion of free vehicles (ϕ) represents the unbunched vehicles with randomly distributed headways. All bunched vehicles are assumed to have the same intrabunch headway (Δ).

The M1 and M2 models can be derived from the M3 model by setting the bunching parameters as follows:

Negative exponential (M1) model:

$\Delta = 0$

$$\phi = 1 \text{ (therefore } \lambda = q_t) \tag{2a}$$

Shifted negative exponential (M2) model:

$$\phi = 1 \tag{2b}$$

A known value of ϕ can be specified for use in the M3 model. For general application purposes, ϕ can be estimated as a function of the arrival flow rate. The following relationship has been derived by the author by generalizing the bunching implied by the negative exponential model:

$$\phi = \exp(-b\Delta q) \tag{3}$$

where b is a bunching factor and q is the arrival flow rate in vehicles per second.

The M3 model with estimates of ϕ obtained from Equation 3 will be referred to as the M3A model and will be used in this paper with the following parameter values:

Single-lane case:

$\Delta = 2.0$

$$b = 1.5 \tag{4a}$$

Multilane case (number of lanes = 2):

$\Delta = 1.0$

$$b = 1.0 \tag{4b}$$

Multilane case (number of lanes > 2):

$\Delta = 0.5$

$$b = 1.0 \tag{4c}$$

The bunching factor of $b = 1.5$ for the single-lane case was derived as an approximation to the values predicted by the following linear model used by Tanner (*13,14*):

$$\phi = 1 - \Delta q \qquad \text{for } q < 1/\Delta \tag{5}$$

The M3 model with estimates of ϕ obtained from Equation 5 will be referred to as the M3T model.

The bunching factors for multilane cases are based on the treatment of arrival flows in all lanes as a single stream. The values given in Equations 4b and 4c were derived through comparison with lane-by-lane treatment of multilane situations.

Research carried out after writing this paper to calibrate the M3A model using real-life and simulation data (*15*) indicated lower levels of bunching than those predicted by Equations 4a through 4c.

Figure 1 shows cumulative distribution functions for the arrival headway models M1, M2, M3A, and M3T for a single-lane traffic stream with arrival flow rate of 900 veh/hr. There are significant differences in the predictions of arrival headways by different models, especially for small arrival headways (up to about 12 sec). Generally, the shifted negative exponential model does not appear to be a satisfactory model. The amount of bunching as represented by parameter ϕ in Model M3 has a major effect on the prediction of arrival headways.

The following formulas provide two fundamental parameters for actuated signals (used for estimating the extension time before a gap change after queue clearance; see Equations 13 and 14):

$$n_g = -1 + (1/\phi)\exp[\lambda(e_o - \Delta)] \tag{6a}$$

$$h_g = (1/n_g)\{-(e_o + 1/\lambda) + (\Delta/\phi + 1/\lambda)\exp[\lambda(e_o - \Delta)]\} \tag{6b}$$

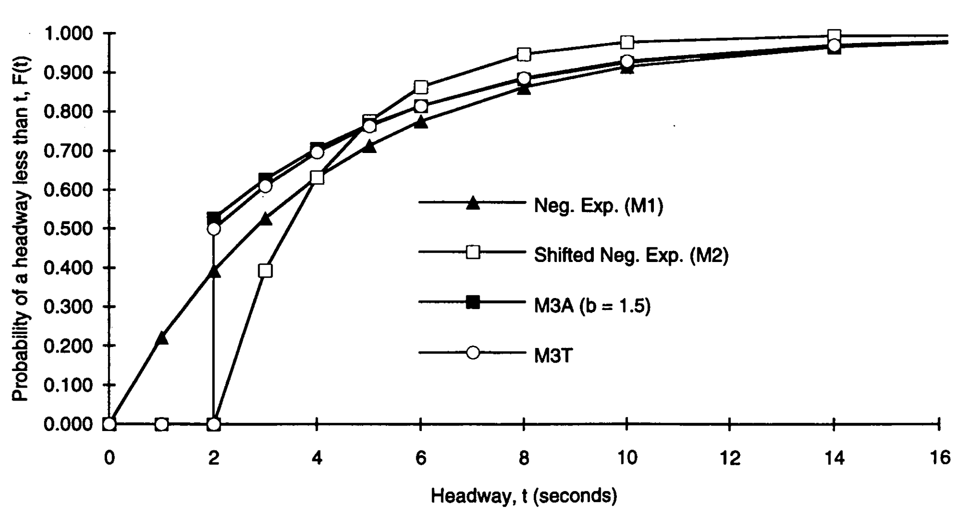

FIGURE 1 Cumulative headway probabilities predicted by Models M1 (Δ = 0), M2 (Δ = 2), M3A (Δ = 2, b = 1.5), and M3T (Δ = 2): single-lane case with arrival flow rate = 900 veh/hr.

where

n_g = average number of arrivals before a gap change after queue clearance (average number of consecutive arrivals with headways $t < e_o$ followed by an arrival with a headway $t \geq e_o$),
h_g = average headway before a gap change after queue clearance (average headway corresponding to n_g), and
e_o = gap time setting.

Equations 6a and 6b can be used for the M1 and M2 models by choosing appropriate parameters as given in Equations 2a and 2b.

Another useful formula is the probability of no arrivals during a given period of T sec (equivalent to the probability of a headway longer than T):

$$p_o = \phi \exp[-\lambda(T - \Delta)] \tag{7}$$

ACTUATED SIGNAL TIMINGS

An analytical method is presented for estimating the average green times and cycle time for a basic actuated controller that uses passage detectors and a fixed gap time setting. Both fully actuated and semiactuated control cases are considered. Most studies of actuated signals reported in the literature assume this type of actuated controller operation. Akçelik further discusses more complex types of actuated signal controllers that use various gap-reduction techniques with passage and presence detectors (*5*).

At vehicle-actuated signals, the green time, and hence the cycle time, is determined according to the vehicle demands registered by detectors. This may be on the basis of phase (stage) control or group (movement) control. Phase sequence may be fixed or variable. A phase can be skipped when there is no demand for it. After a minimum green period, the running phase waits in the rest position when no conflicting demand is present (an automatic call for another phase has the same effect as a conflicting demand).

For a given phasing system, efficient operation of vehicle-actuated signals depends on the values of various controller settings. The three basic controller settings that determine the length of the green period are minimum green, gap time (the terms "vehicle interval," "vehicle extension," and "unit extension" are also used), and maximum extension (or maximum green) settings. Modern controllers have additional settings that differ according to the type of controller: minimum gap time, time to reduce, waste time, headway time, and so on. The location, number and other characteristics of detectors affect the choice of vehicle-actuated settings also.

Figure 2 shows the operation of a basic actuated controller that uses passage detectors and a fixed gap time setting for terminating the green time.

To facilitate easier analytical formulation:

1. The early cut-off and late start intervals are ignored;
2. Actuated signal operation is expressed in terms of movements rather than phases (although the discussion is valid for both group controllers and phase controllers); and
3. The formulas for estimating the green and cycle times are expressed in terms of effective green times rather than displayed green times.

Refer to Akçelik (*1*,pp. 1–5) for introductory discussions related to Items 2 and 3. Appropriate conversion of some controller settings to effective values is required before the formulas given in this paper can be used. For example, the displayed maximum green time

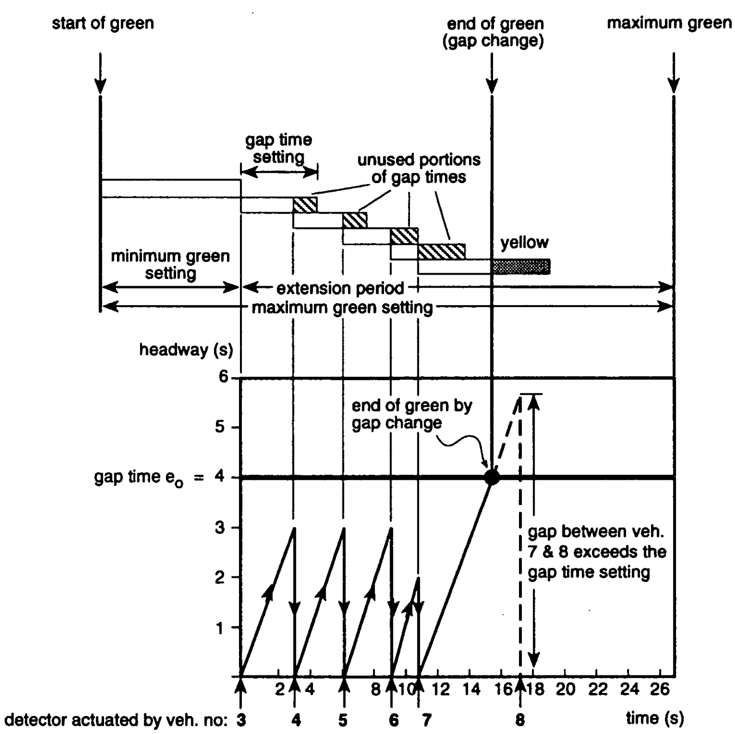

FIGURE 2 Extension sequence in basic vehicle-actuated control using passage detectors and a fixed gap time setting (*6*).

setting (G_{max}) needs to be converted to an effective maximum green time value (g_{max}) using $g_{max} = G_{max} + I - l$ where I is the intergreen time (yellow plus all-red) and l is the lost time. In most cases, $I \cong l$, and therefore $g \cong G$ can be assumed. However, all incremental settings such as the gap time, waste time, and vehicle increment settings can be used as controller settings without any adjustment for effective green time calculations.

Average Green Time for Fully Actuated Signals

The green time (g) allocated to a movement (group) under actuated control comprises a minimum green time (g_{min}) and a green extension time (g_e):

$$g = g_{min} + g_e \tag{8}$$

subject to

$$g \leq g_{max}$$

or

$$g_e < g_{emax}$$

where g_{max} is the maximum green setting and g_{emax} is the maximum extension time setting.

The minimum green time consists of a fixed minimum green time and a variable initial green time. The fixed minimum green time is determined as a safe minimum green that should be long enough for the first vehicle to start moving and enter the intersection (typically 4 to 6 sec). The variable initial green time is an additional variable period determined by the number of vehicle actuations (after the first vehicle) during the red period. Vehicle increment and maximum initial green settings are used in relation to this. The sum of the fixed minimum green time and maximum initial green time must be long enough to clear the vehicles waiting in the critical lane between the detection point and the stop line. For this purpose, the critical lane is defined as the lane with the highest flow rate (5).

The value of the maximum green time (or maximum extension time) setting to be used in practice must be chosen with due consideration to traffic flows at different times (morning peak, evening peak, day off-peak, night off-peak, weekend, and shopping periods) and to the peaking characteristics of traffic. The choice of the design period as a basis of green time calculations is important in this respect. The objective should be to obtain green times that are not too restrictive for maximum possible flow rates (e.g., during a peak 15-min period). On the other hand, long maximum settings coupled with a bad choice of the gap time and other controller settings can lead to unduly long green and cycle times, resulting in inefficient operation during a larger proportion of the time.

Traditionally, the green time calculation methods for fixed-time signals are used for determining suitable maximum green settings. However, the method given in this paper for the analysis of actuated operations could be used to determine appropriate values of the maximum green settings directly without resorting to fixed-time signal analysis.

The method for estimating the green extension time (g_e) for the basic actuated controller operation is given in the following. It assumes that a conflicting demand is registered before the termination of the minimum green period, and therefore, the extension period starts immediately after the expiration of the minimum green period (see Figure 2).

A basic actuated controller uses a fixed value of the gap time (vehicle interval or unit extension) setting (e_o) for terminating the green time (typically e_o = 2.5 to 4 sec). As seen in Figure 2, detection of each additional vehicle extends the green period by an amount equal to the gap time (e_o). The controller starts timing a new gap time at each vehicle actuation. The green period terminates when

1. The time between successive vehicle actuations exceeds the gap time setting, $h > e_o$ (gap change), or
2. The total green extension time after the expiration of minimum green time equals the maximum extension setting, $g - g_{min} = g_{emax}$ or $g = g_{max}$ (maximum change).

During a gap change (see Figure 2), the green period terminates after the gap time expires. In some controllers, a passage time setting (e_p) is used instead of the gap time for the last vehicle to be able to travel the distance between the detector and the stop line before the start of yellow signal. Thus, the terminating time (e_t) at gap change is either the gap time ($e_t = e_o$) or passage time ($e_t = e_p$). The gap timing logic operates from the start of the green period to enable a green termination at the end of the minimum green time.

During the saturated portion of the green period (i.e., during the queue clearance period), the headways are assumed to be equal $h = h_s = 1/s$, where h_s is the saturation headway and s is the combined saturation flow rate (in vehicles per second) for all lanes, allowing for any lane underuse. The standard methods for the calculation of saturation flow can be used (*1–4*). For a single lane, typically s = 1,800 veh/hr = 0.5 veh/sec, therefore h_s = 2.0 sec. For multilane cases, $s = s_c/p_c$ can be used, where p_c is the proportion of total flow in the critical lane and s_c is the critical lane saturation flow. This is a simplistic formula that assumes the same saturation flow (s_c) for all lanes but allows for unequal lane flows. For equal lane flows, $p_c = 1/n_l$, where n_l is the number of lanes, thus $s = n_l s_c$.

A gap change during the saturated portion of green period (after the expiration of the minimum green period) is theoretically possible, at least for a single-lane movement. This would occur if $h_s > e_o$ (for example, for a turning movement with s = 1,200 veh/hr, h_s = 3.0 sec, and e_o = 2.5 sec). Gap change during the saturated portion of the green period indicates an inefficient operation (insufficient green to clear the queue). Therefore, e_o should be set to ensure that a gap change does not occur during the saturated portion of the green period, particularly for single-lane movements. The analyses presented in the rest of this paper assume that the gap time is set to ensure that a gap change does not occur during queue clearance.

A gap change during the unsaturated portion of green—that is, after the queue clearance period—corresponds to conditions when the vehicles in the arrival stream pass through the intersection without queueing. The arrival headway distributions discussed in the previous section are applicable in this case.

The green time in the case of a gap change after queue clearance is

$$g = g_s + e_g \tag{9}$$

subject to

$$g_{min} \leq g \leq g_{max}$$

where g_s is the saturated portion of green period (queue clearance time) and e_g is the extension time by gap change after queue clearance.

From Equation 8, the green extension time can be calculated from

$$g_e = g - g_{\min} = g_s + e_g - g_{\min} \tag{10}$$

The saturated portion of green period can be estimated from

$$g_s = f_q(n_q/s + yr)/(1 - y) \tag{11}$$

where

- f_q = calibration factor to allow for variations in queue clearance time,
- n_q = residual queue from previous green period in case of two green periods per cycle ($n_q = 0$ for the more common case of a single green period per cycle),
- s = saturation flow rate (veh/sec),
- r = red time (sec), and
- y = flow ratio ($y = q/s$ where q = arrival flow rate).

In multilane cases, the saturated portion of green should represent the time to clear the queue in the critical lane (i.e., the longest queue for any lane) considering all lanes of all approaches in the signal group (or phase). Ideally, g_s should be calculated for each lane in each approach using parameters relevant to each lane (e.g., flow, saturation flow, and effective red time for the lane). This method is used by SIDRA (*2*). Alternatively, g_s can be calculated for each lane group (or approach) by appropriate adjustments to flow, saturation flow, and effective red times to ensure that g_s for the lane group approximates the critical lane value.

Equation 11 allows for two green periods per cycle. For the more common case of a single green period per cycle, n_q is 0 (this should not be confused with overflow queues due to oversaturation in the cycle), and the green time can be expressed in terms of the cycle time (c) rather than the red time (r):

$$g = f_q yc + (1 - y)e_g \tag{12}$$

subject to

$$g_{\min} \leq g \leq g_{\max}$$

This is an approximate equation (exact if $f_q = 1$), and the use of Equations 9 and 11 is preferred.

The average extension time by gap change can be estimated from

$$e_g = n_g h_g + e_t \tag{13}$$

where

- n_g = average number of arrivals before a gap change after queue clearance (due to a headway $h > e_o$), given by Equation 6a;
- h_g = average headway before a gap change after queue clearance (due to a headway $h > e_o$), given by Equation 6b; and
- e_t = terminating time at gap change (equals the gap time setting, $e_t = e_o$, or the passage time setting, $e_t = e_p$).

For the case when $e_t = e_o$, Equation 13 is equivalent to

$$e_g = \frac{\exp[\lambda(e_o - \Delta)]}{\phi q} - \frac{1}{\lambda} \tag{14}$$

See Equations 1 through 5 for parameters in this formula. For negative exponential distribution of headways, set $\Delta = 0$, $\phi = 1.0$, and $\lambda = q_t$ (total arrival flow). For shifted negative exponential distribution, set $\phi = 1.0$. The resulting formula is then similar to that given by Lin (*7,8*) except that Lin recommends $\Delta = 1$ sec for the single-lane case and $\Delta = 0$ sec for the multilane case (therefore equivalent to the negative exponential model).

When the gap timer operates from the start of the green period (including the minimum green period) and non-stop-line detectors are used, it is necessary to reduce n_g by the number of vehicles that arrive early during the green period, cross over the detector, and join the back of the queue downstream of the detection point. These vehicles are counted as part of the vehicles departing during queue clearance (sg_s vehicles) as well as part of n_g.

Figure 3 shows an example of average extension time by gap change after queue clearance (e_g) as a function of the total arrival flow (q) for a single-lane case with $e_o = 3$ sec (from Equation 14). The extension times are predicted using the arrival headway models M1 (negative exponential), M2 (shifted negative exponential), and bunched exponential models M3A and M3T. It is seen that there are sizable differences in the predictions of extension times by different headway models. The amount of bunching as represented by parameter ϕ in M3 has a significant effect on the prediction of extension times.

Figure 4 shows average extension time by gap change after queue clearance (e_g) as a function of the total arrival flow (q) using the M3A model for a single-lane case ($\Delta = 2.0$ sec, $b = 1.5$) and a four-lane case ($\Delta = 0.5$ sec, $b = 1.0$) with $e_o = 3.0$ and 4.5 sec. It is seen that the difference between extension times (e_g) for gap time settings of $e_o = 3.0$ and 4.5 sec increases with increasing flows to substantial levels at very high flows.

Semiactuated Signals

Semiactuated signal operation as a simple two-phase system controlling a major-minor road intersection is considered. For the sake of notations, the minor road will be referred to as the "side street" and the major road will be called the "main road." The side street vehicles are detected and controlled as in the case of fully actuated control. On the other hand, the main road has no detections. It receives only a minimum green time after a change of phase to the main road (e.g., by a gap change or maximum change). The main road phase is terminated after a conflicting demand is registered on the side street. Therefore, this type of operation is suitable only when the side street flows are low.

The formulas given here are close to those by Lin (*8*), but the more general M3 arrival headway distribution is used, the saturated part of the green period is dealt with differently, and the assumption about how a conflicting demand is registered in deriving the durations of main road and side street green periods is slightly different.

The average green time for the main road can be estimated from

$$g_M = g_{\min M} + (\phi_s/\lambda_s)\exp[-\lambda_s(e_{tS} - \Delta_s + l_M + g_{\min M})] \tag{15}$$

where

- $\phi_s, \lambda_s, \Delta_s$ = headway distribution parameters calculated from Equations 1 through 5 considering total flow in all lanes of all side street movements;

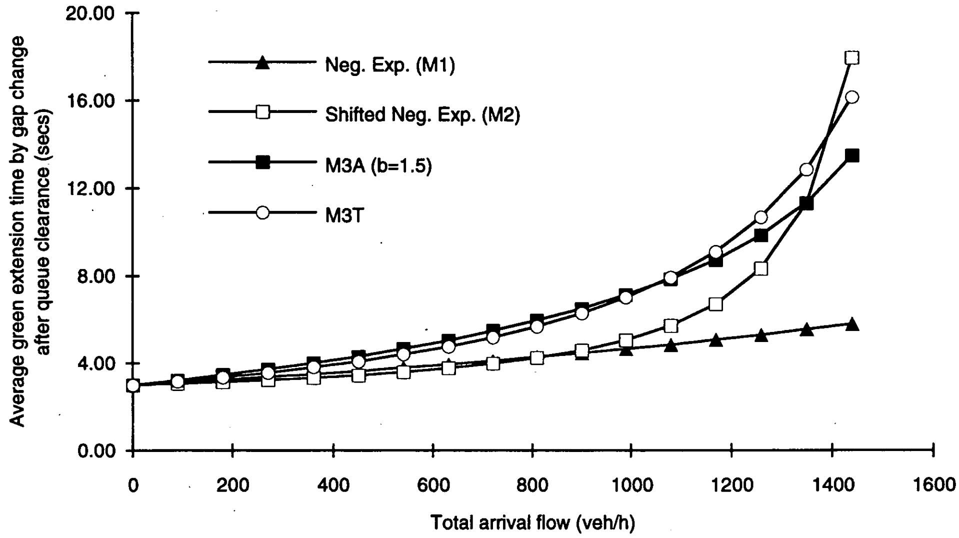

FIGURE 3 Average extension time by gap change after queue clearance (e_g) as a function of the total arrival flow (q) predicted by arrival headway Models M1 ($\Delta = 0$), M2 ($\Delta = 2$), M3A ($\Delta = 2$, $b = 1.5$), and M3T ($\Delta = 2$): single-lane case with $e_o = 3.0$.

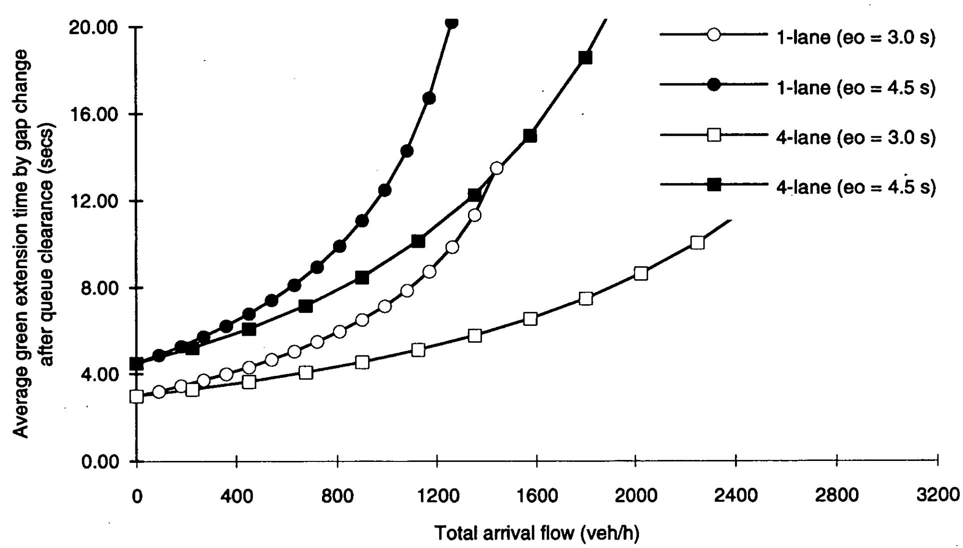

FIGURE 4 Average extension time by gap change after queue clearance (e_g) as a function of the total arrival flow (q) predicted by Model M3A for $e_o = 3.0$ and 4.5: single-lane case with $\Delta = 2$, $b = 1.5$, and multilane case with $\Delta = 0.5$, $b = 1.0$.

e_{tS} = terminating time for side street movements [this can be the gap time (e_o), passage time (e_p), or 0 in the case of maximum change];

l_M (lost time) and $g_{\min M}$ (minimum green time) = main road movements.

In this formula, ($l_M + g_{\min M}$) is used as an approximation to ($I_M + G_{\min M}$) where I_M is the intergreen and $G_{\min M}$ is the displayed minimum green time for the main road. Note that there is no maximum green time constraint for the main road.

The average green time for the side street (g_S) can be estimated from Equations 9 through 14.

The saturated portion of the side street green period (g_{sS}) can be estimated by using the red time for the side street (r_S) calculated from

$$r_s = g_M + L \tag{16}$$

where L is the total intersection lost time, which is the sum of lost times for the main road and the side street ($L = l_M + l_S$).

The cycle time is given by

$$c = g_M + g_s + L \tag{17}$$

The operation of pedestrian-actuated signalized pedestrian crossings (without any actuations for vehicle traffic) is similar to the semiactuated signal operations and can be analyzed by simply replacing the side street with the pedestrian movement. If the vehicle stream is also detected, the analysis method for fully actuated signals applies. Pedestrian green time is always $g = g_{\min P}$, but $g_{\min P}$ can be modified to allow for no pedestrian arrivals during some signal cycles.

Linked Actuated Signals

Although the method for analyzing actuated signal operations described in this paper is applicable to isolated intersections, many aspects of the method also apply to linked actuated operations (i.e., to the operation of actuated signals that are part of a coordinated system). However, several important issues should be taken into consideration; some of them will be discussed briefly.

The headway distribution models described by Equations 1 through 5 are based on the assumption of random arrivals. For platooned arrivals as they occur at linked signals, separate arrival flow rates for main platoons and between main platoons could be used. The arrival headways within platoons can be used as constant headways, but they are not necessarily the same as the minimum arrival headway (Δ).

Similarly, proportions of traffic arriving during green and red periods can be used separately. For example, in a well-coordinated system, a higher proportion of traffic will arrive during green (and in a well-defined platoon). Whereas this high flow rate is relevant to the analysis of extension times, the lower arrival flow rate during the red period is relevant to the determination of minimum green time. Residual queues that can form at the end of the green period because of a particular signal offset should also be considered in determining the minimum green times and extension times.

Linked actuated systems work under a master cycle time with certain amounts of green time preallocated to some phases to guarantee signal offsets for uninterrupted progression of main platoons. Methods given in previous sections need to be extended to deal with this type of operation. One particular extension of the method is the allocation of any excess green time to nominated movements (phases). This excess green time would result from the imposition of a master cycle that may be longer than the cycle time under isolated actuated operation. The green split priority method used in the SIDRA program (*2*) is suitable for this purpose.

The semiactuated signals and pedestrian-actuated crossings provide simple cases for which all excess green time is allocated to the main road. For example, the excess green time (g_x) for a semiactuated signal site can be calculated from

$$g_x = c - (g_s + g_M + L) \tag{18}$$

where c is the master cycle time.

If g_x is positive, the main road green time can be adjusted by this amount ($g'_M = g_M + g_x$ so that $g'_M + g_s + L = c$). This would increase the queue clearance time for the side street (g_{sS}) and therefore the green time would increase. As a result, an iterative method would be required for estimating the green times. A negative g_x indicates that the master cycle time is insufficient for the operation of this site, but this could be accommodated by adjusting the side street green time down, for example, by setting a lower maximum green time.

Similarly, all excess green time at a pedestrian-actuated crossing can be allocated to the main road, but a negative excess time is not acceptable since the pedestrian movement operates at minimum green time.

Cycle Time for Actuated Signals

The green times estimated using the methods described previously can be used to calculate the cycle time. For this purpose, it is necessary to identify the critical movements, that is, the movements that require the longest green (and lost) times. The critical movement identification method described by Akçelik (*1*) for fixed-time signals and implemented in the SIDRA program (*2*) can be adapted for this purpose. The method allows for complex cases of movement overlaps and accommodates the movements whose green times are set to minimum or maximum values.

For application of the critical movement identification method to the case of actuated signals, average green time estimates can be used as required green times. The method compares the sums of required green and lost times for all combinations of movements and determines the critical movements as the set of movements that require the greatest sum of required green and lost times. This total time is equivalent to the cycle time (except in the case of linked actuated signals):

$$c = \Sigma(g_i + l_i) \tag{19}$$

where g_i and l_i are the green time and lost time for ith critical movement.

The formulas given in previous sections for determining the queue clearance time indicate that the required green time for a movement depends on the red time (or the cycle time); this effect is stronger with controllers using gap-reduction methods. Therefore, the green and red times for conflicting movements become interdependent, which requires iterative computations. In fact, even the critical movements may change as cycle time changes. Furthermore, saturation flows may change with green time and cycle time because of such factors as opposed turns, lane blockages and short lanes. This situation also necessitates the use of an iterative method. However, it is no different from the analysis of fixed-time signal operations, and such a method is already implemented in SIDRA (*2*).

The method to estimate the average cycle time at actuated signals can be enhanced by using adjusted minimum green times for all movements to allow for the possibility of no vehicle arrivals during the signal cycle (phase-skipping under low flow conditions), as with pedestrian movements.

For single green periods for all movements (i.e., no residual queues), the following formula can be derived from Equations 12 and 19 for estimating the average actuated signal cycle time as a more direct but limited version of Equation 19:

$$c = \frac{L + G_m + E}{1 - Y'}$$

subject to

$$Y' < 1.0 \tag{20}$$

where

L = intersection lost time (sum of all critical movement lost times)

$= \Sigma l_i$

G_m = sum of minimum and maximum green times (g_{imin}, g_{imax}) for critical movements whose green times are set to $g_i = g_{imin}$ or $g_i = g_{imax}$ (therefore not included in the summation for Y' or E)

$$= \Sigma(g_{imin} + g_{imax}) \quad (22)$$

E = adjusted extension time for intersection

$$= \Sigma(1 - y_i)\, e_{gi} \quad (23)$$

where $y_i = q_i/s_i$ is the flow ratio for ith critical movement (q_i = arrival flow, s_i = saturation flow) and the summation is for critical movements excluding those with green times set to g_{imin} or g_{imax}; and

Y' = adjusted flow ratio for intersection

$$= \Sigma f_{qi} y_i \quad (24)$$

where f_{qi} is a queue clearance time calibration factor as in Equations 11 and 12 and the summation is for critical movements excluding those with green times set to g_{imin} or g_{imax}.

For linked actuated signals, a predetermined cycle time is used and excess time is allocated to specified movements (Equation 18).

Fixed-Time Signal Settings

All of these parameters, except for extension time (e_g), are used for calculating cycle time and green times for fixed-time (pretimed) signals (*1–4*). For comparison, the method for fixed-time signals is also given here.

The practical cycle and green time method for computing fixed-time signal settings (*1,2*) tries to achieve specified target (practical) degrees of saturation (x_p) for critical movements. Usually, the principle of equal degrees of saturation (the same x_p value for all movements) is used. A more general method is to allow for different x_p values to be specified for different movements (e.g., 0.90 for the main road and 0.95 for the side road). This method can be expressed in a form consistent with the method given for estimating actuated signal timings:

$$g_i = u_{pi} c_p$$

subject to

$$g_{imin} \leq g_i \leq g_{imax} \quad (25)$$

$$c_p = \frac{L + G_m}{1 - U_p'}$$

subject to

$$U_p' < 1.0 \quad (26)$$

where

c_p = practical cycle time;

G_m = sum of minimum and maximum green times as in Equation 22;

U_p' = adjusted green time ratio for intersection (sum of required green time ratios for critical movements excluding those with green times set to g_{imin} or g_{imax}):

$$= \Sigma u_{pi} (27)$$

where $u_{pi} = y_i/x_{pi}$ is the green time ratio for ith critical movement (y_i = flow ratio, x_{pi} = practical degree of saturation) and the summation is for critical movements excluding those with green times set to g_{imin} or g_{imax}.

Analyses of actuated signal timings using the method described in this paper indicate that equal degrees of saturation do not necessarily result in actuated signal cases, and the choice of 0.95 as a practical degree of saturation for actuated signals (x_p = 0.95) recommended by the 1985 HCM (*3*) is not substantiated. The latter point is important in relation to the development of an appropriate delay model for actuated signals.

The cycle time formula given in the HCM is a simpler version of Equation 26, obtained by ignoring the existence of minimum and maximum green times (a major shortcoming) and assuming the use of equal degree of saturation for all movements.

Example

As a very simple example for vehicle-actuated and fixed-time cycle times, a two-phase case is considered with a single movement in each phase. This occurs at the intersection of two one-way streets with three lanes each. Equal lane utilization is assumed, and equal conditions are assumed for both approaches:

Δ = 0.5 sec

b = 1.0 (multilane case)

s = 1,500 veh/hr/lane

l = 5 sec

g_{min} = 8 sec

g_{max} = 50 sec

e_o = 3.5 sec

x_p = 0.90

Therefore,

$L = 2 \times 5 = 10$ sec

$c_{min} = 2 \times g_{min} + L = 2 \times 8 + 10 = 26$ sec

$c_{max} = 2 \times g_{max} + L = 2 \times 50 + 10 = 110$ sec

Figure 5 shows the cycle time (c) as a function of the total intersection flow (twice the approach flow) for vehicle-actuated as well as two fixed-time signal settings (practical cycle time with x_p = 0.90, and minimum delay cycle settings obtained using SIDRA). With all methods in this example, the green times for the

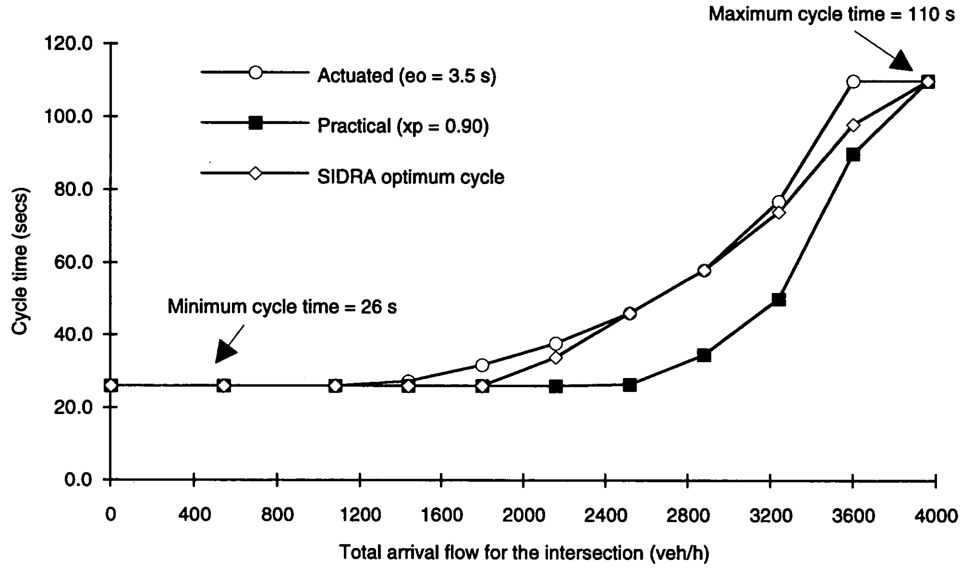

FIGURE 5 Cycle time (c) as a function of the total intersection flow for vehicle-actuated (e_o = 3.5) and fixed-time signals (practical cycle with x_p = 0.90, and SIDRA minimum-delay cycle).

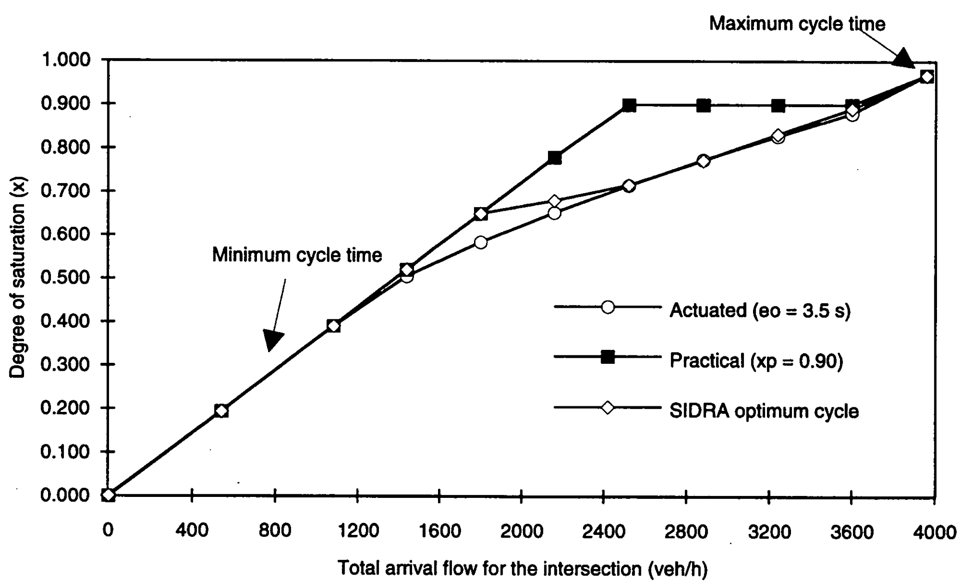

FIGURE 6 Intersection degree of saturation (x) as a function of total intersection flow for vehicle-actuated (e_o = 3.5) and fixed-time signals (practical cycle with x_p = 0.90, and SIDRA minimum-delay cycle).

two phases are equal. The minimum green times are used for low flows, and maximum green times are used for the high flow point in the graph. The intersection degree of saturation (largest x for any movement; $x = q/Q$ where capacity $Q = s_g/c$) corresponding to Figure 5 is shown in Figure 6.

CONCLUSION

An analytical method for estimating average green times and cycle time at actuated signals has been presented. The method can be seen as an extension of the current Australian, HCM, United Kingdom, and similar methods (*1–4*) for the analysis of fixed-time signals. The analysis method presented in this paper can be implemented manually. However, implementation through computer software such as SIDRA (*2*) is useful for dealing with complex intersection geometry and phasing arrangements and for obtaining solutions that require iterations.

The discussion in this paper is limited to the operation of a basic actuated controller that uses passage detectors and a fixed gap time setting (fully and semiactuated control cases). However, the method can be applied to the operation of more complex actuated signal controllers using various gap reduction techniques with passage and presence detectors (*5*).

Model M3A for the proportion of bunched traffic has been calibrated for single-lane and multilane cases using real-life and simulation data after the writing of this paper (*15*). Validation and calibration of the green time and cycle time formulas given in this paper as well as the formulas for estimating intersection performance (delay, queue length, number of stops) are also needed.

The method given in this paper provides an analytical tool for investigating the optimization of actuated signal operations (to minimize delay, queue length, number of stops, or a performance index). This can be done with relative ease compared with the use of a simulation model, which is the most common method used for this purpose.

Many actuated controller parameters can be considered for optimization, namely, minimum green, gap time, maximum extension (or maximum green), additional parameters for gap reduction, and the location and other characteristics of detectors. Some suggestions are available on the effect of a fixed gap time setting in a basic actuated controller based on a limited amount of research published in the literature (*4,6,16–18*). These are summarized by Akçelik (*5*), and work is in progress to investigate the validity of these suggestions by means of simple examples reported in the literature.

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